Conical RCWT Manual

Seung-Hune Yang

College of Optical Sciences,
University of Arizona,
Tucson, AZ
United States, 85721

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1 Introduction

Rigorous Coupled Wave Theory (RCWT) is considered as one of several methods to rigorously analyze the optical systems. RCWT is a rigorous solution to Maxwell's equations for calculating diffracted orders from a plane wave incident on periodic structures. Based on permittivity expressed as a Fourier series, a set of coupled differential equations are solved by using eigenvalue-eigenvector approach and boundary conditions at each interface of the structure. *ConicalRCWT.m* is the core program of RCWT in OPTISCAN. It calculates complex amplitudes of reflected and transmitted diffracted orders from a grating illuminated by a plane wave. In Section 2, it is explained how to use *ConicalRCWT.m*.

2 How to use *ConicalRCWT.m*

The way to use *ConicalRCWT.m* is shown below,

```
[Rs,Rp,Ts,Tp]   = ConicalRCWT(Grating,max_m,phi,psi,theta,lambda);
```

For arguments, *ConicalRCWT.m* requires grating, number of diffracted orders, and incident plane wave informations. `Grating` is a structure containing all information about gratings. With given arguments, *ConicalRCWT.m* calculates normalized reflectance and transmittance from a plane wave. `max_m` is number of diffracted orders interested in. Thus, returning variables, `Rs, Rp, Ts` and `Tp` consist of $2*$`max_m`$+1$ elements, starting from $-$`max_m` order to $+$`max_m` order. The larger `max_m` provides higher calculation accuracy with slower calculation speed. Incident plane wave to the object is described as

$$\mathbf{U}_{inc}(\mathbf{r}) = \mathbf{U}_{inc} \exp\left[ik_0 n_{inc}(\alpha\hat{\mathbf{x}} + \beta\hat{\mathbf{y}} + \gamma\hat{\mathbf{z}})\right], \quad (1)$$

where k_0 is $2\pi/\lambda_0$, n_{inc} is refractive index of incident medium. Direction cosines are expressed with ϕ and θ.

$$\alpha = \sin\theta\cos\phi \quad (2)$$

$$\beta = \sin\theta\sin\phi \quad (3)$$

$$\gamma = (1 - \alpha^2 - \beta^2)^{1/2}, \quad (4)$$

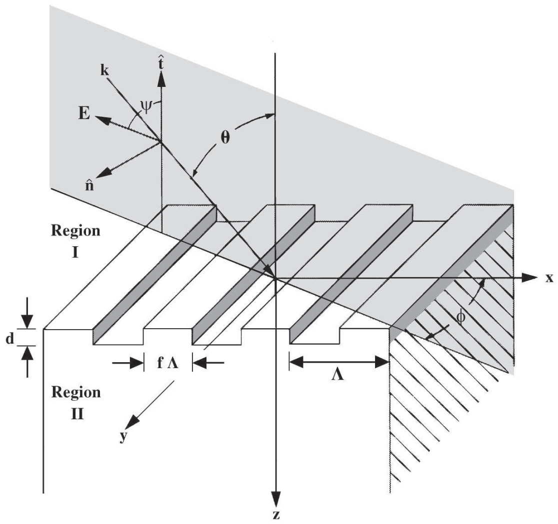


Figure 1: RCWT Geometry. (M. G. Moharam, E. B. Grann, D. A. Pommet, and T. K. Gaylor. Formulation for stable and efficient implementation of the rigorous coupled-wave analysis of binary gratings. *Journal of the Optical Society of America A*, 12:1068.1076, May 1995).

and ϕ, θ and ψ are defined in Fig. 1. Note that ϕ, θ and ψ are expressed in degree and unit of length is meter.

$\mathbf{U}_{inc}$ is described in local polarization with respect to $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}$.

$$\mathbf{U}_{inc} = U_{inc} \begin{bmatrix} \cos\psi \\ \sin\psi \end{bmatrix}, \tag{5}$$

where U_{inc} is a complex amplitude of incident plane wave.

θ, ϕ and ψ are given from Eqns. (2), (3), (4), (5) and used for arguments of *ConicalRCWT.m.*

`Rs` and `Ts` are normalized reflectance and transmittance of electric field while `Rp` and `Tp` are those of magnetic field. Thus `Rp` and `Tp` are multiplied by conversion factors. Thus, diffracted orders are

$$\mathbf{U}_{reflected} = \sum_{j=-m}^{m} \mathbf{R}(j)\mathbf{U}_{inc} \exp\left[i2\pi n_{inc}(\alpha_j\hat{\mathbf{x}} + \beta\hat{\mathbf{y}} - \gamma_j\hat{\mathbf{z}})\right] \tag{6}$$

$$\mathbf{U}_{transmitted} = \sum_{j=-m}^{m} \mathbf{T}(j)\mathbf{U}_{inc} \exp\left\{i2\pi n_{sub}[\alpha'_j\hat{\mathbf{x}} + \beta'\hat{\mathbf{y}} + \gamma'_j(\hat{\mathbf{z}} - d)]\right\}, \tag{7}$$

where

$$\mathbf{R}(j) = \begin{bmatrix} r_s(j) & 0 \\ 0 & \dfrac{r_p(j)}{\text{conj}(n_{inc} \cdot i)} \end{bmatrix} \tag{8}$$

$$\mathbf{T}(j) = \begin{bmatrix} t_s(j) & 0 \\ 0 & \dfrac{t_s(j)}{-\text{conj}(n_{sub} \cdot i)} \end{bmatrix}. \tag{9}$$

And α_n and α'_n are determined from the Floquet condition and are given by

$$\alpha_j = \alpha - j(\lambda_0/\Lambda) \tag{10}$$

$$\alpha'_j = \alpha' - j(\lambda_0/\Lambda), \tag{11}$$

where α' and β' are determined from Snell's law and are given by

$$n_{inc}\sqrt{\alpha^2 + \beta^2} = n_{sub}\sqrt{\alpha'^2 + \beta'^2}, \tag{12}$$

where n_{sub} is refractive index of the substrate, d is grating height and Λ is grating period. Therefore γ_j and γ'_j are given by

$$\gamma_j = \sqrt{1 - \alpha_j^2 - \beta^2} \tag{13}$$

$$\gamma'_j = \sqrt{1 - \alpha_j'^2 - \beta'^2} \tag{14}$$

In Appendix, one example is provided in MATLAB^{TM} code to show how to use *ConicalRCWT.m*.

Appendix

Contents

Initialize Program

```
clear all; close all;

%Add RCWT folder into Matlab path
addpath C:\OSCAN75_V0\workfunc\accessories\rcwt_calculator\;
nm          = 1e-9;         % nano meter
```

Define refractive indices of material to be used in simulation

```
SIL         = 2.0;
Air         = 1.0;
Glass       = 1.5;
Cr          = 2.314+1i*3.136;
```

Define RCWT Geometry variables

```
%Grating Period
period      = 320*nm;

%Grating heights
grating_h   = [ 30;
                50]*nm;

%Refractive index of incident medium
```

```
n_incident  = SIL;

%Refractive index of substrate
n_substrate = Glass;

%Refractive indices of material to the left side.
n_grating1  = [ Air;
                Cr  ];

%Refractive indices of material to the right side.
n_grating2  = [ Air;
                Air ];

%First column for rising edge, second column for falling edge
% and third column should be 1.00
duty_cycle  = [ 0.25   0.75     1.00;
                0.25   0.75     1.00    ];
```

Define incident plane wave

```
lambda      = 405*nm;   % wavelength
psi         = 45;       % in degree
phi         = 45;       % in degree
theta       = 60;       % in degree
U_inc       = 1;        % Complex amplitude of incident plane wave
```

Input RCWT Grating

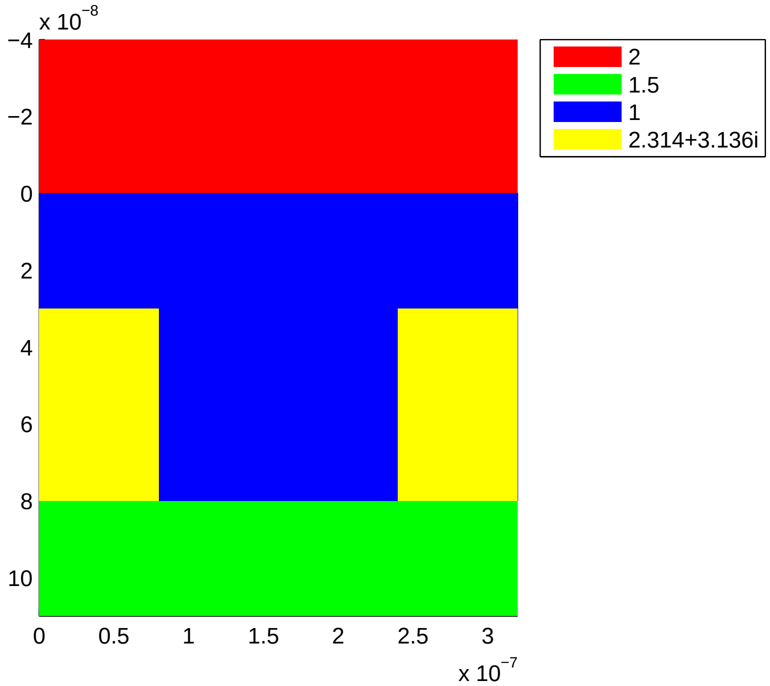
```
Grating.n1      = n_incident;
Grating.n2      = n_substrate;
Grating.ng1     = n_grating1;
Grating.ng2     = n_grating2;
Grating.h       = grating_h;
Grating.d       = period;
Grating.cperd   = duty_cycle;

%# of orders to be used in RCWT
max_m           = 3;
```

```
%========Dummy part but this is required to show geometry plot===========
Grating.incident_flag  = 0;
Grating.substrate_flag = 0;
Grating.inc_filename = '';
Grating.sub_filename = '';
Grating.inc_dispers_file = '';
Grating.sub_dispers_file = '';
load layers.mat;
for ii=1:length(duty_cycle),
    mlayers(ii,1)     = layers;
end
%========================================================================
```

RCWT Geometry Plot

```
DrawGratF2(Grating,mlayers);
```



Conical RCWT

```
[Rs,Rp,Ts,Tp]   = ConicalRCWT(Grating,max_m,phi,psi,theta,lambda);

U_rs    = Rs*U_inc;                           %Complex amplitude of U_rs
U_rp    = Rp*U_inc/(conj(n_incident)*1i);   %Complex amplitude of U_rp
U_ts    = Ts*U_inc;                           %Complex amplitude of U_ts
U_tp    = Tp*U_inc/(-conj(n_substrate)*1i); %Complex amplitude of U_tp
```