LAB 2: ABERRATIONS

In ideal imaging, rays originating from one object point converge to a single image point. Every point in the object is mapped to a unique point in the image. The result is a perfect image of the object (with the appropriate change in magnification). In reality, this is never quite achieved. The ideal image point becomes a small blur of light. When all of the image points are considered, the final image may now appear blurry or distorted. The image is said to be *aberrated*, or to contain aberrations.

Aberrations may be classified into two general types: **monochromatic** and **chromatic**. The monochromatic aberrations occur even if just one color of light is present in the optical system, while chromatic aberration occurs when two or more wavelengths are present. The five monochromatic aberrations are:

1. Spherical aberration
2. Coma
3. Astigmatism
4. Field Curvature
5. Distortion

In this lab we will investigate:
spherical aberration, chromatic aberration, coma, and astigmatism.

It is important to understand that an image becomes aberrated due simply to the nature of the curved surfaces in the optical system used to form the image. It is not a manufacturing defect or glass defect that gives rise to aberrations. A perfectly-made lens or mirror will still inherently produce some aberration. It is the goal of the optical engineer to understand the origins of aberrations, and to understand how to design the surfaces in an optical system to reduce the aberrations.

The main assumption of paraxial optics was the small-angle approximation, namely that $\sin \bar{u} \approx \bar{u}$ and $\sin u \approx u$. This allowed for paraxial ray tracing to find the paraxial properties of a system. In order to determine the actual properties of an optical system and its associated image, the sines of the angles need to be calculated. The sine of an angle may be represented as a series expansion:

$$\sin u = u - \frac{u^3}{3!} + \frac{u^5}{5!} - \frac{u^7}{7!} + \qquad (2.1)$$

If only the first term is used, we have the case of paraxial, or *first-order optics*. If the next term is also used, we have an example of *third-order optics*. The monochromatic aberrations begin to show up when rays are traced under the more-correct assumptions of third-order optics. It is interesting to consider the history behind these third-order aberrations:

Aberrations based on third-order theory are often called **von Seidel aberrations**, named after Ludwig Philipp von Seidel (1821-1896), German mathematician and astronomer, professor at the University of Munich. After working on divergent and convergent series, von Seidel became interested in stellar photometry, probability theory, and the method of least squares, studied the trigonometry of skew rays, and became the first to establish a rigorous theory of monochromatic third-order aberrations which led to the construction of much improved astronomical telescopes. L. Seidel, "Ueber die Entwicklung der Glieder 3ter Ordnung, welche den Weg eines ausserhalb der Ebene der Axe gelegenen Lichtstrahles durch ein System brechender Medien bestimmen," *Astron. Nachr.* **43** (1856), 289-304, 305-20, 321-32." (from Jurgen R. Meyer-Arendt, *Introduction to Classical & Modern Optics,* 3rd ed., 1989, p. 109).

Spherical Aberration

Consider a lens or mirror and imagine its clear aperture to be divided up into many rings shaped like donuts, all centered on the axis. The width of each ring can be made arbitrarily small, but not exactly zero. Each of these rings is termed a **zone** of the lens or mirror. If a mask is made with an opening around only one zone, then only light passing through that zone will appear in the final image. This mask is termed a **zone plate**. In this way, the ability of the lens or mirror to focus light from outside of the paraxial region may be studied.

Spherical aberration arises from the fact that different zones of a spherical surface have slightly different focal lengths. Rays from outside of the paraxial region come to focus at different points, giving rise to an aberrated image. The cross-section of the beam at any of these focal points will be a circularly symmetric blur.

When rays from all zones are traced through the lens or mirror, the outer portion of the ray bundle in image space defines a surface known as the **caustic**. The caustic surface arises from the fact that rays in adjacent zones cross each other before coming to their respective focus. Figure 2.1 shows the caustic surface. The point along the caustic where the beam diameter is a minimum is known as the **circle of least confusion**. This location coincides with the point where the marginal (peripheral) ray crosses the caustic surface after passing through its focus. It may be shown that *the distance between the paraxial focus and the circle of least confusion is three-fourths the distance between the paraxial and marginal foci*. It may also be shown that *the ratio of beam diameter at the circle of least confusion to the beam diameter at the paraxial focus is 0.25*.

Positive spherical aberration is illustrated in Fig. 2.1 and 2.2.

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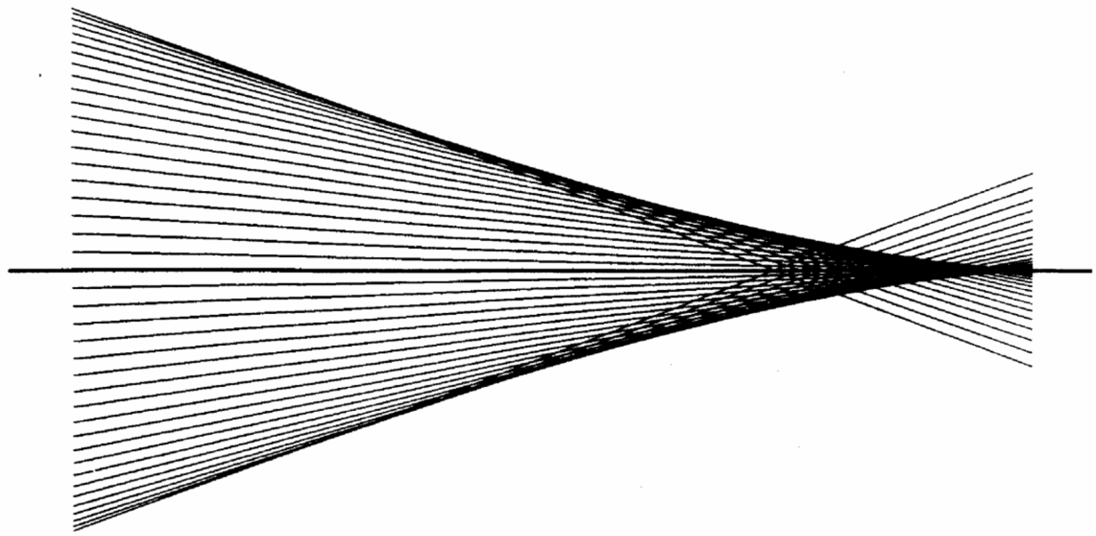
Optical Sciences Center, University of Arizona



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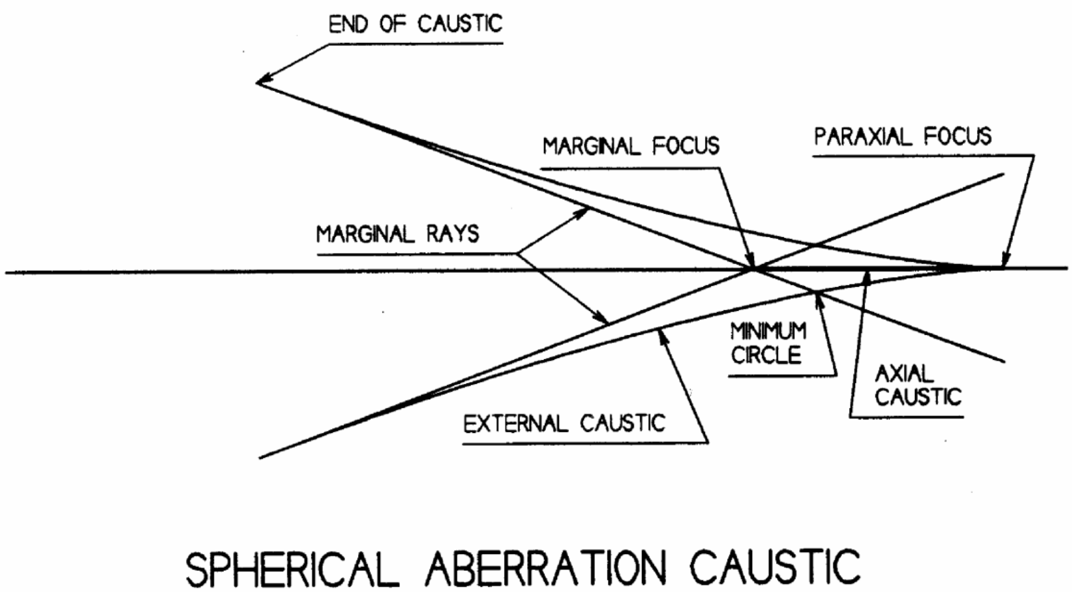


Figure 2.1. Spherical aberration and the caustic surface.

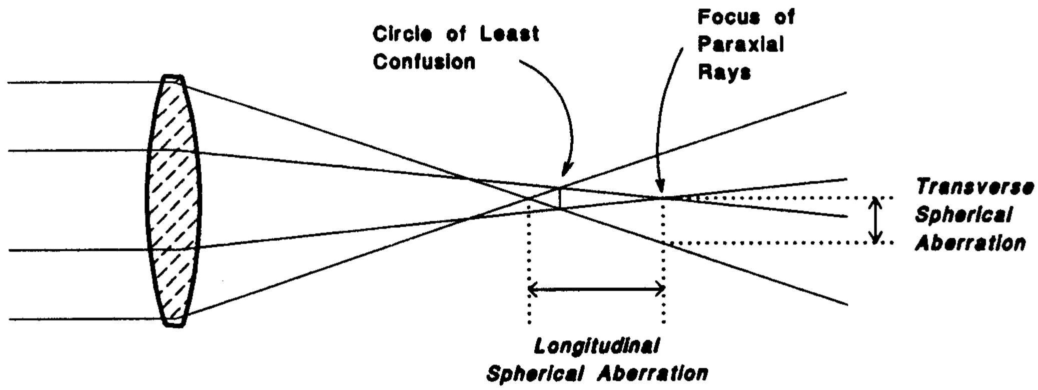


Figure 2.2. Longitudinal and transverse spherical aberration.

Begin your study of spherical aberration by considering Fig. 2.3 (a)-(f). In each part, the notation used is as follows where C_1 and C_2 represent the curvatures of the first and second lens surfaces, respectively.

$$X \equiv \text{SHAPE FACTOR} \qquad = \frac{C_1 + C_2}{C_1 - C_2}$$

$$P \equiv \text{POSITION FACTOR} \qquad = \frac{z' + z}{z' - z} \qquad (2.2)$$

K $\equiv$ CONIC CONSTANT

Part (a) shows a collimated beam of light incident on a spherical mirror. As already described, different zones of the mirror bring rays to focus at different points along the axis. In effect, each zone has its own focal length. The result is spherical aberration. A solution to this problem is to use a parabolic surface. A parabola has the unique property that every zone has the same focal point. The result is that a parabola produces *no* spherical aberration, demonstrated in part (b).

The proper use of a plano-convex lens in reducing spherical aberration is shown in parts (c) and (d). For both examples, a collimated beam is incident on the lens from the left. With the planar side towards the source, part (c), all of the refraction occurs at the second surface. The large variation in angle of incidence along this surface causes large amounts of spherical aberration. By turning the lens around so the curved surface is toward the source, part (d), the spherical aberration is reduced. This is because the overall refraction of each ray is distributed between both surfaces.

The same principle of distributing the refraction applies when a lens is used at 2f-2f conjugates (object distance = image distance = 2f, P=0). In this case, minimum spherical aberration occurs when a biconvex lens is used. This is illustrated in parts (e) and (f).

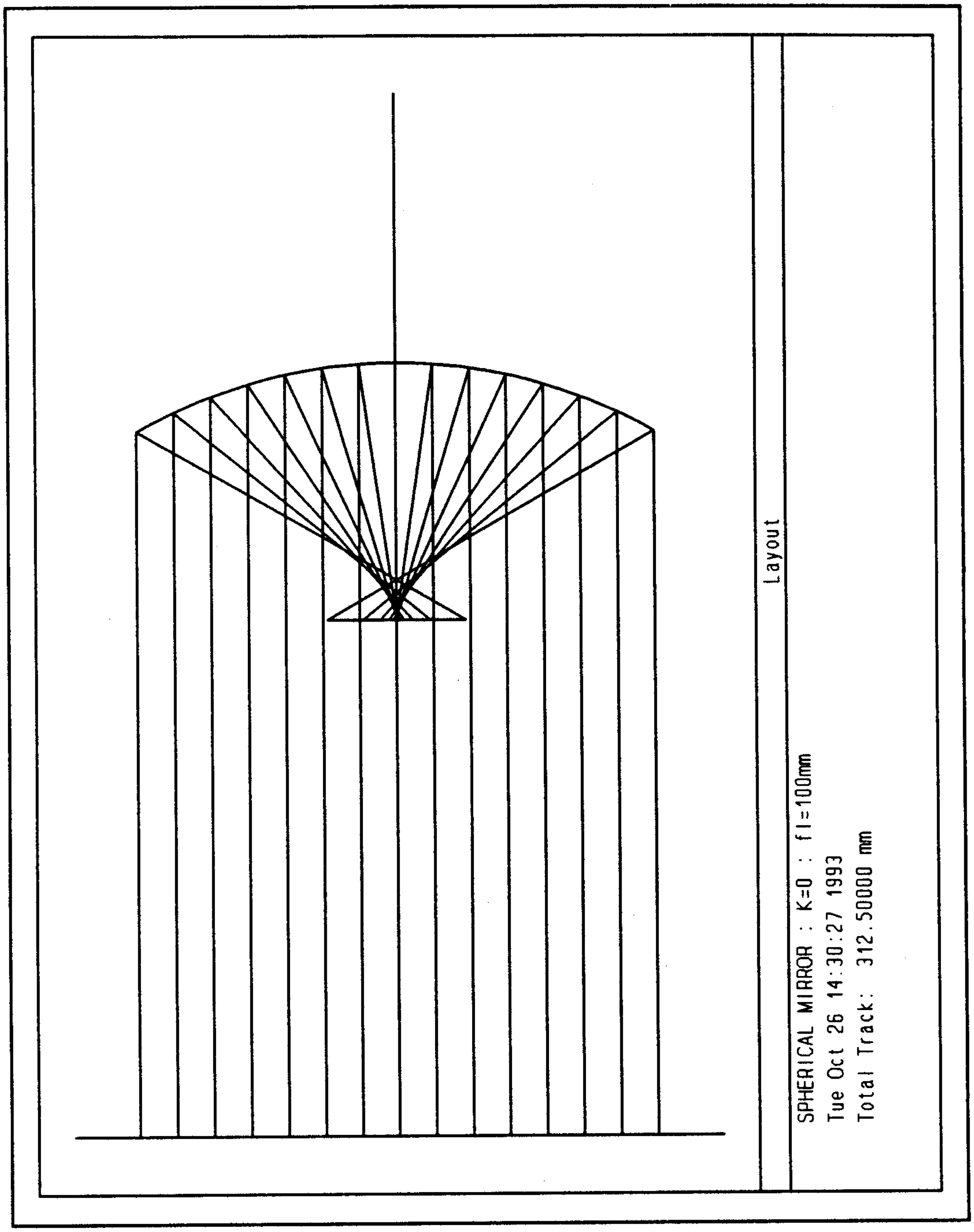


Figure 2.3(a) Spherical mirror.

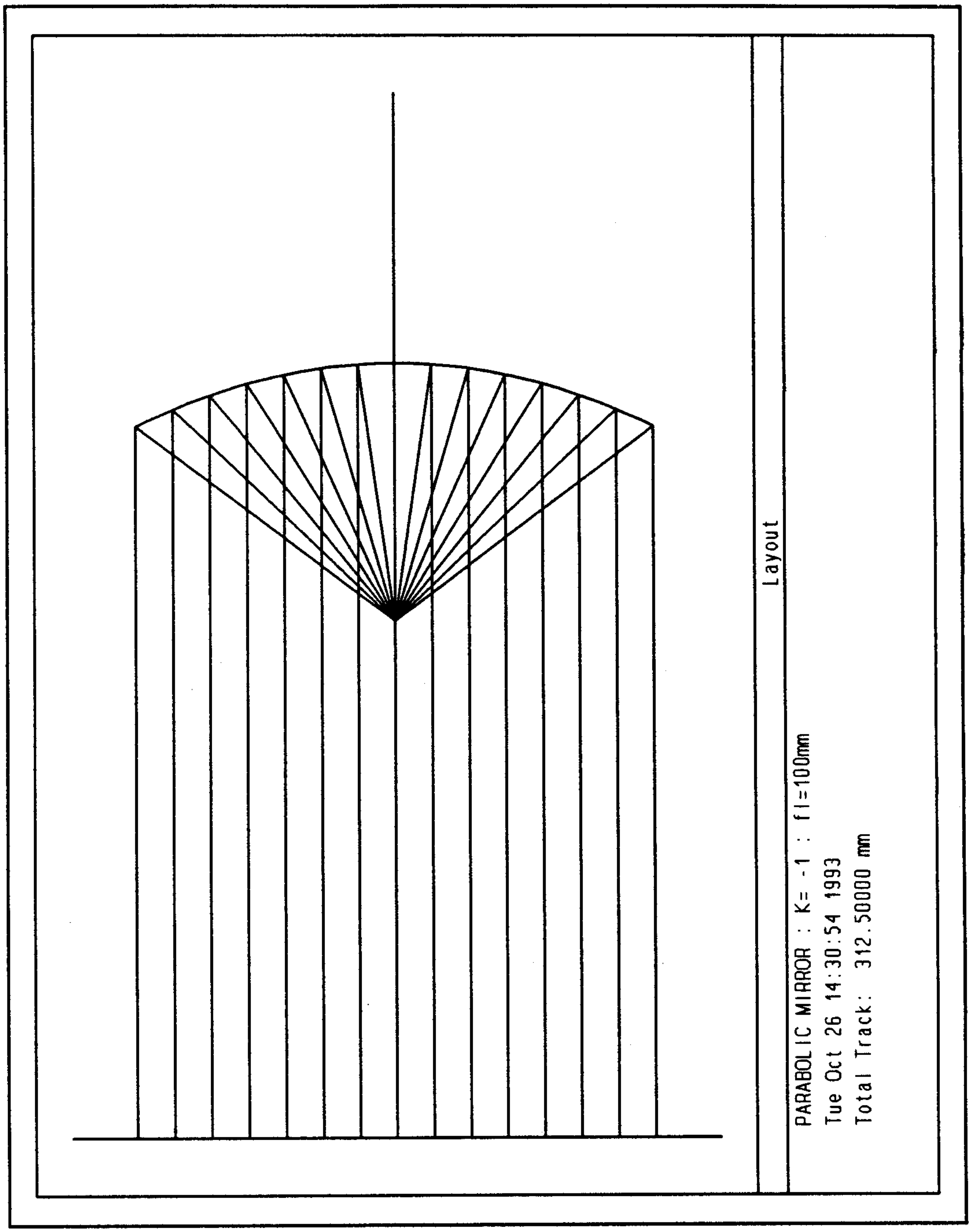


Figure 2.3(b) Parabolic mirror.

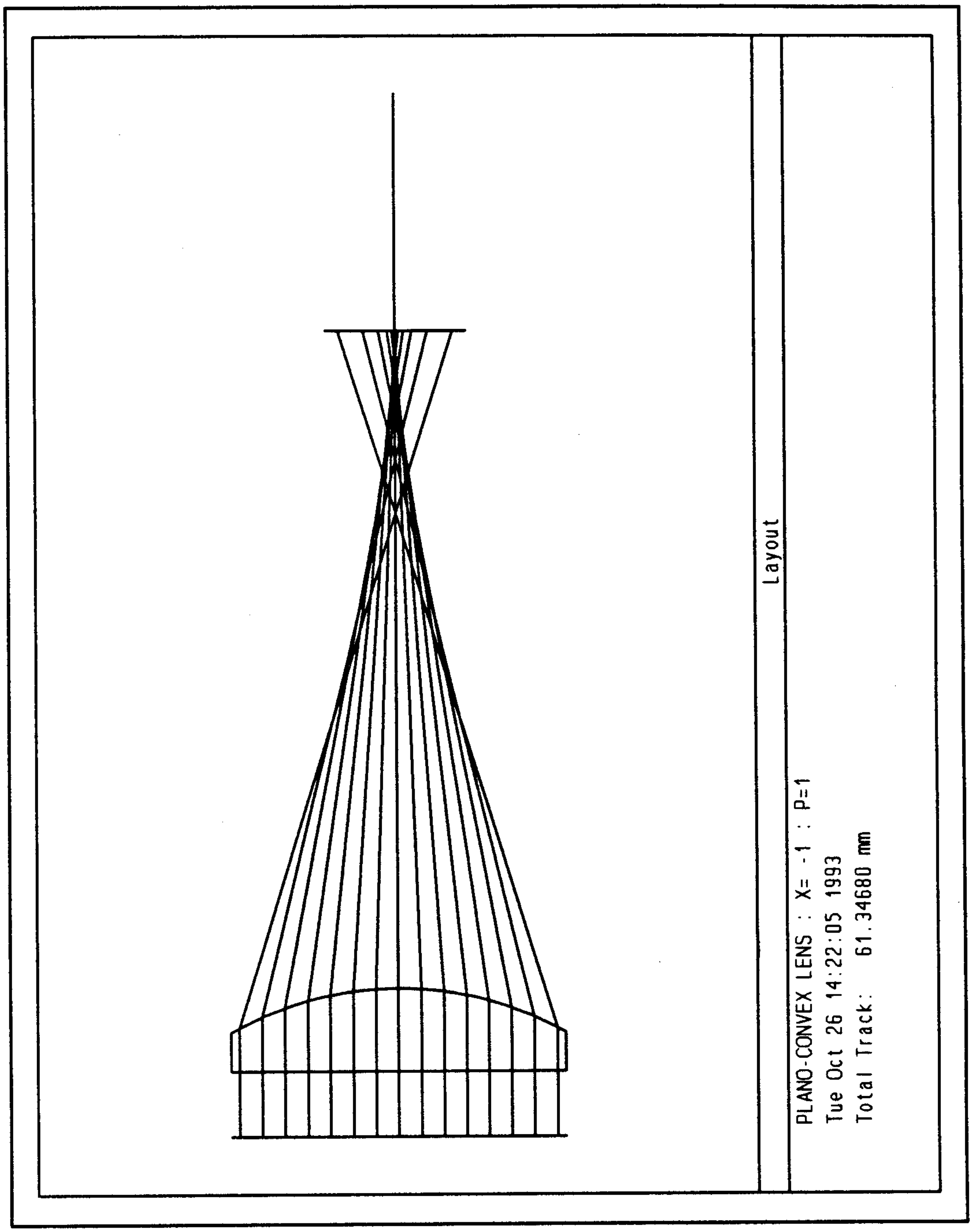


Figure 2.3(c) Plano convex lens -- used incorrectly.

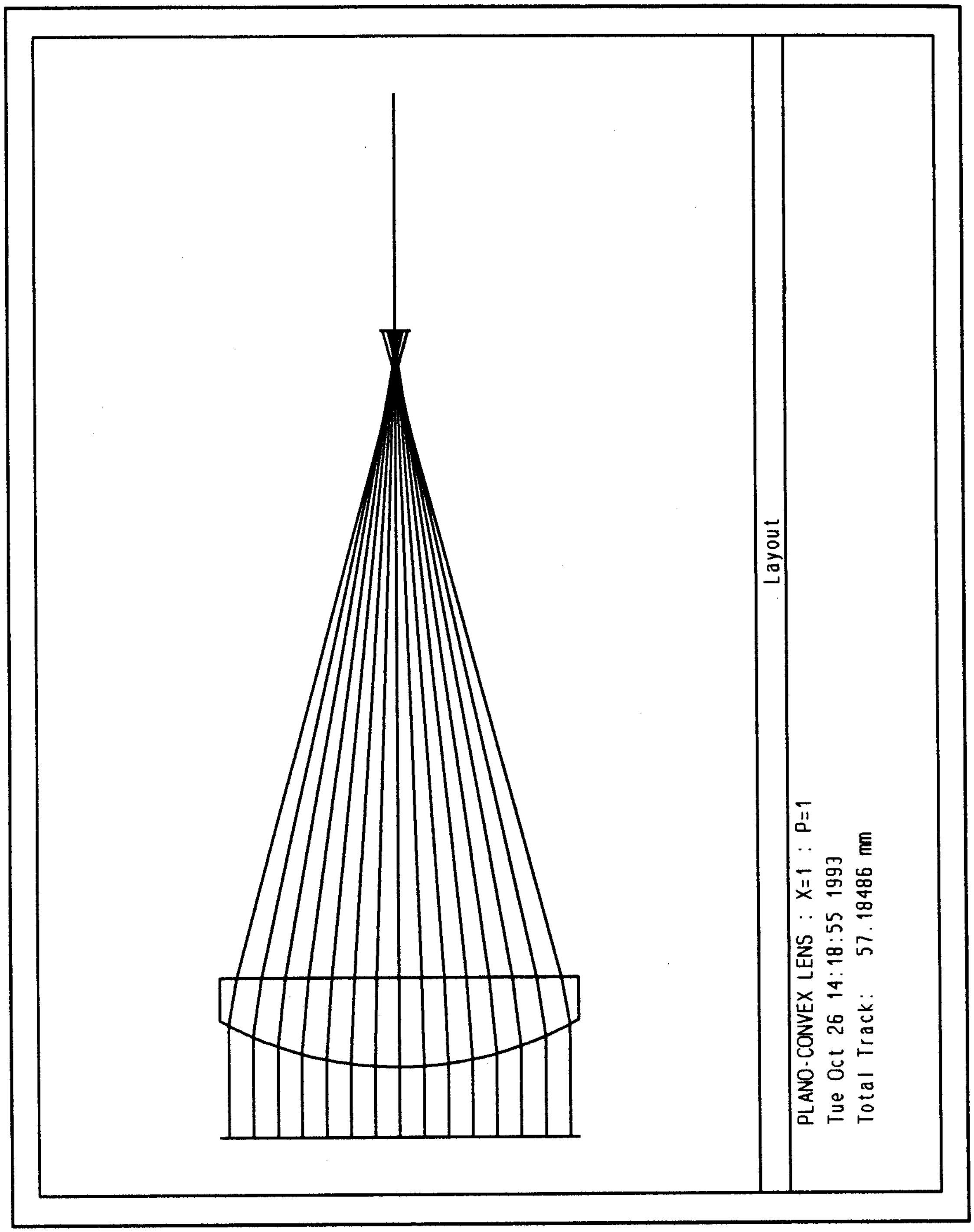


Figure 2.3(d) Plano convex lens -- used correctly

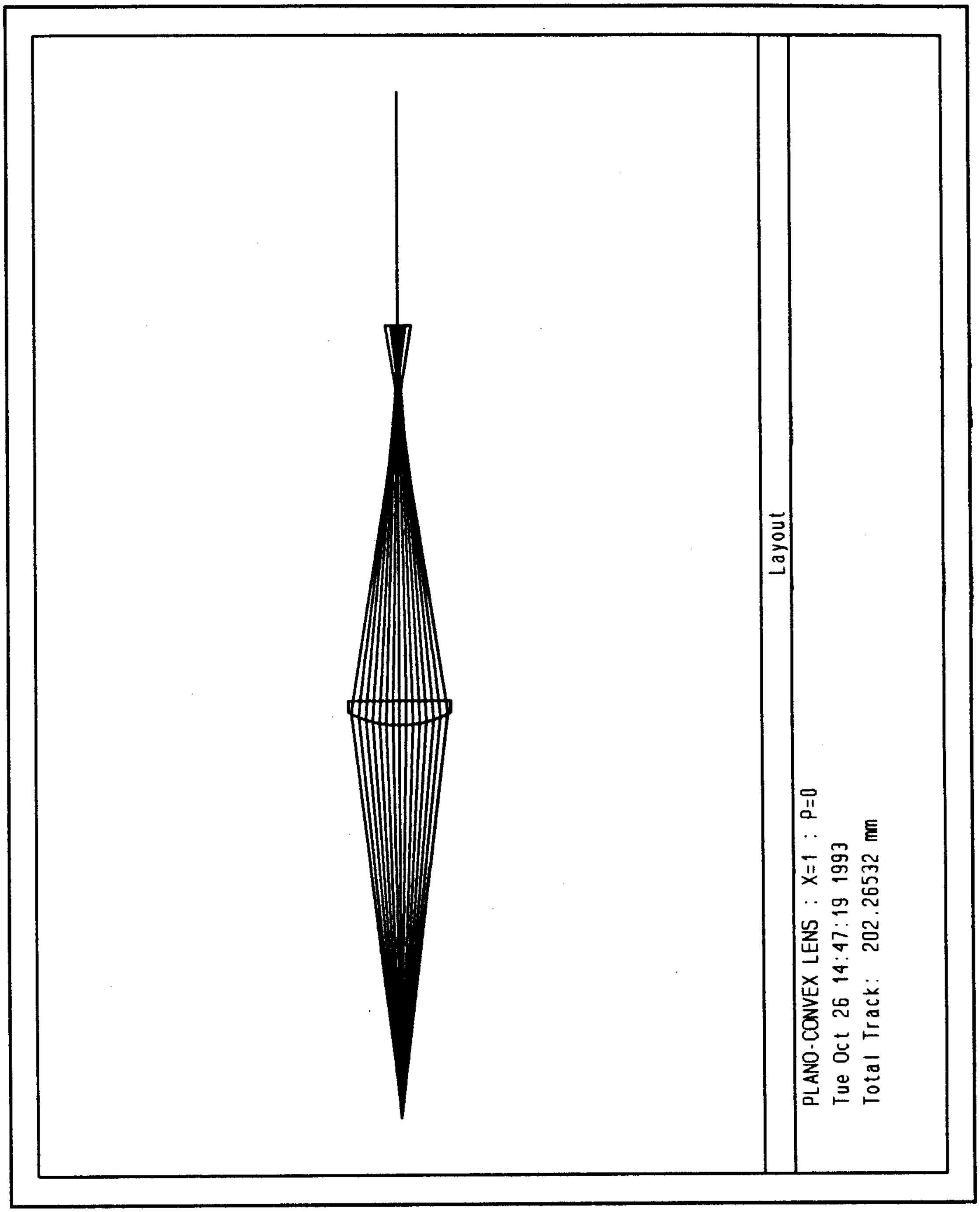


Figure 2.3(e) Plano convex lens.
2f-2f Imagery.

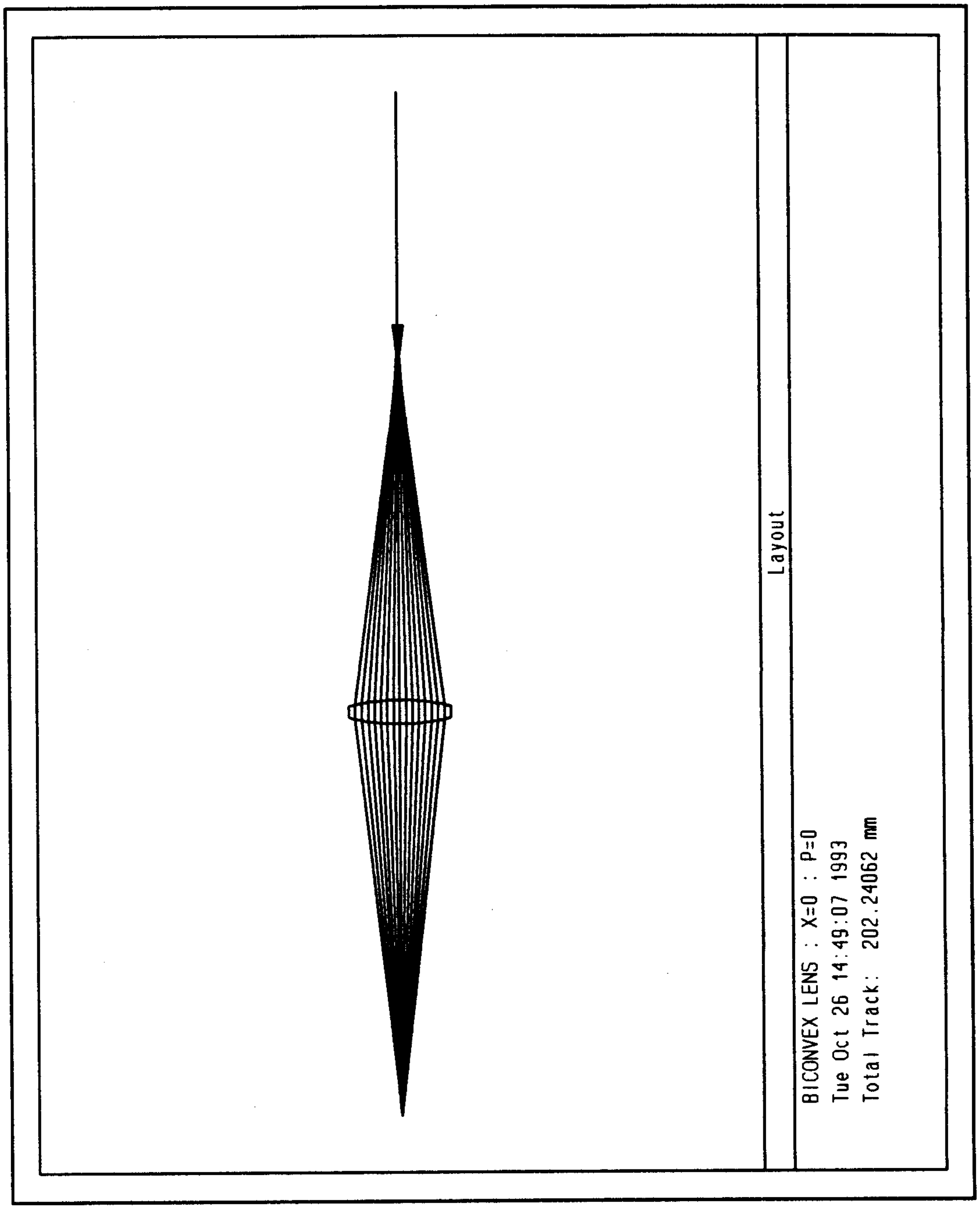


Figure 2.3(f) Plano convex lens
2f-2f Imagery

The Caustic: Graphical Exercise

Use part Fig. 2.3(c) to study the properties of the caustic surface produced by spherical aberration.

Questions:

(**Q1**) Locate and label the paraxial focus.
(**Q2**) Locate and label the marginal (peripheral) focus.
(**Q3**) On the bottom half of the ray bundle, trace out the caustic surface.
(**Q4**) On the top half locate and label the point where the marginal ray crosses the caustic at the point of minimum beam diameter. This is the circle of least confusion.
(**Q5**) Measure the ratio of beam diameters at the circle of least confusion to the paraxial focus. Compare to the theoretical value of 1/4.
(**Q6**) Measure the distance between the circle of least confusion and the marginal focus. Measure the distance between the paraxial focus and marginal focus. Calculate the ratio and compare to the theoretical value of 1/4.

Shape Factor: Computer Exercise

Use the computer program ZEMAX to study the effect of lens shape factor on spherical aberration.

Consider an object at infinity $z = -\infty$, position factor $P = \dfrac{z' + z}{z' - z} = -1$

For the first example, consider a 100 mm focal length lens (n = 1.5168) used at a wavelength of 0.587 μm.

- Using the following shape factors (and associated files listed in table 2.1), calculate the geometrical spot size (in μm) in the paraxial focal plane.
- Plot the spot size (in μm) vs. shape factor.

For the second example, consider a 100 mm focal length lens (n = 4) used in the infrared at the CO_2 laser wavelength of 10.6 μm.

- Using the following shape factors (and associated files listed in table 8.2), calculate the geometrical spot size (in μm) vs. shape factor.
- On the same graph, plot the spot size (in μm) vs. shape factor. Label each curve.
- Locate and label the point of minimum spherical aberration for each curve.
- What is the shape factor at these minimum points? Compare your results to the expected values (for P = -1) calculated from:

$$X\ (\text{minimum spherical}, P = -1) = \frac{2 \cdot \left(n^2 - 1\right)}{n + 2} \tag{2.3}$$

Table 2.1 ZEMAX files showing spherical aberration for various shape factors.

X	VIS LENS FILE
-2	V1
-1.5	V2
-1.0	V3
-.5	V4
0	V5
.5	V6
.714	V7
1.0	V8
1.5	V9
2	V10

Table 2.2 ZEMAX files showing spherical aberration for various shape factors.

X	IR LENS FILE
-7	IR1
-6	IR2
-5	IR3
-4	IR4
-3	IR5
-2	IR6
-1	IR7
0	IR8
1	IR11
2	IR13
3	IR14
4	IR15
5	IR16
6	IR17
7	IR18

Lab Exercises:

Shape Factor

Use a 100-mm focal length plano-convex lens to investigate spherical aberration. Use the microscope and ground glass screen assembly to study the amount of spherical aberration vs. the orientation of the lens. The source is at infinity, so your results should verify the ray traces of Figures 2.3(c) and 2.3(d).

- Focus the microscope on the ground glass screen by sliding the microscope back and forth in its holder. Once focused, whatever pattern of light falls on the screen will automatically be in focus in the microscope. You can then move the entire assembly along the z-axis to study the aberrated patterns of light seen through focus. (This method 'forces' the point of focus of the microscope to be in just one plane along the z-axis. With the ground glass screen, it is the ground (frosted) side of the plate. Without the ground glass screen, it is through a region along the z-axis defined by the microscope's depth of focus).

- For reference, note which side of the lens is planar, and which side is curved.

- Orient the lens so the planar side is towards the source:
● Make drawings of the patterns of light at paraxial focus, marginal focus, and at the minimum blur circle.
● Measure the distance between the marginal and paraxial focus, using the microscope.

- Orient the lens so the curved side is towards the source:
● Make drawings of the patterns of light at paraxial focus, marginal focus, and at the minimum blur circle.
● Measure the distance between the marginal and paraxial focus, using the microscope.

● Which lens orientation gave minimum spherical aberration?
● If you are going to use a plano-convex lens to focus a laser beam (which is essentially a collimated beam of light), how should you orient the lens to produce the smallest spot size at focus?

Chromatic Aberration

The refractive index of a material depends on the wavelength of light. Generally, for glass in the visible region, the index for blue light is significantly higher than for red light. We saw this when we measured the dispersion curve for the prism. For a positive lens, the result is that it has a longer focal length for red light than for blue light. This results in a separation of colors along the optical axis, termed ***longitudinal chromatic aberration***, **(LCA)**. The corresponding separation of colors transverse to the optical axis is called ***transverse chromatic aberration***, **(TCA)**.

In this part of the lab you will qualitatively study both LCA and TCA of the positive 100 mm focal length lens and a 100 mm focal length achromat.

- Use the 100-mm focal length **plano-convex** lens, and the microscope/ground glass screen assembly to study the distribution of colors through the region between marginal focus and paraxial focus.

- Make drawings of what you see at marginal focus, paraxial focus, and at the minimum blur circle.
- Explain the order of colors that you observe both in the longitudinal (LCA) and transverse (TCA) directions.

- Repeat these observations using the 100-mm focal length lens **achromat** lens.

- Make drawings of what you see at marginal focus, paraxial focus, and at the minimum blur circle.

- Which lens produces less chromatic aberration? Why?

Coma

This is difficult to demonstrate in isolation and is usually mixed with other aberrations. Pure coma is shown in Fig. 2.4. Use the achromatic doublet lens (100mm FL) illuminated by a point source to observe a coma pattern on a screen (turn the lens so that the flatter side is towards the source). Investigate the nature of the image as a function of location along the optical axis (z-axis) and off-axis angle (the field angle $\bar{u}$, which is equivalent to the rotation angle of the lens).

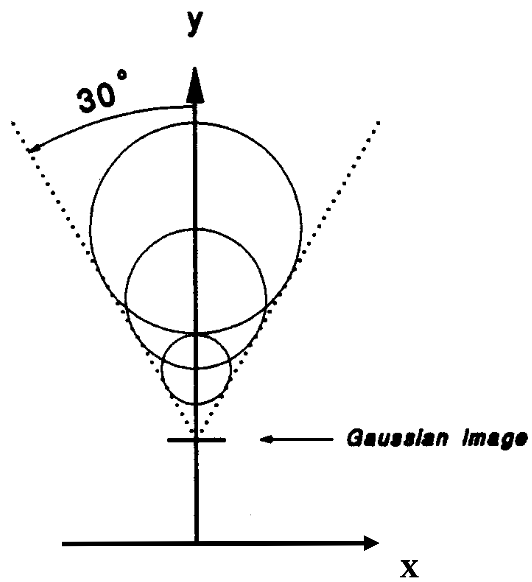


Figure 2.4. Illustration of coma.

- Describe and carry out a simple means of knowing whether or not the lens is normal to the optical axis when the rotation stage reads "0°" [HINT: consider the <u>symmetry</u> of the patterns for spherical aberration]
- Make drawings of the image seen at three locations along the optical axis, when the lens rotation angle is 0° (lens axis is normal to the beam). Record the three z-axis values to use in the following steps.
- Does this look like pure spherical aberration? Should it? Why or why not?

- Change the lens rotation angle (field angle $\bar{u}$) of +10°. Make three drawings at the same z-axis locations as before.
- Change the lens rotation angle (field angle $\bar{u}$) of -10°. Make three drawings at the same z-axis locations as before.
- Change the lens rotation angle (field angle $\bar{u}$) of +20°. Make three drawings at the same z-axis locations as before.
- Change the lens rotation angle (field angle $\bar{u}$) of -20°. Make three drawings at the same z-axis locations as before.

(NOTE: At this point, you should have made a total of 15 drawings.)

- Were the observed aberration patterns symmetrical with respect to the image plane location? Were they symmetric with respect to the field angle $\bar{u}$?

Astigmatism

Astigmatism is caused by the fact that the radius of curvature of a spherical surface is different in two orthogonal planes for an off-axis object point. The result is that a point object gets imaged into two orthogonal lines. Figure 2.5 shows an off-axis object point and two orthogonal fans of rays that traverse obliquely through the lens.

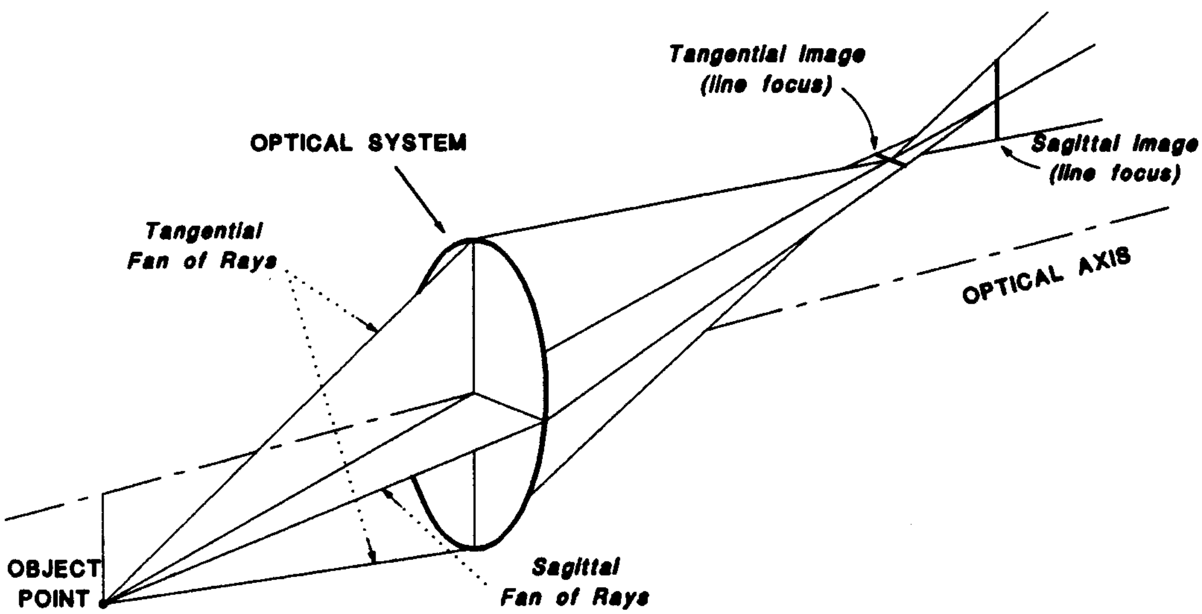
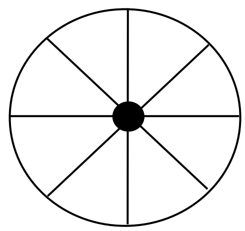


Figure 2.5. Sagittal and tangential foci.

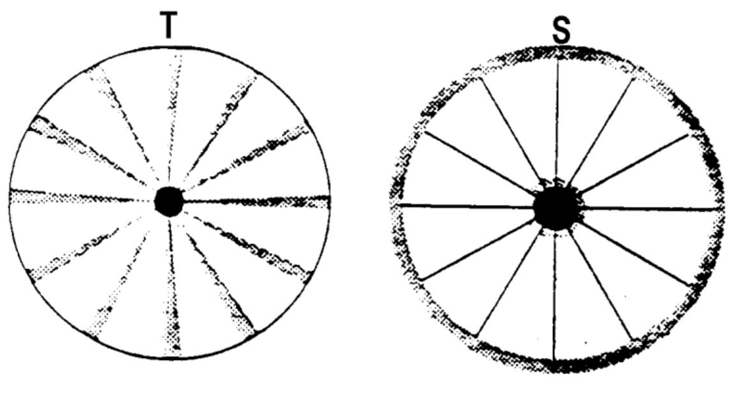
The **tangential plane** is defined as the plane containing the object point and the optical axis. The **sagittal plane** is defined as the plane which is perpendicular to the tangential plane and contains the optical axis. Notice that the line images from rays in these two planes are rotated 90 degrees from their respective plane. In other words, the tangential image is formed in the sagittal plane, and the sagittal image is formed in the tangential plane. Study Fig. 2.5 to make sure you understand this geometry. As with spherical aberration, there is a circle of least confusion located midway between the tangential and sagittal line images. The circle of least confusion will be a circular blur if no other aberrations are present. Refer to Fig. 2.5 to answer the following questions:

- Where do rays in the tangential plane come to focus?
- Where do rays in the sagittal plane come to focus?
- Which rays produce the tangential line image?
- Which rays produce the sagittal line image?

- Explain the astigmatic images of a spoked wheel, shown in Fig.2.6.



Spoked Wheel Object



Images

Figure 2.6. Astigmatic images of a spoked wheel.

Astigmatism is easily demonstrated. Use the same setup as for coma, but turn the achromat lens around so that the <u>flatter side is towards the image</u>.

- Measure the separation of the line foci as a function of field angle.
- Describe and explain any differences in your experimental setup, compated to Fig. 2.5.

- Occlude the lens with the aperture shown in Fig. 2.7 (commonly called a Hartman screen). Alternately open and block the apertures and look at the line foci.

- Which holes contribute to the tangential focus?
- Which holes contribute to the sagittal focus?