Measuring Depolarization Properties of Everyday Materials

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1 Introduction

The polarization of light is the preferential direction of its electric field oscillation. Polarization measurements can contain information regarding the geometry, texture, and material of an object not given by conventional irradiance measurements. Scattering by optical components is generally minimized by design, but everyday materials such as fabric or opaque plastics tend to have strongly diffuse scattering properties. Depolarization, which goes hand-in-hand with scattering, is therefore a useful property to consider when studying such materials. For coherent light such as collimated laser light, a 2x1 complex vector called a Jones vector is sufficient to describe the electric field vector. A 2x2 complex matrix called a Jones matrix describes how the electric field vector changes when the light interacts with a medium.

In cases of polychromatic, incoherent, and/or partially polarized light, Jones calculus is insufficient because it does not describe depolarization. Instead, the polarization state of light is described by a 4x1 real Stokes vector and the change of a Stokes vector upon interaction with a medium is described by a 4x4 real Mueller matrix. The resulting Stokes vector after an interaction described by a Mueller matrix is given by the matrix-vector product with the initial Stokes vector

$$\begin{pmatrix} S_0' \\ S_1' \\ S_2' \\ S_3' \end{pmatrix} = \begin{pmatrix} M_{00} & M_{01} & M_{02} & M_{03} \\ M_{10} & M_{11} & M_{12} & M_{13} \\ M_{20} & M_{21} & M_{22} & M_{23} \\ M_{30} & M_{31} & M_{32} & M_{33} \end{pmatrix} \begin{pmatrix} S_0 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix}. \tag{1}$$

The first value in a Stokes vector, S_0, is the total flux. S_1 is the difference in horizontally and vertically polarized flux, S_2 is the difference in 45° and 135° polarized flux, and S_3 is the difference in right and left circularly polarized flux.

The degree to which a Stokes vector describes fully polarized, partially polarized, or unpolarized light is its degree of polarization given by the equation

$$DoP = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0}. \tag{2}$$

The DoP ranges from 0 to 1, where 0 corresponds to unpolarized light and 1 is fully polarized light. Billings offers a helpful definition[1]: "Unpolarized light is light in which the state of polarization changes more rapidly than can be followed by the particular detector which is being used." The set of physically realizable Stokes vectors is the Poincaré sphere where circular states exist at the poles, linear states exist along the equator, fully polarized states exist on the surface of the sphere, and partially polarized states exist within the sphere. The Poincaré sphere is analogous to the Bloch sphere used in quantum mechanics to describe a two-state system. The coherent basis states exist on the poles and the surface of the Bloch sphere represent the possible superpositions of those two basis states with varying weights and phase. The space within the surface of the Bloch sphere contains the mixed quantum states which require statistical interpretation, just the same as partially polarized light. In quantum computing, the decoherence which results in mixed states is undesirable because information is lost. However, when considering polarization, the interest is in probing the depolarizing properties of a material rather than preserving the intitial polarization states.

The polarization properties of a Mueller matrix are retardance, diattenuation, polarizance, and depolarization. Retardance is the difference in phase between two orthogonal polarization states, and can cause a rotation in the polarization orientation and ellipticity without affecting the flux. In the Mueller matrix, retardance appears in the lower-right 3x3 submatrix. Diattenuation is the preferential transmission of one polarization state versus the orthogonal state. It appears in the top row of the Mueller matrix which is called the diattenuation or analyzer vector. The dot product a of Stokes vector with the top row of a Mueller matrix gives the S_0 or irradiance component, and the analyzer vector is the polarization state which has highest throughput. The first column of a Mueller matrix is called the polarizance vector, which indicates the Stokes vector that would be produced for unpolarized input. Depolarization causes a reduction in the degree of polarization of a Stokes vector. It does not have a simple appearance in the Mueller matrix. The ideal depolarizer which completely depolarizes all input states has the form $\mathrm{diag}[(1, 0, 0, 0)]$.

2 Depolarization Physics

2.1 Time Averaging

There are three mechanisms by which a light-matter interaction can cause light to be depolarized: incoherent addition over time, spectrum, or space. The original definition of unpolarized light applied to light where the polarization state changes randomly in time. The instantaneous electric field may have a well defined orientation, but this manifests as unpolarized light when integrated over a finite time interval at a detector. Depolarization due to time averaging can occur for narrowband, small-area beams. In a gas with fast-moving particles this may be relevant, but this case is not of as much interest when considering everyday materials because they typically do not change with time.

2.2 Spectral Averaging

The second situation in which depolarization can occur is when the polarization state varies with respect to wavelength. For this reason, narrowband filters are typically used in light measuring polarimeters and monochromators are used in sample measuring polarimeters. It can also lead to a need for achromatic retarder designs for broadband applications. Depolarization due to integration over spectrum can occur for short integration times and small-area beams. For example, when an interaction is dominated by Fresnel reflection, dispersion in the refractive index over a large enough spectrum can cause depolarization. The molecules which give paint pigments their color experience different scattering regimes depending on the wavelength can also lead to depolarization when integrating over a large spectrum.

2.3 Spatial Averaging

The final case, and the primary case of interest for everyday materials, is when the polarization state varies rapidly over some spatial dimension, typically an area or solid angle. At a given spatial location the interaction may leave the light fully polarized, but when integrated over a detector element's field of view, the measured light is depolarized. This is the mechanism by which measurements of scattered light are depolarized. Light from first-surface scattering off of rough surfaces will undergo different geometric polarization changes, different diattenuation magnitudes and orientations, and have different optical path lengths. Each of these contributes to spatially varying polarization states which average to a depolarizing effect. The more optically rough a surface is, the more it will depolarize. Light penetrates into the material and excites motion in the material's electrons, causing the electrons to re-radiate. Everyday materials typically have irregular arrangements of atoms, so this re-emission process also causes spatially varying polarization. Depolarization by spatial averaging can occur for short integration times and narrowband beams.

3 Measuring Depolarization

There are two broad classes of polarimeters: light measuring polarimeters and sample measuring polarimeters. As the name suggests, a light measuring polarimeter can only determine the specific polarization properties of light reaching the acquisition optics. There is no control of illumination, so there is less characterization of the scene. In order to fully determine the depolarizing properties of a material, a sample measuring polarimeter is necessary. The specific example discussed here is that of a rotating-retarder Mueller matrix imaging polarimeter (RRMMIP) [2].

3.1 RRMMIP Optics

Sample measuring polarimeters consist of a polarization state generator (PSG) and a polarization state analyzer (PSA). For a RRMMIP, the PSG is made of a light source, linear polarizer, and rotating retarder in that order. One option for the linear polarizer would be a dichroic film. The dichroic film has long molecules that are oriented parallel to each other. Electrons excited by light polarized orthogonal to the the molecules re-radiates, but electrons excited along the orientation of the molecules lose their energy to heat and do not re-radiate. Since only one polarization state is able to transmit, the beam has become linearly polarized.

The retarder consists of a birefringent material, meaning it has different indices of refraction for orthogonal polarization states. Due to anisotropy in the atomic structure, the electric susceptibility in one direction is different than in the orthogonal direction which results in relative phase delays in the electrons stimulated by the electric field. This causes the speed of light to differ for orthogonal polarization states, transforming the superposition polarization state. By rotating the retarder, the linear polarization is converted to a known sequence of other states based on the orientation of the fast-axis. The optimal retardance has been shown to be approximately 1/3 of a wave[3].

The PSA is made of, in order, a camera lens, rotating retarder, linear polarizer, and the camera. The optimal retarder for the PSA also has approximately 1/3 of a wave of retardance and the same achromatic requirements as described above. For an evenly spaced angular sequence of the PSG retarder, the optimal PSA retarder rotation is 4.91 times the rate of the PSG retarder[4]. The linear polarizer has a fixed orientation, and the combination of the rotating retarder and polarizer have a known sequence of polarization states for which transmission is highest.

3.2 Design Considerations

The light source should be narrowband so the beam is not spectrally depolarized. For measuring large samples, the beam should also be large and of uniform spatial and temporal irradiance. Uniform spatial irradiance is important because darker regions in illumination will correspond to different regions on the detector when measuring a sample versus calibration. Temporal uniformity is important because fluctuations in the source irradiance will erroneously appear as modulation due to the polarization properties of the sample. The source also must be bright because the initial linear polarizer automatically removes half of the light.

If the light source has multiple operating wavelengths, then the retarder must be achromatized to have 1/3 of a wave of retardance at those operating wavelengths. Additionally, achromatizing the retarder can reduce the effects of spectral depolarization if the source bandwidth is not small. Misalignment and wedge between the faces of the retarder also degrades performance because beam-walk due to refraction will cause artifacts in the polarimetric reconstruction. A wedged retarder will also have a spatially varying retardance, but this effect is most likely small compared to the issue of beam-walk.

For non-flat or non-normal-incidence measurement configurations, the stop size must be balanced between collecting enough light at darker PSG/PSA positions while not blurring the scene, as blurring can appear as spatial depolarization if the polarization features in the scene are unfocused. The detector must have a high dynamic range and large bit-depth because irradiance can vary dramatically between the images in the sequence for samples with strong polarization signals. Any systemic polarization effects such as diattenuation on lens surfaces are typically corrected in calibration.

4 Polarimetric Data Reduction

The Mueller matrix of a sample is determined by performing a series of intermediate measurements with different PSG/PSA states. The equation for a single flux measurement (the S_0 component of the Stokes vector at the detector) by a polarimeter is

$$p_n = \mathbf{a}_n^\dagger \mathbf{M} \mathbf{g}_n, \tag{3}$$

where $\mathbf{g}_n$ is the polarizance vector of the PSG and $\mathbf{a}_n$ is the analyzer vector of the PSA. This equation can be rewritten as

$$p_n = \mathbf{w}_n \mathbf{m} \tag{4}$$

where $\mathbf{w}_n = \mathbf{a}_n \otimes \mathbf{g}_n^\dagger$ and $\mathbf{m}$ is 16x1 column vector of the elements of Mueller matrix $\mathbf{M}$. To fully reconstruct the Mueller matrix, there must be at least 16 generator/analyzer pairs although more are typically used to form an overdetermined system

$$\mathbf{P} = \mathbf{W}\mathbf{m}. \tag{5}$$

where the rows of $\mathbf{W}$ are the vectors $\mathbf{w}_n$. The approximate Mueller matrix $\widetilde{\mathbf{m}}$ is reconstructed from the vector of flux measurements by

$$\widetilde{\mathbf{m}} = \mathbf{W}^+ \mathbf{P}. \tag{6}$$

where $\mathbf{W}^+$ is the pseudoinverse of $\mathbf{W}$. The pseudoinverse is chosen as the reconstruction algorithm because it provides the minimum-norm least squares solution to Eq. 5. This matrix-vector product is performed pixel-wise to form a Mueller matrix image. The condition number, defined as the ratio of the largest to the smallest singular value, of $\mathbf{W}$ is used as a metric for the performance of a polarimeter and is the figure of merit used to determine the optimal retarder specifications described in the previous section [4].

5 Analyzing Depolarization

Mueller matrices have 16 degrees of freedom: 1 for throughput, 3 for diattenuation, 3 for retardance, and 9 for depolarization[5]. 9 values are therefore required to fully describe depolarization, although for some applications or certain classes of Mueller matrices, fewer values may be sufficient. There are three groups of parameterizations for depolarization described in this paper: single-parameter metrics, higher-dimensional metrics, and Mueller matrix decompositions. The single-parameter metrics summarize the magnitude of depolarization, but there is no unified definition for this magnitude. The higher-dimensional metrics give some insight into the structure of depolarization, but do not specify that structure. The single- and higher-dimensional metrics do not uniquely parameterize a depolarizing Mueller matrix, i.e. two different depolarizing Mueller matrices may appear identical when only considering these metrics. The third group, Mueller matrix decompositions, are used to uniquely describe depolarizing Mueller matrices. While all of the decompositions presented can be used to describe a given Mueller matrix, some offer more physical insight as will be discussed below.

5.1 Single-Parameter Metrics

5.1.1 Depolarization Index

One of the popular single-parameter metrics for depolarization is the depolarization index. It was introduced by Gil and Bernabeu in 1985, making it the earliest depolarization metric. The depolarization index is the Euclidean distance from a normalized Mueller matrix to the ideal depolarizer as given by the equation[6, 7]

$$DI(\mathbf{M}) = \left|\left| \frac{\mathbf{M}}{M_{00}} - \mathbf{ID} \right|\right|. \tag{7}$$

This quantity ranges from 0 to 1, with 0 being a fully depolarizing Mueller matrix and 1 being a non-depolarizing Mueller matrix.

5.1.2 Average Degree of Polarization

Another single-parameter metric is the average DoP. In general, the DoP of exiting light will depend on both the orientation and DoP of the incoming light. The average DoP is therefore defined as the arithmetic mean of the DoP of exiting states for input states averaged over the Poincaré sphere[7]

$$ADoP(\mathbf{M}) = \frac{\int_0^{\pi}\int_{-\pi/2}^{\pi/2} DoP(\mathbf{M}\cdot\mathbf{S}(\theta,\eta))\cos(\eta)\,d\eta\,d\theta}{4\pi}. \tag{8}$$

Similarly to the depolarization index, the average DoP ranges from 0 to 1, where 0 corresponds to complete depolarization for every input state and 1 corresponds to completely output polarized for every input state.

5.2 Higher-Dimensional Metrics

5.2.1 Coherency Eigenspectrum

Both of the higher-dimensional metrics presented here are related to the coherency eigenspectrum which is an indication of how fundamental the eigenspectrum is to depolarization structure. Once multiple values are used to describe depolarizing Mueller matrices, it becomes possible to describe spaces occupied by those Mueller matrices.

The first and simplest metric is the coherency eigenspectrum itself, called the natural depolarization space by Ossikovski [8]. The coherency matrix is linearly related to the Mueller matrix by

$$\mathbf{C} = \frac{1}{2}\sum_{i,j=0}^{3} M_{ij}\mathbf{U}\left[\boldsymbol{\sigma}_i \otimes \boldsymbol{\sigma}_j^*\right]\mathbf{U}^\dagger \tag{9}$$

where M_{ij} are the elements of the Mueller matrix, $\boldsymbol{\sigma}_i$ and $\boldsymbol{\sigma}_j$ are the Pauli spin matrices, and the $\mathbf{U}$ matrix is given by

$$\mathbf{U} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{pmatrix}. \tag{10}$$

When the eigenvalues of $\mathbf{C}$ are normalized to sum to unity and are indexed in descending order, they can be written $0 \le \xi_3 \le \xi_2 \le \xi_1 \le 1 - \xi_3 - \xi_2 - \xi_1$. In the space defined by (ξ_3, ξ_2, ξ_1), this inequality bounds an irregular tetrahedron which contains all physically realizable Mueller matrices. Non-depolarizing Mueller matrices exist at the point $(0,0,0)$ and the ideal depolarizer exists at $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$.

5.2.2 Indices of Polarimetric Purity

Indices of polarimetric purity (IPP) are linearly related to the coherency eigenspectrum[9]

$$P_n = \sum_{k=1}^{n} k\Delta\xi_k \tag{11}$$

where $\Delta k = \xi_k - \xi_{k+1}$ and $n = 1, 2, 3$. In the space defined by (P_3, P_2, P_1), $(0,0,0)$ is the ideal depolarizer and $(1,1,1)$ is a non-depolarizing Mueller matrix. In IPP space, the tetrahedron of physically realizable Mueller matrices is larger than in the space defined by the eigenspectrum which increases the separation between two points representing nearby Mueller matrices. This is potentially useful for comparing experimentally measured Mueller matrices.

5.3 Decompositions

5.3.1 Lu-Chipman

The Lu-Chipman decomposition is an order-dependent serial decomposition of a depolarizing Mueller matrix into a sequence of a pure diattenuator, retarder, and depolarizer[10]

$$\mathbf{M} = m_{00}\begin{pmatrix} 1 & \mathbf{D}^T \\ \mathbf{P} & \mathbf{m} \end{pmatrix} = \mathbf{M}_\Delta \mathbf{M}_R \mathbf{M}_D. \tag{12}$$

The choice of order is arbitrary, but this one is chosen so that non-depolarizing components occur before the depolarizing one. The diattenuation is separated out by right multiplication with the matrix inverse and the remaining Mueller matrix is written as a product of a depolarizing and a retarding Mueller matrix

$$\mathbf{M}\mathbf{M}_D^{-1} = \mathbf{M}_\Delta \mathbf{M}_R = \begin{pmatrix} 1 & \mathbf{0}^T \\ \mathbf{P}_\Delta & \mathbf{m}_\Delta \end{pmatrix}\begin{pmatrix} 1 & \mathbf{0}^T \\ \mathbf{0} & \mathbf{m}_R \end{pmatrix}, \tag{13}$$

where the polarizance vector $\mathbf{P}_\Delta$ of $\mathbf{M}_\Delta$ is calculated using the polarizance and diattenuation vectors of $\mathbf{M}$. $\mathbf{m}_\Delta$ is determined by the conventional polar decomposition of $\mathbf{m}' = \mathbf{m}_\Delta \mathbf{m}_R$. Lastly, $\mathbf{M}_R$ is calculated by

$$\mathbf{M}_R = \mathbf{M}_\Delta^{-1}\mathbf{M}\mathbf{M}_D^{-1}. \tag{14}$$

When $\mathbf{m}'$ is singular, an alternate algorithm involving the singular value decomposition is employed. In this case, there are an infinite number of retarder matrix solutions, so the one with the minimum retardance magnitude is chosen.

Because of the inherent order-dependence, Lu-Chipman decomposition does not necessarily give insight into physical processes behind depolarization. Unless the specific order of diattenuator-retarder-depolarizer corresponds to the optical system in question, this decomposition will describe a different system which has the same net polarization properties. Despite this, the Lu-Chipman remains a popular way to analyze Mueller matrices.

5.3.2 Matrix Roots

Mueller matrix root decomposition is another serial decomposition, but it is order-independent[11]. It is used to describe Mueller matrices which satisfy $\lim_{p\to\infty} \sqrt[p]{\mathbf{M}} = \mathbf{I}$, termed uniform Mueller matrices. The Mueller matrix $\mathbf{M}$ is divided into p identical, infinitesimal slices. Because the slices approach the identity matrix by definition, $\sqrt[p]{\mathbf{M}}$ can be parameterized in terms of Mueller matrix generators and parameters, and transmittance

$$\sqrt[p]{\mathbf{M}} = e^{-d_0} \left(\prod_{i=0}^{15} \mathbf{G}_i(d_i) \right) \tag{15}$$

so that the uniform Mueller matrix $\mathbf{M}$ itself can be recovered by taking the p^{th} matrix power. The d_i terms are extremely small in magnitude, so they are presented as D_i where $D_i = pd_i$.

The matrix root approach is appealing because the parameters of the decomposition have physical interpretations and correspond to the 16 degrees of freedom in a Mueller matrix. In addition to D_0 which corresponds to overall transmittance, the first 6 parameters refer to non-depolarizing polarization properties.

The depolarization degrees of freedom lend specific insight into the structure and sources of depolarization. The amplitude depolarization parameters D_7, D_8, and D_9 are named as such because they both depolarize and affect the flux of an incident Stokes vector. They share off-diagonal elements with diattenuator matrices. The phase depolarization parameters D_{10}, D_{11}, and D_{12} depolarize but do not affect the flux and share off-diagonal elements with retarder matrices. Each of the diagonal depolarization parameters has a unique interpretation. D_{13} is the relative strength between diagonal depolarization on linear axes (horizontal/vertical vs 45/135). D_{14} is the relative strength between linear and circular depolarization. D_{15} is the isotropic depolarization power. In order to be a physical Mueller matrix, all depolarizing matrices must contain some D_{15} component.

5.3.3 Cloude Spectral Decomposition

In contrast with Lu-Chipman and Mueller matrix roots approaches, Cloude spectral decomposition is a parallel decomposition which parameterizes a depolarizing Mueller matrix as a weighted sum of four non-depolarizing Mueller matrices weighted by the coherency eigenspectrum[12]

$$\mathbf{M} = \xi_0 \widehat{\mathbf{M}}_0 + \xi_1 \widehat{\mathbf{M}}_1 + \xi_2 \widehat{\mathbf{M}}_2 + \xi_3 \widehat{\mathbf{M}}_3 \tag{16}$$

where the hat indicates a Mueller matrix with a rank 1 coherency matrix and ξ_n are the eigenvalues.

The same eigendecomposition as in Eq. 9 is performed here, but the eigenvectors are now also considered. The components of the eigenvectors have the interpretation of coefficients in the Pauli spin expansion of a Jones matrix so the eigenvectors are also referred to as the Pauli expansion vectors. The Mueller matrices in the sum can then be calculated by the equation

$$\widehat{\mathbf{M}}_n = \mathbf{U} \left(\mathbf{J}_n \otimes \mathbf{J}_n^* \right) \mathbf{U}^\dagger, \tag{17}$$

where $\widehat{\mathbf{M}}_n$ is the n^{th} non-depolarizing Mueller matrix, $\mathbf{J}_n$ is the equivalent Jones matrix, and $\mathbf{U}$ is found in Eq. 10. Since the coherency matrix is Hermitian, its eigenvectors are orthogonal. The Jones or Mueller matrices in the sum therefore have a sense of orthogonality as well.

Any depolarizing Mueller matrix can be parameterized in terms of the dominant Jones matrix, the coherency eigenvalues, and the six depolarization angles. Matrices 1-3 can be parameterized in terms of matrix 0. Matrix 0 is chosen as the "parent" matrix because it is the largest contributing coherent process and therefore most likely to be known a priori. Using the Pauli expansion vector for matrix 0, an arbitrary orthonormal reference basis $\{\mathbf{e}_0, \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ in $\mathbb{C}^4$ space is formed.

This orthonormal basis set forms a reference unitary matrix $\mathbf{U}_{4R}^{\dagger} = \begin{pmatrix} \mathbf{e}_0 & \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 \end{pmatrix}$. The orthonormal basis undergoes a unitary transformation which preserves $\mathbf{e}_0$ to transform the reference unitary matrix to the unitary matrix $\mathbf{U}_4$ whose columns are the eigenvalues of the depolarizing Mueller matrix of interest

$$\mathbf{U}_4 = \mathbf{U}_{4R}^{\dagger} \begin{pmatrix} 1 & \mathbf{0}^T \\ \mathbf{0} & \mathbf{U}_3(\phi_i, \zeta_i) \end{pmatrix} \tag{18}$$

where $\mathbf{U}_3(\phi_i, \zeta_i)$ is a 3x3 submatrix given by

$$\begin{pmatrix} \cos(\phi_1) & 0 & -\sin(\phi_1)e^{-i\zeta_1} \\ 0 & 1 & 0 \\ \sin(\phi_1)e^{i\zeta_1} & 0 & \cos(\phi_1) \end{pmatrix} \begin{pmatrix} \cos(\phi_2) & -\sin(\phi_2)e^{-i\zeta_2} & 0 \\ \sin(\phi_2)e^{i\zeta_2} & \cos(\phi_2) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_3) & -\sin(\phi_3)e^{-i\zeta_3} \\ 0 & \sin(\phi_3)e^{i\zeta_3} & \cos(\phi_3) \end{pmatrix}.$$

The final step is to solve for the 3 depolarization angles ϕ_i which are bounded on the interval $[0, \frac{\pi}{2}]$ and 3 angles ζ_i which are bounded on the interval $[-\pi, \pi]$.

6 Applications of Depolarization Measurement

Measurement and analysis of depolarization are important for understanding light-matter interactions in scattering applications. Depolarization is typically characterized by reconstructing the full Mueller matrix from several polarimetric measurements performed by sample-measuring polarimeters, such as the rotating retarder Mueller imaging polarimeter. From the Mueller matrix, any of the several parameters describing depolarization can be calculated depending on the application.

Characterization of the depolarizing properties of materials is important in a wide range of fields. Depolarization due to scattering is used in metrology to identify defects that are transparent to an imaging system[5]. Biological tissue segmentation can also be improved when considering depolarization [13].

For everyday materials, acquiring the polarized bi-directional scattering distribution function (pBRDF) via measurement is of interest to the Polarization Lab, and interpreting a pBRDF in terms of one or more depolarization metrics is a useful available tool. For example, a common assumption in computer vision polarized light scattering models is that interactions are a combination of Fresnel reflection off of a microfacet and some depolarizing term–an ideal or partial depolarizer depending on the model. Li (2021) also showed that a useful approximation for depolarizing Mueller matrices exists when considering the coherency eigenspectrum [14]. Using the different metrics available to compare measurement to models improves accuracy and understanding of the polarization processes involved.

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