Aberration fields of a combination of plane symmetric systems

Lori B. Moore, Anastacia M. Hvisc and Jose Sasian*

College of Optical Sciences, University of Arizona, 1630 E. University Boulevard, Tucson, AZ 85721, USA
**Corresponding author: jose.sasian@optics.arizona.edu*

Abstract: By generalizing the wave aberration function to include plane symmetric systems, we describe the aberration fields for a combination of plane symmetric systems. The combined system aberration coefficients for the fields of spherical aberration, coma, astigmatism, defocus and distortion depend on the individual aberration coefficients and the orientations of the individual plane symmetric component systems. The aberration coefficients can be used to calculate the locations of the field nodes for the different types of aberration, including coma, astigmatism, defocus and distortion. This work provides an alternate view for combining aberrations in optical systems.

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OCIS codes: (080.1005) Aberration expansions; (080.2740) Geometric optical design.

References and links

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1. Introduction

Optical systems that do not have an axis of rotational symmetry have been and continue to be of interest in optical design. In his Ph. D. dissertation, Buchroeder [1] described a class of non-axially symmetric systems, called "tilted component optical systems" constructed from axially-symmetric components. These components may be tilted about their nodal points in such a way that a particular ray, which defines the reference axis, remains undeviated. The aberrations of tilted component optical systems then can be expressed using Shack's [2] vector aberration function where the field vector $\vec{H}$ is modified by a displacement term to account for each component tilt. With vector notation, the final system aberration fields can be found and analyzed as shown by Thompson [3,4].

A plane symmetric system is a system that has a plane of symmetry: that is, one half of the system is a mirror image of the other. Axially symmetric and double-plane symmetric systems belong to the class of plane symmetric systems. Sasian [5,6] developed an aberration function for describing plane symmetric optical systems that uses a vector $\vec{i}$ to define the direction of plane symmetry. The question is: can the plane symmetric formalism be extended to a combination of plane symmetric systems which do not necessarily share the same orientation for their respective planes of symmetry? Using the vector notation developed by Thompson [3,4] we extend the plane symmetric formalism to a combination of plane symmetric systems. The final, or global, system is not necessarily plane symmetric. Instead it may not have any identifiable symmetry.

The result of the work in this paper is a more general theory for non-axially symmetric systems than "tilted component optical systems" given that the component systems are not restricted to being axially symmetric. For example, this theory can be applied to both systems comprised of off-axis aspheres, as long as a plane of symmetry can be defined, and tilted plane symmetric optical components if they are tilted in the plane of symmetry. However this theory is not applicable to all asymmetric systems. It cannot be applied to systems with components that can not be simplified as plane symmetric.

There are several methods to concatenate asymmetric systems which can be used to combine plane symmetric systems. For example, Andrews [7], Forbes [8,9] and Stone [9] have developed methods for the concatenation of asymmetric systems using Hamiltonian methods. Our concatenation methodology differs in that it is based in component system rotation about an optical axis ray.

In addition to these methods, modern commercial lens design programs are capable of analyzing combinations of optical systems. Our approach provides an intuitive understanding of the combination of plane symmetric systems that can guide a lens designer using commercial lens design software. We show which of the aberrations are dependent on the orientation of the plane of symmetry. And we show that for these aberrations, the combined system aberration is simply the sum of the *i* vectors, denoting the direction of the plane of symmetry of the subsystem, weighted by the aberration coefficient for each subsystem. Conceptually it follows that a given aberration can be canceled by adding an equal and opposite amount of the aberration if the orientation of the plane of symmetry is chosen properly. It also follows that the aberration cannot be canceled if the plane of symmetry orientations are not chosen properly. To aid in this conceptual understanding, we have included figures for each of the aberrations and have grouped similar types of aberration fields.

For completeness, we find the nodes of each of the aberration fields. This provides a conceptual example of how the aberration fields for a combination of plane symmetric systems contributes to the overall system aberrations. We build on previous work [3,4] in aberration theory, add graphics to help clarify concepts, and contribute to the theory by including plane symmetric component systems. Specifically the new contributions to the theory are: 1) component system addition is specified by angular displacement rather than by component tilt, resulting in an alternate view of system concatenation, 2) a wider class of optical systems regarding the symmetry of the component systems can be treated, 3) we point out and discuss the occurrence of line nodes, and 4) the graphics contribute to the understanding of vector aberration theory.

2. Aberration function and fields

The wave aberration function for a rotationally symmetric optical system in vector form [2] is

$$W(\vec{H},\vec{\rho}) = \sum_{k,m,n}^{\infty} W_{2k+n,2m+n,n} (\vec{H}\cdot\vec{H})^k (\vec{\rho}\cdot\vec{\rho})^m (\vec{H}\cdot\vec{\rho})^n \tag{1}$$

where $\vec{H}$ is the field vector and $\vec{\rho}$ is the aperture vector, as shown in Fig. 1.

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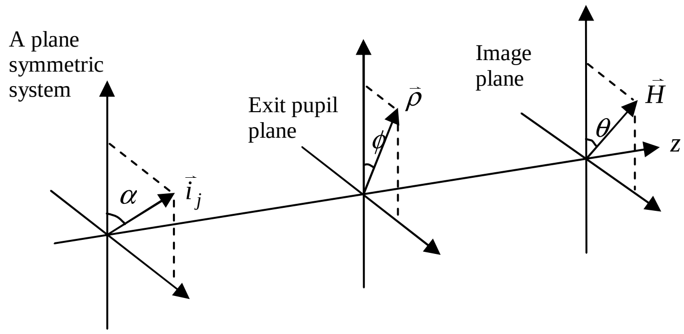

Fig. 1. Conventions for the plane symmetry, field and aperture vectors.

The aberration function can be modified for a plane symmetric system [5]:

$$W(\vec{H},\vec{\rho},\vec{i}) = \sum_{k,m,n,p,q}^{\infty} W_{2k+n+p,2m+n+q,n,p,q} (\vec{H}\cdot\vec{H})^k (\vec{\rho}\cdot\vec{\rho})^m (\vec{H}\cdot\vec{\rho})^n (\vec{i}\cdot\vec{H})^p (\vec{i}\cdot\vec{\rho})^q \tag{2}$$

where $\vec{i}$ is a unit vector that specifies the direction of plane symmetry. The indices *k*, *m*, *n*, *p* and *q* are integer numbers. On the right hand side of Eq. (2), the *W*'s represent the aberration coefficients and convey the magnitude of a given aberration. Note that the first subscript $2k+n+p$ is the algebraic power of the field vector $\vec{H}$, and the second subscript $2m+n+q$ is the algebraic power of aperture vector $\vec{\rho}$. To combine several systems, one can use a field displacement term $\vec{H}-\sigma_j$ for each of the tilted component systems as Thompson did. We instead combine several plane symmetric systems using the vector $\vec{i}_j$ that indicates the relative orientation among each of the *j* plane symmetric systems.

The aberration function describes the aberrations about the optical axis. The center of the field of view, the center of the aperture stop and the pupils lie on the optical axis ray [1]. Optically, the optical axis ray is a straight line; that is, looking at the optical axis from image space, the axis appears like a straight line, while in reality it is a ray that is reflected, refracted or diffracted by the system surfaces. In effect, the system can be unfolded such that the optical axis ray is a straight line. The vector $\vec{i}$ is perpendicular to the optical axis ray. Parametric expressions for the aberration coefficients of a plane symmetric system are given by Sasian [5,6]. Using the notation established by Sasian, the aberration function for a single plane symmetric system up to fourth-order is:

$$W(\vec{H},\vec{\rho},\vec{i}) = \begin{pmatrix} W_{00000} + W_{01001}(\vec{i}\cdot\vec{\rho}) + W_{10010}(\vec{i}\cdot\vec{H}) + \\ W_{02000}(\vec{\rho}\cdot\vec{\rho}) + W_{11100}(\vec{H}\cdot\vec{\rho}) + W_{20000}(\vec{H}\cdot\vec{H}) + \\ W_{02002}(\vec{i}\cdot\vec{\rho})^2 + W_{11011}(\vec{i}\cdot\vec{H})(\vec{i}\cdot\vec{\rho}) + W_{20020}(\vec{i}\cdot\vec{H})^2 + \\ W_{03001}(\vec{i}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{12101}(\vec{i}\cdot\vec{\rho})(\vec{H}\cdot\vec{\rho}) + W_{12010}(\vec{i}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho}) + \\ W_{21001}(\vec{i}\cdot\vec{\rho})(\vec{H}\cdot\vec{H}) + W_{21110}(\vec{i}\cdot\vec{H})(\vec{H}\cdot\vec{\rho}) + W_{30010}(\vec{i}\cdot\vec{H})(\vec{H}\cdot\vec{H}) + \\ W_{04000}(\vec{\rho}\cdot\vec{\rho})^2 + W_{13100}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{22200}(\vec{H}\cdot\vec{\rho})^2 + \\ W_{22000}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho}) + W_{31100}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho}) + W_{40000}(\vec{H}\cdot\vec{H})^2 \end{pmatrix}. \tag{3}$$

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The total or global aberration function of a number j of plane symmetric systems with relative orientation $\vec{i}_j$ and with aberration coefficients $W_{2k+n+p,2m+n+q,n,p,q,j}$ is the sum of the individual aberration functions. The fact that the total aberration is still a sum of the individual surface aberrations, even when there are tilted or decentered components, was discussed by Buchroeder [1]. The sum to fourth-order is:

$$W(\vec{H},\vec{\rho}) = \sum_{1}^{j} \begin{pmatrix} W_{00000j} + W_{01001j}(\vec{i}_j \cdot \vec{\rho}) + W_{10010j}(\vec{i}_j \cdot \vec{H}) + \\ W_{02000j}(\vec{\rho}\cdot\vec{\rho}) + W_{11100j}(\vec{H}\cdot\vec{\rho}) + W_{20000j}(\vec{H}\cdot\vec{H}) + \\ W_{02002j}(\vec{i}_j \cdot \vec{\rho})^2 + W_{11011j}(\vec{i}_j \cdot \vec{H})(\vec{i}_j \cdot \vec{\rho}) + W_{20020j}(\vec{i}_j \cdot \vec{H})^2 + \\ W_{03001j}(\vec{i}_j \cdot \vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{12101j}(\vec{i}_j \cdot \vec{\rho})(\vec{H}\cdot\vec{\rho}) + W_{12010j}(\vec{i}_j \cdot \vec{H})(\vec{\rho}\cdot\vec{\rho}) + \\ W_{21001j}(\vec{i}_j \cdot \vec{\rho})(\vec{H}\cdot\vec{H}) + W_{21110j}(\vec{i}_j \cdot \vec{H})(\vec{H}\cdot\vec{\rho}) + W_{30010j}(\vec{i}_j \cdot \vec{H})(\vec{H}\cdot\vec{H}) + \\ W_{04000j}(\vec{\rho}\cdot\vec{\rho})^2 + W_{13100j}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{22200j}(\vec{H}\cdot\vec{\rho})^2 + \\ W_{22000j}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho}) + W_{31100j}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho}) + W_{40000j}(\vec{H}\cdot\vec{H})^2 \end{pmatrix}. \quad (4)$$

This combination of plane symmetric systems still shares the same optical axis ray. The relative orientation of each of the systems is conveyed by the vectors $\vec{i}_j$. Figure 2 shows an example of possible orientations of these vectors as seen looking down the optical axis ray.

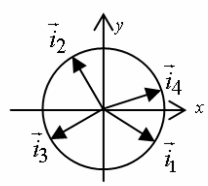

Fig. 2. Possible orientation of vectors $\vec{i}_j$ as seen looking down the optical axis. Notice that the vectors are all unit vectors in different directions. (The optical axis is in and out of the page at (x,y) = (0,0).)

To organize the aberrations in Eq. (4), we first define the variables listed in Table 1. These variables label each of the aberration coefficients by the aberration type and field dependence. This table also shows which of the aberrations have no dependence on $\vec{i}$ or are dependent on $\vec{i}$ or $\vec{i}^{\,2}$. It shows how the aberration coefficients from each component are combined into the system aberration coefficient. For the aberrations that depend on $\vec{i}$, the combined system aberration coefficient is simply the sum of the aberration coefficient for the individual components multiplied by the $\vec{i}$ vector denoting the orientation of plane of symmetry. In general, the plane of symmetry for each aberration coefficient of the combined system will have a different plane of symmetry so the combined $\vec{i}$ vectors are redefined. To minimize the combined system aberrations, either the individual subcomponent aberration coefficients can be minimized or, like with axially symmetric systems, the combined aberration can be reduced by balancing aberrations with equal but opposite amounts of the individual component aberrations, as long as the proper orientation of the individual planes of symmetry is chosen.

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Table 1. Coefficient and vector definitions

Uniform Piston: $W_{up} = \sum_j W_{00000j}$	Field Displacement: $W_{fd}\vec{i}_{fd} = \sum_j W_{01001j}\vec{i}_j$	Linear Piston: $W_{lp}\vec{i}_{lp} = \sum_j W_{10010j}\vec{i}_j$
Defocus: $W_d = \sum_j W_{02000j}$	Magnification: $W_m = \sum_j W_{11100j}$	Quadratic Piston I: $W_{qpI} = \sum_j W_{20000j}$
Defocus from Constant Astigmatism: $W_{dca} = \frac{1}{2}\sum_j W_{02002j}$	Magnification from Anamorphism: $W_{ma} = \frac{1}{2}\sum_j W_{11011j}$	Quadratic Piston IIa: $W_{qpIIa} = \frac{1}{2}\sum_j W_{20020j}$
Constant Astigmatism: $W_{ca}\vec{i}_{ca}^{\,2} = \frac{1}{2}\sum_j W_{02002j}\vec{i}_j^{\,2}$	Anamorphism: $W_a\vec{i}_a^{\,2} = \frac{1}{2}\sum_j W_{11011j}\vec{i}_j^{\,2}$	Quadratic Piston IIb: $W_{qpIIb}\vec{i}_{qpIIb}^{\,2} = \frac{1}{2}\sum_j W_{20020j}\vec{i}_j^{\,2}$
Constant Coma: $W_{cc}\vec{i}_{cc} = \sum_j W_{03001j}\vec{i}_j$	Linear Astigmatism: $W_{la}\vec{i}_{la} = \frac{1}{2}\sum_j W_{12101j}\vec{i}_j$	Field Tilt: $W_{ft}\vec{i}_{ft} = \sum_j W_{12010j}\vec{i}_j$
Quadratic Distortion I: $W_{qdI}\vec{i}_{qdI} = \sum_j W_{21001j}\vec{i}_j$	Quadratic Distortion II: $W_{qdII}\vec{i}_{qdII} = \sum_j W_{21110j}\vec{i}_j$	Cubic Piston: $W_{cp}\vec{i}_{cp} = \sum_j W_{30010j}\vec{i}_j$
Spherical Aberration: $W_{sa} = \sum_j W_{04000j}$	Linear Coma: $W_{lc} = \sum_j W_{13100j}$	Quadratic Astigmatism: $W_{qa} = \frac{1}{2}\sum_j W_{22200j}$
Field Curvature: $W_{fc} = \sum_j W_{22000j}$	Cubic Distortion: $W_{cd} = \sum_j W_{31100j}$	Quartic Piston: $W_{qp} = \sum_j W_{40000j}$

In Table 1, the terms "constant," "uniform," "linear," "quadratic" and "cubic" describe the algebraic power of the field dependence (first subscript). (We chose the name "uniform piston" for the aberration that would typically be called "constant piston" in this paper so that the subscript would be different from cubic piston, which uses "cp".)

Each time the direction unit vector $\vec{i}$ occurs in Table 1, it is always normalized to one. For example, in the constant coma case, we have:

$$\vec{i}_{cc} = \frac{\sum_j W_{03001j}\vec{i}_j}{\left|\sum_j W_{03001j}\vec{i}_j\right|} . \tag{5}$$

Therefore, the W coefficient accounts for the entire weight, such as in the constant coma term:

$$W_{cc} = \left|\sum_j W_{03001j}\vec{i}_j\right| . \tag{6}$$

Multiplying Eq. (5) and Eq. (6) results in the constant coma equation in Table 1.

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In order to better illustrate the field dependence of the aberration function in Eq. (4), we group the aberration coefficients by their dependence on the aperture vector $\vec{\rho}$. As shown in Table 2 this groups the aberration function into six aberration fields: spherical aberration, coma, astigmatism, defocus, distortion, and piston.

Table 2. Aberration fields of a combination of plane symmetric systems

Field of Piston	$\left\{\begin{matrix} W_{up} + W_{lp}\left(\vec{i}_{lp}\cdot\vec{H}\right) + \left(W_{qpI} + W_{qpIIa}\right)\left(\vec{H}\cdot\vec{H}\right) + \\ + W_{qpIIb}\left(\vec{i}^{\,2}_{qpIIb}\cdot\vec{H}^2\right) + W_{cp}\left(\vec{i}_{cp}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{H}\right) + W_{qp}\left(\vec{H}\cdot\vec{H}\right)^2 \end{matrix}\right\}\vec{\rho}^{\,0}$
Field of Distortion	$\left\{\begin{matrix} W_{fd}\vec{i}_{fd} + \left(W_m + W_{ma}\right)\vec{H} + W_a\vec{i}^{\,2}_a\vec{H}^* + \\ W_{qdI}\left(\vec{H}\cdot\vec{H}\right)\vec{i}_{qdI} + W_{qdII}\left(\vec{i}_{qdII}\cdot\vec{H}\right)\vec{H} + W_{cd}\left(\vec{H}\cdot\vec{H}\right)\vec{H} \end{matrix}\right\}\cdot\vec{\rho}$
Field of Defocus	$\left\{W_d + W_{dca} + W_{ft}\left(\vec{i}_{ft}\cdot\vec{H}\right) + W_{la}\left(\vec{i}_{la}\cdot\vec{H}\right) + \left(W_{qa} + W_{fc}\right)\left(\vec{H}\cdot\vec{H}\right)\right\}\left(\vec{\rho}\cdot\vec{\rho}\right)$
Field of Astigmatism	$\left\{W_{ca}\vec{i}^{\,2}_{ca} + W_{la}\vec{i}_{la}\vec{H} + W_{qa}\vec{H}^2\right\}\cdot\vec{\rho}^{\,2}$
Field of Coma	$\left\{W_{cc}\vec{i}_{cc} + W_{lc}\vec{H}\right\}\cdot\vec{\rho}\left(\vec{\rho}\cdot\vec{\rho}\right)$
Field of Spherical Aberration	$W_{sa}\left(\vec{\rho}\cdot\vec{\rho}\right)^2$

Some of the terms in Table 2, involve the product of vectors, such as $\vec{i}\vec{H}$ or $\vec{H}^2$. This operation, named "vector multiplication" by Shack [2], is different from both a vector dot product and a vector cross product. If we have two vectors $\vec{A}$ and $\vec{B}$ expressed as

$$\vec{A} = a\exp(i\alpha) = a\left(\sin\alpha\hat{i} + \cos\alpha\hat{j}\right) \tag{7}$$

$$\vec{B} = b\exp(i\beta) = b\left(\sin\beta\hat{i} + \cos\beta\hat{j}\right), \tag{8}$$

then the vector product $\vec{A}\vec{B}$ is defined as:

$$\vec{A}\vec{B} = ab\exp\left(i(\alpha+\beta)\right) = ab\left(\sin(\alpha+\beta)\hat{i} + \cos(\alpha+\beta)\hat{j}\right). \tag{9}$$

The conjugate of a vector is a reflection of the vector across the y-axis:

$$\vec{B}^* = b\exp(-i\beta) = b\left(\sin(-\beta)\hat{i} + \cos(-\beta)\hat{j}\right) = b\left(-\sin\beta\hat{i} + \cos\beta\hat{j}\right). \tag{10}$$

The following vector identities were also used to generate Table 2 [3]:

$$\left(\vec{A}\cdot\vec{B}\right)\left(\vec{A}\cdot\vec{C}\right) = \tfrac{1}{2}\left[\left(\vec{A}\cdot\vec{A}\right)\left(\vec{B}\cdot\vec{C}\right) + \vec{A}^2\cdot\vec{B}\vec{C}\right], \tag{11}$$

$$\text{and } \left(\vec{A}\cdot\vec{B}\vec{C}\right) = \left(\vec{A}\vec{B}^*\cdot\vec{C}\right). \tag{12}$$

For an explanation of vector multiplication, see Thompson [3].

The vector identities shown in Eqs. (11) and (12) are used to split the anamorphic distortion and each of the astigmatism terms into a combination of two aberrations. This is discussed in the appendix.

The aberrations discussed here are to the fourth-order of approximation, as discussed by Sasian [5], and do not include higher-order terms. For example, terms like cylindrical field curvature are not considered. The use of higher-order terms would impact the nature of the aberration fields and their point and nodal lines where they vanish.

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3. Graphical view of aberration field components

All of the components of the aberration fields in graphical form are shown in Tables 3 and 4 (except for piston which is a phase error that does not affect the image). These are similar to those presented by Thompson for tilted component systems but they correspond to a combination of plane symmetric systems which includes tilted component systems. In our graphical representation, anamorphism is described with respect to the average image size. Astigmatism is described with respect to the medial surface; see Appendix.

Table 3. Aberration field components that contribute to distortion mapping errors, shown in both grid and vector plot forms. In the grid form, the dotted lines show the nominal mapping positions of a square grid. The solid line shows the distortion of the square grid. In the vector form, the vectors show the magnitude and direction of the mapping distortion. All *i* vectors used in creating these graphs are pointing to the right.

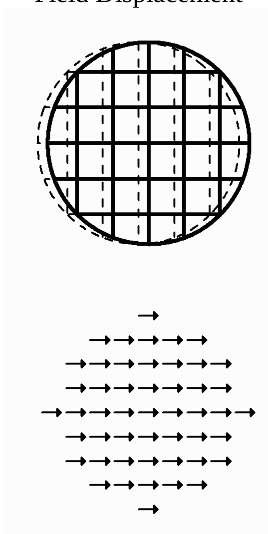
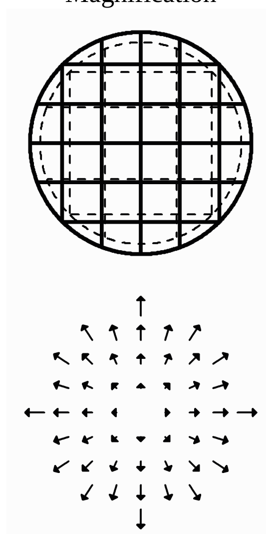
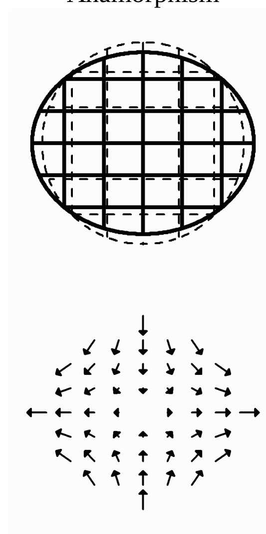
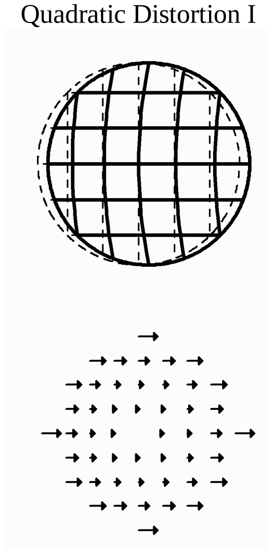
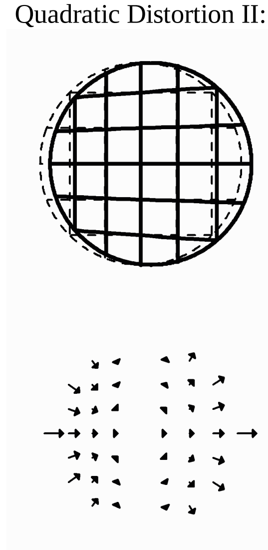
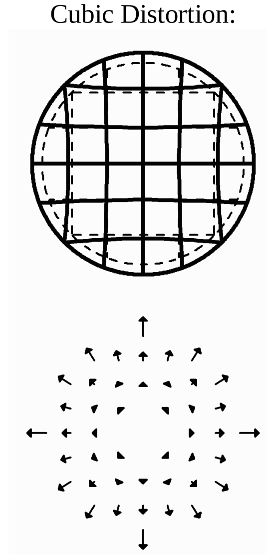
Field Displacement	Magnification	Anamorphism
		
Quadratic Distortion I	Quadratic Distortion II:	Cubic Distortion:
		

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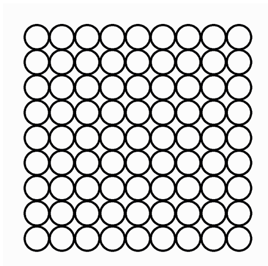
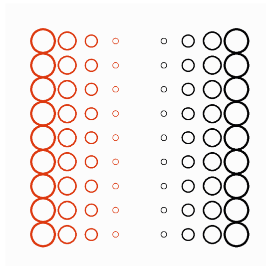
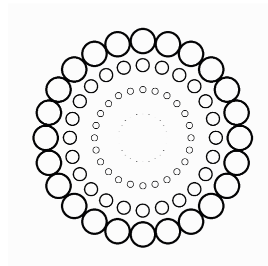
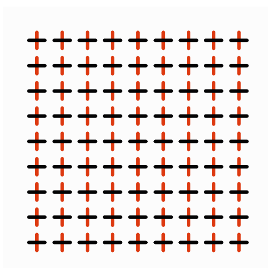
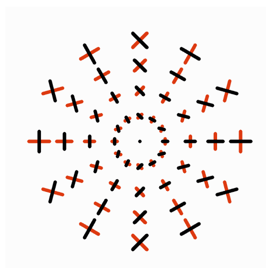
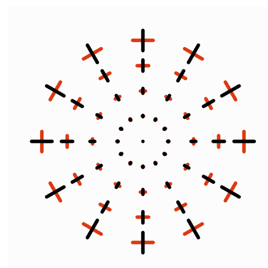
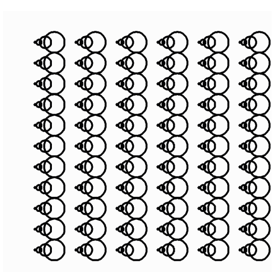
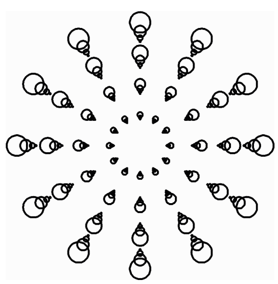
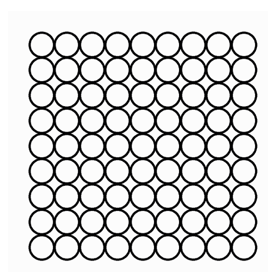
Table 4. Aberration field components. The field of defocus is represented by circles that convey the size of the defocused image across the field. Astigmatism is represented by a projection of the astigmatic focal lines. Coma is represented by a collection of circles. Each circle represents a fixed magnitude of the aperture vector, ρ, thus the collection of circles show the magnitude and orientation of the aberration in the field. Spherical aberration is represented by circles that show the magnitude of the aberration. All *i* vectors used in creating these graphs are pointing to the right. (Color online: Red denotes locations in the field where the focus position is before the image plane)

Defocus	Field Tilt	Field Curvature
		
Constant Astigmatism	Linear Astigmatism	Quadratic Astigmatism
		
Constant Coma	Linear Coma	Spherical Aberration
		

4. Aberration field nodes

Since each of the aberration fields listed in Table 2 is the sum of multiple terms, each with a different field dependence, it is possible that these terms sum to zero at certain locations in the field. These special locations are called field nodes and geometrically can be points, lines or circles. Following the work of Thompson [3,4] for tilted component systems, we find the nodes for a combination of plane symmetric systems. In many cases the location of the nodes is similar, but because we use different vector definitions than Thompson the equations used to find the nodes are not always the same.

To find the nodes in the field of view, we treat each aberration field independently. Notice that the pupil dependence for each field in Table 2 has been factored out. This leads to a term in brackets that depends only on the field vector, plane of symmetry vectors, and the

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aberration coefficients. In order to find the location of the nodes, we set this bracketed term equal to zero and solve for $\vec{H}$ as a function of the aberration coefficients and *i*.

4.1 Spherical aberration

The field of spherical aberration is uniform; it does not vary as a function of the field of view, so there are no field nodes.

4.2 Coma

The field of coma can be linear or constant with respect to the field of view:

$$W_{cc}\vec{i}_{cc} + W_{lc}\vec{H} = 0\,. \tag{13}$$

Thus, there can be one field node:

$$\vec{H} = -\frac{W_{cc}}{W_{lc}}\vec{i}_{cc}\,. \tag{14}$$

4.3 Astigmatism

The magnitude of the astigmatism can be uniform, linear or quadratic as a function of the field of view. This field can have one or two nodes where the astigmatism vanishes. Shack [2] was the first to recognize that the field of astigmatism could have two nodes and he called this case binodal astigmatism. In the presence of constant, linear, and quadratic astigmatism the node positions satisfy the equation:

$$W_{ca}\vec{i}_{ca}^{\,2} + W_{la}\vec{i}_{la}\vec{H} + W_{qa}\vec{H}^{2} = 0\,. \tag{15}$$

The locations of the nodes are at the two field points that can be found solving Eq. (15) and are given by:

$$\vec{H} = -\frac{1}{2}\frac{W_{la}}{W_{qa}}\vec{i}_{la} \pm \sqrt{\frac{1}{4}\left(\frac{W_{la}}{W_{qa}}\right)^{2}\vec{i}_{la}^{\,2} - \frac{W_{ca}}{W_{qa}}\vec{i}_{ca}^{\,2}}\,. \tag{16}$$

There are some special cases where the location of the node is simplified:

1) If there is no constant or linear astigmatism ($W_{ca} = 0$ and $W_{la} = 0$), there is one field node at the optical axis ray ($\vec{H} = 0$).

2) When there is no constant astigmatism ($W_{ca} = 0$), there are two nodes:

$$\vec{H} = 0 \text{ and } \vec{H} = -\frac{W_{la}}{W_{qa}}\vec{i}_{la}\,. \tag{17}$$

3) If there is no linear astigmatism ($W_{la} = 0$), then there are two nodes at:

$$\vec{H} = \pm\sqrt{-\frac{W_{ca}}{W_{qa}}}\vec{i}_{ca}\,. \tag{18}$$

4) If there is no quadratic astigmatism ($W_{qa} = 0$), the field of astigmatism is linear in field and there is only one node located at:

$$\vec{H} = -\frac{W_{ca}}{W_{la}}\frac{\vec{i}_{ca}^{\,2}}{\vec{i}_{la}}\,. \tag{19}$$

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4.4 Defocus

The nodes of defocus can be point nodes, line nodes, or circle nodes. A single point node is a circle node of radius zero. The defocus nodes must satisfy the following equation:

$$W_d + W_{dca} + W_{ft}\left(\vec{i}_{ft} \cdot \vec{H}\right) + W_{la}\left(\vec{i}_{la} \cdot \vec{H}\right) + \left(W_{qa} + W_{fc}\right)\left(\vec{H} \cdot \vec{H}\right) = 0\,. \tag{20}$$

Note that the astigmatism terms in the field of defocus describe the defocus required to move from the image plane to the medial astigmatic surface.

A line node will occur if the quadratic astigmatism term is balanced by the field curvature term ($W_{qa} = -W_{fc}$). As an example, if the defocus also balances the defocus from constant astigmatism ($W_d = -W_{dca}$), then the line node satisfies the following equations:

$$W_{ft}\left(\vec{i}_{ft} \cdot \vec{H}\right) + W_{la}\left(\vec{i}_{la} \cdot \vec{H}\right) = 0 \tag{21}$$

$$\text{or } \left(W_{ft}\vec{i}_{ft} + W_{la}\vec{i}_{la}\right) \cdot \vec{H} = 0\,. \tag{22}$$

Thus, the line node occurs when the field vector $\vec{H}$ is perpendicular to the vector $W_{ft}\vec{i}_{ft} + W_{la}\vec{i}_{la}$. If the two defocus terms do not cancel, then

$$\left(W_{ft}\vec{i}_{ft} + W_{la}\vec{i}_{la}\right) \cdot \vec{H} = -\left(W_d + W_{dca}\right). \tag{23}$$

The line node will shift and it no longer crosses through the center of the field of view. It is possible to shift the node outside the field of view. An example of a line node is plotted in Fig. 3.

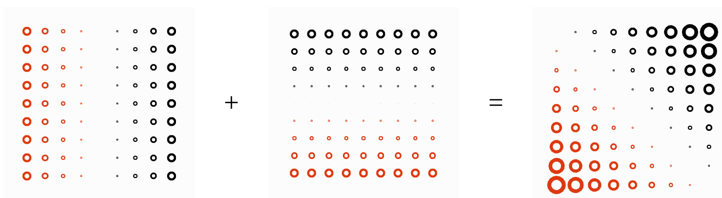

Fig. 3. A line node in the field of defocus from a combination of field tilts. (Color online: Red is focused before the image plane.)

However in general, the nodes from defocus will be circular. The values of the coefficients and the directions of the *i* vectors will determine the location of the circle and where it is centered. Figure 4 shows an example circular node in the field of defocus.

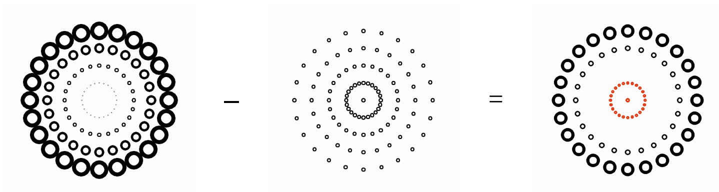

Fig. 4. A circular node in the field of defocus from a combination of field curvature and constant defocus. (Color online: Red is focused before the image plane.)

If the field tilt is balanced by the linear astigmatism ($W_{ft}\vec{i}_{ft} = -W_{la}\vec{i}_{la}$), then the circular node is centered on the on-axis field point. The radius of this circular node is derived as follows:

$$W_d + W_{dca} + \left(W_{qa} + W_{fc}\right)\left(\vec{H} \cdot \vec{H}\right) \Rightarrow W_d + W_{dca} + \left(W_{qa} + W_{fc}\right)H^2 = 0 \tag{24}$$

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$$H = \sqrt{-\left(\frac{W_d + W_{dca}}{W_{qa} + W_{fc}}\right)}. \tag{25}$$

Note that the quantity in parenthesis must be negative or else the field radius where the circular node exists will be imaginary and there will not be a circular node at all.

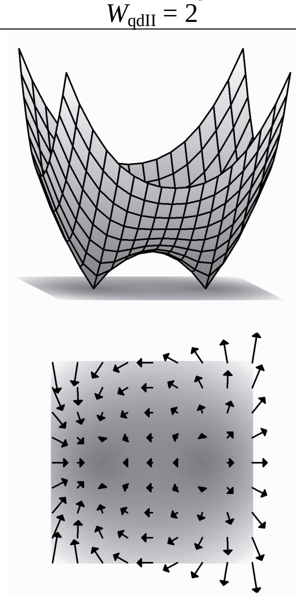
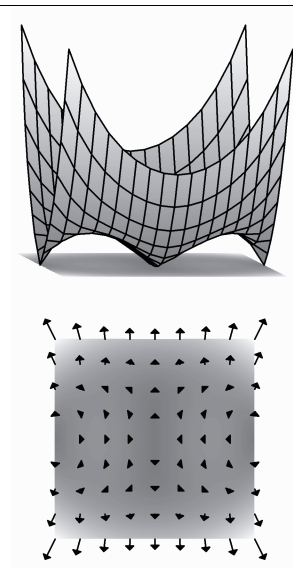
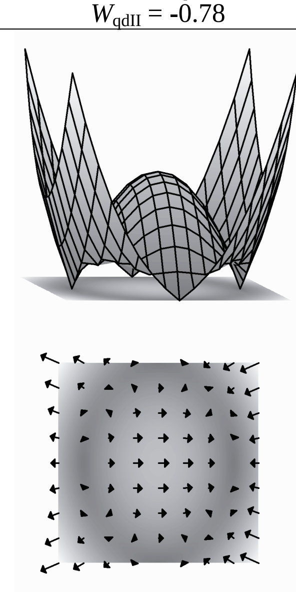
4.5 Distortion

Distortion is purely a mapping error which does not cause blurred images. Therefore it is possible to correct distortion with post-processing of the image. There are still applications where post-processing is not possible or distortion should be minimized to reduce the error in the post-processing. For this reason we describe the types of nodes found in the field of distortion. The distortion nodes satisfy the following equation:

$$W_{fd}\vec{i}_{fd} + (W_m + W_{ma})\vec{H} + W_a\vec{i}_a^{\,2}\vec{H}^* + (W_{qdII}\vec{i}_{qdII} \cdot \vec{H})\vec{H} + (\vec{H}\cdot\vec{H})(W_{qdI}\vec{i}_{qdI} + W_{cd}\vec{H}) = 0. \tag{26}$$

Table 5 shows some sample distortion vector plots with two, three, and four nodes and list the contributing coefficient amounts.

Table 5. Some distortion plots showing 2, 3, and 4 nodes. The surface maps represent the magnitude of the distortion. The points where the surfaces meet the plane are the nodes. In the vector plots, the vectors represent both the magnitude and direction of the mapping distortion. The shading represents the magnitude of the distortion. Darker shades have less distortion. All *i* vectors used in creating these graphs are pointing to the right.

2 nodes	3 nodes	4 nodes
W_{fd} = -0.25; W_{qdI} = -1 W_{qdII} = 2	W_{a} = -1; W_{cd} = 1	W_{fd} = 1; W_{qdI} = -1 W_{qdII} = -0.78
		

As with the field of defocus, it is also possible to get circular nodes and line nodes with the field of distortion. For example, if all other terms cancel except field displacement and quadratic distortion I, then the field of distortion simplifies to

$$\left\{W_{fd}\vec{i}_{fd} + W_{qdI}(\vec{H}\cdot\vec{H})\vec{i}_{qdI}\right\}\cdot\vec{\rho}. \tag{27}$$

This leads to a circular node (shown in Table 6), with a radius derived as follows:

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$$W_{fd}\vec{i}_{fd} + W_{qdI}\vec{i}_{qdI}\left(\vec{H}\cdot\vec{H}\right) = 0 \tag{28}$$

$$H = \sqrt{-\frac{W_{fd}}{W_{qdI}}} \,. \tag{29}$$

Like with the case of circular defocus nodes, W_{fd} and W_{qdI} must have opposite signs or the radius of the circle node will be imaginary and there will not be a node in the field.

Another circular node can be created from magnification and cubic distortion. This set also has a node on axis. If:

$$\left(W_m + W_{ma}\right)\vec{H} + W_{cd}\left(\vec{H}\cdot\vec{H}\right)\vec{H} \Rightarrow \left(\left(W_m + W_{ma}\right) + W_{cd}\left(\vec{H}\cdot\vec{H}\right)\right)\vec{H} = 0\,, \tag{30}$$

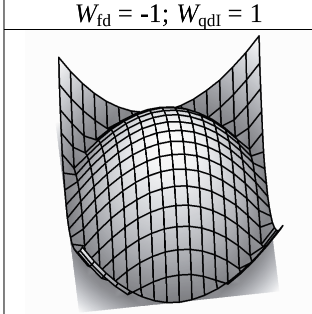
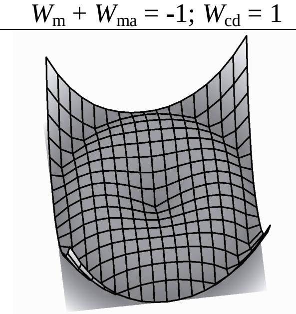
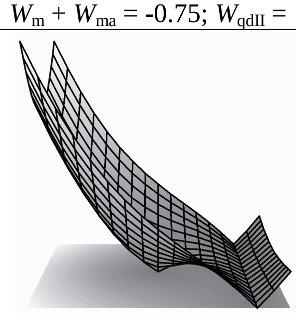
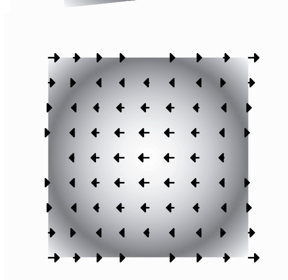
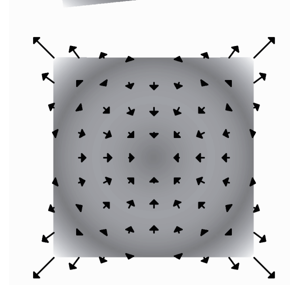
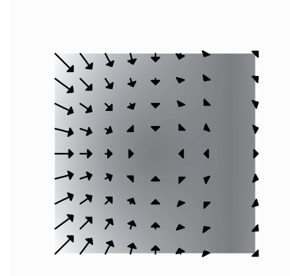
then there is one node on axis and a circular node (shown in Table 6) at

$$H = \sqrt{-\frac{\left(W_m + W_{ma}\right)}{W_{cd}}} \,. \tag{31}$$

Again the term (W_m + W_{ma}) and W_{cd} must have different signs for there to be a node.

A line node in the distortion field may be found when there is no cubic distortion. One simple example of a line node is the case of quadratic distortion II. Additionally, by adding an equal, but opposite, amount of quadratic distortion I to quadratic distortion II, (in effect $W_{qdI} = -W_{qdII}$ and $\vec{i}_{qdI} = \vec{i}_{qdII}$) a line node will be created. A line node may also be found by adding complementary amounts of magnification and anamorphism. It is also possible to get a line node and a point node using a combination of quadratic distortion II and magnification.

Table 6. Some distortion plots showing line and circular nodes. The surface maps represent the magnitude of the distortion. In the vector plots, the vectors represent both the magnitude and direction of the mapping distortion. The shading represents the magnitude of the distortion. Darker shades have less distortion. All *i* vectors used in creating these graphs are pointing to the right.

a) Circular distortion node	b) Circular distortion node with on-axis node	c) Line distortion node with on-axis node
W_{fd} = -1; W_{qdI} = 1	W_{m} + W_{ma} = -1; W_{cd} = 1	W_{m} + W_{ma} = -0.75; W_{qdII} = 1
		
		

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5. Summary

This paper adds to the theory of non-axially symmetric systems. Specifically, we extended the plane symmetric vector aberration function to determine the aberration fields for a combination of plane symmetric systems that do not necessarily share the same orientation for their respective planes of symmetry. Noteworthy is that the system combination is carried out by rotations about the optical axis ray of each system component. This paper provides mathematical expressions for the resulting aberration fields: spherical aberration, coma, astigmatism, defocus, and distortion. To help the conceptual understanding of the aberrations we defined and plotted the individual aberration terms that contribute to each field. In addition, the paper furthers the concept of field nodes by using the equations for the aberration fields to calculate and illustrate the locations of the field nodes, which may be point nodes, line nodes, or circle nodes depending on the aberration field. Although this theory is applicable only to the range of asymmetric systems that can be considered plane symmetric, it is in principle more general than the previous theories that apply only to axially symmetric system components.

Appendix A

The anamorphic distortion and astigmatism terms in Eq. (4) can each be split in to two terms. For the case of anamorphic distortion, the split terms change the reference to an average magnification. For the case of astigmatism, the split terms change the reference to the medial astigmatic surface.

1. Anamorphism

Anamorphic distortion is given by:

$$\sum_j W_{11011j}(\vec{i}_j \cdot \vec{H})(\vec{i}_j \cdot \vec{\rho}) \,. \tag{32}$$

This equation can be split into:

$$\begin{aligned}\sum_j W_{11011j}(\vec{i}_j \cdot \vec{H})(\vec{i}_j \cdot \vec{\rho}) &= \sum_j \left(\tfrac{1}{2}W_{11011j}\left(\vec{H} \cdot \vec{\rho}\right) + \tfrac{1}{2}W_{11011j}\vec{i}_j^{\,2}\vec{H}^* \cdot \vec{\rho}\right) \\ &= W_{ma}\left(\vec{H} \cdot \vec{\rho}\right) + W_a \vec{i}_a^{\,2}\vec{H}^* \cdot \vec{\rho} \,. \end{aligned} \tag{33}$$

The first term in Eq. (33) represents a magnification change and the second term represents a mapping change in two orthogonal directions. It is the second term that is used in Table 3. Thus in Table 3 anamorphism is described with respect to the average magnification rather than as the usual anamorphic, one-directional mapping stretch. This is shown graphically in Fig. 5.

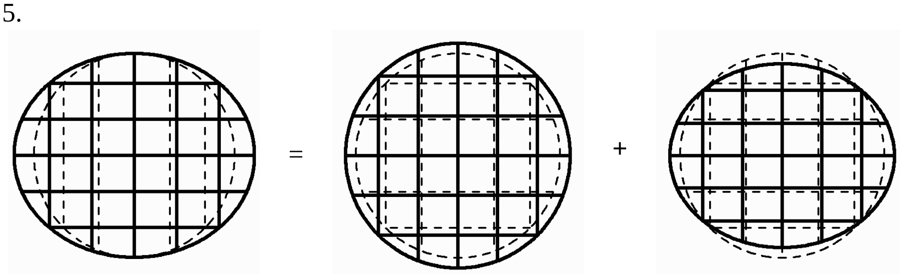

Fig. 5. Anamorphism can be described as an average magnification (center figure) plus an anamorphic term (figure on the right).

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2. Astigmatism

For the case of astigmatism, each of the terms in Eq. (4) related to the field of astigmatism can be split into two aberration components. For example, the term that describes linear astigmatism is split into:

$$\sum_j W_{12101j}(\vec{i}_j \cdot \vec{\rho})(\vec{H} \cdot \vec{\rho}) = W_{la}\left(\vec{i}_{la} \cdot \vec{H}\right)\left(\vec{\rho} \cdot \vec{\rho}\right) + W_{la}\vec{i}_{la}\vec{H} \cdot \vec{\rho}^{\,2}. \tag{34}$$

The first term of the split $W_{la}\left(\vec{i}_{la} \cdot \vec{H}\right)\left(\vec{\rho} \cdot \vec{\rho}\right)$ has the same functional form as field tilt. This locates the medial astigmatic surface on a tilted plane. In Table 2, the field of astigmatism is described from the medial astigmatic surface by the second term of the split $W_{la}\vec{i}_{la}\vec{H} \cdot \vec{\rho}^{\,2}$.

Figure 6 shows the components of the field of astigmatism, constant, linear and quadratic astigmatism, and their relation to the image plane as expressed in Eq. (4). The surfaces shown are the locus of the astigmatic focal lines. The medial surface is shown in blue online. In contrast, Fig. 7 shows the astigmatic surfaces with respect to the medial surface as mathematically represented in Table 2 and graphically shown in Table 4. This representation is used by Thompson [3,4].

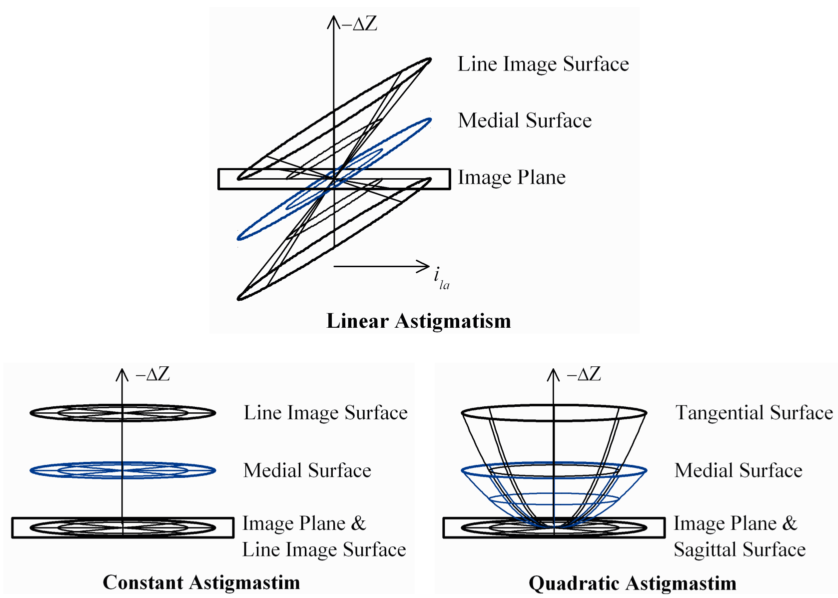

Fig. 6. Astigmatic surfaces and their relationship according to the astigmatism terms in Eq. (4). Defocus from the image plane is in the ΔZ direction. (Color online: The medial surface is shown in blue)

The surfaces in Figs. 6 and 7 are the locus of the astigmatic line images. These surfaces are called sagittal and tangential astigmatic surfaces for the case of quadratic astigmatism. However because the orientation of the line images in linear astigmatism depart from the radial symmetry of quadratic astigmatism, the terms sagittal and tangential are not quite appropriate for describing the line images of linear astigmatism. Instead we will refer to them with the more general term – line image astigmatic surfaces [4]. As shown in Fig. 7, for the case of linear astigmatism the line image astigmatic surfaces are along a cone, and astigmatic lines with the same orientation are located along a line. For example, note the same orientation of the astigmatic lines along the dashed red or green lines in Fig. 7 for linear astigmatism.

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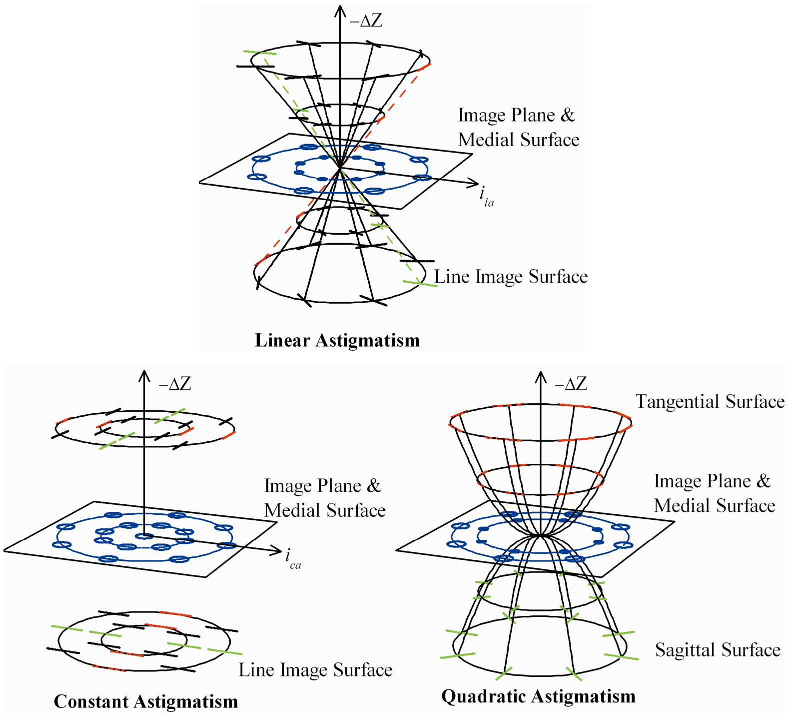

Fig. 7. Location and orientation of the astigmatic surfaces for the linear, constant, and quadratic astigmatism with respect to the medial surface and as mathematically represented in Table 2. Defocus from the medial surface is in the ΔZ direction. The dashed lines in the linear astigmatism figure highlight the locations of the sagittal and tangential foci. (Color online: The medial surface is blue. The tangential foci are red. The sagittal foci are green.)

3. Transverse ray aberrations

The transverse ray aberration vector $\vec{\varepsilon}$ was used to make some of the figures in this paper. Table 7 provides the transverse ray aberrations derived from the standard relationship,

$$\vec{\varepsilon} = \frac{1}{nu'} \nabla_{\rho} W\left(\vec{H}, \vec{\rho}\right) \tag{35}$$

where *n* is the index of refraction and *u'* is the marginal ray slope in image space.

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Table 7. Transverse ray aberrations

Field of Distortion	$nu'\cdot\vec{\varepsilon} = W_{fd}\vec{i}_{fd} + (W_m + W_{ma})\vec{H} + W_a\vec{i}_a^{\,2}\vec{H}^* +$ $W_{qdI}(\vec{H}\cdot\vec{H})\vec{i}_{qdI} + W_{qdII}(\vec{i}_{qdII}\cdot\vec{H})\vec{H} + W_{cd}(\vec{H}\cdot\vec{H})\vec{H}$
Field of Defocus	$nu'\cdot\vec{\varepsilon} = 2\{W_d + W_{dca} + W_{ft}(\vec{i}_{ft}\cdot\vec{H}) + W_{la}(\vec{i}_{la}\cdot\vec{H}) + (W_{qa} + W_{fc})(\vec{H}\cdot\vec{H})\}\vec{\rho}$
Field of Astigmatism	$nu'\cdot\vec{\varepsilon} = 2\{W_{ca}\vec{i}_{ca}^{\,2}\vec{\rho}^* + W_{la}\vec{i}_{la}\vec{H}\vec{\rho}^* + W_{qa}\vec{H}^2\vec{\rho}^*\}$
Field of Coma	$nu'\cdot\vec{\varepsilon} = W_{cc}((\vec{\rho}\cdot\vec{\rho})\vec{i}_{cc} + 2(\vec{i}_{cc}\cdot\vec{\rho})\vec{\rho}) + W_{lc}((\vec{\rho}\cdot\vec{\rho})\vec{H} + 2(\vec{H}\cdot\vec{\rho})\vec{\rho})$
Field of Spherical Aberration	$nu'\cdot\vec{\varepsilon} = 4W_{sa}(\vec{\rho}\cdot\vec{\rho})\vec{\rho}$

The following vector identities were used:

$$\vec{\nabla}(\vec{a}\cdot\vec{\rho}) = \vec{a} \tag{36}$$

$$\vec{\nabla}(\vec{\rho}\cdot\vec{\rho}) = 2\vec{\rho} \tag{37}$$

$$\vec{\nabla}(\vec{a}\cdot\vec{\rho}^2) = 2\vec{a}\vec{\rho}^* \tag{38}$$

$$\vec{\nabla}(\vec{a}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) = (\vec{\rho}\cdot\vec{\rho})\vec{a} + 2(\vec{a}\cdot\vec{\rho})\vec{\rho} \tag{39}$$

$$\text{and } \vec{\nabla}(\vec{\rho}\cdot\vec{\rho})^2 = 4(\vec{\rho}\cdot\vec{\rho})\vec{\rho} \tag{40}$$

where $\vec{a}$ is any vector ($\vec{i}, \vec{H}, \vec{i}^{\,2}\vec{H}^*$, etc) that does not depend on $\vec{\rho}$.

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