

## LAB 4: AFOCAL SYSTEMS

### REFRACTING TELESCOPES

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History: (from Hecht and Zajac, *Optics*, 1976, p. 151-155)

“It is not at all clear who actually invented the telescope. In point of fact, it was probably invented and reinvented many times. Do recall that by the seventeenth century spectacle lenses had been in use in Europe for about three hundred years. During that long span of time, the fortuitous juxtapositioning of two appropriate lenses to form a telescope seems almost unavoidable. In any event, it is most likely that a Dutch optician, possibly even the ubiquitous Zacharias Jansen of microscope fame, first constructed a telescope and in addition had inklings of the value of what he was peering into. The earliest indisputable evidence of the discovery, however, dates to October 2, 1608 when Hans Lippershey petitioned the States-General of Holland for a patent on a device for seeing at a distance (which is what *teleskopos* means in Greek). Incidentally, as you might have guessed, its military possibilities were immediately recognized. His patent was therefore not granted; instead the government purchased the rights to the instrument and he received a commission to continue research. Galileo heard of this work and by 1609 he had fashioned a telescope of his own using two lenses and an organ pipe as a tube. It was not long before he constructed a number of greatly improved instruments and began to astound the world with the forthcoming astronomical discoveries for which he is so justly famous.”

For a wonderful history of the telescope, including an image of the first-known sketch of a telescope, point your Internet web browser to:

<http://galileo.rice.edu/sci/instruments/telescope.html>

A great collection of photos of antique telescopes may be found at:

<http://www.antiquetelescopes.org/>

### Theory:

A telescope is used to image and magnify a distant object. In its simplest form, a telescope consists of 2 optical elements—an objective and an eyepiece separated by the sum of their focal lengths. These may be either refractive or reflective elements (lenses or mirrors, respectively). While the objective always has positive power, the eyepiece may have either positive or negative power. If the eyepiece has positive power, the telescope is a **Keplerian or Astronomical telescope**, shown in Fig. 4.1 (a). If the eyepiece has negative power, the telescope is a **Galilean telescope**, shown in Fig. 4.1 (b). Note that in both cases the distance between the two elements is equal to the sum of the focal lengths of the elements (positive or negative signs taken into account).

For this lab, we will consider objects to be near or at infinity, so that the rays in object space are parallel. The concept of infinity does not strictly mean the object is at an infinite distance, as is essentially the case when looking at astronomical objects. Rather,

an “infinite” object distance is one for which the object distance  $z$  is much greater than  $z'$  in the imaging equation:

$$\frac{1}{f} = \frac{1}{z'} - \frac{1}{z} \quad (4.1)$$

In this case, the object distance  $z$  may be ignored and the equation reduces to:

$$\frac{1}{f} \approx \frac{1}{z'} \quad (4.2)$$

which says that  $z' \approx f$ . In other words, an image is located at the rear focal point,  $F^*$ , of the objective. This intermediate image in turn becomes the object for the eyepiece. Because this intermediate image is, in turn, located at the front focal point,  $F$ , of the eyepiece, the eyepiece forms the final image at infinity. Stated another way, the rays leave the eyepiece parallel, just like the rays in the original object space. The relaxed eye can then bring these parallel rays to focus on the retina. In actual telescopes, this is done by moving (“focusing”) the eyepiece with a mechanical stage until the front focal point  $F$  coincides with the intermediate image.

An optical system for which the rays in object and image space are parallel, or equivalently a system for which the object is at  $-\infty$  and the image at  $+\infty$  (infinite conjugates) is termed an **afocal** system. Afocal literally means without a focal length. The telescopes you will build in this lab are all afocal systems. The following theory applies for afocal telescopes.

## MAGNIFICATION

The magnification of a telescope (afocal optical system) is described in terms of the angular magnification  $M_\theta$ , also called the magnifying power  $M_p$ . The angular magnification is defined as the ratio of the angular size of the image subtended at the eye (seen through the telescope) to the angular size of the object subtended at the eye (seen without the telescope). Stated another way, the angular magnification is the ratio of the apparent size of the image seen looking through the telescope to the apparent size of the object seen with the unaided eye. For an object at infinity, the angular size of the object subtended at the eye is essentially the same as the angular size of the object subtended at the aperture stop of the telescope (the objective). It is the angular subtense at the objective that will be used for the derivation of angular magnification.

Figures 4.2 and 4.3 show the marginal rays (MR) and chief rays (CR) for Keplerian and Galilean telescopes, respectively. By definition, the angular magnification

$M_\theta$  is given as the ratio of the chief ray angle in image space to the chief ray angle in object space:

$$M_\theta = \frac{\bar{u}'}{\bar{u}} \quad (4.3)$$

While this is fine as a definition, it does not lend itself well as a working definition of angular magnification. The following derivation relates angular magnification to physical parameters that are easily measured for an actual telescope.

Consider the chief ray height  $\bar{y}$  at the eye lens, or eyepiece, in Fig. 4.2:

$$\tan \bar{u} = -\frac{\bar{y}}{z}; \tan \bar{u}' = -\frac{\bar{y}}{z'} \quad (4.4)$$

In order to find where the eye lens forms the exit pupil  $E$ , use the imaging equation (4.1), substituting  $z = -(f_o + f_e)$  to give:

$$\frac{1}{z'} = \frac{f_o}{f_e \cdot (f_o + f_e)} \quad (4.5)$$

Further substitution gives:

$$\tan \bar{u} = \frac{\bar{y}}{f_o + f_e}; \tan \bar{u}' = -\frac{\bar{y} \cdot f_o}{f_e \cdot (f_o + f_e)} \quad (4.6)$$

For small angles (i.e. for small fields of view),  $\tan \bar{u} \approx \bar{u}$  and  $\tan \bar{u}' \approx \bar{u}'$ . The final result may now be obtained:

$$M_\theta = -\frac{f_o}{f_e} \quad (4.7)$$

The magnifying power of a telescope is the ratio of the focal length of the objective to the focal length of the eyepiece. If  $f_e$  is positive,  $M_\theta$  is negative and the image is inverted (Keplerian). If  $f_e$  is negative,  $M_\theta$  is positive and the image is upright (Galilean).

Finally, consider the entrance and exit pupils as shown in Fig. 4.2. If the diameters of the entrance and exit pupils are  $E$  and  $E'$  respectively, then by similar triangles:

$$\left| -\frac{f_o}{f_e} \right| = \frac{E}{E'} \quad (4.8)$$

The final equations for angular magnification are then:

$$|M_\theta| = \left| -\frac{f_o}{f_e} \right| = \frac{E}{E'} \quad (4.9)$$

where the absolute value signs are used for consistency with the fact that  $E$  and  $E'$  are both positive quantities.

### The Keplerian or Astronomical Telescope:

In its simplest form, this consists of a long focal length objective and a short focal length eye lens (ocular or eyepiece) separated by distance equal to the sum of the focal lengths, as shown in Fig. 4.1 (a).

Construct the following series of Keplerian telescopes as shown in Fig. 4.4 ([1]-[8]). Note that distant objects are enlarged (magnified) but inverted. There are several problems with this simple design, which we will also investigate.

### Magnification

Begin by constructing telescopes [1], [2], and [3] shown in Fig. 4.4. In each case, the focal length of the objective is the same but the focal length of the eye lens is different. For each telescope, first *calculate*  $M_\theta$  as the ratio of the focal lengths and then *measure*  $M_\theta$  as the ratio of the diameters of the pupils. This is easily done by placing an illuminated piece of diffuse mylar against the front side of the objective (the aperture stop and entrance pupil  $E$ ) and locating its image. This is the exit pupil  $E'$  itself. Measure the diameter of the exit pupil in order to calculate  $M_\theta$ . It should be noted that the proper use of a telescope is to place your eye in the exit pupil when looking through the instrument.

- Compare your results:  $|M_\theta| = \left| \frac{f_o}{f_e} \right|$  with  $|M_\theta| = \frac{E}{E'}$

The **eye relief** is defined as the distance from the rear vertex of the eye lens to the exit pupil  $E'$ . For each of these three telescopes, measure the eye relief.

- How does the eye relief seem to vary with  $M_\theta$ ?
- For the three telescopes, calculate where  $E'$  is located.

Use the optical design program *ZEMAX* and files listed below to compare your results to predicted values for these three telescopes:

- (Computer) What is  $M_0$ , as calculated from  $\bar{u}' / \bar{u}$ ? (use files Kep1, Kep2, and Kep3).
- (Computer) Where is the exit pupil  $E'$  located? (use files Kep1, Kep2, and Kep3).

## Chromatic Aberration

Consider telescope [4]. Observe the edge of a white object and notice the rainbow. This is chromatic aberration, and will be discussed in a later lab. Sketch the orders of the colors vs. field position. Replace the objective with an achromat by constructing telescope [5]. An achromat is a cemented pair of lenses constructed of two different glasses so that chromatic aberration is decreased. Observe the difference and make another sketch. Even more improvement could be obtained by replacing the eye lens with an achromat or color-corrected eyepiece.

## Vignetting

Vignetting in an optical system occurs when not all of the rays in the original on-axis bundle make it to the final image plane. This happens as the object point moves off-axis, i.e. as the field of view increases. The result is that towards the edge of the field of view the brightness of the image decreases. While this may not be readily apparent visually, a piece of film recording the image can easily show the unwanted effects of vignetting. Figure 4.6 illustrates the vignetting caused by the eye lens in a Keplerian telescope as the field of view increases (as  $\bar{u}$  increases).

The problem is overcome by placing a field lens at the intermediate focus between the objective and eye lens. This will produce a wider field of view, and the field will be uniformly illuminated. This is shown in Fig. 4.7. The field lens is at the image plane so the power of the telescope is unchanged. However, any dust or scratches on the surface of the field lens would be seen in the final image.

Construct telescope [6]. Note that this is the same as telescope [1] except for the field lens. Observe an image with and without the field lens in place. Describe any differences. Measure the location of the exit pupil with and without the field lens in place.

- What happens to the location of  $E'$  with the use of a field lens?
- What happens to the vignetting with the use of a field lens?

Now investigate the effect the field lenses have on the field of view, as shown in Fig. 4.7. Measure the full field of view with and without the field lenses in place.

This is easily done by positioning a meter stick in the field of view a large distance away from the telescope. Measure the distance between the meter stick and telescope,  $d$ , and measure the full height of the meter stick,  $h$ , seen looking through the telescope. The full field of view is then calculated as:

$$\text{FOV} \equiv 2 \bar{u} = 2 \cdot \arctan \left( \frac{h}{2d} \right) \quad (4.10)$$

- What was the measured full field of view (in degrees) with and without the field lens in place?
- (C) What is the full field of view calculated using ZEMAX? (for files Kep1 and Kep6).
- Calculate the magnifying power  $M_\theta$  for the telescopes, with and without the field lens. How does the field of view change as  $M_\theta$  changes?

### Inverted Image

In all of the Keplerian telescopes you have constructed so far, the image has been inverted. This is not a problem for stargazing, but it is for many terrestrial applications. Binoculars use a Porro-Abbe prism combination to invert the image. This is why the axis of the eyepiece is displaced from the objective axis in most binoculars.

Another approach is to use a **relay lens**. A relay lens is used to relay an image from one point in space to another. Most relay lenses are designed to work at  $2f$ - $2f$ , the condition of minimum conjugates. As part of a complete optical system, this extra image relay has the effect of inverting the final image to an upright orientation. Figure 4.1 (c) shows a Keplerian telescope with an erector, or relay lens in place.

Investigate the effect of using a relay lens by constructing telescope [7]. This is done by placing the relay lens a distance twice its focal length behind the first intermediate image plane. Reposition the eye lens so that the second image plane is at the front focal point of the eye lens. Note the upright orientation of the final image seen through the eye lens.

The vignetting problems still exist. They can be overcome by adding a field lens to the system. Using a 75mm field lens, construct a Keplerian having *both* a field lens and a relay lens. This configuration is telescope [8] in Fig. 4.4.

- When you have correctly built this telescope and understand how it works, show it to the lab instructor for credit!

## The Galilean Telescope:

To see an image of Galileo's original lens that he used to discover the moons of Jupiter, point your Internet web browser to:

<http://galileo.imss.firenze.it/news/cielimedicei/01/egalileo.html>

A Galilean telescope is a long focal length positive objective and a short focal length negative eye lens separated by the sum of the focal lengths. Note that because the focal length of the eye lens is negative, the length of the system is shorter than for the Keplerian. This too is an afocal system. Unlike the Keplerian, the image is upright, so no relay lens is required. This design is often used in field glasses and opera glasses. A more common use is in high power laser beam expanders. (An afocal telescope may be "turned around" and used to expand a beam as well. The objective and eye lenses are interchanged). The Galilean design is ideal for high power laser beams because there is no intermediate focus as in the Keplerian. Such a focus can easily concentrate the laser power high enough to break down the surrounding air and cause, in effect, lightning streaks (interesting, but nasty stuff!). The Galilean design avoids this problem.

### **The major drawback of the Galilean telescope is the location of the exit pupil.**

*Assuming that the aperture stop is at the objective (a different assumption than used in Fig. 4.3), derive an equation that locates the exit pupil  $E$  with respect to the eye lens. For the Galilean telescopes in Fig. 4.5, calculate where the exit pupil is located. How does the location change as the power of the objective is changed?*

Construct the Galilean telescopes [9] and [10] in Fig. 4.5. For these examples, the power of the eye lens remains fixed while the power of the objective is changed.

- Is there as much chromatic aberration as in the Keplerian?
- Where does it seem to be the best position to place your eye while looking through these telescopes?

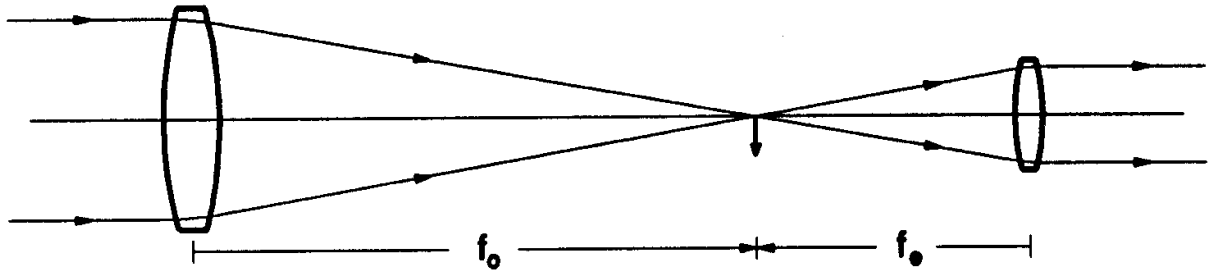
Calculate the magnifying power  $M_0$  of each of the Galilean telescopes.

- How does the field of view change as  $M_0$  changes?

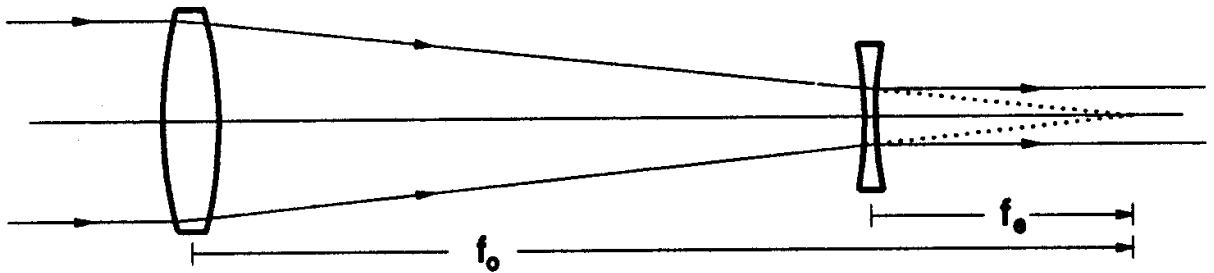
### **Questions:**

Use ZEMAX and files Gal9 and Gal10 to answer the following questions:

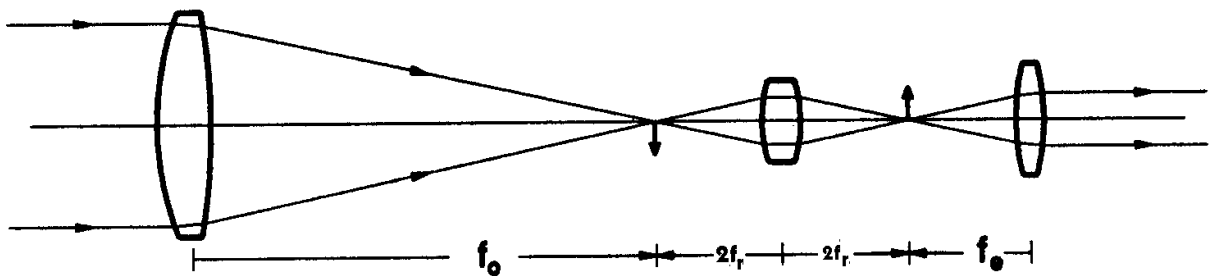
- (C) What is  $M_0$ ? (from the top of the screen).
- (C) Where is the exit pupil  $E'$  located?
- (C) What is the full field of view?



(a) Keplerian Telescope  
(astronomical)



(b) Galilean Telescope



(c) Field Telescope  
(terrestrial)

Figure 4.1. The three basic types of telescopes.



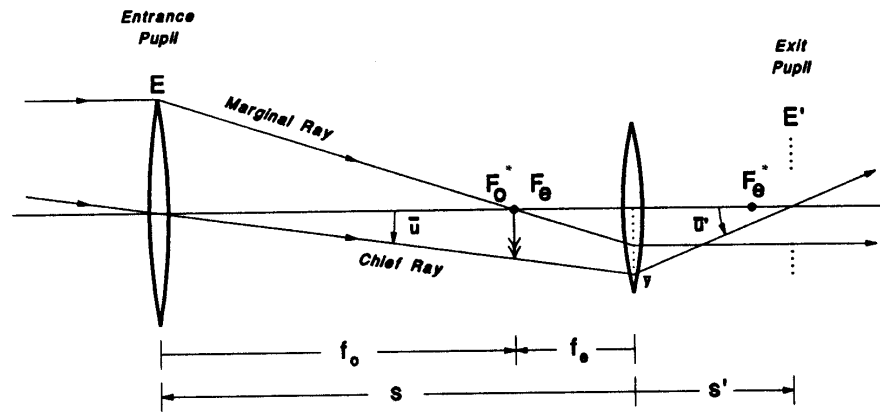


Figure 4.2. Keplerian telescope: marginal and chief rays.

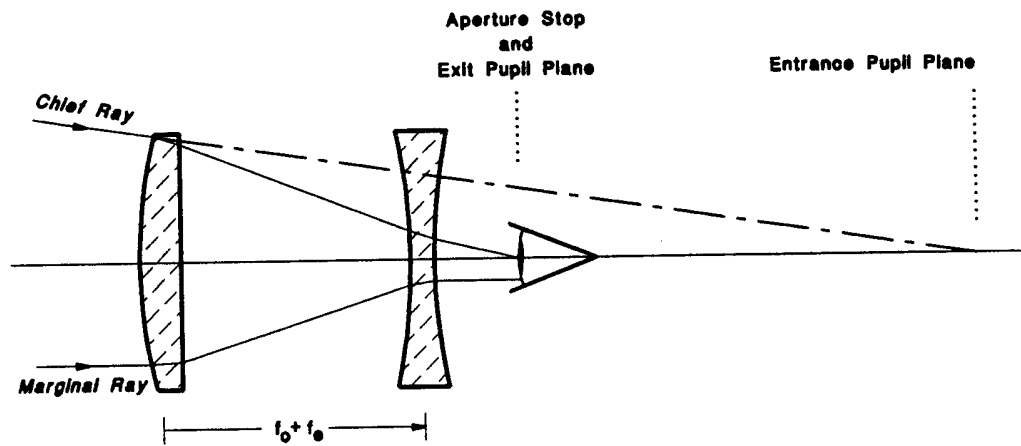
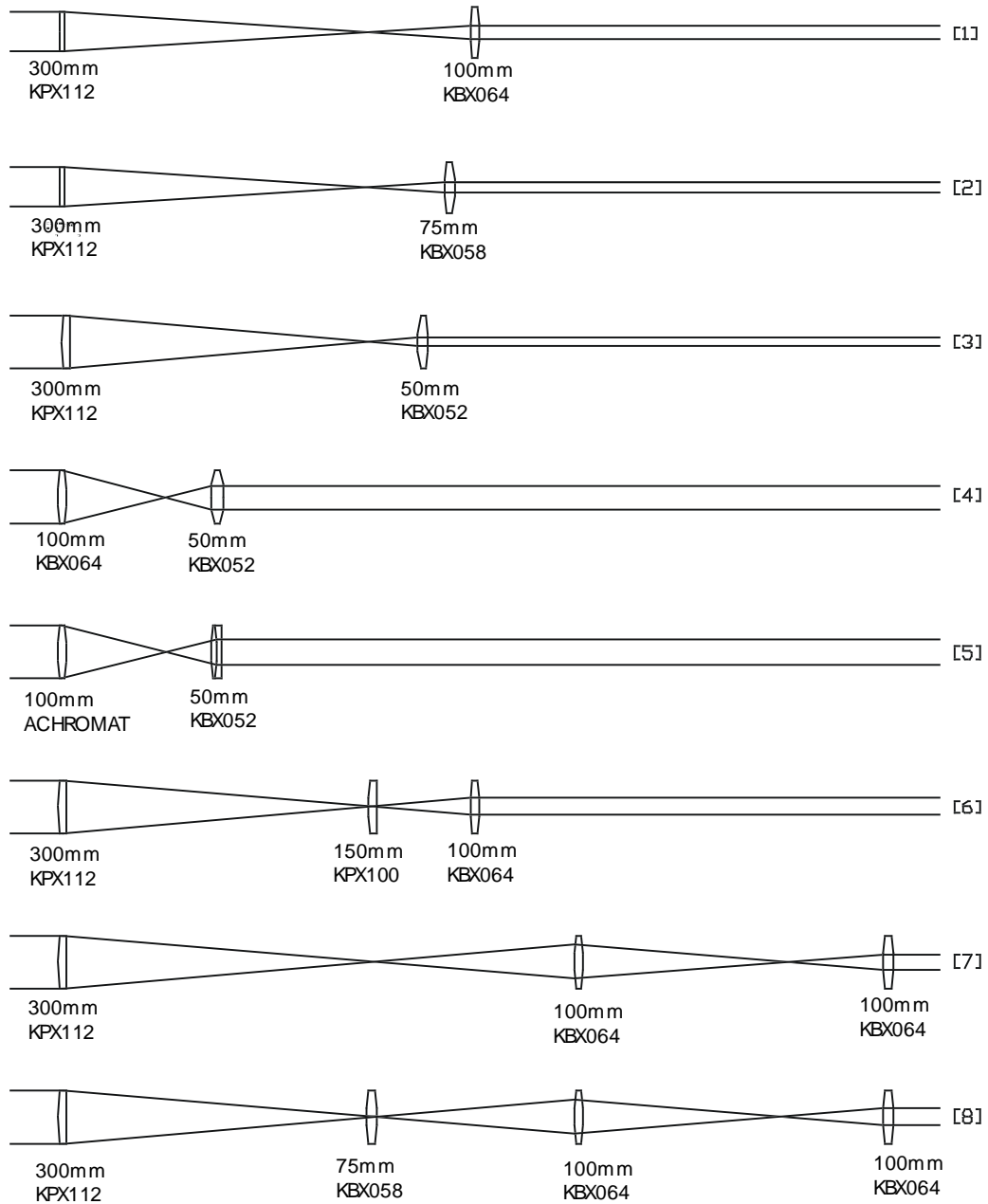


Figure 4.3. Galilean telescope: marginal and chief rays.



**Figure 4.4.** Keplerian telescope: experimental variations.

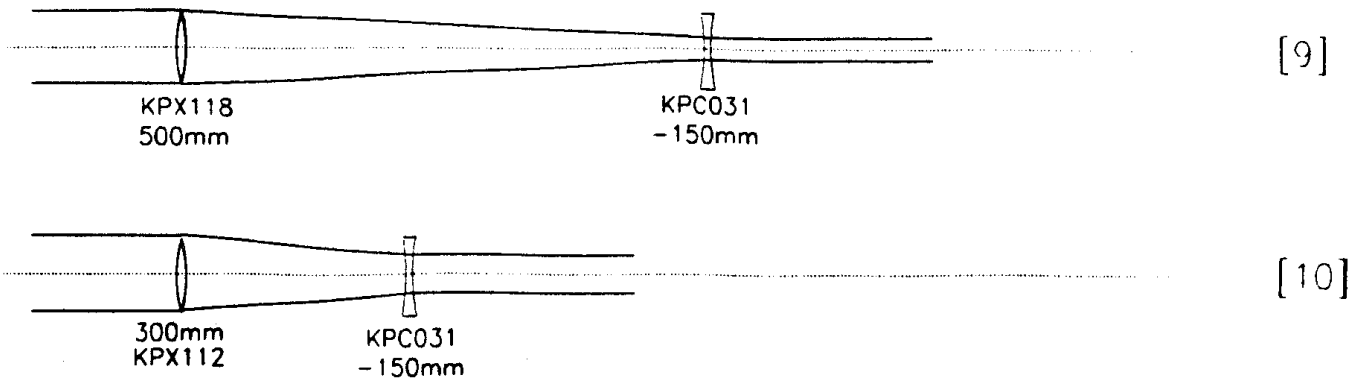


Figure 4.5. Galilean telescope: experimental variations.

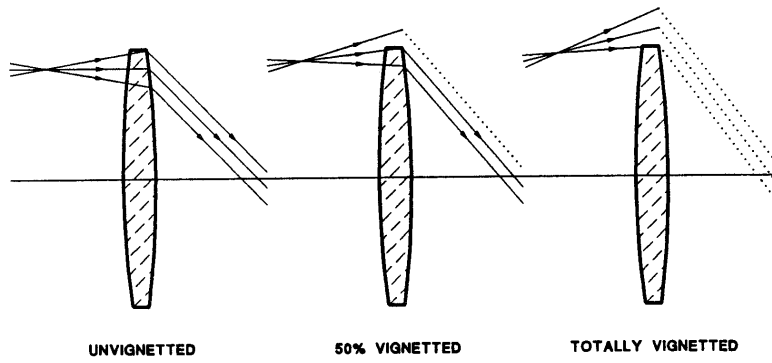


Figure 4.6. Vignetting: eye lens of a keplerian telescope.

