Beam Quality Factor of Higher Order Modes in a Step-Index Fiber

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***Abstract*—The beam quality factor (or M^2-parameter) for linearly polarized (LP)-modes of a step-index fiber is calculated in a closed form, as a function of the fiber V-number. It is shown that M^2 sharply peaks for all fiber modes when they are close to cutoff. Particularly simple expressions are derived in the limit $V \to \infty$. Two practically important coherent superpositions of modes are considered for which the degree of degradation of the beam quality due to the higher order mode content is calculated. The reported results can be useful for designing large-core high-power fiber lasers, amplifiers, and fiber-based beam delivery systems, when preservation of the spatial beam quality is important.**

***Index Terms*—Beam quality factor, fiber modes, multimode fiber.**

I. Introduction

The BEAM quality factor (or M^2-parameter) of an optical beam is defined as the ratio of divergence of the beam to that of an imaginary fundamental Gaussian beam such that the two beams have the same second-order intensity moment at the waist [1]. Since the fundamental Gaussian beam has the least divergence, $M^2 > 1$ for real beams. In modern laser research and development, the M^2-parameter has become a universal standard of the laser beam quality and the measured value of M^2 is nearly always specified whenever a new laser system is reported. A high-power fiber laser is not an exception. The field profile in a robustly guiding single-mode fiber can be closely approximated by a Gaussian, and therefore, the beam quality factor for a fiber laser based on a single-mode active fiber is typically very close to one [2].

As the output power of fiber lasers grows into the kilowatt range [3], [4], the detrimental effects of nonlinear processes in the fiber such as stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS) become the major factors that limit a further power upscaling. It is therefore desirable to increase the size of the active fiber core while maintaining a high beam quality of the laser output. The same applies to the high-power fiber-based beam delivery systems. Increasing the core size of a fiber eventually will allow the fiber to support several spatial modes. Various methods have been devised in order to achieve high spatial beam quality even with the large-core multimode step-index fiber-based laser systems [4]–[7].

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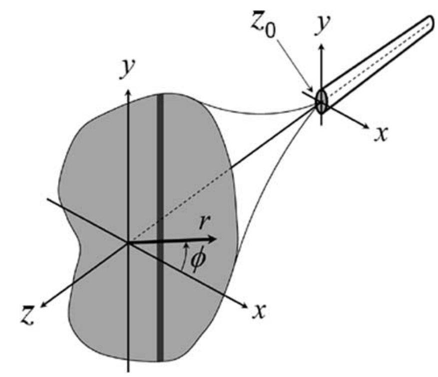

Fig. 1. Choice of the rectangular and polar coordinate systems. The scanning slit of the beam analyzer is shown with the black vertical stripe in the $x-y$ plane.

However, if the fiber is not single mode, the higher order mode content will be inevitably present in the output laser beam which will degrade the beam quality to some extent.

In this paper, we report a calculation of the beam quality factor for a multimode step-index fiber. In particular, the analytical result for the fundamental fiber mode that has been reported previously [8] is extended to the higher order modes. It is shown that the M^2-parameter of the pure linearly polarized (LP)-modes sharply peaks when the modes are near cutoff. Similar peaking is expected in the case of a general beam emerging from a multimode fiber when one of the modal components of the beam is close to its cutoff. Particularly simple analytical formulas for the M^2-parameter for the fiber modes are found in the limit $V \to \infty$. Further, two practically important coherent superpositions of modes are considered, and the quantitative degree of degradation of the beam quality is found as a function of the higher order mode content of the beam. The results reported here will be helpful in designing practical fiber laser systems when a specific value of M^2 is targeted.

II. Basic Formulas

Consider a lightwave exiting from a cleaved facet of a step-index optical fiber. The cleave is supposed to be perfect, i.e., the plane of the facet is orthogonal to the geometrical axis of the fiber. A rectangular coordinate system is chosen, as shown in Fig. 1. The coordinate z-axis is parallel to the geometrical axis of the fiber, and the origin of the coordinate system is located at the center of the circular fiber core. We further assume that the optical signal exiting the fiber is a monochromatic lightwave with angular frequency ω, and it is passed through a linear polarizer which is placed right after the fiber end facet. Thus, the field diverging in free space can be described in terms of a scalar, normalized electric field amplitude

$$E(x,y,z,t) = E(x,y,z) \cdot e^{i\omega t}$$

where

$$\iint dx\, dy\, |E(x,y,z)|^2 = 1. \tag{1}$$

Suppose that the intensity distribution of the beam in free space is analyzed using a scanning-slit device, with the slit parallel to the y-axis. Accordingly, define the z-dependent center coordinate $\langle x\rangle(z)$ and variance $\sigma_x^2(z)$ of the beam as follows:

$$\begin{aligned}\langle x\rangle(z) &= \iint dx\, dy\, x\, |E(x,y,z)|^2 \\ \sigma_x^2(z) &= \iint dx\, dy\, (x - \langle x\rangle(z))^2\, |E(x,y,z)|^2.\end{aligned} \tag{2}$$

Assuming the paraxial beam propagation and using the Kirchhoff–Fresnel diffraction formula, we find

$$\begin{aligned}\langle x\rangle(z) =& \langle x\rangle(z_0) - i\frac{\lambda}{4\pi}\cdot(z - z_0) \\ &\cdot \iint dx\, dy \left[E(x,y,z_0)\cdot\frac{\partial E^*}{\partial x}(x,y,z_0) - \text{c.c.}\right] \\ \sigma_x^2(z) =& \sigma_x^2(z_0) - i\frac{\lambda}{2\pi}\cdot A\cdot(z - z_0) \\ &+ \frac{\lambda^2}{(2\pi)^2}\cdot B\cdot(z - z_0)^2\end{aligned} \tag{3}$$

where λ is the wavelength of light in free space, and the parameters A and B are defined as follows:

$$\begin{aligned}A =& \iint dx\, dy\, (x - \langle x\rangle(z_0)) \\ &\times \left[E(x,y,z_0)\cdot\frac{\partial E^*}{\partial x}(x,y,z_0) - \text{c.c.}\right] \\ B =& \iint dx\, dy \left|\frac{\partial E}{\partial x}(x,y,z_0)\right|^2 + \frac{1}{4} \\ &\times \left\{\iint dx\, dy \left[E(x,y,z_0)\cdot\frac{\partial E^*}{\partial x}(x,y,z_0) - \text{c.c.}\right]\right\}^2.\end{aligned} \tag{4}$$

The equations in (3) express the z-dependent center coordinate and variance of the beam in terms of the amplitude distribution at the fiber facet (at z_0). They are three-dimensional generalizations of the results obtained in [9] for two-dimensional beams (without y-dependence). Note that z_0 is the coordinate of the output facet of the fiber which does not necessarily coincide with the position of the beam waist.

Following Siegman [9], we introduce the position of the beam waist $\tilde{z}_{0x}$ and the beam-quality factor M_x^2 such that the formulas (3) can be rewritten in the Gaussian-beam-like form

$$\sigma_x^2(z) = \sigma_x^2(\tilde{z}_{0x}) + M_x^4\cdot\frac{\lambda^2}{16\pi^2\sigma_x^2(\tilde{z}_{0x})}\cdot(z - \tilde{z}_{0x}). \tag{5}$$

By comparing (5) and (3), we find the following formulas for the position of the beam waist, the second-order intensity moment at the waist, and the beam-quality factor:

$$\tilde{z}_{0x} = z_0 + i\frac{\pi}{\lambda}\frac{A}{B} \tag{6}$$

$$\sigma_x^2(\tilde{z}_{0x}) = \sigma_x^2(z_0) + \frac{A^2}{4B} \tag{7}$$

$$M_x^2 = \sqrt{4B\sigma_x^2(z_0) + A^2}. \tag{8}$$

All three parameters of the beam (6)–(8) are expressed in terms of the field distribution at the fiber exit facet.

Expression (8) for the beam quality factor will be used in the rest of this paper with specific field distributions at the fiber end facet. Note that for fields that are real at z_0, both A and the last term in expression for B in (4) vanish which simplifies the calculation of the M_x^2-parameter.

III. Beam-Quality Factor for the Pure LP-Modes of a Step-Index Fiber

An optical field propagating in a weakly guiding step-index fiber can be locally expressed in terms of the so-called LP fiber modes (or LP-modes). Strictly speaking, all LP-modes except for the fundamental mode LP_{01}, are not the true eigenmodes of a step-index fiber. Instead, they are linear superpositions of the true eigenmodes TE, TM, EH and HE, which do not have a uniform polarization across the fiber [10]. In practical weakly guiding fibers, the true eigenmode components of a particular LP-mode are nearly degenerate, and the beatlength between the components is typically of the order of a few centimeters. The reason for using the LP-mode basis instead of the basis of the true eigenmodes is that the spatial field distribution of the LP-modes is described by considerably simpler expressions than that of the true modes. In addition, if a particular LP_{nm}-mode is launched into an ideal, perturbation-free fiber, it will evolve so that the field exiting from the fiber will still be in the form of the LP_{nm}-mode, with the same indexes n and m as that of the input mode, but rotated around the fiber axis and with different (but still uniform across the fiber) polarization. In this section, we will derive expressions for the beam-quality factor of pure LP-modes exiting from a step-index fiber.

Accordingly, consider a step-index fiber with the core radius a and core refractive index n_{core}. The cladding is assumed to be infinitely large with the refractive index n_{clad}. The refractive index step between the core and the cladding, $(n_{\text{core}} - n_{\text{clad}})$ is assumed to be small, so that the fiber can be considered as weakly guiding. Then, in the LP-approximation, the field distribution across the fiber end facet for a particular LP$_{nm}$-mode of the fiber can be written as [10]

$$E_{nm}(r,\phi,\xi) = E_{nm}(r) \cdot \cos\left[n(\phi+\xi)\right] \tag{9}$$

where (r,ϕ) are the polar coordinates, and parameter ξ determines the orientation of the field pattern of a particular mode with respect to the scanning slit of the beam analyzer. The radial dependence of the approximately transverse field amplitude in (9) is given by

$$\begin{aligned} E_{nm}(r) &\propto J_n\left(\frac{u_m}{a}r\right) \qquad (0 \le r < a) \\ &\propto K_n\left(\frac{w_m}{a}r\right) \qquad (r \ge a) \end{aligned} \tag{10}$$

where J_n and K_n are the nth-order Bessel function of the first kind and the modified Bessel function of the first kind, respectively, and the pair (u_m, w_m) is an mth solution of the following system of equations

$$\begin{aligned} \frac{J_n(u)}{uJ_{n+1}(u)} &= \frac{K_n(w)}{wK_{n+1}(w)} \\ u^2 + w^2 &= V^2 \end{aligned} \tag{11}$$

where V is the so-called V-parameter of the fiber defined as follows:

$$V = \frac{2\pi a}{\lambda}\sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2}. \tag{12}$$

Since the field distribution of LP-modes is real, the location of the beam waist for individual modes coincides with that of the exit fiber facet, as follows from (6). Further, both second-order intensity moment at the fiber end facet (7) and the beam quality factor (8) for the LP-modes can be calculated in a closed form. By using the field amplitudes from (9) and (10) in the general formula (8), and by applying known relations between Bessel functions and their integrals [11], for the beam-quality factor of the individual LP-modes, we find

$$\left(M_x^2\right)_{nm}^2 = -2C_n \cdot \frac{\sigma_x^2(z_0)}{a^2} \cdot \frac{w_m^2 J_n^2}{J_{n-1}J_{n+1}} \tag{13}$$

where u_m and w_m are solutions of (11) for a particular LP$_{nm}$ mode. The second-order intensity moment at the fiber end facet in the above formula is explicitly given by

$$\begin{aligned} \left(\frac{\sigma_x^2(z_0)}{a^2}\right)_{nm} = C_n \cdot \Bigg\{ &-\frac{J_n}{3u_m J_{n-1}}\left[1 + \frac{n(n-1)}{u_m}\frac{J_n}{J_{n+1}}\right] \\ &+ \frac{1}{6}\left[1 - 2(n^2-1)\left(\frac{1}{w_m^2} - \frac{1}{u_m^2}\right)\right]\Bigg\}. \end{aligned} \tag{14}$$

In the above formulas, we used, for brevity

$$J_i \equiv J_i(u_m). \tag{15}$$

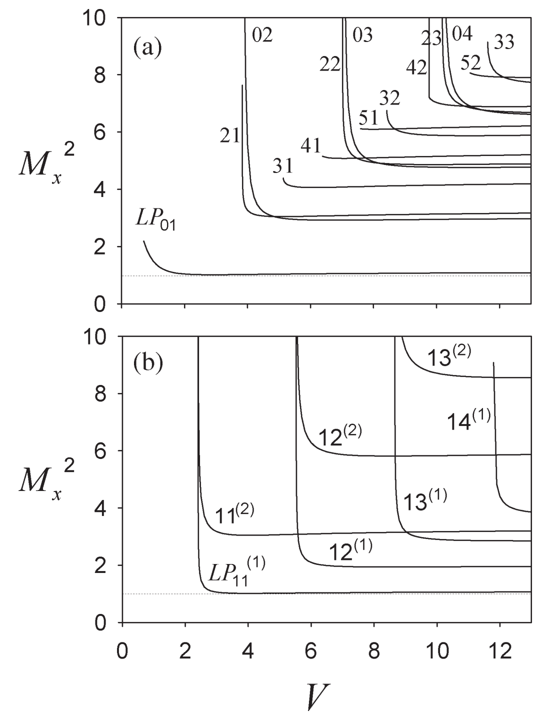

Fig. 2. Dependence of the beam-quality factor on the value of the fiber V-number, for several lowest order LP$_{nm}$-modes. (a) LP$_{nm}$-modes with $n \neq 1$. (b) LP$_{1m}$-modes. $1m^{(1)}$ stands for the LP$_{1m}$-mode with $\propto \sin(\phi)$-dependence, and $1m^{(2)}$ stands for the LP$_{1m}$-mode with $\propto \cos(\phi)$-dependence.

In addition, the dimensionless parameter C_i is defined as follows:

$$\begin{aligned} C_n &= 1, \qquad \text{for LP}_{nm}\text{-modes with } n \neq 1 \\ C_n &= 1 + \frac{1}{2}\cos(2\xi), \qquad \text{for LP}_{1m}\text{-modes} \end{aligned} \tag{16}$$

where ξ is the orientation parameter for a particular LP-mode in (9). The validity of (13)–(16) has been verified by confirming that the results obtained by using these formulas are identical to those of a direct numerical integration using the general formulas (4) and (8), for several randomly picked LP-modes. Note that the value of the beam-quality factor depends on the mode orientation with respect to the scanning slit of the beam analyzer (i.e., on the parameter ξ), only for the LP$_{1m}$-modes, and is independent of ξ for all modes with $n \neq 1$. Further, by using $n = 0$ and $J_{-1}(x) = -J_1(x)$ in the above general expressions, we recover the result reported in [8] for the fundamental mode LP$_{01}$.

The M_x^2 for several lowest order LP-modes as a function of the fiber V-number is shown in Fig. 2. In particular, Fig. 2(a) shows the combined data for the LP$_{nm}$-modes with $n \neq 1$, and the data for several LP$_{1m}$-modes is shown in Fig. 2(b). In the latter figure, the two extreme cases for each LP$_{1m}$-mode are shown which correspond to the two orientations of the lobes of the field distribution: parallel to the scanning slit of the beam analyzer ($\xi = \pi/2$) and perpendicular to the slit ($\xi = 0$). Note that the mode LP$_{11} \propto \sin(\phi)$ far from the mode cutoff has a slightly lower value of the M_x^2-parameter than that of the fundamental fiber mode LP$_{01}$. Of course, this does not imply that the higher order mode $LP_{11} \propto \sin(\phi)$ has a "higher" beam quality than the fundamental mode. Instead, this means that the beam analyzer with a single scanning slit and the corresponding definitions (2)–(8) are not universally adequate for quantifying the beam quality of an arbitrary beam, and the two-dimensional beam analysis is in general, necessary (e.g., with two orthogonal scanning slits).

Further, as seen from the Fig. 2, the beam-quality factor for all modes sharply peaks near the corresponding cutoff values of the V-number. It is known that the fraction of the mode energy that is propagating in the cladding of the fiber has a similar property of sharply peaking near the mode cutoff [12]. Since in practice the fiber cladding has a finite size, the modes that are too close to cutoff will be severely attenuated. Therefore, the peaking of the beam quality factor near mode cutoffs in real fibers will be somewhat less pronounced than that shown in Fig. 2.

As the V-number increases, the M_x^2-parameter for the LP-modes approaches constant values. Using known properties of Bessel functions, these limiting values can be found in a closed form as follows:

$$\lim_{V\to\infty}\left(M_x^2\right)_{nm} = C_n\sqrt{\frac{\left(V_{\text{cutoff}}^{(n+1),m}\right)^2 + 2(n^2-1)}{3}} \qquad (17)$$

where $V_{\text{cutoff}}^{(n+1),m}$ is the cutoff value of V-number for the next higher order mode $LP_{(n+1),m}$. The ratios of the second-order intensity moments at the fiber end facet to the fiber core radius squared for the LP-modes also approach constant values that can be found in the following form:

$$\lim_{V\to\infty}\left(\frac{\sigma_x^2(z_0)}{a^2}\right)_{nm} = \frac{2C_n}{3}\left[1+\frac{2(n^2-1)}{\left(V_{\text{cutoff}}^{(n+1),m}\right)^2}\right]. \qquad (18)$$

Table I summarizes the numerical limiting values of the beam-quality factor for the lowest order LP-modes, together with the cutoff values of the V-number.

TABLE I
SUMMARY OF THE CUTOFF VALUES OF THE V-NUMBER AND THE LIMITING VALUES OF THE BEAM-QUALITY FACTOR FOR SEVERAL LOWEST ORDER LP-MODES

	$V_{cut-off}$	$M^2(V\to\infty)$
LP_{01}	0	1.123
LP_{02}	3.832	3.081
LP_{03}	7.016	4.929
LP_{04}	10.173	6.759
$LP_{11}\propto\cos(\phi)$	2.405	3.318
$LP_{11}\propto\sin(\phi)$	2.405	1.106
$LP_{12}\propto\cos(\phi)$	5.520	6.076
$LP_{12}\propto\sin(\phi)$	5.520	2.025
$LP_{13}\propto\cos(\phi)$	8.654	8.810
$LP_{13}\propto\sin(\phi)$	8.654	2.937
$LP_{14}\propto\cos(\phi)$	11.792	11.539
$LP_{14}\propto\sin(\phi)$	11.792	3.846
LP_{21}	3.832	3.285
LP_{22}	7.016	5.061
LP_{23}	10.173	6.856
LP_{24}	13.323	8.659
LP_{31}	5.136	4.348
LP_{32}	8.417	6.090
LP_{33}	11.620	7.861
LP_{34}	14.796	9.647
LP_{41}	6.379	5.403
LP_{42}	9.760	7.128
LP_{43}	13.017	8.880
LP_{44}	16.224	10.651

IV. MIXED-MODE CASES

In the remainder of the paper, we will consider two practically important examples of beams that are coherent superpositions of particular LP-modes. Such coherent superpositions can result from, for example, injecting a narrowband laser light into a multimode step-index fiber of moderate length.

A. *Mixture of LP_{01} and LP_{11}*

This case is practically important because out of all guided fiber modes, LP_{11} has the closest propagation constant to that of the fundamental mode LP_{01}. Thus, if the pure LP_{01}-mode is launched into a slightly multimode fiber and the fiber is subjected to a disturbance, some of the intensity will be predominantly transferred to the LP_{11} modes as the light propagates down the fiber [13]. Accordingly, consider a normalized field distribution at the exit fiber facet in the form

$$E(r,\phi) = \sqrt{1-\alpha}\cdot E_{01}(r) + \sqrt{\alpha}\cdot e^{i\psi}E_{11}(r,\phi,\xi) \qquad (19)$$

where the notation is the same as in (9), and $0<\alpha<1$ is a fraction of the total intensity carried by the LP_{11}-mode. As before, we assume that the light is linearly polarized and consider two extreme cases of the excited mode orientation with respect to the scanning slit of the beam analyzer: $\xi=\pi/2$ and $\xi=0$. By using the general formula (8) for the beam-quality factor with the above field distribution (19), we find that in the case of $\xi=\pi/2$, the value of M_x^2-parameter is independent of ψ, which is the phase shift between the two modes at the fiber end facet. The calculated beam-quality factor in this case is shown in Fig. 3(a), as a function of a fraction of the total intensity of the beam carried by the excited mode. For concreteness, in the calculation, we assumed that the step-index fiber has the V-number equal to 7. (Such as a fiber with $\text{NA}=0.1$ and the core diameter of 22 μm at a wavelength of 1 μm, for example.) From Fig. 3(a), the value of the M_x^2-parameter is very close to 1 over the entire range of α. As in the case of pure LP_{11}-mode, this does not mean that the beam necessarily has a high quality but that analyzing the beam with a single scanning slit is not adequate in this case, and the analysis in two dimensions is necessary.

In the case of the orthogonal orientation of the LP_{11}-mode ($\xi=0$), the beam-quality factor of the beam depends on the phase ψ between the two modes at the fiber end facet.

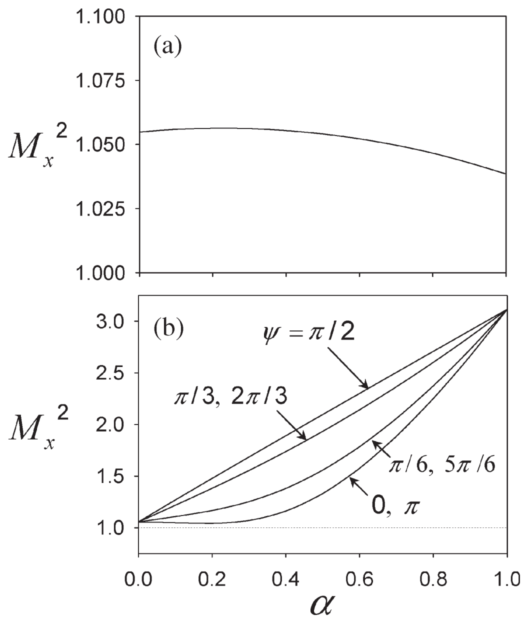

Fig. 3. Beam quality factor of the coherent superposition (19) of LP_{01} and LP_{11} modes, as a function of the higher order mode content. The step-index fiber has V-number equal to 7. (a) Case when the lobes of the LP_{11}-mode are parallel to the scanning slit of the beam analyzer ($\xi = \pi/2$). The M_x^2-parameter of the beam in this case is independent of ψ, the phase shift between the modes at the fiber end facet. (b) The lobes of the LP_{11}-mode are orthogonal to the scanning slit ($\xi = 0$). Different curves correspond to different values of the phase shift between the modes.

The corresponding curves at several different values of ψ are shown in Fig. 3(b). The multimode step-index fiber is again assumed to have the V-number equal to 7. From the figure, the most severe degradation of the M_x^2-parameter due to the mode-mixing occurs when the phase between the two spatial modes at the fiber end facet equals to $\pi/2$. For example, in this case 20% of the total intensity carried by the excited mode LP_{11} degrades the beam quality factor from ~ 1.05 for the pure fundamental mode, to ~ 1.5.

The dependence of the beam quality factor on the excited mode content of the beam is represented by smooth curves that naturally start at the M_x^2 value for the fundamental mode at $\alpha = 0$ and end at the value for the LP_{11}-mode at $\alpha = 1$. In the case when the V-number approaches the value of 2.405 from above (2.405 is of course the cutoff value for the LP_{11}-mode), the degradation of the beam quality due to the mode mixing becomes severe. In Fig. 4, we plot the M_x^2-parameter as a function of the excited mode content in a fiber with V-number equal to 2.42 so that the LP_{11} mode is allowed to propagate in the fiber, but it is close to cutoff. As before, the M_x^2 depends on the spatial orientation of the excited mode with respect to the scanning slit of the beam analyzer. However, the dependence on ψ, the phase shift between the two modes in this case becomes negligible. Note that in practice, the modes that are too close to cutoff experience severe attenuation in the fiber because a large fraction of the mode energy propagates in the fiber cladding. Therefore, the degradation of the beam quality due to the excited modes that are close to cutoff will be less pronounced in real fibers, and the actual value of the M_x^2-parameter in such cases will depend on how far the excited modes can propagate in the fiber.

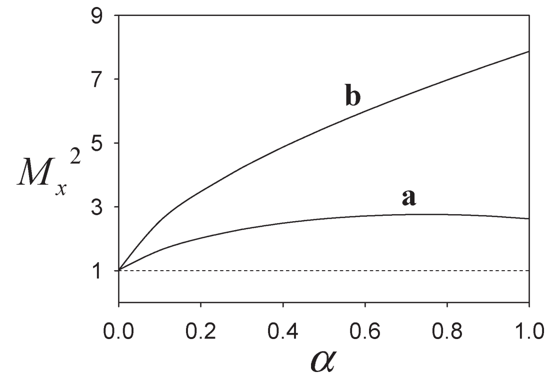

Fig. 4. Beam quality factor of the coherent superposition (19) of LP_{01} and LP_{11} modes, as a function of the higher order mode content, when the LP_{11} mode is close to cutoff (V-number equals to 2.42). The two curves correspond to two different orientations of the excited mode with respect to the scanning slit of the beam analyzer. (a) $\xi = \pi/2$ and (b) $\xi = 0$. Note that when LP_{11}-mode is close to cutoff, the M_x^2-parameter becomes independent of ψ, which is the phase shift between the modes, for both orientations of the excited mode.

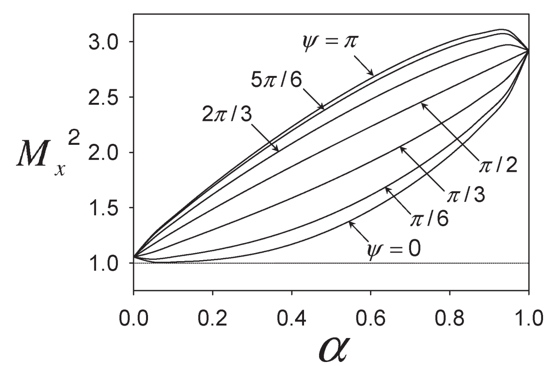

Fig. 5. Beam quality factor of the coherent superposition (20) of LP_{01} and LP_{02} modes, as a function of the higher order mode content. The step-index fiber has V-number equal to 7. Different curves correspond to different values of ψ, which is the phase shift between the modes at the fiber end facet.

B. Mixture of LP_{01} and LP_{02}

As a second practically important mixed-mode case consider a coherent superposition of adjacent axially symmetric modes LP_{01} and LP_{02}

$$E(r) = \sqrt{1-\alpha} \cdot E_{01}(r) + \sqrt{\alpha} \cdot \mathrm{e}^{i\psi} E_{02}(r). \qquad (20)$$

The notation above is the same as in the previous example (19). Dependence on ϕ in this case is absent because both modes are axially symmetric. The output field in the form (20) can approximate a situation when the pure fundamental mode is launched into a multimode step-index fiber, and the fiber is subjected to an axially symmetric disturbance such as tapering or splicing to a fiber with a different core size or numerical aperture [14]. As before, the field is assumed to be linearly polarized. The calculated beam-quality factor as a function of the fraction of energy carried by the excited mode is shown in Fig. 5. Different curves in the figure correspond to several fixed values of ψ, the phase shift between the two modes at the exit facet of the fiber. In the calculation, we assumed that the V-number of the fiber equals 7. From the figure, the case of the most severe degradation of the beam-quality factor due to

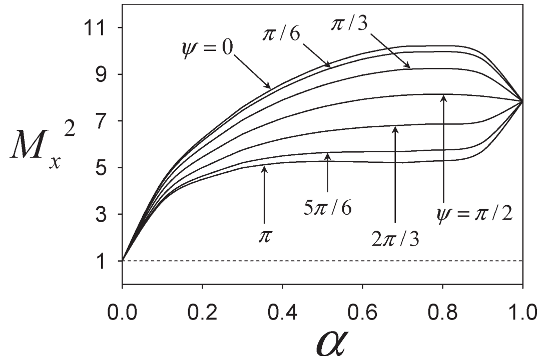

Fig. 6. Beam quality factor of the coherent superposition (20) of LP_{01} and LP_{02} modes, as a function of the higher order mode content, when LP_{02} mode is close to cutoff (V-number equals to 3.94). Different curves correspond to different values of ψ, which is the phase shift between the modes at the fiber end facet. Note that the most severe degradation of the beam quality factor in this case occurs when $\psi = 0$, and not $\psi = \pi/2$, as is in the case when the excited mode is far from cutoff.

the excited-mode content of the beam occurs when the two modes are out-of-phase ($\psi = \pi$). In this case, 20% of intensity carried by the excited mode degrades the M_x^2-parameter to ~ 1.7.

Further, consider the degradation of the beam quality due to the mode mixing when the excited mode is close to cutoff. In Fig. 6, we plot the M_x^2-parameter as a function of the excited mode content of the beam for a fiber with V-number equal to 3.94, which is slightly above the cutoff value for the LP_{02} mode. The degradation of the beam quality becomes severe, as expected. Note that the worst case now results when the two modes at the exit fiber facet are in phase.

V. Conclusion

The beam-quality factor (M^2) for the modes of a step-index fiber has been found in a closed form as a function of the fiber parameters. It has been shown that this quantity sharply peaks close to the mode cutoffs. Similar peaking is expected when a superposition of modes is exiting from the fiber, with one of the modes being close to its cutoff. Further, simple formulas for the beam quality factor have been found in the limiting case of a fiber with a large V-number. Two practically important coherent superpositions of modes in a multimode fiber have been considered, and the quantitative degree of degradation of the beam quality due to the higher order mode content of the beam in these cases have been found.

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