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Magnetization reversal in thin magnetic films with perpendicular anisotropy

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A one-dimensional theory of magnetization reversal in thin, perpendicularly anisotropic magnetic films is presented. It is postulated that the presence of a defective point creates an infinitely deep, infinitely narrow potential well which inhibits the rotation of local magnetization. The interval D between neighboring defects, the saturation magnetization M_s, the anisotropy constant K_u, and the exchange energy constant A are assumed to be finite and uniform across the film. Starting with an initial state where the film is uniformly magnetized to saturation in the easy direction, we show that a discontinuous change of state occurs when the reverse external field H reaches a critical value H_c. The domains thus nucleated at the critical field expand to cover the entire area of the film as H increases beyond H_c. Using the normalized values of H and D, defined, respectively, as $h = H/(2K_u/M_s)$ and $d = D/(4A/K_u)^{1/2}$, we show that the critical field h_c is a function only of d and that its value decreases significantly as d increases.

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I. INTRODUCTION

Magnetic films with perpendicular anisotropy have now been the subject of study for a few years. The interest in them arises from their potential applications in high-density storage devices for computers.[1] In such applications one is always concerned with the behavior of magnetization in the presence of external magnetic fields. In particular, one is interested in the mechanism of magnetization reversal and in the hysteresis characteristics of the film. Experimentally, it has been observed that the reversal process starts with the nucleation of reverse magnetized domains at certain locations; the process then continues with the expansion of these domains to the entire area of the film.[2,3]

In this paper we present a one-dimensional model for the magnetic film for which the minimization of free energy leads to results similar to the aforementioned experimental observations. The model assumes the existence of infinitely deep, infinitely narrow potential wells at certain locations in the film. The magnetic dipole at the site of a potential well is, therefore, obliged to remain in a fixed configuration, while other dipoles can assume different orientations in response to variations of the external field. One can think of this model as an idealization of the physical situation in which an inhomogeneous film structure creates a nonuniform potential field across the film.

In writing the expression for free energy we will use the micromagnetic theory[4] and include the effects of anisotropy, exchange and external field, neglecting the contributions from all other sources. Our neglect of the demagnetizing field, in particular, can be a serious drawback unless we restrict attention to temperatures close to the Curie temperature where the demagnetizing energy is, in fact, negligible.

In Sec. II the Euler–Lagrange equation for the extremum points of free energy will be derived and solved; the various solutions of this equation will be examined in Sec. III and the solution corresponding to the absolute minimum of free energy will be identified. Numerical results of this analysis are presented in the final section.

II. MINIMIZATION OF FREE ENERGY VIA THE EULER–LAGRANGE EQUATION

A one-dimensional array of magnetic dipoles, located between $x = -D/2$ and $x = +D/2$, and subjected to a reverse external field H is shown in Fig. 1. The dipoles at $x = \pm D/2$ are located in deep, narrow potential wells and are thus obliged to remain in the direction of the Z axis; other dipoles are free to assume any orientations as long as they remain in the XZ plane. Let us assume that the film has uniaxial anisotropy along Z, and that initially all dipoles are aligned in the direction of the easy axis. The free energy E_T of the system can be written as[3]

$$E_T = \int_{-D/2}^{D/2} [A(d\Theta/dx)^2 + K_u \sin^2\Theta + HM_s \cos\Theta]\, dx, \qquad (1)$$

where A is the exchange energy constant, K_u is the anisotropy constant, M_s is the saturation magnetization and $\Theta(x)$ is the angle between the dipole at x and the Z axis. Because of symmetry, $\Theta(x)$ must satisfy the following conditions:

$$\Theta(x = \pm D/2) = 0; \quad (d\Theta/dx)|_{x=0} = 0;$$
$$\Theta(x) = \Theta(-x).$$

To find the extremum points of E_T one must solve the following Euler–Lagrange equation

$$2K_u \operatorname{Sin}\Theta \cos\Theta - HM_s \sin\Theta - 2A(d^2\Theta/dx^2) = 0 \qquad (2)$$

which, upon defining $\lambda = (A/K_u)^{1/2}$ and $h = H/(2K_u/M_s)$, reduces to

$$(d/dx)[(d\Theta/dx)^2 - (2h/\lambda^2)\cos\Theta - (1/\lambda)^2 \sin^2\Theta] = 0. \qquad (3)$$

Noting that $(d\Theta/dx)|_{x=0} = 0$ and defining Θ_0 as the value of Θ at $x = 0$ we arrive at

$$(\lambda d\Theta/dx)^2 = (\cos\Theta - \cos\Theta_0)(2h - \cos\Theta - \cos\Theta_0). \qquad (4)$$

The trivial solution $\Theta(x) = 0$ always satisfies Eq. (4). To obtain other solutions let $Y(x) = \cos\Theta(x)$ and $Y_0 = \cos\Theta_0$ and restrict attention to the region $0 \leqslant x \leqslant D/2$ where $(dY/dx) \geqslant 0$. One obtains

$$dx = \{\lambda dY / [(Y - Y_0)(2h - Y - Y_0)(1 - Y)(1 + Y)]^{1/2}\}$$
$$0 \leqslant x \leqslant D/2. \tag{5}$$

The trivial solution $Y(x) = 1$ can no longer be obtained from Eq. (5); nevertheless it is still an acceptable solution of the Euler–Lagrange (E–L) equation. As for the other solutions, Eq. (5) can be integrated to yield x as an explicit function of Y; noting that $Y(D/2) = \cos \Theta(D/2) = 1$ we arrive at[5]

$$x = D/2 - \lambda (h - Y_0)^{-1/2} \mathsf{F}(\zeta, q) \tag{6}$$

where $\mathsf{F}(\zeta,q) = \int_0^\zeta (1 - q^2 \sin^2\theta)^{-1/2} d\theta$ is the elliptic integral of the first kind and

$$\zeta = \arcsin[(2(1 - Y)(h - Y_0)/[(2h - Y - Y_0)(1 - Y_0)])^{1/2}],$$
$$q = 0.5[(1 - Y_0)(2h - Y_0 + 1)/(h - Y_0)]^{1/2}.$$

From Eq. (6) the value of Y_0 can be determined self consistently. Setting $x = 0$ and $Y = Y_0$ leads to the following equation for Y_0:

$$(h - Y_0)^{-1/2} \mathsf{K}(q) = d, \tag{7}$$

where $\mathsf{K}(q) = \mathsf{F}(\pi/2, q)$ is the complete elliptic integral of the first kind and where $d = D/(2\lambda)$ is the normalized interval between defects.

Plots of $(h - Y_0)^{-1/2}\mathsf{K}(q)$ vs Y_0 for several values of h are shown in Fig. (2). It is readily observed that, depending on the values of h and d, Eq. (7) can have zero, one, or two solutions. For each solution Y_0 of this equation, Eq. (6) yields a consistent solution of the E–L equation. When there are two solutions, the one corresponding to the right side of Fig. 2 will be denoted S_1 and the one corresponding to the left side S_2. Taking the trivial solution $Y(x) = 1$ into account (denoted by S_0), one concludes that for a given set of parameters the E–L equation can have one, two, or three solutions. The next step is, therefore, to evaluate E_T for each solution and identify the one that corresponds to the absolute minimum of free energy.

III. EVALUATION OF FREE ENERGY

The value of the free energy corresponding to the trivial solution $\Theta(x) = 0$ can be easily obtained from Eq. (1) as

$$[E_T/2d(AK_u)^{1/2}] = 2h. \tag{8a}$$

For the states represented by Eqs. (6) some tedious algebra leads to

$$\begin{aligned}[E_T/2d(AK_u)^{1/2}] &= (1 + Y_0)(1 + Y_0 - 2h) \\ &\quad + (4/d)(h - Y_0)^{1/2}\mathsf{E}(q) \\ &\quad + 2h[1 - 2(h - Y_0)^{1/2}\mathsf{Z}(\eta, q)],\end{aligned} \tag{8b}$$

where $\mathsf{E}(q) = \int_0^{\pi/2}(1 - q^2\sin^2\theta)^{1/2} d\theta$ is the complete elliptic integral of the second kind; q is the same as defined in Eq. (6); $\mathsf{Z}(\eta,q) = \mathsf{E}(\eta,q) - \mathsf{E}(q)\mathsf{F}(\eta,q)/\mathsf{K}(q)$ is the Jacobi zeta function; and $\eta = \mathrm{Arcsin}[(h + 1/2 - Y_0/2)^{-1/2}]$.

To illustrate the behavior of the solutions of the E–L equation we have plotted in Fig. 3 the corresponding curves of free energy versus h for a film with $d = 4$. The curve S_0 represents the trivial solution and the curves S_1 and S_2 correspond to the solutions represented by Eq. (6). We make the following observations:

(i) For small values of h the trivial solution is the only solution of the E–L equation and, therefore, the state $\Theta(x) = 0$ represents the stable state of the system.

(ii) At a certain value of h, two other solutions appear simultaneously. One of these solutions (S_1) corresponds to a local *maximum* of free energy and cannot represent a stable state. The other solution (S_2) corresponds to a local minimum of E_T but initially it has a larger free energy than the trivial solution. Therefore, for a certain range of h the system remains in the state $\Theta(x) = 0$, even though other solutions to the E–L equation exist.

(iii) Beyond some critical field h_c, the solution S_2 becomes the state of minimum free energy. For $h > h_c$, therefore, the solution S_2 represents the stable state of the system.

The above remarks, although made in connection with Fig. 3 which corresponds to the special case of $d = 4$, remain valid for most values of d. In fact, it can be shown that the curves of Fig. 3 are characteristic of the solutions of the E–L equation for $d > d_0 = \pi/(2\sqrt{3})$.

For $d \leqslant d_0$ a different set of curves applies; Fig. 4 is typical of the behavior of solutions of the E–L equation in the region $d \leqslant d_0$. The general features of this region are (i) The only nontrivial solution (S_1) exists for relatively large values of h. (ii) The free energy for S_1 is never greater than the free energy for S_0. (iii) The transition from S_0 to S_1 at the critical point is smooth, i.e., the change in magnetization occurs continuously.

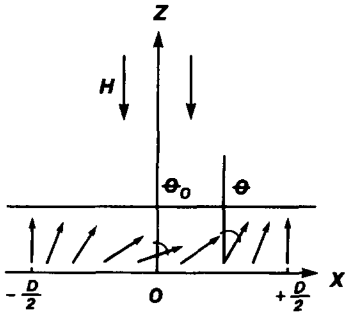

FIG. 1. One-dimensional model of a thin magnetic film.

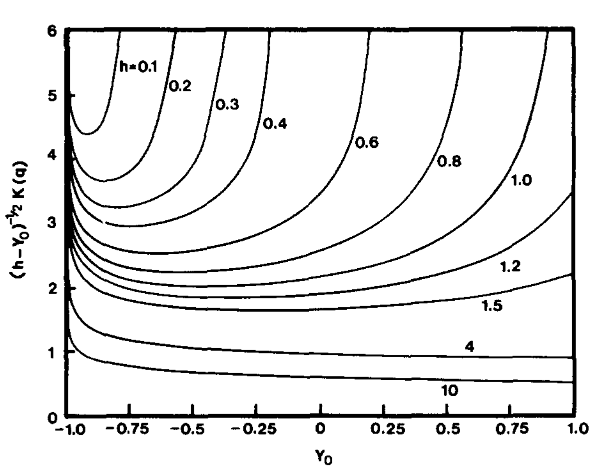

FIG. 2. Plots of $(h - Y_0)^{-1/2}\mathsf{K}(q)$ vs Y_0 for several values of h.

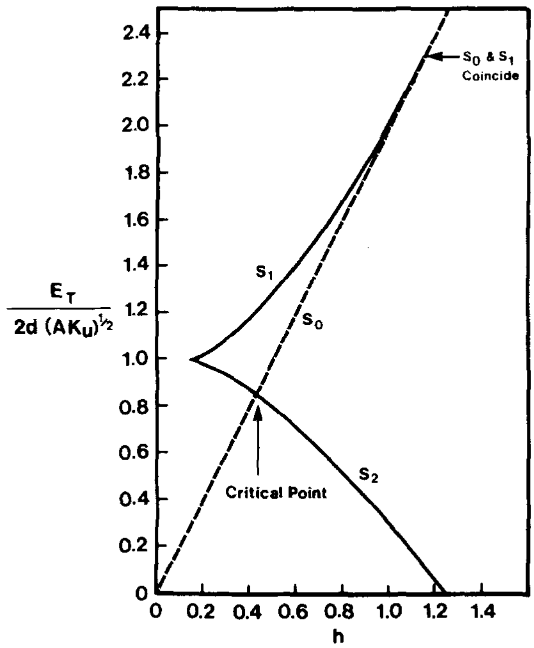

FIG. 3. Free energy vs h for solutions of the Euler–Lagrange equation ($d = 4$).

As a result, films with $d \leqslant d_0$ must exhibit extraordinarily large coercive forces and nonsquare hysteresis loops. We shall not pursue the case of small d any further in this paper. In the next section attention will be focussed on situations with relatively large values of d where predictions of the theory are qualitatively similar to effects observed experimentally in MnBi films.

IV. NUMERICAL RESULTS AND CONCLUSIONS

Figure 3 shows that for a film with $d = 4$ the transition from S_0 to S_2 occurs at $h_c = 0.42$. For this film, Fig. 5 shows plots of local magnetization $Y(x) = \cos \Theta(x)$ vs x at the critical field $h = h_c$ and at a much larger field $h = 10h_c$. The trivial solution $Y(x) = 1$ which corresponds to $h < h_c$ is also shown in this figure. The curve for $h = h_c$ shows the dramatic change at the transition point; the nucleated domain has already covered a significant portion of the interval between defects. The curve for $h = 10h_c$ shows that at large values of h the entire area of the film (except for the immediate neighborhood of the defective points) must be reverse magnetized.

The total magnetization

$$M(h) = \int_{-D/2}^{D/2} Y(x)dx$$

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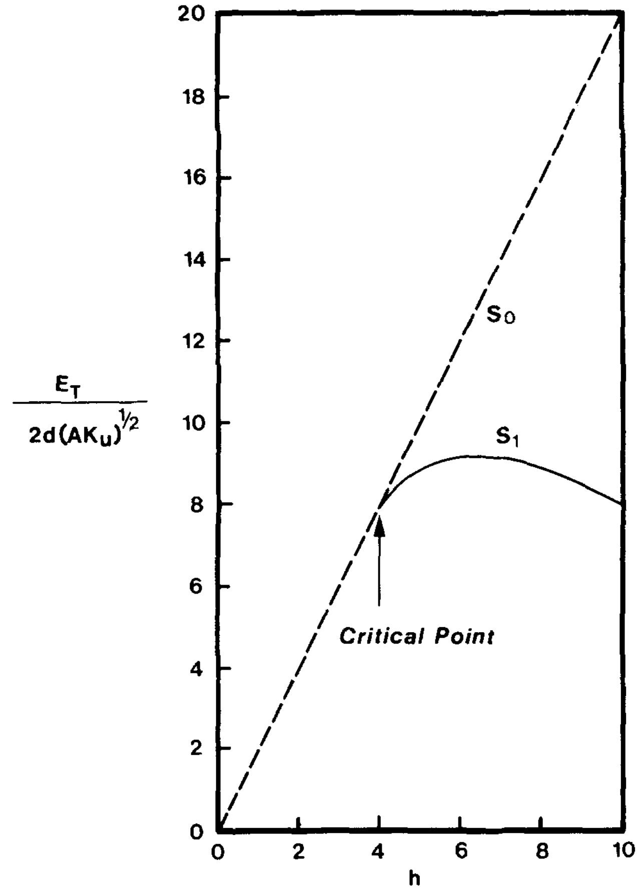

FIG. 4. Free energy vs h for solutions of the Euler–Lagrange equation [$d = \pi/(2\sqrt{3})$].

$$M(h) = (1/D)\int_{-1/2D}^{1/2D} Y(x)dx$$

can be obtained from Eq. (6) by integration and has the following closed form

$$M(h) = 1 - 2(h - Y_0)^{1/2} Z(\eta, q). \quad (9)$$

where η, q, and the Jacobi zeta function Z have already been defined in the previous sections. Plots of $M(h)$ vs h for several values of d are shown in Fig. 6. (The complete hysteresis loops have been obtained by arguing that the magnetization at $x = \pm D/2$ must be reversed in the limit when $h \to \infty$.) The dotted curve in Fig. 6 shows the terminii of the first order jumps for all values of d. Note in particular that the

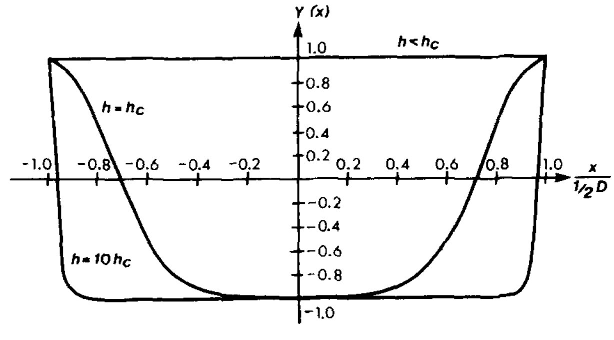

FIG. 5. Local magnetization in the region between neighboring defects for different values of h. ($d = 4$).

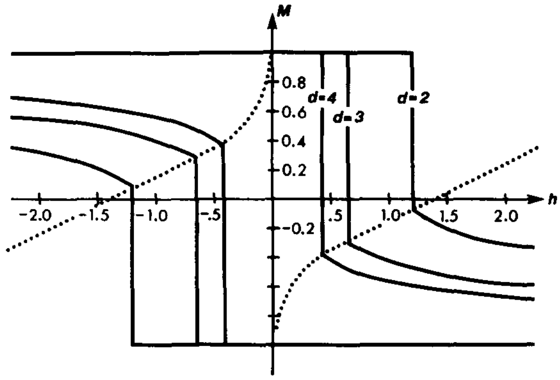

FIG. 6. Hysteresis loops for different values of d. (The dotted curve is the terminii of the first order jumps for all values of d.)

squareness of the loops increases with d and that films with larger values of d exhibit smaller coercivities. The jumps vanish at $h = 4$ which corresponds to $d = \pi/(2\sqrt{3})$.

Typically, for films with $K_u \sim 10^6$ ergs/cm^3 and $M_s \sim 10^2$ G the measured coercivity is of the order of 10^3 Oe. This means that $h_c \sim 10^{-1}$ which, according to our theory, corresponds to large values of d. The theory can, therefore, predict hysteresis loops with a high degree of squareness, in accordance with experimental data. It can also explain the experimentally observed tails of the hysteresis loops,[6,7] suggesting that pinning may indeed control the hysteretic behavior in these films. For a typical film $A \sim 10^{-6}$ ergs/cm which, together with the above value of K_u, results in $\lambda \sim 10^2$ Å. The interval between defects D can thus assume values in the range of a few hundred to a few thousand angstroms.

A final remark, concerning the thermodynamic stability of the solutions S_0, S_1, and S_2 of the E–L equation, is in order here. As was mentioned earlier, S_1 corresponds to a local maximum of free energy and is, therefore, an unstable state of the system. This maximum is located between the two stable states S_0 and S_2 and the energy barrier that separates S_0 from S_2 is characterized by the free energy of S_1. The dynamics of the transition between stable states, which we did not discuss in this paper, is affected by the height of this energy barrier.

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