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Diffraction Effects in Interferometry

Ping Zhou, James H. Burge
College of Optical Science, University of Arizona, 1630 E. University Blvd., Tucson, AZ, USA 85721
pzhou@optics.arizona.edu

Abstract: Besides the geometrical errors, interferometry suffers errors due to diffraction, because the wavefront aberrations of the test and reference beams change as they propagate. This paper addresses errors due to diffraction effects in interferometry.

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1. Introduction

Errors in interferometric measurement are either random or systematic. Random errors can be reduced by averaging many measurements. Systematic errors cannot be averaged out. Errors due to diffraction effects, like the geometric errors, are systematic and cannot be reduced by averaging. They will be left in the measurement if not calibrated. Diffraction effects include phase smoothing and edge diffraction. Phase smoothing means the attenuation of the high spatial frequency components, and edge diffraction refers to the diffraction "ripple" around the edge of the test surface [1].

Errors due to diffraction can be studied using the Talbot imaging theory. Talbot imaging is a diffraction phenomenon that occurs for any wavefront with a periodic structure. If a phase ripple with a period of p is illuminated by collimated light, then that same phase ripple is formed by free space diffraction at integer multiples of the Talbot distance $z_T = 2p^2/\lambda$. As a sinusoidal phase pattern propagates, it will cycle through a reverse contrast amplitude pattern, a conjugate phase pattern, a pure amplitude pattern, then back to the original phase pattern. This paper discusses the phase smoothing using the Talbot effect in Section 2 and the edge diffraction in Section 3.

The Talbot effect decomposes the phase object into sinusoidal ripples. The phase object can also be described using the Zernike decomposition, which is quite common in optical testing. In Section 4, errors due to diffraction are studied using numerical simulation for Zernike polynomials.

2. Diffraction effects: phase smoothing

If an object with small phase ripples of W ($W << 1$) waves is illuminated by a collimated beam, the magnitude of the phase ripples, after propagating a distance of L, follows a cosine function [2]

$$W' = W\cos\left(2\pi\frac{L}{z_T}\right) = W\cos\left(2\pi\frac{L\lambda}{p^2}\right). \quad (1)$$

The attenuation of the phase ripple depends on the propagation distance and the spatial frequency of the object. Smaller p (high spatial frequency) causes more attenuation in magnitude, and we call this phenomenon phase smoothing.

The Talbot distance z_T is evaluated for a collimated illumination. For a spherical illumination, the replication of the periodic object will be amplified and not occur at integer multiples of the Talbot distance. It is convenient to convert the spherical illumination into an equivalent collimated one, and then use Eq. (1) to calculate the phase smoothing for a certain spatial frequency. The diffraction pattern for a spherical beam is the same as that observed for a collimated beam, except that the diffraction pattern occurs at the *effective propagation distance* L_e, and it is scaled in the transverse dimension.

For a converging wavefront starting with radius of curvature R_1, diameter $2a_1$, and ripples with period p_1, propagates to a position where it has radius of curvature R_2. The effective propagation distance is

$$L_e = \frac{R_1(R_1 - R_2)}{R_2}, \quad (2)$$

where R_1 and R_2 should have the sign information. To avoid the scaling issue, the ripple period can be normalized by $2a_1$, thus the normalized frequency $f_{\text{normalized}} = 2a_1/p_1$ [cycles/diameter] remains unchanged as the wavefront propagates. By replacing L and p with L_e and $f_{\text{normalized}}$, Eq. (1) becomes

$$W' = W \cos\left(\frac{\pi f_{normalized}^2}{4N_f}\right), \quad (3)$$

where $N_f = a_1^2 / \lambda L_e$ is the Fresnel number [3].

In interferometry, phase smoothing can be discussed from three aspects: diffraction effects from the test wavefront, the reference wavefront, and the common wavefront.

Diffraction effects from the test wavefront: Errors in the test wavefront are caused by the null optics (if they exist) and the test surface. A simple case is to consider only errors from the test surface itself. Interferometers usually focus the test surface onto the detector to avoid diffraction effects from the test wavefront and correctly measure the surface under test.

Diffraction effects from the reference wavefront: Errors in the reference wavefront are caused by imperfections from the optics in the reference arm. A simple case is to assume that the reference surface figure is the sole source of errors in the reference wavefront. Any errors in the reference wavefront will appear as errors in the test surface unless the interferometer is calibrated. The reference wavefront also suffers from diffraction effects since the reference surface is usually not in focus. However, errors from the reference wavefront, including diffraction effects, can be calibrated with an absolute test.

Diffraction effects in the common wavefront: The common wavefront refers to the wavefront from the illumination optics in an interferometer. Reference and test beams carry the same common wavefront information right before they are split from each other. Figure 1 shows a Fizeau interferometer testing a plane mirror. The common wavefront keeps propagating a distance *L* forward to the test optic, so that the total round-trip propagation distance difference between the test and reference arms is 2*L*. Because the diffraction effect varies with the propagation distance, the propagation distance between the two beams will therefore introduce errors to the final wavefront map.

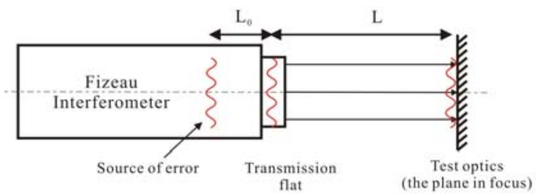

Fig. 1 A plane mirror, a distance *L* away from the transmission flat, is tested with a Fizeau interferometer.

There are a variety of methods to calibrate the reference wavefront errors, such as the three-flat test, the three-position test etc. It is best to perform the calibration after testing the surface of interest without changing the zoom or focus of the interferometer. This is because when the interferometer is used to test an optic, the imaging focus or zoom will be adjusted accordingly so that the interferometer images the test optic on the detector to correctly measure its error.

The reference wavefronts are the same in both the surface measurement and the calibration test since no optics inside the interferometer are changed. Therefore, the diffraction effects on the reference wavefronts can be calibrated out. For the test and common wavefronts, calibration will have residual errors if their propagation distances are different. The errors will depend on the spatial frequency and the effective propagation distance.

3. Diffraction effects: Edge diffraction

Edge diffraction can be seen when the aperture of an interferometer is not in focus. There are many apertures inside interferometers that cannot be in focus since interferometers often image the surface under test on the detector to correctly represent the errors in the test surface. However, edge diffraction from the limiting aperture usually has the most dominant effect in a measurement. The limiting aperture of the interferometer itself is the transmission sphere or flat, which will generally not be in focus. The edge diffraction pattern from the aperture of the transmission optics can be calculated as a spherical wavefront propagating from the aperture to the test optic (plane of focus). The severity of edge diffraction from the transmission optics depends on the Fresnel number, which represents the propagation geometry from the transmission optic to the test optic. The larger the Fresnel number, the denser the diffraction ring pattern and the less edge diffraction effect is observed. Figure **2** illustrates that systems with a smaller Fresnel number have larger RMS phase errors due to edge diffraction.

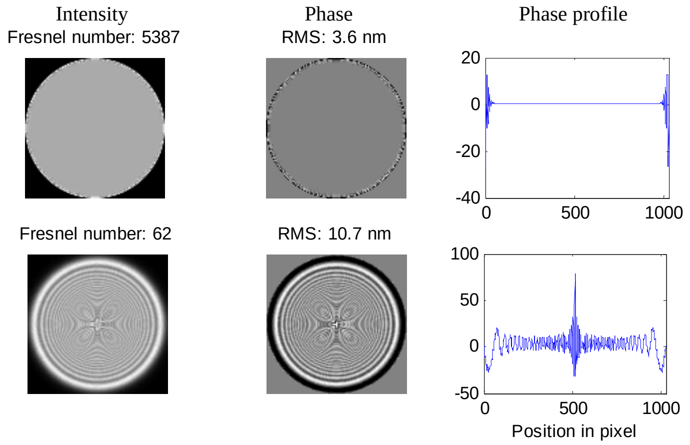

Fig. 2 Intensity pattern (left), corresponding phase pattern (middle) and the line cross-section along the diameter (right) due to edge diffraction at different Fresnel numbers. The wavelength is 632.8 nm.

4. Zernike propagation

Unlike sinusoidal ripples, Zernike polynomials do not have a single spatial frequency. Therefore, it is difficult to use the Talbot effect to estimate the phase smoothing of Zernike polynomials. We simulated the behavior of Zernike polynomials due to wave propagation in a collimated beam with MATLAB.

The simulation includes both diffraction effects: phase smoothing and edge diffraction. As shown in Fig. 3, if the original input of the Zernike polynomial Z_i has a magnitude α_{in}, the output Zernike term will have a magnitude α_{out} after propagating a distance of *L*, and this is the smoothing effect. Because each Zernike polynomial has more than one spatial frequency and each frequency component has a different smoothing effect, there will also be residual errors which could not be fit by the original Zernike term. Figure 4 shows the simulation results for the standard Zernike term 39 with a Fresnel number of 50.

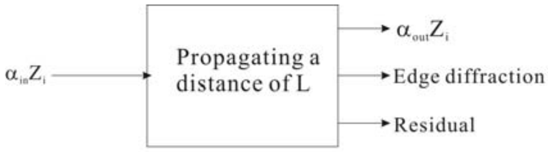

Fig. 3 As a Zernike term propagates a distance *L*, there will be smoothing effect, edge diffraction and some residual errors.

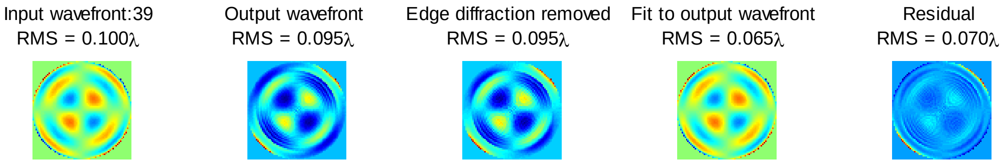

Fig. 4 Computer simulation of wavefront propagation for the standard Zernike term 39 with a Fresnel number of 50. The first column is the input Zernike functions with an error of 0.1λ rms. The second column is the output wavefront after propagating a distance of *L* in a collimated space. The third column shows the output wavefront after removing the edge diffraction effect. The fourth column is a fit of the input Zernike polynomial to the output wavefront. The fifth column shows the difference between the third and the fourth columns

5. Conclusion

Interferometric measurements suffer from errors due to diffraction. Diffraction effects include phase smoothing in the mid/high spatial frequencies and edge diffraction. Errors due to diffraction effects can be calibrated out if the propagation distances are the same for the reference, test and common wavefronts in the surface measurement and the calibration test. Edge diffraction is often observed when the limiting aperture of the interferometer is not in focus. Edge diffraction also affects the accuracy of interferometric measurements.

6. References

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