Problem 1-2)

$$\frac{d}{dx}f(x) = a_1 + 2a_2x + 3a_3x^2 + \cdots + na_nx^{n-1} + \cdots$$

$$= a_0 + a_1x + a_2x^2 + \cdots + a_{n-1}x^{n-1} + \cdots.$$

Therefore,

$$a_1 = a_0, \quad 2a_2 = a_1, \quad 3a_3 = a_2, \quad na_n = a_{n-1}, \cdots.$$

Or, equivalently,

$$a_1 = a_0, \quad a_2 = \frac{1}{2!}a_0, \quad a_3 = \frac{1}{3!}a_0, \quad a_4 = \frac{1}{4!}a_0, \quad a_n = \frac{1}{n!}a_0, \cdots.$$

The function $f(x)$ is thus given by

$$f(x) = a_0\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots\right) = a_0e^x.$$