Monday, December 4, 2023 1:27 PM

Ch. 14: ~~10~~, ~~11~~, ~~12~~, ~~14~~, ~~15~~, ~~16~~, ~~18~~, ~~28~~, ~~29~~, ~~30~~, 33

#1: Si pn junction solar cell, $T = 300K$,

$$N_a = 10^{16}\,cm^{-3} \qquad N_d = 10^{15}\,cm^{-3}$$
$$D_p = 10\,cm^2/s \qquad D_n = 25\,cm^2/s$$
$$\tau_{p0} = 5\times10^{-7}\,s \qquad \tau_{n0} = 10^{-6}\,s$$

$$A = 5\,cm^2$$

Entire junction uniformly illuminated s.t. $G_L = 5\times10^{21}\,cm^{-3}\,s^{-1}$

a) Calc. short-circuit photocurrent in space charge region

First need to determine reverse sat. current (density)

$$J_s = en_i^2\left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)$$

$$L_n = \sqrt{D_n \tau_{n0}} = 5\times10^{-3}\,cm$$
$$L_p = \sqrt{D_p \tau_{p0}} = 2.236\times10^{-3}\,cm \quad \Rightarrow \quad J_s = 1.792\times10^{-10}\,\frac{A}{cm^2}$$

$$\Rightarrow I_s = A J_s = 8.96\times10^{-10}$$

We also have to determine V_{bi}.

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = 0.635\,V$$

$$\Rightarrow W = \left[\frac{2\epsilon_s V_{bi}}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right]^{1/2} = 9.5\times10^{-5}\,cm$$

Then

$$\boxed{I_L = eG_L A W = 0.38\,A = 380\,mA}$$

b) calc. open circuit voltage

$$\boxed{V_{oc} = V_t \ln\left(1 + \frac{I_L}{I_s}\right) = 0.5145\,V}$$

c) $\boxed{\frac{V_{oc}}{V_{bi}} = 0.81}$

#2 Si pn junc. solar cell w/ same parameters at 14.10. $I_L = 120\,mA$.

a)
$$V_{oc} = V_t \ln\left(1 + \frac{I_L}{I_s}\right) = 0.4847\,V$$

b) determine voltage across junc. that will produce $I = 100\,mA$

$$I = I_L - I_s\left[e^{V/V_t} - 1\right] \Rightarrow -\frac{(I - I_L)}{I_s} = e^{V/V_t} - 1$$

$$\Rightarrow \left(\frac{I_L - I}{I_s} + 1\right) = e^{V/V_t} \Rightarrow \boxed{V = V_t \ln\left(\frac{I_L - I}{I_s} + 1\right) = 0.4383\,V}$$

c) max. power output

$$P = IV = I_L V - I_s V\left[e^{V/V_t} - 1\right] = I_L V + I_s V - I_s V e^{V/V_t}$$

$$\frac{dP}{dV} = (I_L + I_s) - I_s e^{V/V_t} - I_s V \cdot \frac{1}{V_t} e^{V/V_t} = 0$$

$$\Rightarrow I_L + I_s = I_s e^{V/V_t}\left(1 + \frac{V}{V_t}\right) \Rightarrow \frac{I_L + I_s}{I_s} = e^{V/V_t}\left(1 + \frac{V}{V_t}\right)$$

$$\frac{}{dV} - (I_L + I_s) - I_s e^{} - I_s V \cdot \frac{1}{V_t} e^{} = 0$$

$$\Rightarrow I_L + I_s = I_s e^{V/V_t}\left(1 + \frac{V}{V_t}\right) \Rightarrow \frac{I_L + I_s}{I_s} = e^{V/V_t}\left(1 + \frac{V}{V_t}\right)$$

This must be solved by either trial and error or graphing. By graphing, I found $V_{max} \simeq .411\ V$. Now,

$$I_{max} = I_L - I_s\left[e^{V_{max}/V_t} - 1\right] = .113\ A$$

$$\Rightarrow P_{max} = I_{max} V_{max} = .0463\ W$$

$$\boxed{P_{max} = 46.3\ mW}$$

d) $V_{max} = I_{max} R_L \Rightarrow \boxed{R_L = \frac{V_{max}}{I_{max}} = 3.64\ \Omega}$

#3: a) $I_s = 8.96 \times 10^{-10}\ A$

i) $V_{oc} = V_t \ln\left(1 + \frac{I_L}{I_s}\right) = .42\ V$

ii) $\left(1 + \frac{V_{max}}{V_t}\right) e^{V_{max}/V_t} = 1 + \frac{I_L}{I_s} \Rightarrow V_{max} = .351\ V$

$I_{max} = I_L - I_s\left[e^{V_{max}/V_t} - 1\right] = 9.31\ mA$

$\Rightarrow P_{max} = I_{max} V_{max} = 3.27\ mW$

b) i) $V_{oc} = .48\ V$

ii) $V_{max} = .407\ V$

$I_{max} = 94\ mA \Rightarrow P_{max} = 38.3\ mW$

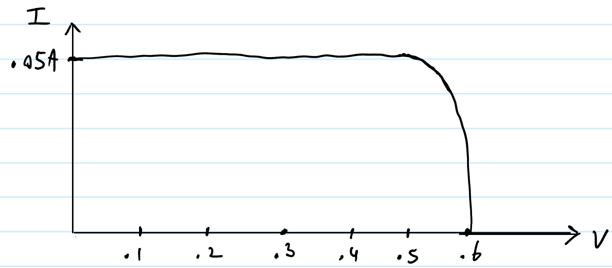
c) $\frac{P_{max2}}{P_{max1}} = \frac{38.3\ mW}{3.27\ mW} = 11.7$

#4: a) $I_L = A J_L = 50 \times 10^{-3}\ A$

$$J_s = e n_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{n_0}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p_0}}}\right] = 2.289 \times 10^{-12}\ \frac{A}{cm^2}$$

$I_s = 4.579 \times 10^{-12}\ A$

$I = I_L - I_s\left[e^{V/V_t} - 1\right]$ (plot)



b) $V_{max} = .52\ V$, $I_{max} = 47.6\ mA \Rightarrow P_{max} = 24.8\ mW$

c) $V = IR \Rightarrow R = \frac{.52\ V}{47.6 \times 10^{-3} A} = 10.9\ \Omega$

#5: a) $V_{oc} = .474\ V$

b) $V_{max} = .402\ V$, $I_{max} = 169\ mA \Rightarrow P_{max} = 67.9\ mW$

c) $R_L = \frac{V_{max}}{I_{max}} = 2.379\ \Omega$

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d) $R_L = 3.568\,\Omega \Rightarrow V_{max} = .444\,V$, $I_{max} = .1244\,A$

$\Rightarrow P_{max} = 55.2\,mW$

#6: a) $V_{oc} = .5367\,V$

b) $V_{max} = .461\,V$, $I_{max} = 94.63\,mA \Rightarrow P_{max} = 43.62\,mW$

c) $N = \frac{10V}{V_{max}} = 21.69 \Rightarrow$ 22 solar cells in series

d) $V = N V_{max} = 10.14\,V \Rightarrow P = IV \Rightarrow I = \frac{P}{V} = \frac{5.2W}{10.14V} = 0.5128A$

then $N' = \frac{I}{I_{max}} = 5.42 \rightarrow N' = 6$ in parallel

e) $I = N' I_{max} = 0.5678\,A$

$$R_L = \frac{V}{I} = \frac{10.14V}{0.5678A} = 17.86\,\Omega$$

#7: 90% absorption:

$$\frac{I(x)}{I(0)} = e^{-\alpha x} = 1 - .9 \Rightarrow -\alpha x = \ln(.1) \Rightarrow x = -\frac{1}{\alpha}\ln(.1) = 2.3\times10^{-4}\,cm$$

so $\boxed{x = 2.3\,\mu m}$

similarly, for $\alpha = 10^5\,cm^{-1}$, $\boxed{x = 230\,nm}$

#8: $Al_x Ga_{1-x} As$

section 14.4.3: $0 < x < 0.45$ for direct gap

use (14.52)

$\Rightarrow E_g = (1.424 + 1.247x)\,eV \Rightarrow (1.424 < E_g < 1.9852)\,eV$

$$\Rightarrow \frac{1.24}{E_{g,max}} < \lambda < \frac{1.24}{E_{g,min}} \Rightarrow \boxed{.625\,\mu m < \lambda < .871\,\mu m}$$

#9: $GaAs_{1-x}P_x$

a) $x = 0.2$

use fig. 14.24(a)

$x = 0.2$: $E_g \simeq 1.64\,eV \Rightarrow \boxed{\lambda \simeq 0.756\,\mu m}$

b) $x = 0.32$: $E_g \simeq 1.75\,eV \Rightarrow \boxed{\lambda \simeq 0.708\,\mu m}$

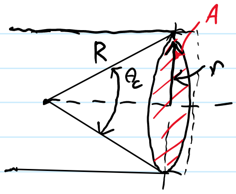
#10: Determine x in $Al_x Ga_{1-x} As$ s.t. $\lambda = .670\,\mu m$

$$E_g = \frac{1.24}{.670\,\mu m} = 1.85\,eV$$

$\Rightarrow \boxed{x \simeq 0.35}$ – from 14.23

or: $E_g = 1.424 + 1.247x \Rightarrow \frac{1.85 - 1.424}{1.247} = \boxed{x = 0.342}$

#11:



$$\sin\frac{\theta_c}{2} = \frac{r/2}{R} \Rightarrow r = 2R\sin\frac{\theta_c}{2}$$

$$A = \pi r^2 = 4\pi R^2 \sin^2\left(\frac{\theta_c}{2}\right)$$

Let T_1 be Fresnel losses and T_c be critical angle losses

$$T_1 = 1 - \Gamma_1 = 1 - \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 \Rightarrow (n_2 + n_1)^2 T_1 = (n_2 + n_1)^2 - (n_2^2 + n_1^2 - 2n_1 n_2)$$

$$\Rightarrow (n_2 + n_1)^2 T_1 = 4 n_2 n_1 \Rightarrow T_1 = \frac{4 n_2 n_1}{(n_2 + n_1)^2}$$ (losses to Fresnel transmission)

$$T_c = \frac{\pi r^2}{4\pi R^2} = \sin^2\left(\frac{\theta_c}{2}\right) = \frac{1}{2}(1 - \cos\theta_c)$$

Thus the external efficiency is given by the product $T_1 T_c$

$$\Rightarrow \boxed{\eta_{ext} = T_1 T_c = \frac{2 n_2 n_1}{(n_2 + n_1)^2}(1 - \cos\theta_c)}$$

which is the fraction of light that is emitted by the LED.

ex: GaAs, $n_2 = 3.8$ at $\lambda = 0.7\,\mu m$ and air

$$\Rightarrow \theta_c = \sin^{-1}\left(\frac{1}{3.8}\right) = 15.3^\circ$$

$$\Rightarrow \eta_{ext} \simeq .012$$