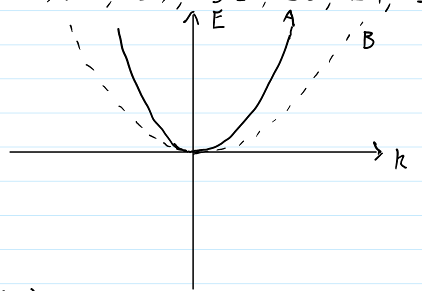
3.13, 14, 16, 17, 26, 27, 29, 32, 33, 34, 35, 39, 40, 45, 46

3.13: $\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$



$m^* = \hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}$

$\boxed{m_B^* > m_A^*}$

Since $\left|\frac{d^2E_A}{dk^2}\right| > \left|\frac{d^2E_B}{dk^2}\right|$

curvature(A) > curvature(B)

More intuitively: $m^* \propto \frac{1}{\text{band curvature}}$ $\Rightarrow$ larger curvature means smaller effective mass

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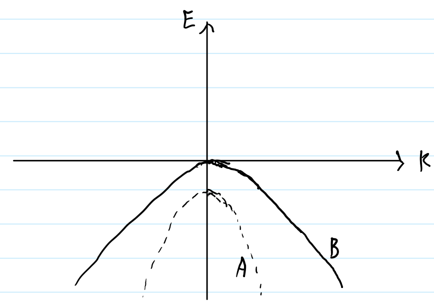
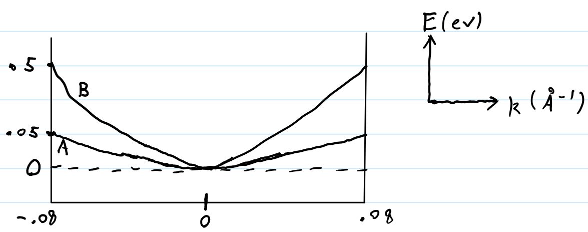
3.14:



$\boxed{m_B^* > m_A^*}$

Same concept as 3.13.

3.16:



$E_A = C_1k^2 \Rightarrow C_1 = \frac{E_A}{k^2} = \frac{.05\text{ eV}}{(.08\text{ Å}^{-1})^2} = 7.8125\text{ eV Å}^2$

convert to mksA $\Rightarrow C_1 \times 1.602\times10^{-19}\text{ J/eV} \times \frac{(1\times10^{-10}\text{ m})^2}{1\text{ Å}^2} = 1.252\times10^{-38}\text{ J m}^2$

Then, using $m_A^* = \frac{\hbar^2}{2C_1} = \frac{\left(\frac{6.626\times10^{-34}}{2\pi}\text{ J}\cdot\text{s}\right)^2}{2(1.252\times10^{-38}\text{ Jm}^2)} = 4.44\times10^{-31}\text{ kg}$

(check dimensions: $\left[\frac{J^{\cancel{2}}s^2}{\cancel{J}m^2}\right] \to \left[J\frac{s^2}{m^2}\right] = \left[kg\frac{m^2}{s^2}\right]\cdot\left[\frac{s^2}{m^2}\right] = [kg]$ ✓)

So, in terms of the electron mass,

$\frac{m_A^*}{m_0} = \frac{4.44\times10^{-31}\text{ kg}}{9.11\times10^{-31}\text{ kg}} = .488$

$\Rightarrow \boxed{m_A^* = .488m_0}$

B: $E_B = C_1k^2 \Rightarrow C_1 = \frac{E_B}{k^2} = \frac{.5\text{ eV}}{(.08\text{ Å}^{-1})^2} = 78.125\text{ eV Å}^2$

mksA: $C_1 \times 1.602\times10^{-19}\frac{J}{eV} \times \frac{(1\times10^{-10}\text{ m})^2}{1\text{ Å}^2} = 1.252\times10^{-37}\text{ J m}^2$

$m^* \quad \frac{\hbar^2}{} = \frac{(1.054\times10^{-34}\text{ J}\cdot\text{s})^2}{} = 4.44\times10^{-32}\text{ kg}$

MKSA: $C_1 \times 1.602\times10^{-19}\ \frac{J}{eV} \times \frac{(1\times10^{-10}m)^2}{1\,\mathring{A}^2} = 1.252\times10^{-37}\ J\,m^2$

$$M_B^* = \frac{\hbar^2}{2C_1} = \frac{(1.054\times10^{-34}\ J\cdot s)^2}{2(1.252\times10^{-37}\ J M^2)} = 4.44\times10^{-32}\ kg$$

$$\frac{M_B^*}{M_0} = \frac{4.44\times10^{-32}\ kg}{9.11\times10^{-31}\ kg} = .0488$$

$$\boxed{M_B^* = .0488\,M_0}$$

3.17: Basically the same calculation as 3.16.

$E - E_V = -C_2 k^2$

$$E_A = -C_2 k^2 \Rightarrow C_2 = \frac{E_A}{k^2} = \frac{.025\,eV}{(.08\,\mathring{A}^{-1})^2} = 3.91\ eV\cdot\mathring{A}^2$$

MKSA: $C_2 \times 1.602\times10^{-19}\ \frac{J}{eV} \times \frac{(1\times10^{-10}m)^2}{1\,\mathring{A}^2} = 6.26\times10^{-39}\ J\,M^2$

$$\Rightarrow M_A^* = \frac{-\hbar^2}{2C_2} = \frac{-(1.054\times10^{-34}\ J\cdot s)^2}{2(6.26\times10^{-39}\ J\,M^2)} = -8.87\times10^{-31}\ kg$$

$$\Rightarrow \frac{M_A^*}{M_0} = -.975 \Rightarrow \boxed{M_A^* = -.975\,M_0}$$

similarly for B:

$$\boxed{M_B^* = -.0813\,M_0}$$

3.26: a)

$$N = \int_{E_1}^{E_2} g(E)\,dE$$

In the conduction band of a bulk semiconductor, the density of states is

$$g_c(E) = \frac{4\pi(2M_c)^{3/2}}{h^3}\sqrt{E-E_c}\,.$$

So the total number of states per unit volume from $E = E_c$ to $E = E_c + 2k_BT$

$$N = \frac{4\pi(2M_c^*)^{3/2}}{h^3}\int_{E_c}^{E_c+2k_BT} dE\,(E-E_c)^{1/2}$$

$E - E_c = u \Rightarrow \frac{du}{dE} = 1 \Rightarrow du = dE$

$E_v - E = u$

$E = E_v - 3kT$

$\Rightarrow$

$$= \frac{4\pi(2M_c^*)^{3/2}}{h^3}$$

at $E = E_c \to u = 0$

at $E = E_c + 2k_BT \to u = 2k_BT$

$$\Rightarrow N = \frac{4\pi(2M_c^*)^{3/2}}{h^3}\int_0^{2k_BT} u^{1/2}\,du = \frac{4\pi(2M_c^*)^{3/2}}{h^3}\left[\frac{2}{3}u^{3/2}\right]\Bigg|_0^{2k_BT} = \frac{4\pi(2M_c^*)^{3/2}}{h^3}\cdot\frac{2}{3}\cdot(2k_BT)^{3/2}$$

So now it's a matter of plugging in values:

– Note: $k_B = 1.38\times10^{-23}\ \frac{J}{K}$ & $M_c^* = 1.08\,M_0$

$T = 300\,K$: $N = 6\times10^{25}\ \frac{\text{states}}{m^3} \times \frac{(.01\,m)^3}{1\,cm^3} \Rightarrow \boxed{N = 6\times10^{19}\ \frac{\text{states}}{cm^3}}$

$T = 400\,K$: $N = 9.22\times10^{25}\ \frac{\text{states}}{m^3} \times \frac{(.01\,m)^3}{1\,cm^3} \Rightarrow \boxed{N = 9.22\times10^{19}\ \frac{\text{states}}{cm^3}}$

b) For GaAs, the only difference is $M_c^* = .067\,M_0$

b) For GaAs, the only difference is $M_c^* = .067 M_0$

$$\Rightarrow T = 300\,K: \quad N = 9.25 \times 10^{23} \frac{\text{states}}{m^3} \times \frac{(.01\,m)^3}{1\,cm^3} \Rightarrow \boxed{N = 9.25 \times 10^{17} \frac{\text{states}}{cm^3}}$$

$$T = 400\,K: \quad N = 1.42 \times 10^{24} \frac{\text{states}}{m^3} \times \frac{(.01\,m)^3}{1\,cm^3} \Rightarrow \boxed{N = 1.42 \times 10^{18} \frac{\text{states}}{cm^3}}$$

3.27: effectively the same calculation as 3.26

$$N = \int_{E_1}^{E_2} g(E)\,dE$$

here, however, the bounds are flipped assuming the top of the valence band is $E = 0$. we have then

$$N = A \int_{E_v - 3k_BT}^{E_v} \sqrt{E_v - E}\,dE \rightsquigarrow A \int_0^{3k_BT} (u)^{1/2}\,du = A \cdot \frac{2}{3} u^{3/2} \Big|_0^{3k_BT}$$

$$\Rightarrow N = A \cdot \frac{2}{3} (3k_BT)^{3/2} = \frac{4\pi (2M_v^*)^{3/2}}{h^3} \cdot \frac{2}{3} (3k_BT)^{3/2} \text{ or } \rightsquigarrow \frac{8\pi}{3} \left(\frac{2M_v^*}{h^2}\right)^{3/2} (3k_BT)^{3/2}$$

$$\left.\begin{array}{l} (M_v^*)_{Si} = .56 M_0 \\ (M_v^*)_{GaAs} = .48 M_0 \end{array}\right\} \begin{array}{l} A_{Si} = 4.45 \times 10^{55} \left[\frac{kg}{J^2 s^2}\right]^{3/2} \\ A_{GaAs} = 3.53 \times 10^{55} \left[\frac{kg}{J^2 s^2}\right]^{3/2} \end{array}$$

$$\Rightarrow Si: \quad T = 300K: \quad N = A_{Si} \cdot \frac{2}{3} \cdot (3k_BT)^{3/2} = 4.11 \times 10^{25} \frac{\text{states}}{m^3} = 4.11 \times 10^{19} \frac{\text{states}}{cm^3}$$

$$T = 400K: \quad N = 6.32 \times 10^{25} \frac{\text{states}}{m^3} = 6.32 \times 10^{19} \frac{\text{states}}{cm^3}$$

$$GaAs: \quad T = 300K: \quad N = 3.26 \times 10^{25} \frac{\text{states}}{m^3} = 3.26 \times 10^{19} \frac{\text{states}}{cm^3}$$

$$T = 400\,K: \quad N = 5.02 \times 10^{25} \frac{\text{states}}{m^3} = 5.02 \times 10^{19} \frac{\text{states}}{cm^3}$$

3.29

$$\frac{g_c}{g_v} = \frac{\cancel{\frac{4\pi}{h^3}} (2M_n^*)^{3/2} \sqrt{E - E_c}}{\cancel{\frac{4\pi}{h^3}} (2M_p^*)^{3/2} \sqrt{E_v - E}}$$

for $E = E_c + k_BT$ & $E = E_v - k_BT$ in the conduction & valence bands respectively:

a) $$\frac{g_c}{g_v} = \frac{(M_n^*)^{3/2}}{(M_p^*)^{3/2}} = \frac{(1.08)^{3/2}}{(0.56)^{3/2}} = 2.68$$

b) similarly, for GaAs:

$$\frac{g_c}{g_v} = \frac{(M_n^*)^{3/2}}{(M_p^*)^{3/2}} = \frac{(.067)^{3/2}}{(.48)^{3/2}} = .052$$

3.32:

For this, it's simply a matter of plugging values into the Fermi function

$$f(E) = \frac{1}{1 + \exp\left[\frac{E - E_F}{k_BT}\right]}$$

$$f(E) = \frac{1}{1 + \exp\left[\frac{E - E_F}{k_B T}\right]}$$

a) $f(E = E_F + k_B T) = \frac{1}{1+e} = .269$

b) $f(E = E_F + 5 k_B T) = \frac{1}{1+e^5} = .0067$

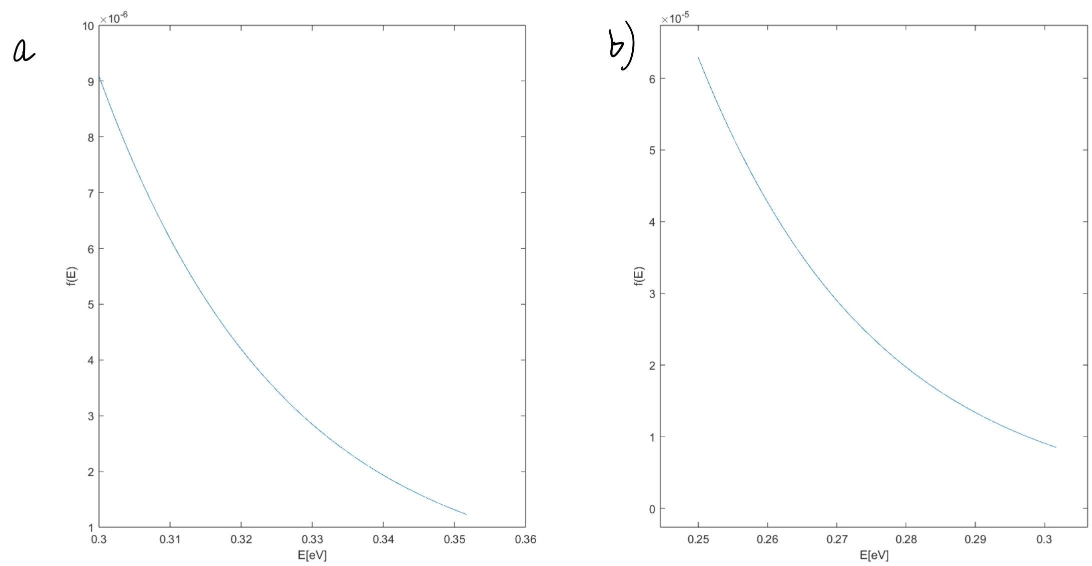
c) $f(E = E_F + 10 k_B T) = \frac{1}{1+e^{10}} = 4.5 \times 10^{-5}$

3.33: Now it is the flipped problem:

P(state empty of electron) $= 1 - f(E)$, but since we flip the sign of E, the answers are the same as 3.32.

3.34:

a) Set $E - E_F = .3 + n k_B T$, where n is just a number ranging from $0 \to 2$, then substitute into $f(E)$.



3.35:

$$f(E = E_c + k_B T) = 1 - f(E = E_v - k_B T)$$

use the Boltzmann approximation:

$$f(E) \simeq \exp\left[-\frac{(E - E_F)}{k_B T}\right] \quad \text{and} \quad 1 - f(E) \simeq \exp\left[-\frac{(E_F - E)}{k_B T}\right]$$

$$\Rightarrow \quad \exp\left[-\frac{(E_c + k_B T - E_F)}{k_B T}\right] = \exp\left[-\frac{(E_F - E_v + k_B T)}{k_B T}\right]$$

$$-E_c - \cancel{k_B T} + E_F = -E_F + E_v - \cancel{k_B T}$$

$$2E_F = E_c + E_v \quad \Rightarrow \quad \boxed{E_F = \frac{E_c + E_v}{2}}$$

so the Fermi energy lies at mid-gap.

3.39:

so the Fermi energy lies at mid-gap.

3.39:

Here we want to use

$$\frac{f_B(E_1)-f_F(E_1)}{f_F(E_1)} = .01$$

or

$$\frac{\dfrac{1}{\exp\left[\frac{(E_1-E_F)}{k_B T}\right]} - \dfrac{1}{1+\exp\left[\frac{E_1-E_F}{k_B T}\right]}}{\dfrac{1}{1+\exp\left[\frac{E_1-E_F}{k_B T}\right]}} = \frac{1+\exp\left[\frac{E_1-E_F}{k_B T}\right]}{\exp\left[\frac{(E_1-E_F)}{k_B T}\right]} - 1 = .01$$

$$\Rightarrow \frac{1}{\exp\left[\frac{(E_1-E_F)}{k_B T}\right]} = .01 \quad \Rightarrow \quad \exp\left[\frac{(E_1-E_F)}{k_B T}\right] = 100$$

Then, taking the natural log,

$$E_1 - E_F = k_B T \ln(100) \Rightarrow \boxed{E_1 = E_F + 4.61\, k_B T}$$

b) At this energy, the probability is

$$\boxed{f(E_1) = \frac{1}{1+e^{4.61}} = .0099 \simeq .01}$$

3.40:

$E_F = 5.5\text{ eV}$ at 300K

a) $$f(E = E_F + .3\text{eV}) = \frac{1}{1+\exp\left[\frac{.3\text{eV}}{.0259\text{eV}}\right]} = 9.32\times10^{-6}$$

b) Increase to $T = 700\text{ K}$, $k_B T = 1.38\times10^{-23}\ \frac{\text{J}}{\text{K}} \times 700\text{K} = 9.66\times10^{-21}\ \text{J} \cdot \frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}$

$$\Rightarrow f(E = E_F + .3\text{eV}) = \frac{1}{1+\exp\left[\frac{.3\text{eV}}{.0603\text{ eV}}\right]} = 6.86\times10^{-3}$$

c) $f(E = E_F - .25\text{eV}) = .02$

We can apply the Boltzmann approximation here

$$\exp\left[\frac{-.25\text{ eV}}{k_B T}\right] = .02 \Rightarrow -.25\text{eV} = k_B T \ln(.02)$$

$$\Rightarrow \boxed{T = \frac{-.25\text{ eV}}{k_B \ln(.02)} = 742\text{ K}}$$

3.45: a) $E_{g,Si} = 1.12\text{ eV}$, $E_{g,Ge} = .66\text{ eV}$, $E_{g,GaAs} = 1.42\text{ eV}$

$$E_c - E_F = \frac{E_g}{2}$$

$$\text{Si: } f = \frac{1}{1+\exp\left[\frac{E_g}{2k_B T}\right]} = 4.07\times10^{-10}$$

Si: $f = \frac{1}{1+\exp\left[\frac{E_g}{2k_BT}\right]} = 4.07\times10^{-10}$

Ge: $f = 2.93\times10^{-6}$

GaAs: $f = 1.24\times10^{-12}$

b) The probabilities are the same

3.46:

a) $f_B(E = E_F + .6eV) = \exp\left[\frac{-.6eV}{k_B T}\right] = 1\times10^{-8}$

$$\Rightarrow \frac{-.6eV}{k_B \ln(1\times10^{-8})} = \boxed{T = 378K}$$

b) $$\frac{-.6eV}{k_B \ln(1\times10^{-6})} = \boxed{T = 504K}$$