Mechanical effects of light on material media: radiation pressure and the linear and angular momenta of photons

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Abstract. Electromagnetic waves carry energy as well as linear and angular momenta. Interactions between light and material media typically involve the exchange of all three entities. In all such interactions energy and momentum (both linear and angular) are conserved. Johannes Kepler seems to have been the first person to notice that the pressure of sunlight is responsible for the tails of the comets pointing away from the Sun. Modern applications of radiation pressure and photon momentum include solar sails, optical tweezers for optical trapping and micro-manipulation, and optically-driven micro-motors and actuators. This paper briefly describes certain fundamental aspects underlying the mechanical properties of light, and examines several interesting phenomena involving the linear and angular momenta of photons.

Linear momentum of the electromagnetic field. It is well known that electromagnetic (EM) fields carry momentum as well as energy. Johannes Kepler (1571-1630) appears to have been the first to notice that a comet's tail always points away from the Sun, a phenomenon he attributed to the pressure of the sunlight on particles that evaporate from the comet's surface; see Fig. 1.

Fig. 1. Johannes Kepler suggested that the comet tails always point away from the Sun because of the pressure exerted by the sunlight on particles that evaporate from the comet.

Perhaps the simplest way to deduce the magnitude of the EM field momentum from first principles is by means of the Einstein box thought experiment. Shown in Fig. 2 is an empty box of length L and mass M, placed on a frictionless rail, and free to move forward or backward. At some point in time, a blob of material attached to the left wall emits a short EM pulse of energy $\mathcal{E}$ and momentum $\boldsymbol{p}$, which remains collimated as it travels the length of the box and gets absorbed by another blob attached to the right-hand wall. The recoil velocity of the box is thus $-\boldsymbol{p}/M$, the time of flight is L/c, and the box displacement along the rail is $-(p/M)(L/c)$.

Associating a mass $m = \mathcal{E}/c^2$ with the EM pulse and assuming that $M \gg m$, it is easy to see that the displacement of the center-of-mass of the system must be proportional to $(\mathcal{E}/c^2)L - M(p/M)(L/c)$. In the absence of external forces acting on the box, however, its center-of-mass is not expected to move. Setting the net displacement in the above expression equal to zero, we find that $p = \mathcal{E}/c$. Thus, in free space, a light pulse of energy $\mathcal{E}$ carries a momentum $p = \mathcal{E}/c$ along its direction of propagation. This result, which is independent of the particular shape of the pulse as a function of time, is accurate provided that the amplitude and phase profiles of the EM wave are smooth and uniform over a large cross-sectional area, thus ensuring that the pulse remains collimated as it traverses the length of the box.

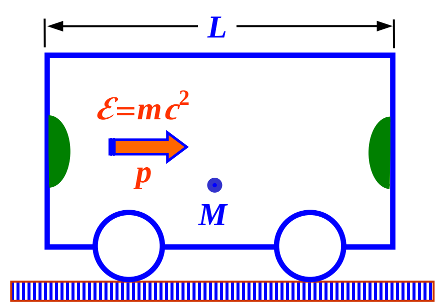

Fig. 2. Einstein box gedanken experiment.

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Electromagnetic fields in free space are defined by their electric field $\boldsymbol{E}(\boldsymbol{r},t)$ and magnetic field $\boldsymbol{H}(\boldsymbol{r},t)$, where $(\boldsymbol{r},t)$ represents the space-time coordinates. The rate of flow of energy (per unit area per unit time) is then given by the Poynting vector $\boldsymbol{S}=\boldsymbol{E}\times\boldsymbol{H}$. In terms of the Poynting vector, one can readily show that the momentum-density of the EM fields in the Einstein box thought experiment is given by $\boldsymbol{S}(\boldsymbol{r},t)/c^2$. To see this, assume a cross-sectional area A for the pulse, and note that the entire pulse moves at constant velocity c from left to right. Choose an arbitrary cross-section of the pulse (perpendicular to its propagation direction), and observe that the EM energy passing through this cross-section during a short time interval Δt is given by $\Delta\mathcal{E}=S(\boldsymbol{r},t)A\Delta t$. This energy, which proceeds to occupy the infinitesimal volume $\Delta v=Ac\Delta t$ to the right of the chosen cross-section, yields an energy density $\Delta\mathcal{E}/\Delta v=\ S(\boldsymbol{r},t)/c$ at point $\boldsymbol{r}$ at time t and, consequently, a momentum density $\Delta\boldsymbol{\wp}/\Delta v=\boldsymbol{S}(\boldsymbol{r},t)/c^2$ at that location.

A straightforward application of radiation pressure is found in the concept of solar sails; see Fig. 3. At 1.0 astronomical unit (i.e., the Sun-Earth distance), sunlight provides $\sim 1.4\ \mathrm{kw/m^2}$ of EM power density. Dividing this by the speed of light c and multiplying by 2 (to account for momentum reversal upon reflection from the sail) yields a pressure of $\sim 9.4\ \mu\mathrm{N/m^2}$. Over a sufficiently long period of time, the continuous force of sunlight exerted on a large-area solar sail can propel a small spacecraft to speeds comparable to or greater than those achievable by conventional rockets.

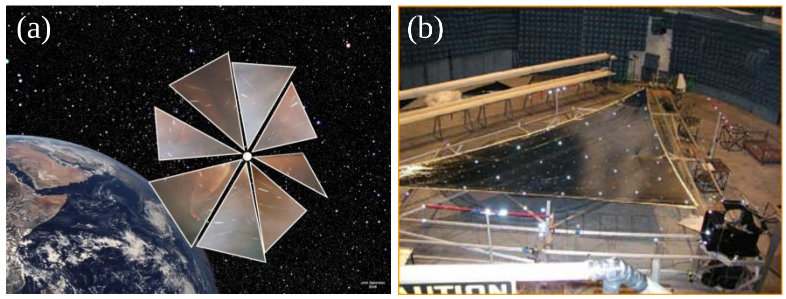

Fig. 3. (a) Artist's conception of a small solar-sail-driven spacecraft traveling away from the sun outside the Earth's atmosphere. (b) A 10-meter solar sail sits fully deployed in a vacuum chamber at NASA's Langley Research Center.

Optical tweezers. The first optical traps were built by Arthur Ashkin at AT&T Bell laboratories in the 1970s. "Levitation traps" used the upward-pointing radiation pressure to balance the downward pull of gravity, whereas "two-beam traps" relied on counter-propagating beams to trap particles. Then, in 1986, Ashkin and colleagues realized that the gradient force alone would be sufficient to trap small particles. They used a single, tightly-focused laser beam to trap a transparent particle in three dimensions. The principle of single-beam trapping is shown in Fig. 4. A small spherical dielectric bead of refractive index n_{bead} is immersed in some liquid of refractive index n_{liquid}. A laser beam is focused from above into the glass bead, with the focal point placed slightly above the center of the sphere. (Only two of the incident rays are shown, but the remaining rays behave essentially in the same way.) The bending of the rays by the glass bead causes them to exit with a smaller deviation from the optical axis. The projection of the exiting rays' momenta on the optical axis is thus greater than that of the incident rays. Stated differently, optical momentum along the z-axis increases upon transmission through the bead. In the process, this change of optical momentum is transferred as a lift force to the glass bead, helping to support it against the downward pull of gravity. Additionally, it is not

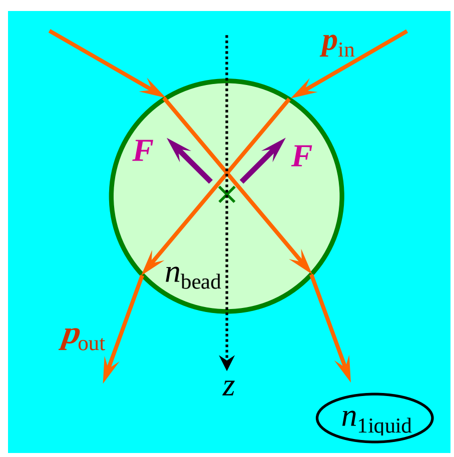

Fig. 4. Single-beam optical trap.

difficult to show that, if the bead is laterally displaced from equilibrium, the resulting gradient force will return it to its original position; in other words, the equilibrium is a stable one.

Electromagnetic spin and orbital angular momenta. It was mentioned earlier that the linear momentum-density (i.e., momentum per unit volume) of an EM field is $\boldsymbol{\wp}(\boldsymbol{r},t)=\boldsymbol{S}(\boldsymbol{r},t)/c^2$, where $\boldsymbol{S}$ is the Poynting vector and c is the speed of light in vacuum. The angular momentum density with respect to the origin of coordinates is thus given by $\boldsymbol{j}(\boldsymbol{r},t)=\boldsymbol{r}\times\boldsymbol{S}(\boldsymbol{r},t)/c^2$. A bullet of light having a finite duration and occupying a finite volume of space will carry, at any given time, a certain amount of linear and angular momenta, which amounts can be determined by integrating the corresponding momentum densities over the region of space occupied by the light bullet at any given instant of time. In the absence of interactions with material media (i.e., when the light bullet resides in free space), one can show, using Maxwell's equations, that the total linear momentum and also the total angular momentum of a given bullet remain constant in time, that is, the linear and angular momenta of the bullet are conserved. If the light enters a region of space where material media reside, it will exert forces and torques on various parts of these media in accordance with the Lorentz force law. Such exchanges between fields and material media cause the EM momenta (linear as well as angular) to vary in time. These variations, however, are always accompanied by corresponding variations in the linear and angular momenta of the material media (i.e., mechanical momenta), in such a way as to conserve the total momentum of the system of fields-plus-media, be it linear or angular, at all times.

The angular momentum of a light pulse (or bullet) in free space could arise as a trivial consequence of its center-of-mass trajectory (i.e., a straight-line along the linear momentum of the pulse) not going through the chosen reference point. Selecting a reference point on the center-of-mass trajectory then eliminates this trivial (extrinsic) contribution. The remaining contributions to angular momentum can be divided into two categories: *spin* and *orbital* angular momenta. In general, spin has to do with the degree of circular polarization of the light pulse, whereas orbital angular momentum arises from spatial non-uniformities of amplitude and phase that render the beam asymmetric around its propagation axis. Vorticity, which is associated with a continuous increase or decrease of phase around closed loops in the beam's cross-sectional plane, is a particularly interesting (and useful) source of orbital angular momentum.

A circularly-polarized light pulse of energy $\mathcal{E}$ propagating along the z-axis carries a spin angular momentum $\boldsymbol{\mathcal{S}}=\pm(\mathcal{E}/\omega)\hat{\boldsymbol{z}}$. The $\pm$ signs indicate the dependence of the direction of $\boldsymbol{\mathcal{S}}$ on the handedness of circular polarization (i.e., right or left). Such a light pulse, upon passing through a half-wave plate, will have its sense of polarization and, consequently, its direction of spin angular momentum (SAM) reversed. Conservation of angular momentum then requires the transfer of $2\boldsymbol{\mathcal{S}}$ units of angular momentum to the half-wave plate, as shown in Fig. 5. The passage of the light pulse thus sets the wave-plate spinning around the z-axis, a phenomenon that has been exploited in optically-driven micro-machines.

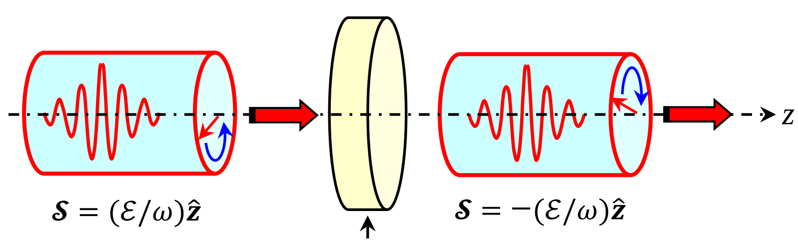

Fig. 5. A circularly-polarized light pulse of energy $\mathcal{E}$ and frequency ω carries a spin angular momentum $\boldsymbol{\mathcal{S}}=\pm(\mathcal{E}/\omega)\hat{\boldsymbol{z}}$. Upon transmission through a half-wave plate, the change in the optical angular momentum ($2\boldsymbol{\mathcal{S}}$) is transferred to the wave-plate, thereby setting the plate spinning around the z-axis.

When a collimated beam of light passes through a transparent spiral ramp, as depicted in Fig. 6, the emergent beam acquires optical vorticity, which carries orbital angular momentum (OAM). Once again, conservation of angular momentum requires the transfer of an equal but opposite angular momentum to the spiral ramp. Both SAM and OAM may be used to drive micro-machines. While the SAM associated with a single photon ($\mathcal{E} = \hbar\omega$) can only have a magnitude of $\pm\hbar$ (i.e., the reduced Planck's constant), the magnitude of OAM could be any integer multiple of $\hbar$.

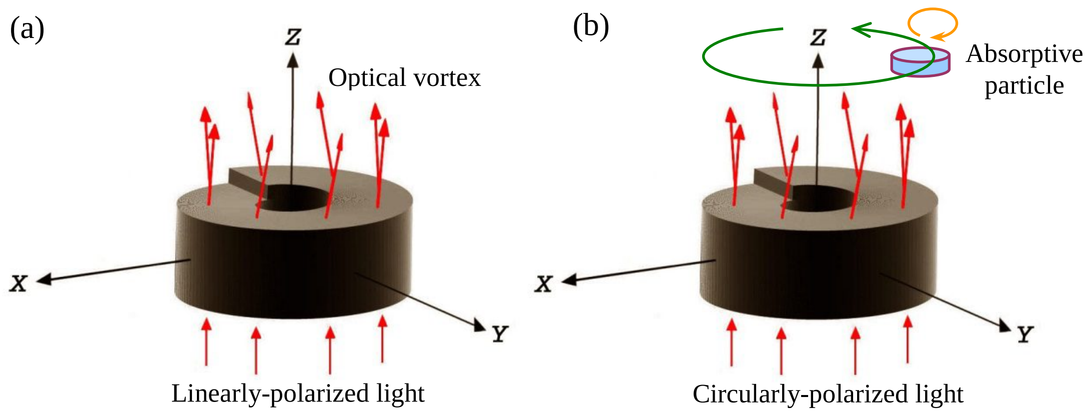

Fig. 6. (a) A transparent spiral ramp endows an incident beam with phase vorticity, which carries a certain amount of orbital angular momentum. (b) When the beam incident on the spiral ramp happens to be circularly polarized, the transmitted beam, a circularly-polarized optical vortex, carries both spin and orbital angular momenta. A small absorbing particle, placed in the path of such a beam, will spin on its own axis while, at the same time, travelling in a circle around the axis z of the spiral ramp.

The Balazs thought experiment. The arguments of the preceding sections do not shed any light on the momentum of EM waves inside material media. However, a different thought experiment, due to N. L. Balazs and dating back to 1953, reveals that the EM momentum-density within a transparent material must also be $\boldsymbol{p}(\boldsymbol{r},t) = \boldsymbol{S}(\boldsymbol{r},t)/c^2 = \boldsymbol{E}(\boldsymbol{r},t) \times \boldsymbol{H}(\boldsymbol{r},t)/c^2$. This particular expression is known as the Abraham momentum-density of EM waves inside material media.

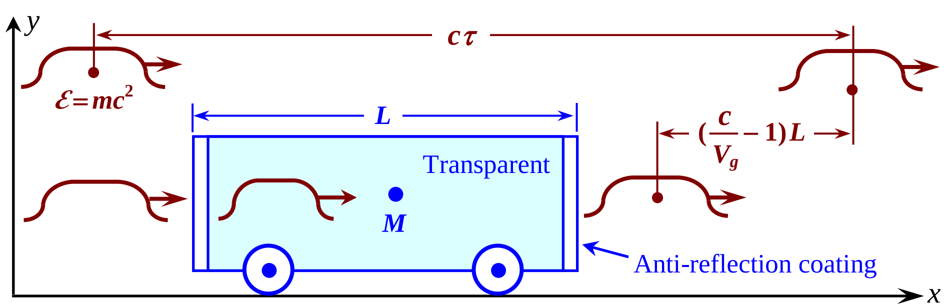

Fig. 7. The thought experiment of Balazs involves the propagation of a light pulse of energy $\mathcal{E}$ through a transparent rod of length L and mass M. The rod can move on a frictionless rail along the x-axis. Since the group velocity $V_g = c/n_g$ of the pulse inside the rod is less than c, the emergent pulse is somewhat behind the location it would have reached had it travelled in vacuum all along.

With reference to Fig. 7, consider a transparent dielectric (e.g., glass) rod of length L, refractive index n, and large mass M. Let a short light pulse enter the rod from the left and exit from the right, without losses due to absorption, scattering, or reflections at the entrance and exit facets. Balazs suggested three different schemes for avoiding reflection at the facet, but, for our purposes, it suffices to assume the existence of perfect anti-reflection coatings on these facets.

When the pulse emerges from the rod it will be delayed by the reduced speed of light within the glass. In other words, had the pulse travelled parallel to its current path but outside the rod, it would have been ahead a distance of $(n-1)L$ compared to where it will be upon emerging from the rod. Since there are no external forces acting on the system of rod plus the light pulse, the center-of-mass of the system should be in the same location irrespective of whether the pulse went through the rod or followed a parallel path outside the rod. Let the energy of the light pulse in vacuum be $\mathcal{E}$, which corresponds to a mass of $\mathcal{E}/c^2$. The delay has caused a leftward shift of the product of mass and displacement by $(n-1)L\mathcal{E}/c^2$. This must be compensated by a rightward shift of the rod itself. Let the light pulse have EM momentum p while inside the rod. Considering that the momentum of the pulse before entering the rod is $\mathcal{E}/c$, the rod must have acquired a net momentum of $(\mathcal{E}/c)-p$ while the pulse travelled inside. Its net mass times forward displacement, therefore, must be $[(\mathcal{E}/c)-p]nL/c$. Equating the rightward and leftward mass × displacement yields $p=\mathcal{E}/(nc)$ for the EM momentum of the pulse inside the rod. In particular, the EM momentum of a single photon inside a transparent dielectric is $p=\hbar\omega/(nc)$. This argument not only assigns the Abraham value to the EM momentum of the light pulse, but also indicates that the refractive index appearing in the Abraham expression for photon momentum is the group index (as opposed to the phase index) of the transparent medium.

Photon momentum deduced from the Fresnel reflection coefficient. A simple argument yields a formula for the *total* photon momentum (i.e., electromagnetic plus mechanical) inside a transparent dielectric. Consider a glass slab of refractive index $n=1.5$ surrounded by vacuum, as shown in Fig.8. The Fresnel reflection coefficient at the entrance facet of the slab being $r=(1-n)/(1+n)=-0.2$, a total of $|r|^2=4\%$ of all incident photons bounce back from the interface. Momentum conservation dictates that a reflected photon must transfer a momentum of $2\hbar\omega/c$ to the slab, while a transmitted photon must maintain its vacuum momentum of $\hbar\omega/c$. Assuming the incident light pulse contains a total of 100 photons, the total momentum of the photons entering the slab plus that of the slab itself must be $(96+4\times 2)\hbar\omega/c=104\hbar\omega/c$. The momentum associated with individual photons that have entered the slab is then given by $(104/96)\hbar\omega/c=1.0833\hbar\omega/c=½(n+n^{-1})\hbar\omega/c$. (This argument holds for any value of n and any number of photons contained in the incident light pulse, provided, of course, that the number of incident photons is sufficiently large to justify statistical averaging.) Recalling that the Balazs thought experiment associates the Abraham momentum $p=\hbar\omega/(nc)$ with the EM component of the photon momentum, the additional contribution to photon momentum in the preceding expression must be mechanical. (A similar argument applied to the angular momentum of circularly-polarized photons reveals the angular momentum of individual photons inside the dielectric to be the same as that in vacuum, i.e., $\hbar$, simply because reflected photons do *not* transfer any angular momentum to the glass slab upon reflection from its front facet.)

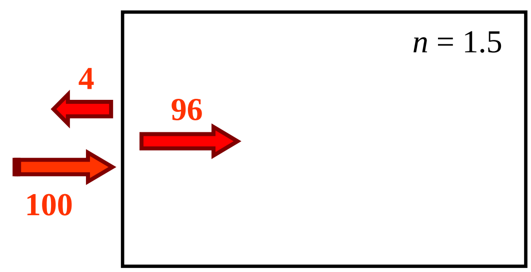

Fig.8. Light pulse containing 100 photons arrives from free space at the entrance facet of a glass slab of refractive index $n=1.5$.

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