

LAB 7: AFOCAL SYSTEMS

REFLECTIVE TELESCOPES

History:

James Gregory (1638-1675), a Scottish mathematician, invented the first reflecting telescope in 1663. He provided a description of the reflecting telescope in "Optica Promota," which was published in 1663. He never actually made the telescope, which was to have used a parabolic (primary) and an ellipsoidal (secondary) mirror.

In 1668, Newton used a concave mirror to actually make the first reflective telescope.

We learn more of this history from two articles published by Louis Bell in 1921. Titled 'Notes on the Early Evolution of the Reflector,' an excerpt is presented here:

"James Gregory attempted to make a Gregorian telescope in 1664 with Reive (presumably Richard Reeve, father or son). This six foot telescope was a failure because Reive used cloth to polish the speculum and was unable to achieve an accurate figure. Robert Hooke showed a Gregorian to the Royal Society 05 February 1674. Newton presented the Royal Society with a 6 inch telescope on 11 January 1672. The minutes of the Royal Society from 25 January, 1672 note 'There was produced a reflecting telescope 4 feet long of Mr. Newton's invention which, though the metaline concave was not duly polished, yet did pretty well, but was under charged'. It was improved for the next meeting, when the minutes note 'The 4 foot telescope of Mr. Newton's invention was produced again, being improved since the last meeting. It was recommended to Mr. Hooke to see it perfected as far as it was capable of being'. No further note is found in the minutes, nor is it certain who fabricated the 4 foot telescope. About 30 years later, Newton experimented with glass telescope mirrors, silvered on the rear surface, an idea also proposed by James Gregory".

(Louis Bell. "Notes on the Early Evolution of the Reflector." 1921, 1922. The Telescope. 1922. 287pp.)
<http://articles.adsabs.harvard.edu/full/1922JRASC..16..179B/0000180.000.html>

Bell further credits John Hadley with the invention of the reflecting telescope, because Hadley parabolized his mirror using effective tests and taught others the figuring skills he had developed."

Wikipedia provides a detailed history of the telescope:

http://en.wikipedia.org/wiki/History_of_the_telescope

Theory:

The optical principles behind a mirror-based, reflective telescope are the same as for a lens-based, refractive telescope--an objective (positive power) forms a real image, which in turn is viewed through an eyepiece. The refractive version uses a single positive lens as the objective (or more likely an achromatic doublet to minimize chromatic aberrations). The reflective version can use either a single concave mirror (Newtonian design), or it can use two mirrors (a variety of designs) to form a real image. The details of these systems will be discussed in a following section. As for the refractive telescope, an eyepiece is used to view the final image. In both cases, the eyepiece forms the final image at infinity, making these afocal optical systems.

Note that in modern-day reflective telescopes used by professional astronomers, an eyepiece is seldom (if ever) used. Instead, the telescope forms a real image onto film, a CCD camera, or the entrance slit of a spectrograph. In this case, the system is NOT afocal.

MAGNIFICATION

The magnification of an afocal reflective telescope is described in terms of the angular magnification M_θ , also called the magnifying power M_p . The angular magnification is defined as the ratio of the angular size of the image subtended at the eye (seen through the telescope) to the angular size of the object subtended at the eye (seen without the telescope). Stated another way, the angular magnification is the ratio of the apparent size of the image seen looking through the telescope to the apparent size of the object seen with the unaided eye. For an object at infinity, the angular size of the object subtended at the eye is essentially the same as the angular size of the object subtended at the aperture stop of the telescope (the objective). It is the angular subtense at the objective that will be used for the derivation of angular magnification.

As we showed for refractive telescopes, the angular magnification M_θ is given as the ratio of the chief ray angle in image space to the chief ray angle in object space:

$$M_\theta = \frac{\bar{u}'}{u}$$

As before, the magnifying power of a reflective telescope may be expressed as the ratio of the focal lengths or the diameters of the pupils:

$$M_\theta = -\frac{f_o}{f_e} \quad ; \quad |M_\theta| = \frac{E}{E'}$$

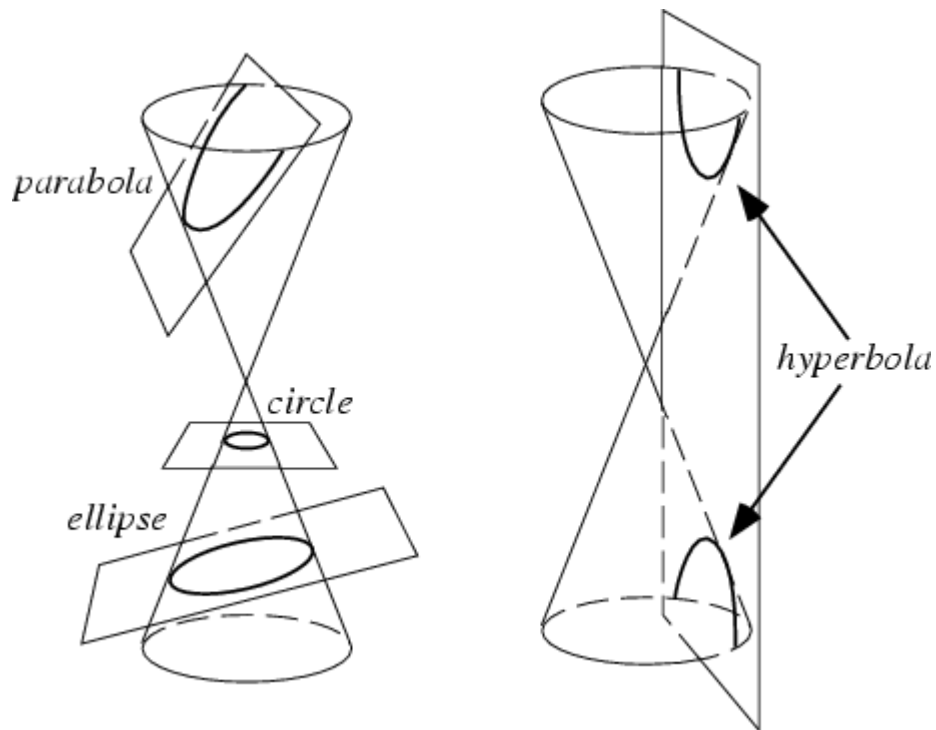
The minus sign denotes an inverted image if both f_o and f_e are positive quantities.

NOTE: If the telescope is used without an eyepiece, the final real image that is formed has a lateral extent given by the height of the chief ray in the focal plane. For a small Field of View:

$$\bar{y}' \approx f_o \cdot \bar{u}'$$

Conic Sections

The cross-sectional shape of any modern-day telescope mirror is that of a conic section. Conic sections are a family of mathematical curves that have been studied for over 2000 years. A conic section is the curve that results from the intersection of a plane with a right-angle cone:



If the plane is perpendicular to the axis of the cone, the conic is a circle. If the plane is parallel to the "side" of the cone, the conic is a parabola. If the plane intersects just one piece of the cone (and not perpendicular to the axis), the conic is an ellipse. Finally, if the plane intersects both pieces of the cone, the resulting conic is a hyperbola.

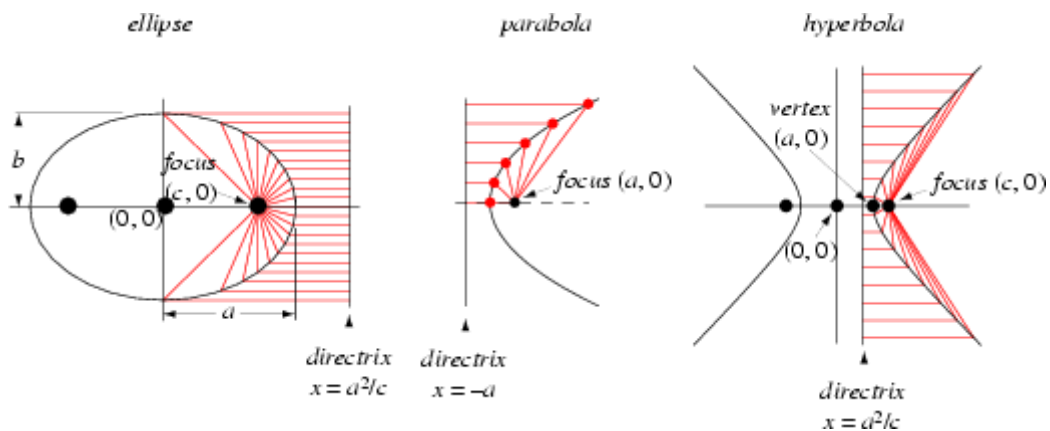
Another geometrical description of the conic sections may be given:

Ellipse -- the set of all (x,y) points in the plane such that the sum of the distances from any point (x,y) to two fixed points is a constant. The two points are the foci of the ellipse. This explains why a string can be used to draw an ellipse (Tack each end down at what will be the two foci, and use a pencil point to stretch out the string. Move the pencil around, drawing out the ellipse. The sum of the distances from the pencil tip to the two foci is a constant, namely the length of the string!)

Parabola -- the set of all (x,y) points in the plane that are the same distance from a fixed line (the directrix) and a fixed point not on the directrix (the focus).

Hyperbola -- the set of all (x,y) points in the plane such that the difference of the distances from any point (x,y) to two fixed points is a constant. The two points are the foci of the hyperbola.

Mathematically, a more formal definition of the conic sections is given by defining a relationship between a point (the focus) and a line (the directrix):



Each conic section is the locus of points whose distance from the focus is proportional to the horizontal distance from the directrix. Call this proportionality e , the eccentricity:

- If $e = 0$, the conic is a circle.
- If $e = 1$, the conic is a parabola.
- If $0 < e < 1$, the conic is an ellipse.
- If $e > 1$, the conic is a hyperbola.

The mathematical equation describing a conic section is given by:

$$y^2 - 2Rz + (1 - e^2)z^2 = 0$$

where R is the radius of curvature at the vertex.

Implied in all of this is that a "3-dimensional" mirror is made by revolving a conic section around the optical, or z-axis. The mathematical equation describing this is found by substituting $\rho^2 = (x^2 + y^2)$ for y^2 :

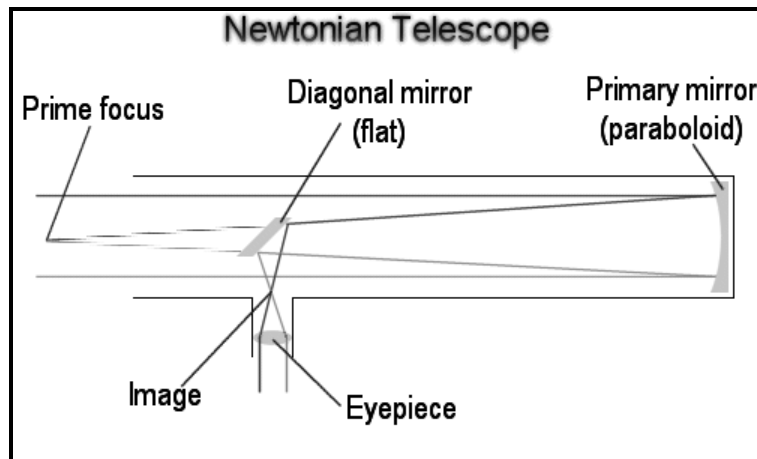
$$\rho^2 - 2Rz + (1 - e^2)z^2 = 0$$

Finally, as is usually done in the optical literature, define the conic constant K as $K = -e^2$ so that:

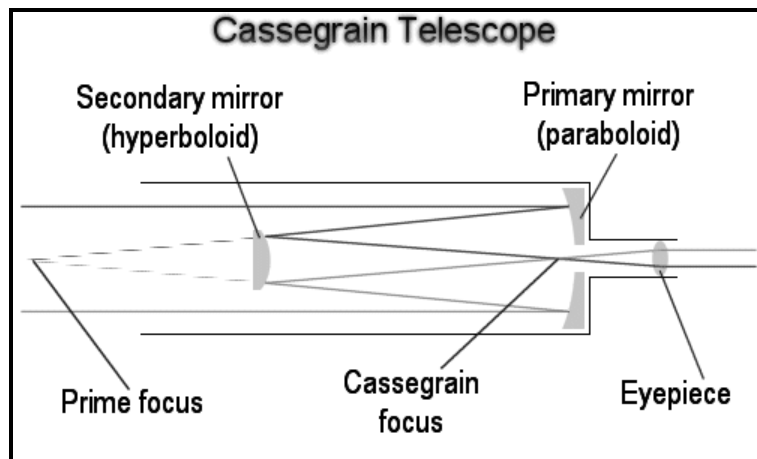
$$\rho^2 - 2Rz + (1 + K)z^2 = 0$$

Types of Telescopes

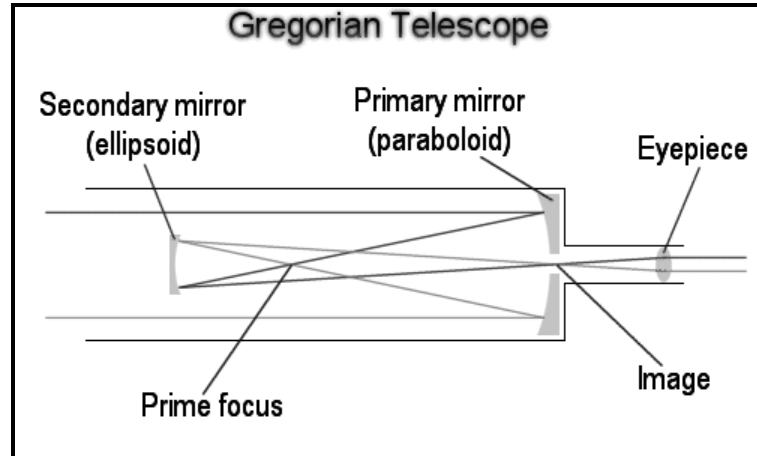
(from <http://www.tivas.org.uk/socsite/scopes.html>)



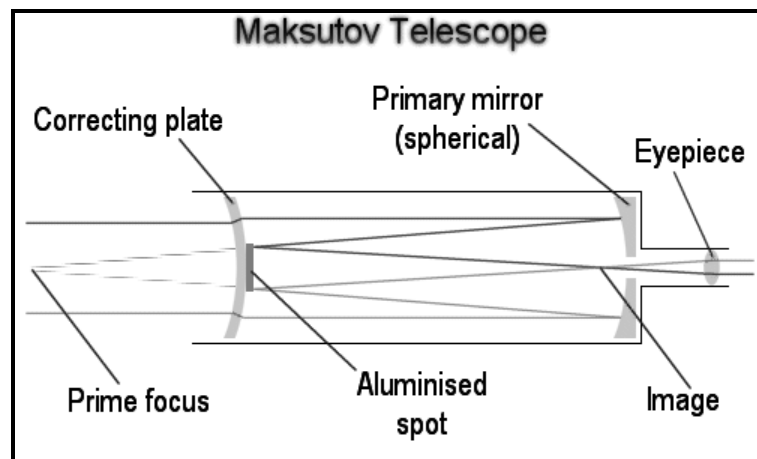
- Invented in 1668 by Sir Isaac Newton.
- A 45° prism can be used in place of the diagonal mirror.



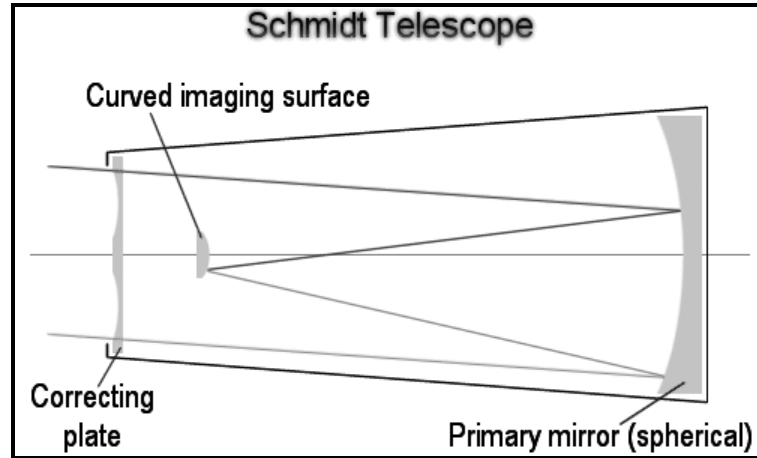
- Invented in 1672 by a Frenchman, Guillaume Cassegrain.
- Small field of view, primarily due to coma.



- Invented in 1663 by the Scottish mathematician James Gregory.
- First compound reflecting telescope.
- Small field of view, primarily due to coma.



- Invented in 1944 by a Soviet.
- Telescope is 'catadioptric': correcting plate is a meniscus lens (deeply curved).
- All surfaces are spherical surfaces.
- The diagram above illustrates Maksutov-Cassegrain optics.



- Invented in 1930 by Bernhard Schmidt, an Estonian.
- Telescope is 'catadioptric': the correcting plate eliminates spherical aberration, and the optics provide an extremely large field of view.

Pre-Lab Questions:

(Q1) What lens system is the Maksutov telescope an equivalent of? (HINT: We've already studied this particular lens system in detail!). Write a paragraph to explain the differences and similarities.

(Q2) For a focal length of 1500 mm, calculate t , the spacing between the primary and secondary mirrors. Use the specifications for the mirrors, found on page 7-10 of this handout.

Lab Exercises:

In this lab we will "dissect" a Maksutov telescope, to measure the radii of curvatures of the primary and secondary mirrors, and the overall focal length of the assembled telescope. One approach to knowing the focal length would be to first measure the radii of curvatures, then measure the vertex-to-vertex separation of the two mirrors. The problem with this is that the primary mirror has a hole in the center of it! As such, we will "work the problem backwards" by first measuring the focal length, then calculating the corresponding mirror separation.

*** NOTE: Any fingerprints left on any of the mirror surfaces will deduct 50 points from the semester's total, for everybody on your lab team! NO EXCEPTIONS.**

Mirror Radii of Curvature

Use the setup provided to measure the radius of curvature of both the primary and secondary mirrors.

- Q1** • Give a complete written description of HOW you did your tests.
- Q2** • Give a complete written description of WHY the test setup works. You must mention something about the wavefront and ray concept of the test beam, and how they relate to the surface under test, for full credit.

Telescope Focal Length

NOTE: *In all of the following steps, do NOT move the pinhole, light source, or collimating lens. The pinhole is collimated and the beam is parallel to the optical axis of the telescope.*

- Q3** • Describe the beam incident on the entrance pupil of the telescope. Is the pinhole source on-axis or off-axis, optically speaking?

(Step 1) Slide the eyepiece away from the back of the telescope. Find the real image of the pinhole on a white card, behind the telescope. The 2-mirror system functions as the objective lens in a refracting telescope, in that it forms a real image of our pinhole "star."

- Q4** Technically speaking, our 2-mirror system is not "yet" a telescope. Describe how to use the eyepiece to make this into an afocal system (i.e. a telescope)!

(Step 2) Look through the eyepiece and move it back and forth along the optical rail, until it is focused on the image of the pinhole.

***** At this point, the telescope is focused at infinity!

- Q5** • Give a complete optical description of how the final image that you are viewing relates to the original pinhole object:
- Where does the copy lens form an image of the pinhole? (real or virtual image?)
 - Where is the object for the telescope located?
 - Where does the telescope form the intermediate image? (real or virtual?)
 - Where does the eyepiece form the final image? (real or virtual?)

(Step 3) To measure the focal length of the telescope, hold a piece of lens tissue in front of the telescope to diffusely illuminate the primary mirror. Use the microscope to measure the diameter of the exit pupil. Then, calculate the focal length of the telescope:

$$\frac{f_{primary}}{f_{eyepiece}} = \frac{E}{E'} \quad ; \quad f_{primary} = f_{eyepiece} \cdot \left(\frac{E}{E'} \right)$$

Use this procedure to measure the focal length of the telescope, for 2 different positions of the adjusting screw, 2mm apart:

- (1) with the threads just concealed into the assembly
- (2) with the threads turned clockwise by $\approx 2\text{mm}$.

- Q6** • Explain how rotating the screw moves the primary mirror.
- Q7** • Report the 2 different focal lengths. How does focal length vary with mirror separation? Does this follow what you'd expect to happen? Explain, and compare to the telephoto lens.
- Q8** • For both focal lengths, calculate the separation of the mirrors, using the equations from Pre-Lab (Q2).
- Q9** • Use the method of propagation of errors to estimate the error in focal length.

Assume:

- a 10% error in knowing the focal length of the eyepiece
- your estimated error in knowing the entrance pupil diameter, E
- your estimated error in knowing the exit pupil diameter, E'

ZEMAX Layout

Use ZEMAX to lay out the telescope:

- Model the system using just the 2 mirrors (leave out the corrector plate).
- Set the thickness between the primary and secondary mirror to be the value of "t" predicted by the two-thin-lens equation for a Gaussian-reduced telephoto lens.
- Set the primary mirror to be the aperture stop.

Answer the following questions for both measured focal lengths:

- Q10** • What are the focal lengths that ZEMAX calculates?
- Q11** • Use ZEMAX to trace a marginal and a chief ray through each system. Print out these pictures. Using a pencil and ruler, locate the rear principal plane. Scale your drawings appropriately, and determine each focal length. Does they match the expected values?
- Q12** • What is the FOV? (trace off-axis rays and look for vignetting).
- Q13** • Print out a picture of the on-axis and off-axis bundle of rays.
- Q14** • Print out the corresponding spot diagrams. What aberrations seem to be present?

Prescription of our Maksutov Telescope (from Meade):

- All surfaces are spherical
- $R_{\text{primary}} = 21.915''$
- Clear aperture = 5.00"
- $R_{\text{secondary}} (\text{concave}) = 5.195''$ (i.e. the outer glass surface)
- $R_{\text{secondary}} (\text{convex}) = 5.48''$ (i.e. the inner glass surface)
- Thickness of the glass meniscus = 0.47"-0.50"

*** NOTE:** Any fingerprints left on any of the surfaces will cost everybody in your entire lab section **50 points from the semester's total! NO EXCEPTIONS.**

More ZEMAX Analysis.....(if time permits)

- Q15** • What problem (discrepancy) have you noticed between your two measured values of focal length, and the values predicted by ZEMAX (without the corrector plate in the system)?
- Q16** • How would the corrector plate account for this?
- Q17** • Using paraxial equations for the corrector plate, calculate the:
- focal length
- power
- principal plane locations
- Q18** • Use ZEMAX to find the same values.
- Q19** • Go back into ZEMAX and add the corrector plate. For both focal lengths, re-trace a marginal and a chief ray through the system. Print out these pictures. Using a pencil and ruler, locate the rear principal plane. Scale your drawings appropriately, and determine each focal length. Now do they match the expected values?