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Demagnetizing field computation for thin films: Extension to the hexagonal lattice

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A previously published method of calculating the magnetic field pattern of thin films using fast Fourier transforms is extended to situations where the magnetization distribution is specified on a hexagonal lattice.

INTRODUCTION

In a previous paper[1] we described a method of calculating the magnetic field distribution for thin magnetic films. The method was based on Fourier transforms and the corresponding numerical computations were performed on square lattices. In this brief paper we extend the results to hexagonal lattices.

Consider a rectangular slab of magnetic film in the xy plane. The slab's dimensions are $L_x \times L_y \times h$, where h is the film thickness. Also let the magnetization distribution $\mathbf{M}(x,y)$ be uniform through the film thickness. Since periodic boundary conditions will be assumed, the magnetization pattern in the xy plane shall be periodic, with L_x and L_y representing the periods along the x and y axes. Figure 1 shows a regular two-dimensional hexagonal lattice. The original slab is the rectangular region of dimension $L_x \times L_y$ having its lower left-hand corner at the origin. Because of the assumed periodicity, a few lattice cells beyond the slab are also shown.

Discrete Fourier transformations cannot be performed easily on the rectangular region since it does not conform to the natural bases of the hexagonal lattice. These bases are the vectors $\mathbf{a}$ and $\mathbf{b}$ with equal length ($|\mathbf{a}| = |\mathbf{b}| = d$) as shown in Fig. 1. The shaded region in Fig. 1 over which the Fourier transform shall be performed has N_1 cells along a and N_2 cells along b where $L_x = N_1 d$ and $L_y = (\sqrt{3}/2)N_2 d$. This new parallelepiped slab has the same number of cells as the original rectangular slab; either slab could be considered as a basic pattern of magnetization which, when replicated in the xy plane, would reproduce the distribution $\mathbf{M}(x,y)$. Note, however, that although periodicity along $\mathbf{a}$ follows from the periodicity in the x direction, the periodicity along y does not guarantee the same along $\mathbf{b}$. Nonetheless, if number N_2 is chosen to be equal to $2N_1$ (or any integer multiple of $2N_1$), periodicity along $\mathbf{b}$ will be ensured. The following restrictions thus apply to the lattice:

$$N_2 = 2N_1, \tag{1a}$$

$$L_y = \sqrt{3}L_x. \tag{1b}$$

Under these conditions, one can express the periodic nature of $\mathbf{M}(x,y)$ in the new coordinate system as follows:

$$\mathbf{M}[(\alpha + mN_1)\mathbf{a} + (\beta + nN_2)\mathbf{b}] = \mathbf{M}(\alpha\mathbf{a} + \beta\mathbf{b}). \tag{2}$$

In Eq. (2) α and β are arbitrary real numbers while m and n are arbitrary integers.

THE FOURIER METHOD

Let us now define the reciprocal vectors $\mathbf{A}$ and $\mathbf{B}$ in the frequency domain, i.e., in the $f_x f_y$ plane. $\mathbf{A}$ and $\mathbf{B}$ must satisfy the following two conditions:

$$\mathbf{a}\cdot\mathbf{B} = \mathbf{b}\cdot\mathbf{A} = 0, \tag{3a}$$

$$\mathbf{a}\cdot\mathbf{A} = \mathbf{b}\cdot\mathbf{B} = 1. \tag{3b}$$

$\mathbf{A}$ and $\mathbf{B}$ have equal lengths $\{|\mathbf{A}| = |\mathbf{B}| = 2/(\sqrt{3}d)\}$ and are shown in the inset in Fig. 1. With the aid of the reciprocal vectors, we write the following expression for sinusoidal patterns of magnetization that have the periodicity of $\mathbf{M}(\alpha\mathbf{a} + \beta\mathbf{b})$ as described by Eq. (2),

$$\begin{aligned}\mathbf{M}_{mn} \exp\left[i2\pi\left(\frac{m}{N_1}\mathbf{A} + \frac{n}{N_2}\mathbf{B}\right)(\alpha\mathbf{a} + \beta\mathbf{b})\right] \\ = \mathbf{M}_{mn} \exp\left[i2\pi\left(\frac{m\alpha}{N_1} + \frac{n\beta}{N_2}\right)\right].\end{aligned} \tag{4}$$

The Fourier series expansion of $\mathbf{M}(\alpha\mathbf{a} + \beta\mathbf{b})$ is now written

$$\mathbf{M}(\alpha\mathbf{a} + \beta\mathbf{b}) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mathbf{M}_{mn} \times \exp\left[i2\pi\left(\frac{m\alpha}{N_1} + \frac{n\beta}{N_2}\right)\right], \tag{5a}$$

where

$$\mathbf{M}_{mn} = \frac{1}{L_x L_y} \iint_{\text{parallelepiped slab}} \mathbf{M}(\alpha\mathbf{a} + \beta\mathbf{b}) \times \exp\left[-i2\pi\left(\frac{m\alpha}{N_1} + \frac{n\beta}{N_2}\right)\right] ds. \tag{5b}$$

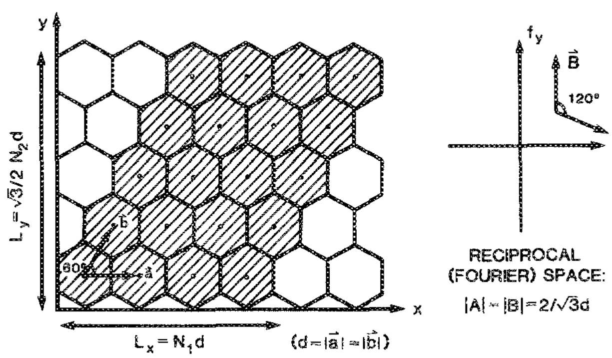

FIG. 1. The hexagonal lattice and its parameters. The inset shows the reciprocal lattice vectors A and B.

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In Eq. (5b) $L_x L_y$ is the surface area of the parallelepiped slab. Approximating the preceding integral with a finite sum over the hexagonal lattice one obtains

$$\mathbf{M}_{mn} \simeq \frac{1}{N_1 N_2} \sum_{I=0}^{N_1-1} \sum_{J=0}^{N_2-1} \mathbf{M}(I\mathbf{a} + J\mathbf{b}) \times \exp\left[-i2\pi\left(\frac{mI}{N_1} + \frac{nJ}{N_2}\right)\right]. \quad (6)$$

A straightforward application of the discrete Fourier transform formula to the magnetization distribution, sampled over the parallelepiped slab, thus produces the coefficients $\mathbf{M}_{mn}$.

Next, we relate the magnetization distribution to the demagnetizing field. It was shown in Ref. 1 that the thickness-averaged $\mathbf{H}$ field for the sinusoidal magnetization of Eq. (4) is given by

$$\mathbf{H}_{mn}^{\text{avg}}(\alpha,\beta) = -4\pi\{[1 - G(hf)](\mathbf{M}_{mn}\cdot\hat{\boldsymbol{\sigma}})\hat{\boldsymbol{\sigma}} + G(hf)(\mathbf{M}_{mn}\cdot\hat{\mathbf{z}})\hat{\mathbf{z}}\} \times \exp\left[i2\pi\left(\frac{m\alpha}{N_1} + \frac{n\beta}{N_2}\right)\right]. \quad (7a)$$

Here h is the film thickness, f is the magnitude of the spatial frequency vector $\mathbf{f}$ which, for the hexagonal lattice, is given by

$$\mathbf{f} = \frac{m}{N_1}\mathbf{A} + \frac{n}{N_2}\mathbf{B} = \frac{m}{L_x}\hat{\mathbf{x}} + \left(\frac{n}{L_y} - \frac{m}{\sqrt{3}L_x}\right)\hat{\mathbf{y}}, \quad (7b)$$

$\hat{\boldsymbol{\sigma}}$ is the unit vector along $\mathbf{f}$, i.e., $\hat{\boldsymbol{\sigma}} = \mathbf{f}/|\mathbf{f}|$, and $\hat{\mathbf{z}}$ is the unit vector perpendicular to the film plane. The function $G(\cdot)$ is a circularly symmetric function in the frequency domain given by

$$G(r) = \exp(-\pi r)[\sinh(\pi r)/\pi r]. \quad (7c)$$

Figure 2 shows a plot of $G(r)$. Notice that the argument of $G(\cdot)$ in Eq. (7a) is the magnitude of frequency scaled by

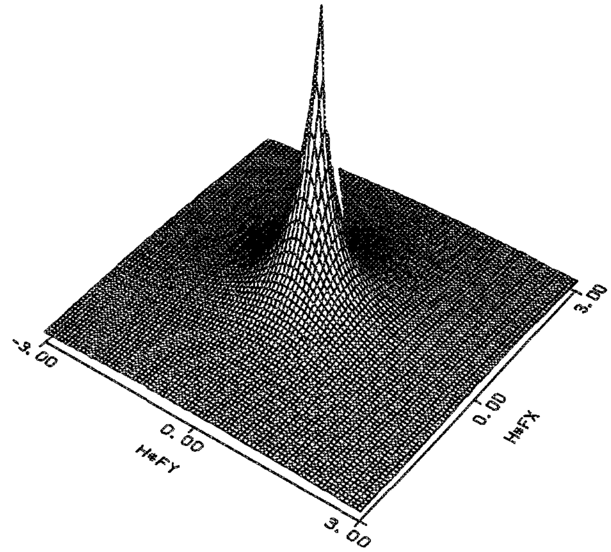


FIG. 2. Plot of $G(hf)$ in the frequency domian.

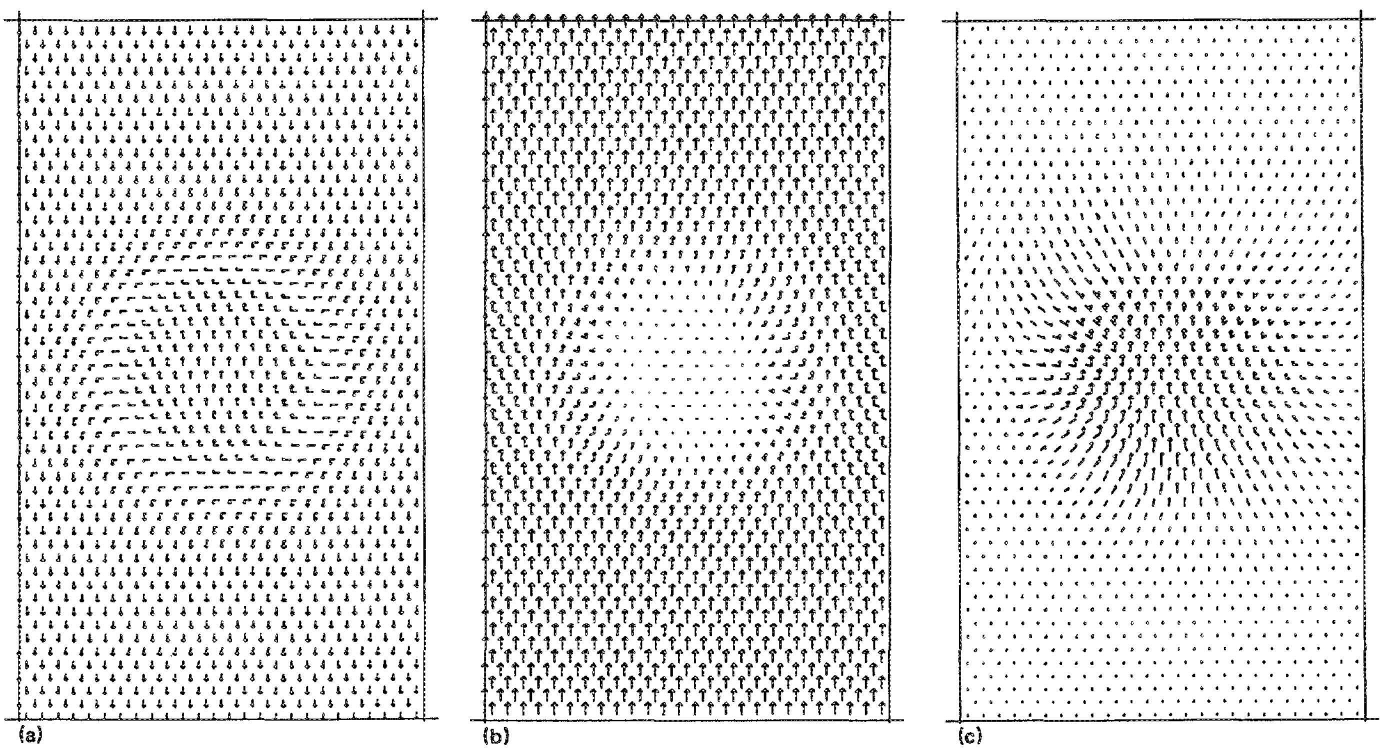
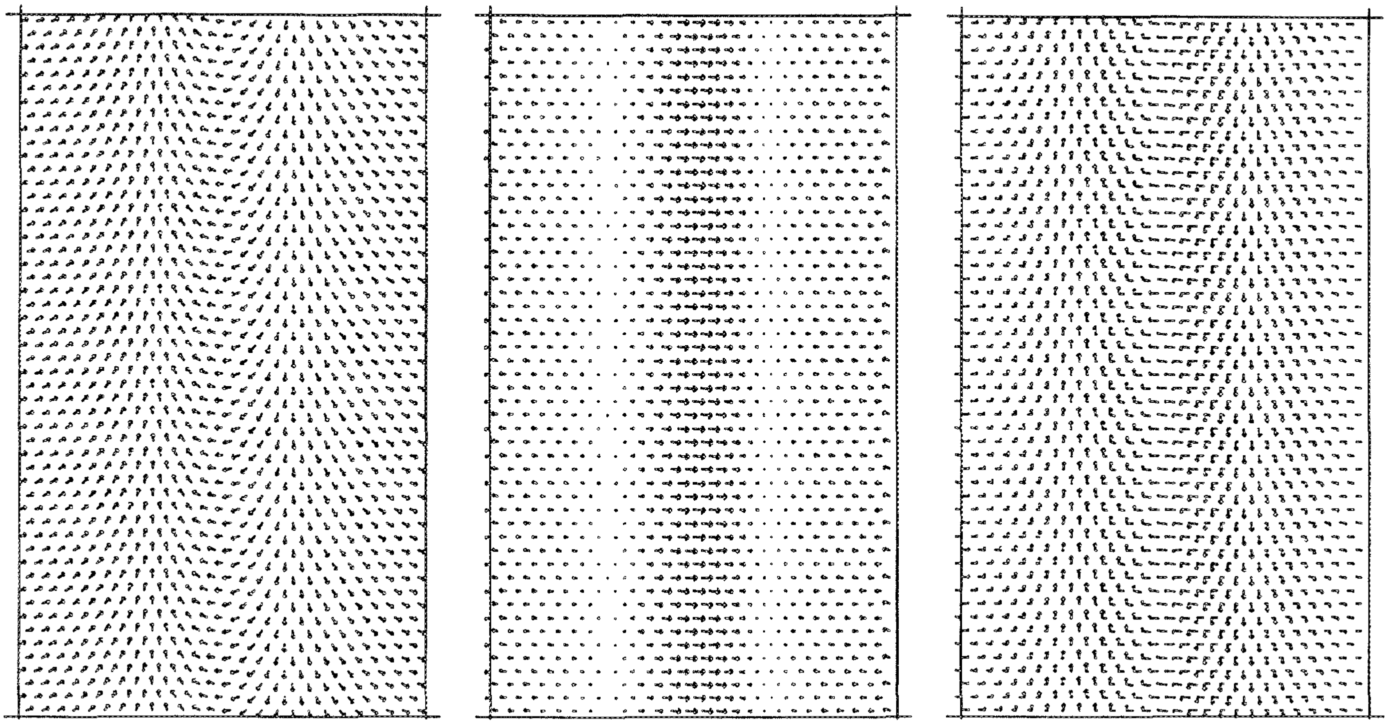


FIG. 3. (a) Assumed magnetization distribution in a perpendicular film with a circular reverse domain. In units of the lattice constant d, the radius is $R_0 = 7$ and the wall thickness parameter is $\Delta_w = 2$. The arrows represent the perpendicular component of magnetization while the appendage to each arrow shows the corresponding in-plane component. (b) Thickness-averaged demagnetizing field for the magnetization distribution of part (a). In units of the lattice constant the film thickness is $h = 10$. (c) Magnetic field distribution outside the film. The film thickness is $h = 50d$ and the distance above the surface at which the field is computed is d.



(a) (b) (c)

FIG. 4. (a) Assumed in-plane magnetization distribution with a pair of head-to-head walls. (b) Thickness-averaged demagnetizing field for the magnetization distribution of part (a) with $h = 10d$. (c) Magnetic field distribution outside the film. The film thickness is $h = 10d$ and the field is computed at a distance of d above the film surface.

the film thickness. This is the only appearance of h in the formula. The perpendicular component of magnetization in Eq. (7a) is subjected to the low-pass spatial filter $G(\cdot)$, while the in-plane component is processed by the high-pass filter, $1 - G(\cdot)$. The spatial distribution of $\mathbf{H}$ in Eq. (7) is nonetheless sinusoidal and has the same frequency as its source, namely the magnetization pattern of Eq. (4). The total demagnetizing field can therefore be obtained by summing over all frequency components, that is, performing the following inverse Fourier transform:

$$\mathbf{H}^{\text{avg}}(\alpha,\beta) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \mathbf{H}_{mn} \exp\left[i2\pi\left(\frac{m\alpha}{N_1} + \frac{n\beta}{N_2}\right)\right]. \tag{8}$$

Similar results can be obtained for the $\mathbf{H}$ field in various planes parallel to the plane of the film; calculations can be done for regions outside of the film as well as those inside. In all cases the results of Ref. 1, which pertain to square lattices, can be applied provided that (i) the parallelepiped slab of Fig. 1, satisfying the constraints of Eq. (1), is taken as the pertinent region and (ii) the frequency vector $\mathbf{f}$, its magnitude f, and the unit vector $\hat{\sigma}$ are associated with the reciprocal lattice space, as indicated in Eq. (7b). Results of numerical computations for certain problems of interest in the areas of magnetic and optical disk data storage are shown in Figs. 3 and 4.

ACKNOWLEDGMENT

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[1] M. Mansuripur and R. Giles, IEEE Trans. Magn. MAG-24, 2326 (1988).