Problem 1-1) The elementary integral can be readily evaluated, as follows:

$$\int_1^{x_0} x^{\varepsilon-1} dx = \frac{x^\varepsilon}{\varepsilon}\bigg|_1^{x_0} = \frac{x_0^\varepsilon - 1}{\varepsilon}.$$

Now, $(x_0^\varepsilon - 1)/\varepsilon = 1$ yields $x_0^\varepsilon = 1 + \varepsilon$, which results in $x_0 = (1+\varepsilon)^{1/\varepsilon}$.

$$\begin{aligned}
\varepsilon &= 1.0 && \rightarrow && x_0 = 2,\\
\varepsilon &= ½ && \rightarrow && x_0 = (1 + ½)^2 = 2.25,\\
\varepsilon &= ⅓ && \rightarrow && x_0 = (1 + ⅓)^3 = 2.37\cdots,\\
\varepsilon &= ¼ && \rightarrow && x_0 = (1.25)^4 = 2.44\cdots,\\
\varepsilon &= 0.1 && \rightarrow && x_0 = (1.10)^{10} = 2.5937\cdots,\\
\varepsilon &= 0.01 && \rightarrow && x_0 = (1.01)^{100} = 2.7048\cdots.
\end{aligned}$$

Clearly, x_0 is approaching e, the base of the natural logarithm ($e = 2.7183\cdots$).

Note that, as $\varepsilon \rightarrow 0$, the function $x^{\varepsilon-1}$ approaches $1/x$. Consequently

$$\int_1^{x_0} \frac{1}{x} dx = \ln x \bigg|_1^{x_0} = \ln(x_0) - \cancelto{0}{\ln(1)} = \ln(x_0),$$

which equals 1 when $x_0 = e$. In fact, all properties of $\ln(x)$ can be derived from $\int_1^x x^{\varepsilon-1} dx$ in the limit when $\varepsilon \rightarrow 0$.