Coherence
(Chapter 5 – Part A)

Diffraction and Interferometry

What is Coherence?

- Many of our expressions to describe interference fringes have the form:

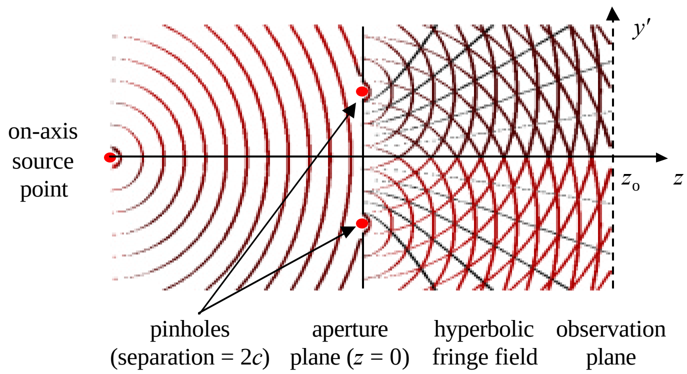
$$I_{total} = I_1 + I_2 + 2\sqrt{I_1 I_2}(coherence\ factor)\cos(\phi_1 - \phi_2)$$

- I_1 and I_2 are the irradiance values from the individual beams that combine to make the fringe field. ϕ_1 and ϕ_2 are beam phases.
- The third term in the expression, called the *cross term*, produces modulation and defines orientation and spacing of fringes.
- When the source emits more than one wavelength, or the source is distributed, like a light bulb filament, an extra factor must be included in the cross term. The extra factor decreases fringe contrast, and it may be a function of the geometry. In this chapter, we learn how to calculate the coherence factor.

Outline

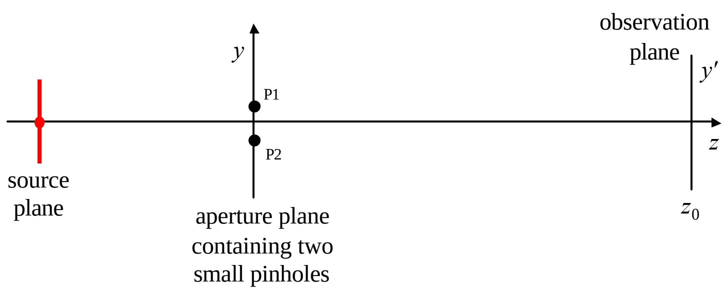
- Categories of Coherence
  - Temporal coherence: What happens when the source emits more than one wavelength?
  - Spatial coherence: What happens when the source is not a point source?
  - Degree of coherence: What contrast reduction is due to coherence effects?
- Temporal Coherence
- Spatial Coherence
- Formal Coherence Development

Young's Double-Pinhole Interferometer (YDPI)



- Diffraction from ideal pinholes creates two point sources.
- Assume $2c << z_o$
- Assume asymptotes describe small region around z axis at observation plane

YDPI Used for Coherence Theory Development



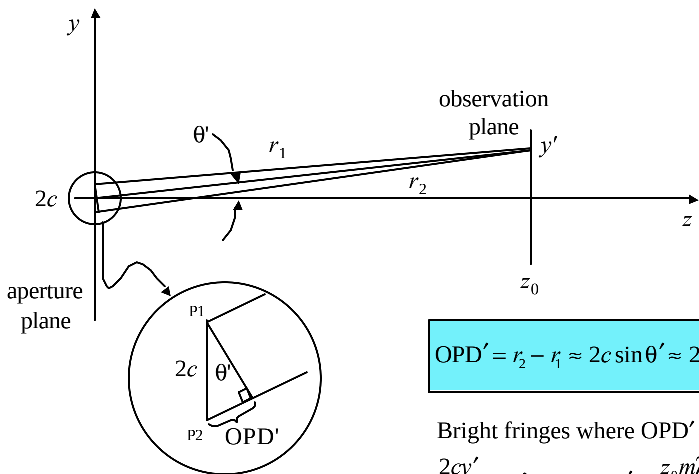
- Observe interference pattern on the observation plane to determine the degree of coherence
- Temporal coherence: point source on axis
- Spatial coherence: extended source

Temporal Coherence

What happens when the source emits more than one wavelength?

- YDPI with a single point source on axis
- YDPI with a two-wavelength source
- YDPI with a polychromatic source
- Interpretation and applications

YDPI Optical Path Difference in Observation Space

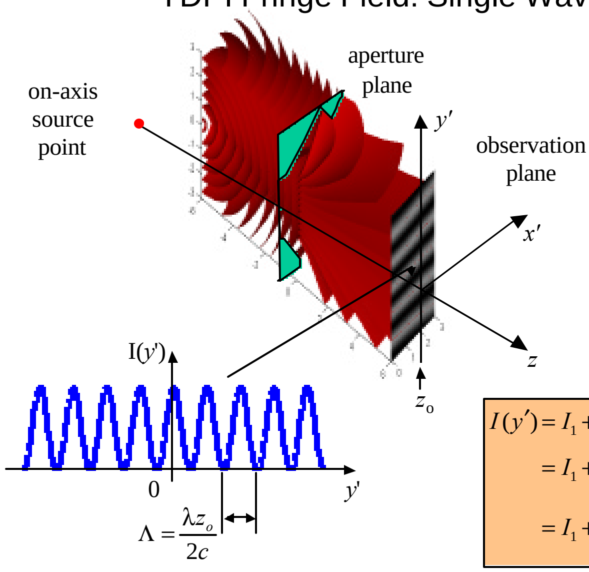


$$\mathrm{OPD}' = r_2 - r_1 \approx 2c\sin\theta' \approx 2c\frac{y'}{z_0}$$

Bright fringes where $\mathrm{OPD}' = m\lambda \Rightarrow$

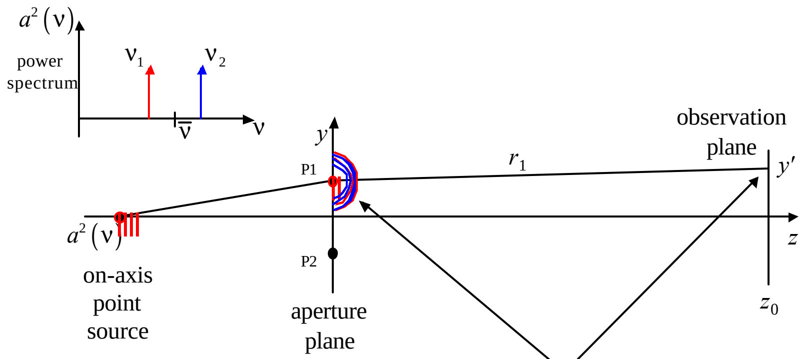
$$\frac{2cy'}{z_0} = m\lambda \quad \text{and} \quad y' = \frac{z_0 m\lambda}{2c}$$

YDPI Fringe Field: Single Wavelength



$$I(y') = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(k\left[r_2 - r_1\right]\right)$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(k\,\mathrm{OPD}'\right)$$
$$= I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\left(k\frac{2cy'}{z_o}\right)$$

YDPI: Temporal Behavior with Two Wavelength Source



The wave arriving at the observation plane from each pinhole is a combination of two frequencies. As a function of time, this wave contains high frequency and beat terms, as described in Chapter 3.

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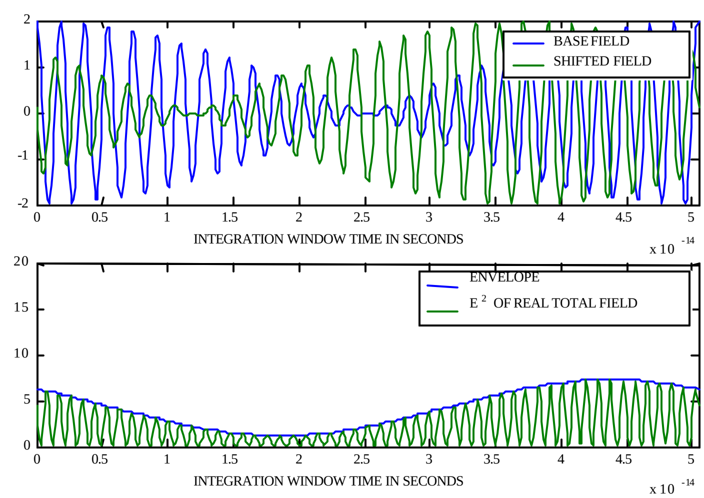
YDPI: Temporal Behavior with Two Wavelength Source



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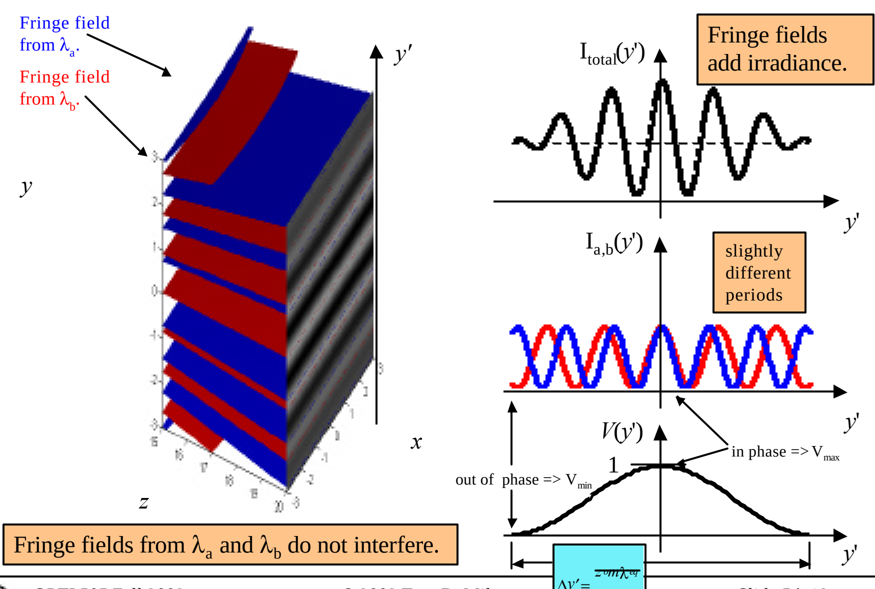
YDPI: Temporal Behavior with Two Wavelength Source

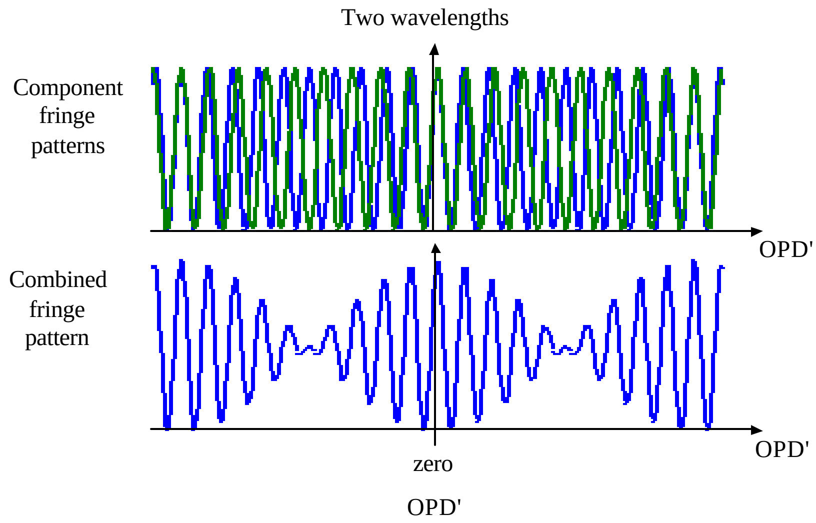
Some important conclusions can be drawn from temporal behavior of the two-wavelength source:

- Two wavelengths, widely separated, produce interference in which inter-modulation between wavelengths is averaged out by the detector.
- Closely-spaced wavelengths produce interference that is equivalent to the pattern from the mean wavelength, except in very special cases.
- The two facts above justify treatment of the fringe pattern resulting from each wavelength independently from the patterns due to other component wavelengths. That is, there is no detectable inter-modulation in the total fringe pattern, and the fringes from each wavelength can be summed to produce the total pattern.

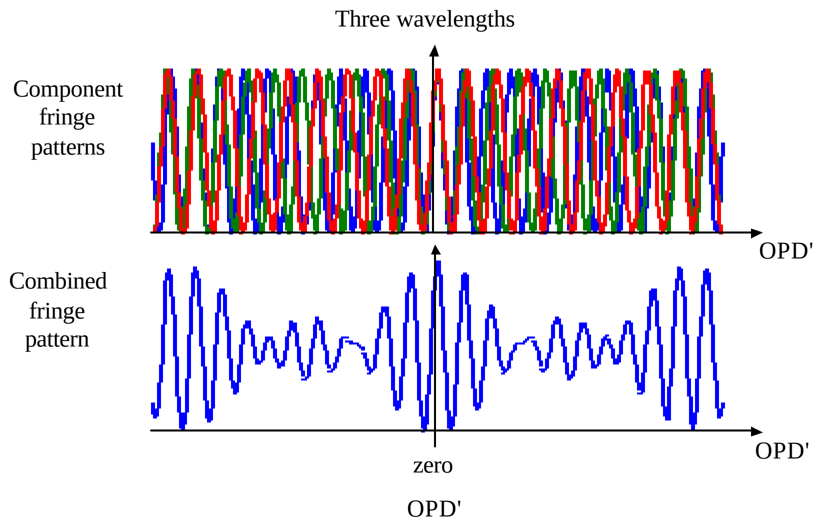
YDPI Fringe Field – Two-Wavelength Source



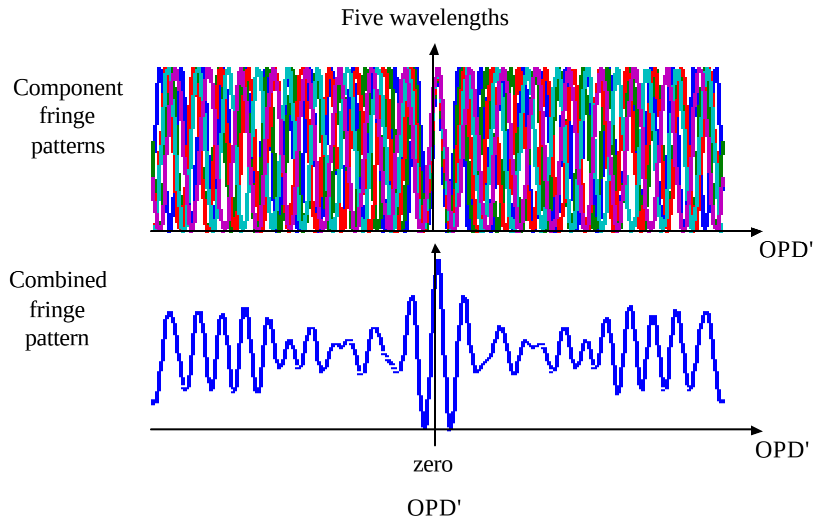
YDPI Fringe Field – Polychromatic Source



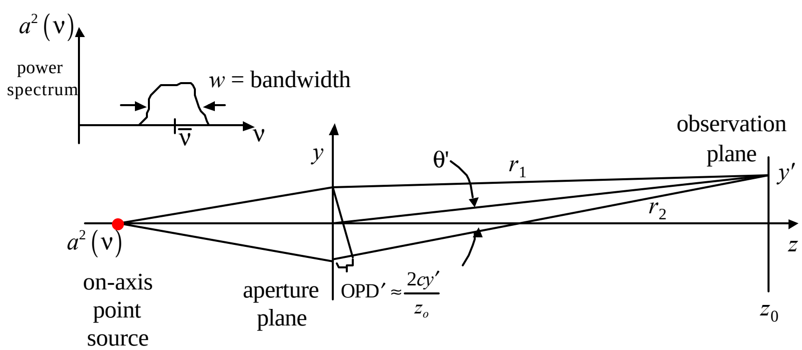
YDPI Fringe Field – Polychromatic Source



YDPI Fringe Field – Polychromatic Source



YDPI Fringe Field – Polychromatic Source



$$I\left(\nu,\frac{\mathrm{OPD}'}{c}\right)=K_1a^2(\nu)+K_2a^2(\nu)+2\sqrt{K_1K_2}\,a^2(\nu)\cos\left(2\pi\nu\frac{\mathrm{OPD}'}{c}\right)$$

$$I\left(\frac{\mathrm{OPD}'}{c}\right)=\int I\left(\nu,\frac{\mathrm{OPD}'}{c}\right)d\nu$$

K's are diffraction constants, c = speed of light

YDPI Fringe Field – Polychromatic Source

$$I\left(\frac{\mathrm{OPD}'}{c}\right)=\int I\left(\nu,\frac{\mathrm{OPD}'}{c}\right)d\nu$$

$$=\int\left[\mathrm{K}_1 a^2(\nu)+\mathrm{K}_2 a^2(\nu)+2\sqrt{\mathrm{K}_1\mathrm{K}_2}\,a^2(\nu)\cos\left(2\pi\nu\frac{\mathrm{OPD}'}{c}\right)\right]d\nu$$

$$=(\mathrm{K}_1+\mathrm{K}_2)\int a^2(\nu)\,d\nu\ +2\sqrt{\mathrm{K}_1\mathrm{K}_2}\int a^2(\nu)\cos\left(2\pi\nu\frac{\mathrm{OPD}'}{c}\right)d\nu$$

$$=(\mathrm{K}_1+\mathrm{K}_2)I_w\ +2\sqrt{\mathrm{K}_1\mathrm{K}_2}\,\mathrm{Re}\left\{\int a^2(\nu)e^{j2\pi\nu\frac{\mathrm{OPD}'}{c}}d\nu\right\}$$

$$=(\mathrm{K}_1+\mathrm{K}_2)I_w\ +2\sqrt{\mathrm{K}_1\mathrm{K}_2}\,I_w\,\mathrm{Re}\left\{\frac{\mathscr{F}\left[a^2(\nu)\right]_{\frac{\mathrm{OPD}'}{c}}}{I_w}\right\}$$

$$=(\mathrm{K}_1+\mathrm{K}_2)I_w\ +2\sqrt{\mathrm{K}_1\mathrm{K}_2}\,I_w m_{12}\left(\frac{\mathrm{OPD}'}{c}\right)\cos\phi_{12}\left(\frac{\mathrm{OPD}'}{c}\right)$$

YDPI Fringe Field – Polychromatic Source

$$I\left(\frac{\mathrm{OPD}'}{c}\right)=(\mathrm{K}_1+\mathrm{K}_2)I_w\ +2\sqrt{\mathrm{K}_1\mathrm{K}_2}\,I_w m_{12}\left(\frac{\mathrm{OPD}'}{c}\right)\cos\phi_{12}\left(\frac{\mathrm{OPD}'}{c}\right)$$

$$m_{12}\left(\frac{\mathrm{OPD}'}{c}\right)=\left|\frac{\mathscr{F}\left[a^2(\nu)\right]_{\frac{\mathrm{OPD}'}{c}}}{I_w}\right|$$

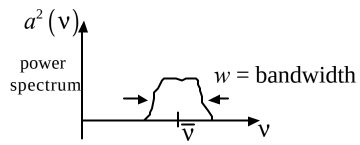
$$\phi_{12}\left(\frac{\mathrm{OPD}'}{c}\right)=phase\left\{\mathscr{F}\left[a^2(\nu)\right]_{\frac{\mathrm{OPD}'}{c}}\right\}$$

$$I_{\max}=(\mathrm{K}_1+\mathrm{K}_2)I_w\ +2\sqrt{\mathrm{K}_1\mathrm{K}_2}\,I_w m_{12}\left(\frac{\mathrm{OPD}'}{c}\right)$$

$$I_{\min}=(\mathrm{K}_1+\mathrm{K}_2)I_w\ -2\sqrt{\mathrm{K}_1\mathrm{K}_2}\,I_w m_{12}\left(\frac{\mathrm{OPD}'}{c}\right)$$

$$V=\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}=\frac{2\sqrt{\mathrm{K}_1\mathrm{K}_2}}{\mathrm{K}_1+\mathrm{K}_2}m_{12}\left(\frac{\mathrm{OPD}'}{c}\right)$$

YDPI Fringe Field – Polychromatic Source



Usually, $w \ll \bar{\nu}$.

$$\Rightarrow \phi_{12}\left(\frac{\text{OPD}'}{c}\right) \approx 2\pi\bar{\nu}\frac{\text{OPD}'}{c} + \beta_{12}\left(\frac{\text{OPD}'}{c}\right)$$

$$\beta_{12}\left(\frac{\text{OPD}'}{c}\right) = phase\left\{\mathscr{F}\left[a^2(\nu)\right]_{\frac{\text{OPD}'}{c}}\right\}$$

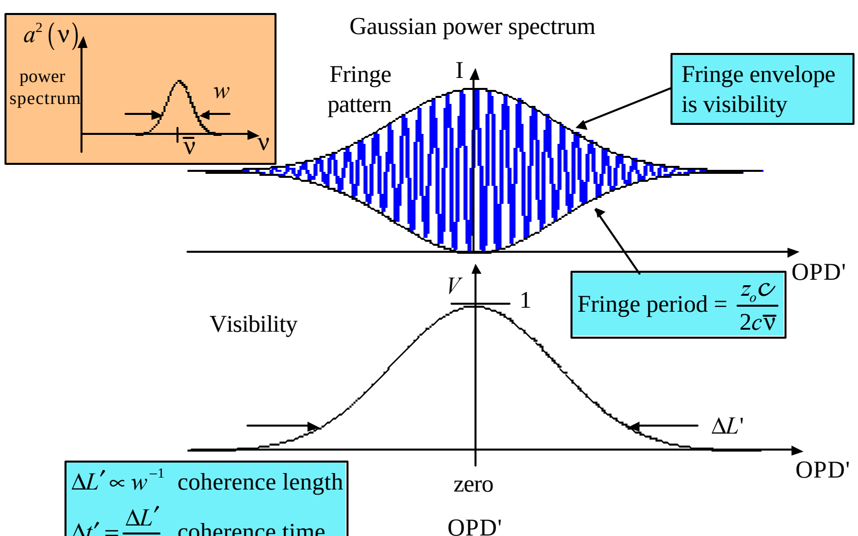
Fringe frequency is mean frequency of the source.

$$I\left(\frac{\text{OPD}'}{c}\right) = (K_1 + K_2)I_w + 2\sqrt{K_1K_2}I_w m_{12}\left(\frac{\text{OPD}'}{c}\right)\cos\left(2\pi\bar{\nu}\frac{\text{OPD}'}{c} + \beta_{12}\right)$$

Fringe modulation is a function of OPD', and goes as the Fourier transform of the power spectrum.

Fringe shift due to bandwidth asymmetry

YDPI Fringe Field – Polychromatic Source



$\Delta L' \propto w^{-1}$ coherence length

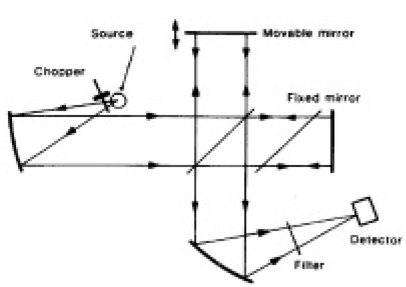
$\Delta t' = \frac{\Delta L'}{c}$ coherence time

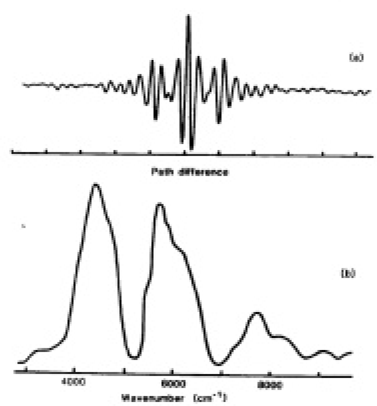
Summary of Temporal Coherence

- The bandwidth components of the source produces multiple fringe fields in the observation space.
- The fringe fields exhibit *different periods.*
- Washout of the fringes occurs for large OPD' .
- Maximum visibility occurs when OPD' is zero.
- The visibility and power spectrum are related by a Fourier transform.
- This analysis can be extended to other types of interferometers, where the OPD' is measured in observation space.
- For example, the *coherence length* is the maximum OPD' in which good contrast fringes can be obtained.

Fourier Transform Spectroscopy

Reference: Hariharan, ch 11





- Measure visibility as a function of OPD'
- Inverse Fourier transform to get $a^2(\nu)$