2.5:

$\phi = h\nu \Rightarrow c = \lambda\nu \Rightarrow \phi = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{\phi}$

$1\,eV = 1.602\times10^{-19}\,J$

Au: $\lambda = \frac{(6.626\times10^{-34}\,\cancel{J}\cdot s)(3\times10^{8}\,m/\cancel{s})}{7.85\times10^{-19}\,\cancel{J}} = 2.53\times10^{-7}\,m = .253\,\mu m$

Cs: $\lambda = 6.53\times10^{-7}\,m = .653\,\mu m$

2.6: $\lambda = 550\,nm$, $\lambda = 440\,nm$

de Broglie relation $p = \frac{h}{\lambda}$

a) $p = \frac{6.626\times10^{-34}\,J\cdot s}{550\times10^{-9}\,m} = 1.205\times10^{-27}\,kg\,\frac{m}{s}$

$v_e = \frac{p}{m} = \frac{h}{m\lambda} = 1322\,\frac{m}{s}$, p is the same since by the de Broglie relation, it is only dependent on λ

b) $p = \frac{6.626\times10^{-34}\,J\cdot s}{440\times10^{-9}\,m} = 1.506\times10^{-27}\,kg\,\frac{m}{s}$

$v_e = \frac{p}{m} = \frac{h}{m\lambda} = 1653\,\frac{m}{s}$

c) Yes

2.7: $p = \sqrt{2mE} \Rightarrow \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

a) i) $\lambda = \frac{6.626\times10^{-34}\,J\cdot s}{\sqrt{2(9.11\times10^{-31}\,kg)(1.2\,eV)(1.602\times10^{-19}\,\frac{J}{eV})}} = 1.12\times10^{-9}\,m = 11.2\,\mathring{A}$

ii) $\lambda = 3.54\times10^{-10}\,m = 3.54\,\mathring{A}$

iii) $\lambda = 1.12\times10^{-10}\,m = 1.12\,\mathring{A}$

b) $m_H = 1.674\times10^{-27}\,kg \Rightarrow \lambda = 2.61\times10^{-11}\,m = .261\,\mathring{A}$

2.10: Given λ_{dB}, find E, p, v

$p = \frac{h}{\lambda}$, $E = \frac{1}{2}mv^2$, $v = \frac{p}{m}$

a) $p = \frac{6.626\times10^{-34}\,J\cdot s}{8.5\times10^{-9}\,m} = 7.795\times10^{-26}\,kg\,\frac{m}{s} \Rightarrow v = 85{,}568\,\frac{m}{s} \Rightarrow E = 3.34\times10^{-21}\,J$

or $E = 2.08\times10^{-2}\,eV$

b) $E = \frac{1}{2}(9.11\times10^{-31}\,kg)(8\times10^{3}\,\frac{m}{s})^2 = 2.92\times10^{-23}\,J = 1.82\times10^{-4}\,eV$

$\Rightarrow p = \sqrt{2mE} = 7.29\times10^{-27}\,kg\,\frac{m}{s} \Rightarrow \lambda = \frac{h}{p} = 9.09\times10^{-8}\,m = 909\,\mathring{A}$

2.12: $\Delta x \Delta p \geq \hbar$, $\Delta x \sim \lambda = 1\,\mu m$

$\Rightarrow \Delta p \geq \frac{\hbar}{\lambda} = 1.055\times10^{-28}\,kg\,\frac{m}{s}$

I would also accept $\Delta x \Delta p \geq \hbar/2 \Rightarrow \Delta p \geq \hbar/2\lambda$

↳ usual convention

2.13: a) $\Delta x \Delta p \geq \hbar \Rightarrow$ i) $\Delta p \geq \frac{\hbar}{\Delta x} = \frac{1.055 \times 10^{-34}\ J \cdot s}{12 \times 10^{-10}\ m} = 8.792 \times 10^{-26}\ kg\ \frac{m}{s}$

ii) $\Delta E = \frac{dE}{dp} \Delta p$, $E = \frac{p^2}{2m} \Rightarrow \frac{dE}{dp} = \frac{p}{m}$

so $\Delta E = \frac{p \Delta p}{m} = \frac{\sqrt{2mE}\, \Delta p}{m} = 2.099 \times 10^{-19}\ J = 1.31\ eV$

b) i) Δp is the same

ii) $\Delta E = 8.902 \times 10^{-21}\ J = 5.56 \times 10^{-2}\ eV$

2.25: e^- in infinite square well

$$\frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

solving yields energy levels

$$\boxed{E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2}}$$

$n = 1$: $E_1 = \frac{\hbar^2 \pi^2}{2ma^2} = \frac{(1.055 \times 10^{-34}\ J \cdot s)^2 (\pi)^2}{2(9.11 \times 10^{-31}\ kg)(75 \times 10^{-10}\ m)^2} = 1.072 \times 10^{-21}\ J = 6.69 \times 10^{-3}\ eV$

$n = 2$: $E_2 = \frac{2\hbar^2 \pi^2}{ma^2} = 4.287 \times 10^{-21}\ J = 2.67 \times 10^{-2}\ eV$

$n = 3$: $E_3 = \frac{9\hbar^2 \pi^2}{2ma^2} = 9.647 \times 10^{-21}\ J = 6.02 \times 10^{-2}\ eV$

2.27: Same energy levels as 2.25

$m = 15\ mg \times \frac{1\ kg}{1 \times 10^6\ mg} = 1.5 \times 10^{-5}\ kg$, $a = 1.2\ cm = .012\ m$

a) $E = 15\ mJ = 15 \times 10^{-3}\ J$

$$n = \sqrt{\frac{2ma^2 E_n}{\hbar^2 \pi^2}} = \frac{a}{\hbar \pi} \sqrt{2mE_n} = 2.4288 \times 10^{28}$$

b) $E_{n+1} = \frac{(n+1)^2 \hbar^2 \pi^2}{2ma^2} = 15\ mJ$

c) NO, quantum effects are not observable for this particle.

2.29:

$$\frac{-\hbar^2}{2m} \frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$V(x) = 0$ for $-\frac{a}{2} < x < \frac{a}{2}$

$$\Rightarrow \frac{d^2\psi(x)}{dx^2} = \frac{2mE}{\hbar^2}\psi(x)$$

can shift origin to lie at $x = a/2$

$\rightarrow V(x) = 0$ for $0 \leq x \leq a$

$$\Rightarrow \psi(x) = A\cos(kx) + B\sin(kx)$$

BC's: $\psi(x = 0) = \psi(x = L) = 0$

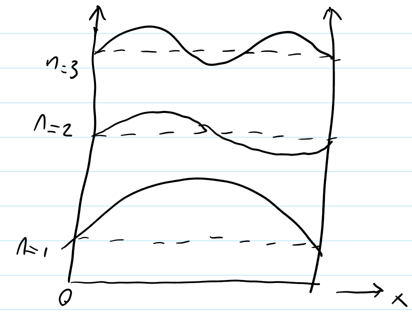
$\psi(x = 0) = A = 0$

similar steps are followed for $\psi(x = -\frac{a}{2}) = \psi(x = \frac{a}{2}) = A\cos(\frac{ka}{2}) + B\sin(\frac{ka}{2}) = 0$

$$\psi(x=L) = B\sin(ka) = 0 \Rightarrow ka = n\pi \rightarrow k = \frac{n\pi}{a}$$

$$\Rightarrow \psi(x) = B\sin\left(\frac{n\pi x}{a}\right) \quad \text{for} \quad n = 1, 2, 3, \ldots$$

not required
$$\left\{\begin{array}{l} |B|^2\int_{-\infty}^{\infty} dx \sin^2\left(\frac{n\pi x}{a}\right) = 1 \rightsquigarrow |B|^2 = \frac{2}{L} \\ \psi(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \end{array}\right.$$



$\boxed{\text{Aside: to find } E_n\text{, plug } \psi(x) \text{ back into TISE.}}$

2.32: a) Same energy levels as before (eq. 2.38)

b) $\Delta E = \frac{\hbar^2\pi^2}{2ma^2}\left[n_2^2 - n_1^2\right]$

for $a = 4\,\text{Å}$: $\Delta E = \frac{3\hbar^2\pi^2}{2ma^2} = 6.16\times10^{-22}\,J = 3.85\times10^{-3}\,eV$

$n_2 = 2,\ n_1 = 1$

$a = .5\,cm$; $\Delta E = 3.93\times10^{-36}\,J = 2.46\times10^{-17}\,eV$