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Parametric geometry analysis for circular-aperture off-axis parabolic mirror segment

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Abstract. We investigated the geometrical characteristics of circular-aperture off-axis parabolic (OAP) mirror segments to clarify the meaning of the loosely defined word “center” used in the literature and in documents to describe OAPs. We proposed the elliptical aperture center of an OAP as the definition of the center. The off-axis distance (OAD) is the vertical distance from the reference optical axis to the aperture center. In addition, the OAD can be varied depending on the desired center of a circular aperture to select the part of a parallel beam for focusing. The radius of the circular aperture becomes the minor-axis semidiameter of the elliptical aperture of the OAP. These geometrical parameters were systematically defined, derived, and/or analyzed in the context of optical engineering applications. Based on a set of those fundamental parameters, an intrinsic datum point utilizing the deepest point on the OAP surface was presented. The datum point provides a well-defined reference co-ordinate frame for locating or aligning an OAP within various astronomical telescope designs, instrument manufacturing and assembly processes, and optical system alignment and testing applications. © The Authors. Published by SPIE under a Creative Commons Attribution 4.0 Unported License. Distribution or reproduction of this work in whole or in part requires full attribution of the original publication, including its DOI. [DOI: [10.1117/1.JATIS.5.2.024010](https://doi.org/10.1117/1.JATIS.5.2.024010)]

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1 Introduction

Off-axis parabolic (OAP) mirrors have been studied for over a century because of their unique property of ideal focus, which does not depend on the relative location of the OAP with respect to the reference optical axis (ROA). Poor considered the aberration,¹ Borodin et al.² concentrated on the radiation of a picosecond laser, Burke³ examined the null test, and Howard⁴ examined the imaging properties of an OAP. Cardona-Núñez et al.^{5–7} investigated and characterized parabolic segments using toroidal and conic surfaces that fit an off-axis conic section.

Despite the numerous analyses of its optical properties, only a limited number of studies have characterized circular-aperture OAPs. A specific list of nomenclatures is often provided by commercial companies,⁸ such as the off-axis distance (OAD), segment axis, and off-axis angle. Although these lists appear as a complete set at first glance, they lack a definition of the geometric center. Avendaño-Alejo and Díaz-Urbe⁹ also used the word “center” for other referenced parameters but gave no mathematical definition for the center.

The absence of clear and commonly accepted geometrical definitions for an OAP mirror becomes more practically challenging in the astronomical optics community. Because of its off-axis nature, the OAP is often used as a solar telescope primary mirror solution, such as the Daniel K. Inouye Telescope (DKIST) in Fig. 1(a). It enables an unobstructed beam path, preventing scattered stray light rays as it does not have a typical on-axis secondary mirror supporting structure masking the incoming beam.¹⁰ In addition, in various astronomical instrument designs, such as spectrometers, OAPs can be utilized in order to collimate, relay, or image beams without causing diffraction

problems from pupil obscurations. For both space and ground telescope applications, many emerging science fields require high-contrast imaging or a well-controlled point spread function (PSF) in order to detect or distinguish a faint exoplanet signal from a bright star. As the diffraction-limited PSF is directly related to the spatial frequency contents of the pupil geometry, the unclipped beam path of the OAP offers one of the best possible PSF performances. As one of the extremely large telescopes, as another application, the Giant Magellan Telescope (GMT) utilizes six OAP mirror segments and an on-axis center segment.¹¹ The 8.4-m OAP segments (shown in Fig. 1) together with the center segment create a primary optical surface with a diameter of about 24 m to achieve its considerable light collecting power, along with a superb diffraction-limited resolution.

Despite all the advantages of using OAPs, an optical system including OAPs often faces practical difficulties in designing, manufacturing, assembling, and/or testing. One of the main challenges is the unclear or poorly defined datum or geometry parameters, which are critical for performing any design, machining, or alignment work. Surprisingly, one can often find an optical designer, fabricator, or optomechanical engineer attempting to explain the exact geometry of an OAP to others using complicated solid models without commonly accepted or reference geometrical parameters or data. Although three-dimensional solid models or mechanical drawings can be used and transferred, they cannot, unfortunately, provide an intuitive geometry or analytical insights of the configuration to an optical designer or mechanical engineer.

It is well known that an OAP can be defined using a circular aperture that selects part of a parallel beam to the ROA for focusing. However, no further information has yet been systematically organized or published, such as whether the center of the circular aperture corresponds to the center of the

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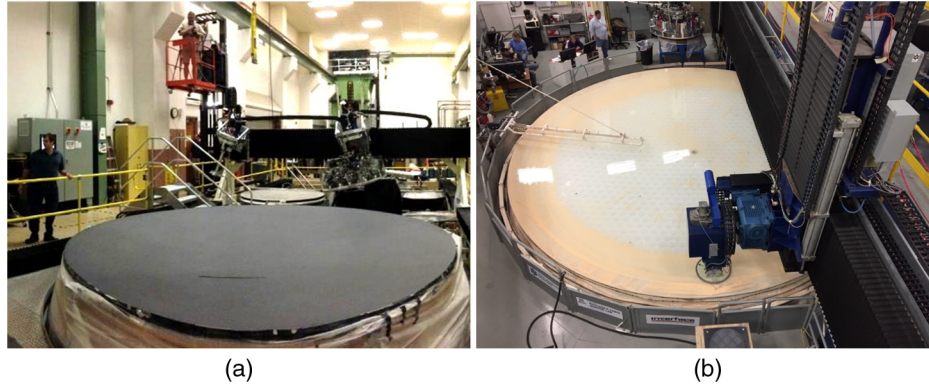


Fig. 1 (a) DKIST 4.2-m off-axis primary mirror ZERODUR substrate with computer-controlled optical surfacing machine¹⁰ and (b) the GMT, 8.4 m in diameter, off-axis segment being polished on the large polishing machine¹¹ at the University of Arizona.

OAP surface to be defined. Thus, we propose a definition for the center as the center of the elliptical aperture of an OAP with a comprehensive set of analytic formulas and provide a possible solution for the circular-aperture OAP mirror segment.

2 Geometrical Parameters of Off-Axis Parabolic Mirror Segment

A section of a paraboloid in the yz -plane of a Cartesian co-ordinate system is depicted in Fig. 2. The z axis and origin of the system coincide with the ROA and vertex of the paraboloid, respectively. The sag of the paraboloid can be expressed as follows:

$$z(x, y) = \frac{x^2 + y^2}{2R} \quad (1)$$

or

$$x^2 + y^2 = 2Rz, \quad (2)$$

where R is the radius of curvature of the paraboloid. The dotted straight line in Fig. 2 represents a plane that intersects with the xy plane of the system at $y = y_C$ with an angle θ to the xz -plane. The

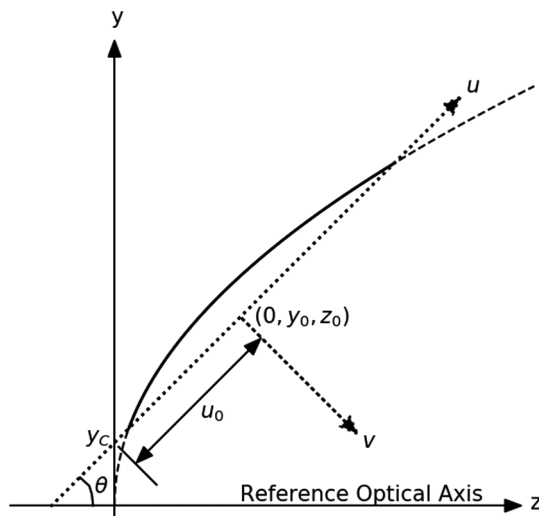


Fig. 2 Parabolic surface (dashed line) in a co-ordinate system with origin and z axis coinciding with the vertex and ROA, respectively. The (x, u, v) co-ordinate system is shown in red.

plane also intersects with the paraboloid and is called the aperture plane. The part of the paraboloid cut by the plane is indicated by a thick black solid line and represents the OAP to be analyzed.

As the xyz co-ordinate system is inconvenient for analyzing the shape and depth of the OAP, the xuv co-ordinate system shown in red in Fig. 2 is considered. The aperture plane now coincides with the xu -plane. The origin of the co-ordinate system is $(0, y_0, z_0)$, where $u_0^2 = (y_0 - y_C)^2 + z_0^2$. The parameter u_0 represents the distance from the point (or line) of $y = y_C$ to the origin of the new co-ordinate system. The transformation matrix between the two systems is as follows:

$$\begin{pmatrix} x \\ y - y_C \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sin \theta & -\cos \theta \\ 0 & \cos \theta & \sin \theta \end{pmatrix} \begin{pmatrix} x \\ u + u_0 \\ v \end{pmatrix}. \quad (3)$$

To find the aperture of the OAP, we let $v = 0$, and the co-ordinates of the intersecting points are expressed as follows:

$$\begin{cases} x = x \\ y = \sin \theta(u + u_0) + y_C \\ z = \cos \theta(u + u_0) \end{cases} \quad (4)$$

After inserting Eq. (4) into Eq. (2), we have

$$x^2 + (\sin \theta(u + u_0) + y_C)^2 = 2R \cos \theta(u + u_0), \quad (5)$$

or, in the equation of an ellipse,

$$\left(\frac{x}{R_x}\right)^2 + \left(\frac{u}{R_u}\right)^2 = 1, \quad (6)$$

where

$$\begin{cases} u_0 = \frac{R \cos \theta - y_C \sin \theta}{\sin^2 \theta} \\ R_x = \sin \theta R_u \\ R_u = \frac{\cos \theta}{\sin^2 \theta} \sqrt{R^2 - 2Ry_C \tan \theta} \end{cases}. \quad (7)$$

Equation (7) indicates that the clear aperture (CA) of an OAP created by cutting the paraboloid with a plane defined by y_C , and θ is an ellipse with the minor semidiameter of R_x parallel to the x axis and the major semidiameter of R_u along the u -axis. The center of the elliptical aperture is located at the origin of the

new co-ordinate system, which is located at a distance u_0 from $y = y_C$. Equation (7) also indicates that the two parameters for the aperture plane can be determined from the center and semi-diameters of the ellipse. This requires the depth profile of the elliptical mirror to be investigated.

3 Intrinsic Datum Defined by the Deepest Point

With the newly defined xuv co-ordinate system, the elliptical mirror surface depth can be obtained using the negative v -component of a point inside the ellipse. Using Eq. (3), Eq. (2) can be rewritten as follows:

$$\begin{aligned} x^2 + (\sin \theta(u + u_0) - \cos \theta v + y_C)^2 \\ = 2R(\cos \theta(u + u_0) + \sin \theta v) \end{aligned} \quad (8)$$

and

$$\begin{aligned} x dx + (\sin \theta(u + u_0) - \cos \theta v + y_C)(\sin \theta(du) - \cos \theta dv) \\ = R(\cos \theta(du) + \sin \theta dv), \end{aligned} \quad (9)$$

where dx , du , and dv are the differentials along the x , u , and v axes, respectively. Equation (8) can then be used to determine the depth for points (x, u) inside the ellipse, as shown in Fig. 3. To characterize the OAP with respect to the depth, we consider two important directions along the major and minor axes of the aperture plane.

The maximum depth and location along the minor axis ($-R_x \leq x \leq R_x$) can be found directly from Eq. (8) with the condition of $u = 0$. Equation (8) can be simplified as follows:

$$x^2 + (\sin \theta(u_0) - \cos \theta v + y_C)^2 = 2R(\cos \theta(u_0) + \sin \theta v), \quad (10)$$

which can be rewritten as a quadratic equation of the form $av^2 - 2bv + c = 0$ with the coefficients:

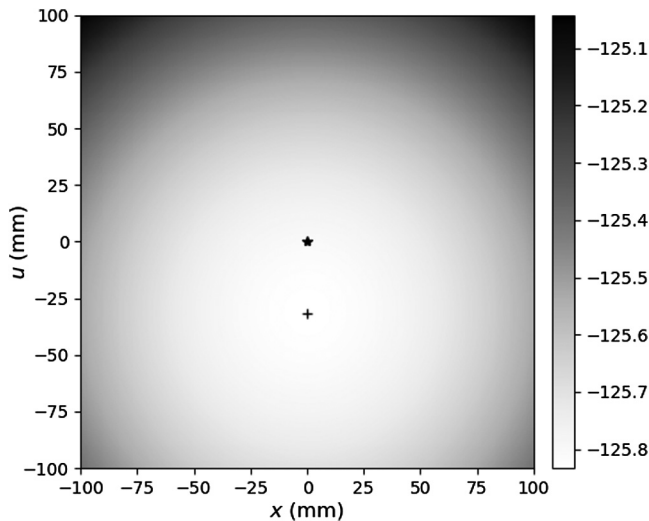


Fig. 3 Calculated depth (i.e., sag) map near the center of an OAP (with $R = 16000$ mm, $OAD = 4000$ mm, and $CA = 4200$ mm). The star marker indicates the location of the center of the elliptical aperture, and the cross marker represents the intrinsic deepest datum point.

$$\begin{cases} a = \cos^2 \theta \\ b = y_C \cos \theta + R \sin \theta + \sin \theta \cos \theta u_0 \\ c = x^2 + y_C^2 + 2(y_C \sin \theta - R \cos \theta)u_0 + \sin^2 \theta u_0^2 \end{cases} \quad (11)$$

With Eq. (7), Eq. (11) can be further simplified as follows:

$$\begin{cases} a = \cos^2 \theta \\ b = \frac{R}{\sin \theta} \\ c = x^2 - (R \cot \theta)^2 + 2Ry_C \cot \theta \end{cases} \quad (12)$$

The depth along the minor axis is a function of the co-ordinate x , which is the negative value of the two roots of the quadratic equation $v(x) = (b - \sqrt{b^2 - ac})/a$. This depth is symmetric for both signs of x , which results in the maximum depth occurring at $x = 0$. This can be derived from the derivative of the depth with respect to the co-ordinate x . Equation (9) can be simplified using the condition of $du = 0$ as follows:

$$\frac{dv}{dx} = \frac{x}{\cos \theta \sin \theta(u + u_0) - \cos^2 \theta v + y_C \cos \theta + R \sin \theta} \quad (13)$$

Setting Eq. (13) equal to zero results in the maximum depth occurring at $x = 0$. The depth along the major axis ($-R_u \leq u \leq R_u$) can be obtained by substituting $x = 0$ into Eq. (8):

$$(\sin \theta(u + u_0) - \cos \theta v + y_C)^2 = 2R(\cos \theta(u + u_0) + \sin \theta v). \quad (14)$$

After some rearranging, another quadratic equation of the form $v(u) = au^2 - 2bu + c$ can be obtained, and the depth can be obtained by the root formula:

$$v(u) = \frac{b - \sqrt{b^2 - ac}}{a}, \quad (15)$$

where

$$\begin{cases} a = \cos^2 \theta \\ b = y_C \cos \theta + R \sin \theta + \sin \theta \cos \theta(u + u_0) \\ c = y_C^2 + 2(y_C \sin \theta - R \cos \theta)(u + u_0) + \sin^2 \theta(u + u_0)^2 \end{cases} \quad (16)$$

Equations (15) and (16) describe the depth directly, but it is nontrivial to determine the maximum depth immediately. To find the location corresponding to the maximum depth, Eq. (9) can be rearranged after substituting in $dx = 0$:

$$\frac{dv}{du} = \frac{\sin \theta(\sin \theta(u + u_0) - \cos \theta v + y_C) - R \cos \theta}{\cos \theta(\sin \theta(u + u_0) - \cos \theta v + y_C) + R \sin \theta} \quad (17)$$

The maximum depth corresponds to the numerator of Eq. (17) being equal to zero:

$$\sin \theta(\sin \theta(u_{\max} + u_0) - \cos \theta v_{\max} + y_C) - R \cos \theta = 0. \quad (18)$$

Therefore, the maximum depth v_{\max} is related to the location u_{\max} as follows:

$$v_{\max} = \frac{\sin^2 \theta (u_{\max} + u_0) + \sin \theta y_C - R \cos \theta}{\sin \theta \cos \theta}. \quad (19)$$

After substituting Eq. (19) into Eq. (14), we can obtain as follows:

$$u_{\max} + u_0 = \frac{R \cos \theta}{2} \left(\frac{1}{\sin^2 \theta} + 1 \right) - y_C \sin \theta. \quad (20)$$

If Eq. (20) is substituted into Eq. (19), the maximum depth can be obtained as follows:

$$v_{\max} = -\frac{R \cos^2 \theta}{2 \sin \theta} + y_C \cos \theta. \quad (21)$$

The location of the maximum depth can also be obtained by substituting the first equation of Eq. (7) into Eq. (20):

$$u_{\max} = \cos \theta \left(y_C \cot \theta - \frac{1}{2} R \cot^2 \theta \right). \quad (22)$$

So far, no approximations or assumptions have been made; however, a plane has been used to define an OAP from a paraboloid. The CA of the OAP is an ellipse with specific minor/major semidiameters that depend on the parameters. Further investigation of the depth profile in two directions shows that the depth is the deepest at the center of the minor axis but not at the center of the major axis. That is, it is not the deepest at the center of the elliptical aperture.

In short, the intrinsic deepest point of an OAP mirror is located at a different point from the center of the elliptical aperture when the OAP is defined by using a plane to cut the parabolic surface. This indicates that the reference point for the distance from the ROA to the OAP can be either the center of the aperture or the deepest point of the mirror. We consider one of two points to be the appropriate definition for the ambiguous center. To determine the answer, we further investigated the details of the OAP for both cases, where we denote the distance from the ROA to the OAP as the OAD, and the major diameter of the elliptical aperture as the CA. These terms are similar to those used in the work on the DKIST and GMT.^{12,13}

4 Parametric Description of Off-Axis Parabolic Mirror Segment

4.1 OAP Segment Definition Using the Intrinsic Datum Point

Figure 4 shows the first method to define an OAP mirror segment, where OAD is the distance from the ROA to the deepest datum point of the OAP mirror. That is, the co-ordinate of the deepest point of the OAP is $(0, \text{OAD}, z_{\text{OAD}})$ in the xyz co-ordinate system, where

$$z_{\text{OAD}} = \frac{(\text{OAD})^2}{2R}. \quad (23)$$

Because the tangential plane at the deepest point is parallel to the aperture plane, the angle of the aperture plane can be obtained by differentiating Eq. (2):

$$\frac{dy}{dz} = \frac{R}{y}. \quad (24)$$

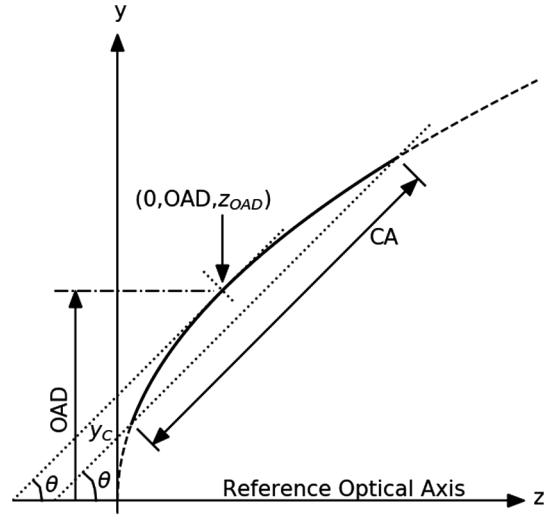


Fig. 4 First definition of OAP using the intrinsic deepest datum point of the mirror.

Then, the tangent of the angle can be expressed as follows:

$$\tan \theta = \frac{R}{\text{OAD}}. \quad (25)$$

As the major diameter of the elliptical aperture of the OAP is called CA, the third equation of Eq. (7) becomes

$$\text{CA} = 2 \frac{\cos \theta}{\sin^2 \theta} \sqrt{R^2 - 2Ry_C \tan \theta}. \quad (26)$$

From Eqs. (25) and (26), the y intersect for the plane can be expressed as follows:

$$y_C = \frac{\text{OAD}}{2} - \frac{\text{CA}^2 R^2}{8\text{OAD}(R^2 + \text{OAD}^2)}. \quad (27)$$

In short, for a set of designed values of OAD and CA, the two parameters, y_C and the angle for the aperture plane to make an appropriate OAP, can be described using Eqs. (25) and (27). Furthermore, using $\tan^2 \theta + 1 = 1/\cos^2 \theta$, the location of the deepest point and the maximum depth can also be described in terms of the designed values of OAD and CA by the direct substitution of Eqs. (25) and (27) into Eqs. (21) and (22) as follows:

$$u_{\max} = -\frac{R\text{CA}^2\text{OAD}}{8(R^2 + \text{OAD}^2)^{3/2}} \quad (28)$$

and

$$v_{\max} = -\frac{R^2\text{CA}^2}{8(R^2 + \text{OAD}^2)^{3/2}}. \quad (29)$$

Figure 5 shows the depth as a function co-ordinate on both the minor (solid line) and major (dashed line) axes of the OAP ($R = 16000$ mm, $\text{OAD} = 4000$ mm, and $\text{CA} = 4200$ mm) generated by the first method. The two parameters for the plane are $\theta = 75.96$ deg and $y_C = 1481.18$ mm. The deepest point u_{\max} is -31.46 mm away from the center of the aperture, and the maximum depth is -125.83 mm, which is about 0.03 mm

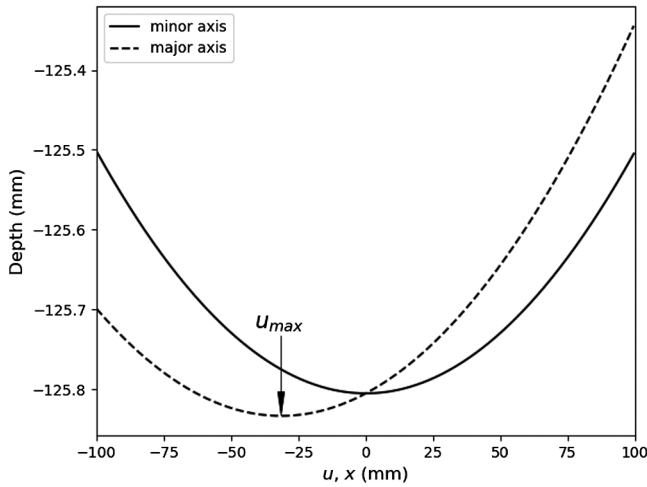


Fig. 5 Depth profile calculated by the first method for the OAP mirror surface of $OAD = 4000$ mm, $CA = 4200$ mm, and $R = 16000$ mm.

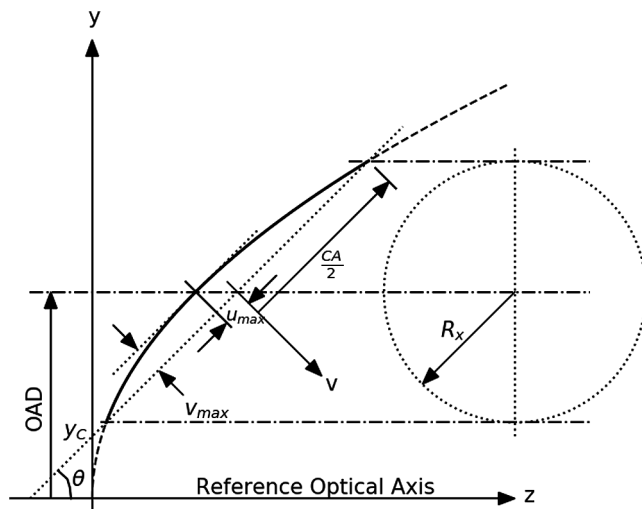


Fig. 6 Second definition of OAP segment with the center of the aperture plane of the mirror. At the same time, this geometry can also be understood as the projection of a circular aperture onto a paraboloid.

deeper than the depth at the center, -125.80 . The computed values have been denoted with two decimal points to emphasize the difference in depth between the two points.

4.2 OAP Segment Definition Using the Center of Clear Aperture Plane

The second method to define an OAP is illustrated in Fig. 6. The major diameter of the clear aperture of the OAP is CA , and the perpendicular distance from the ROA to the center of the clear aperture of the OAP is OAD . In this method, no hint is given for the angle of the aperture plane, unlike in the first method. Thus, the two parameters for the plane must be obtained from the two conditions of CA and OAD simultaneously. That is,

$$\begin{cases} OAD - y_C = \sin \theta u_0 \\ CA = 2R_u \end{cases}, \quad (30)$$

or after the substitution of u_0 of Eq. (7) into Eq. (3):

$$\begin{cases} OAD = \frac{R \cos \theta}{\sin \theta} \\ CA = 2 \frac{\cos \theta}{\sin^2 \theta} \sqrt{R^2 - 2Ry_C \tan \theta} \end{cases}. \quad (31)$$

Interestingly, Eq. (31) is identical to Eqs. (25) and (26). This indicates that both definitions result in the same OAP surface, or the distance from the ROA to the center of the clear aperture of the OAP is the same as that from the ROA to the deepest point of the OAP. As a side note, this interesting result applies only to the OAP, not to off-axis aspheric mirrors in general.¹⁴

4.3 Summary of the Parametric OAP Segment Description

To clarify the nomenclatures and the corresponding definitions for OAPs, we first investigated a plane to cut a paraboloid to define an OAP. We found that the intrinsic deepest point of the OAP is not at the center of the elliptical aperture of the OAP. This indicates that it is possible to set the distance from the ROA to the OAP, regardless of whether it refers to the elliptical aperture center of the OAP or to the deepest point of the OAP mirror. Unlike off-axis aspheric surface segment mirrors,¹⁴ the two choices result in the same OAP, and we can propose that the center of the elliptical aperture of the OAP is more intuitive and a clearer definition.

The fact that both definitions result in the same OAP can be extended to another interesting property, the y component of the major diameter of the OAP, which can be expressed as follows:

$$R_y = R_u \sin \theta. \quad (32)$$

After substituting in Eq. (7), we can obtain $R_y = R_x$. This indicates that the CA of the OAP can be considered a result of the circular aperture projected onto the paraboloid, as shown in Fig. 6. The circular aperture is perpendicular to the ROA, and the distance from the ROA to the center of the circular aperture can be expressed as follows:

$$y_0 = u_0 \sin \theta + y_C. \quad (33)$$

It is easy to show that this equals OAD . This property verifies the alternative and practical definition for the OAP. Consider a circular aperture that is perpendicular to the ROA. The aperture has a designed radius of R_x with the center at a distance from the ROA equivalent to the OAD . The circular aperture is then projected onto a parabolic surface to define an OAP.

5 Concluding Remarks

We proposed the use of a plane to define a circular OAP and found that the aperture of the OAP is an ellipse with a set of formulas for the center location, minor axis, major axis, and the deepest datum point and that the center does not correspond to the deepest point. We thoroughly analyzed the two possible consequences for a given set of OAD s and CA s to determine whether the two consequences are equal. Based on our investigation, we can conclude that an intuitive and convenient definition for the word “center” can be the center of the elliptical aperture of an OAP.

An OAP can be defined by a circular aperture that selects part of a parallel beam from the facts that (1) the y axis component of the major-axis semidiameter of the elliptical aperture of an OAP is equal to the minor-axis semidiameter and (2) both the deepest point of the OAP mirror and center of the elliptical aperture of

the mirror are a vertical distance away from the ROA. Therefore, the center of the circular aperture corresponds to the center of the elliptical aperture of the OAP and to the deepest point of the OAP mirror.

Unlike other fitting-based approaches, we do not make any assumptions or approximations during mathematical derivations and treatments. All the parameters and equations are exact and represent ideal OAP segment geometry. In reality, various measurement errors and practical assumptions/approximations during data analysis may affect the analyzed surface geometry data accuracy. The presented work focuses on the analytical investigation and parameterization of a mathematical OAP geometry without any approximation and assumptions. Thus, there is no fundamental error associated with the presented work.

We acknowledge that there might be a number of equally good but different descriptions to define an OAP geometry using different parameters and approaches. In the meantime, the presented geometrical definition comprising a set of thoroughly derived parameters and the intrinsic deepest datum point can be a good option as a referenceable and retraceable definition between scientists and engineers. For instance, the definition will provide a fully derived and documented referencing origin and orientation of an OAP mirror (or multiple OAPs) in an actual optical system, such as a cross-dispersion spectrometer or a multiconjugate adaptive optics system. The use of common terminology and a datum point will greatly minimize any ambiguous geometrical dimensioning discussions and improve the technical communication efficiency between optical engineers and astronomical scientists.

Currently, there is a lack of clear and commonly accepted geometrical definitions for OAP mirrors, especially the geometric center of a circular aperture. The method proposed in this study provides a suitable definition for the center that is clear, less ambiguous, intuitive, and convenient. The relevance of this study to the wider research field and impact on industrial applications has been presented. The method developed in this study has the potential of improving the location or alignment of OAPs within various astronomical telescope designs, instrument manufacturing and assembly processes, and optical system alignment and testing applications. It can also be used to describe a circular-aperture OAP in various contexts of optical engineering applications. This will make an impact on industrial applications that utilize these instruments, such as astronomical observations, the construction of new telescopes, and the development of new or existing related techniques. We hope this analytical work can serve our optical engineering and astronomical optics societies as a readily available option to describe a circular-aperture OAP in various contexts of optical and optomechanical engineering fields.

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