OPTI510R: Photonics

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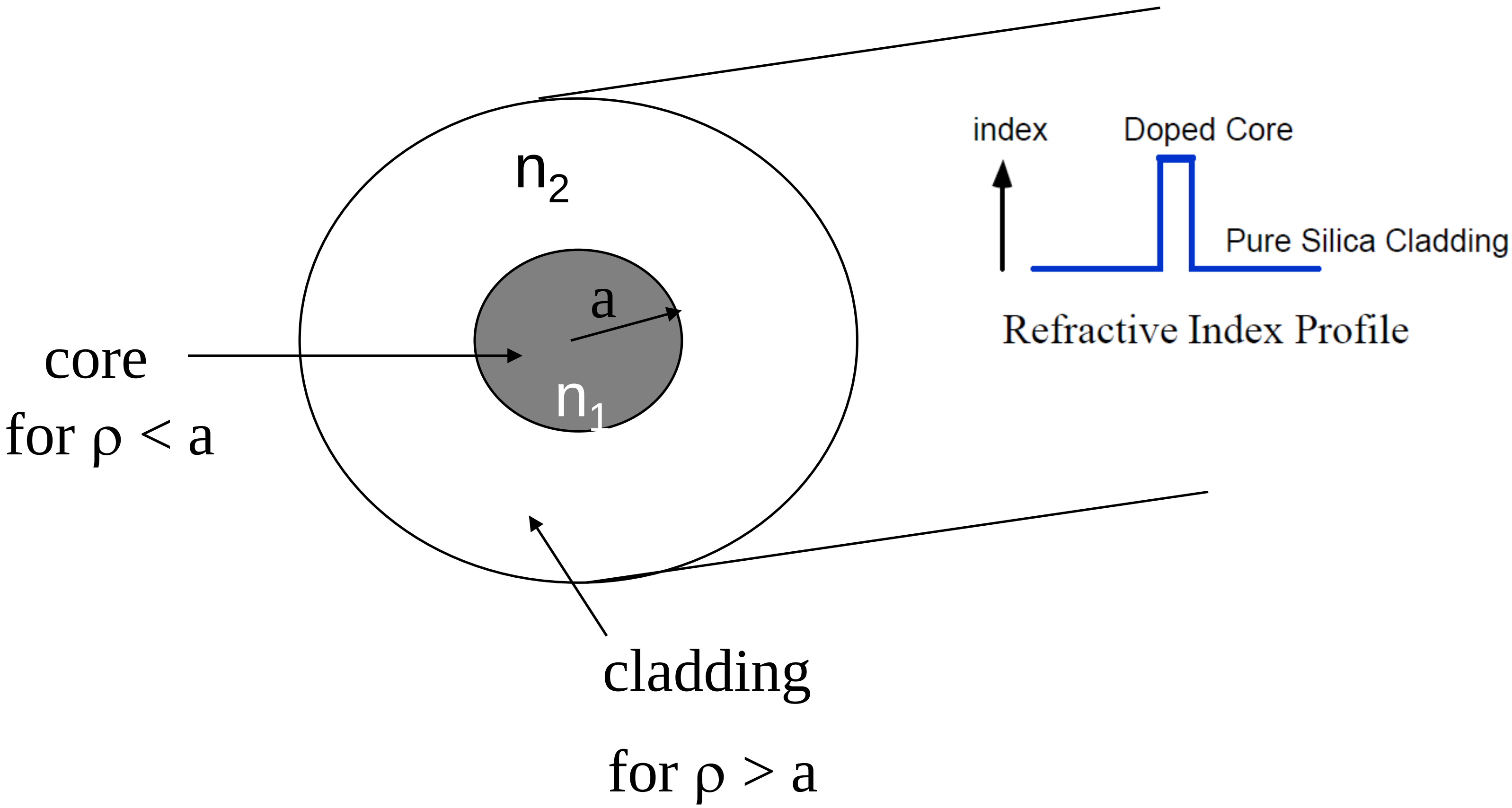
Announcements

- Homework #4 is assigned, due March 25th

Fiber dispersion and compensation

- ➢ Review of fiber modes
- ➢ Sources of dispersion
  - » Modal dispersion
  - » Material dispersion
  - » Waveguide dispersion
  - » Polarization dispersion
  - » Nonlinear dispersion
- ➢ Dispersion compensation techniques
  - » Dispersion engineered fibers
  - » Pre-chirp technique
  - » Dispersion compensating fibers
  - » Chirped fiber Bragg grating
  - » Tunable dispersion compensation

Guided-wave analysis



Guided-wave analysis

- Maxwell's equations in the Fourier domain lead to

$$\nabla^2 \tilde{\mathbf{E}} + n^2(\omega) k_0^2 \tilde{\mathbf{E}} = 0.$$

  $n = n_1$ inside the core but changes to n_2 in the cladding.

- Useful to work in cylindrical coordinates ρ, ϕ, z.

  Common to choose E_z and H_z as independent components.

- Equation for E_z in cylindrical coordinates:

$$\frac{\partial^2 E_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial E_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 E_z}{\partial \phi^2} + \frac{\partial^2 E_z}{\partial z^2} + n^2 k_0^2 E_z = 0.$$

- H_z satisfies the same equation.

(credit: G. Agrawal)

Guided-wave analysis

- Use the method of separation of variables:

$$E_z(\rho,\phi,z) = F(\rho)\Phi(\phi)Z(z).$$

- We then obtain three ODEs:

$$d^2Z/dz^2 + \beta^2 Z = 0,$$

$$d^2\Phi/d\phi^2 + m^2\Phi = 0,$$

$$\frac{d^2F}{d\rho^2} + \frac{1}{\rho}\frac{dF}{d\rho} + \left(n^2k_0^2 - \beta^2 - \frac{m^2}{\rho^2}\right)F = 0.$$

- β and m are two constants (m must be an integer).

  First two equations can be solved easily to obtain

$$Z(z) = \exp(i\beta z), \qquad \Phi(\phi) = \exp(im\phi).$$

- $F(\rho)$ satisfies the Bessel equation.

(credit: G. Agrawal)

Guided-wave analysis

- General solution for E_z and H_z:

$$E_z = \begin{cases} AJ_m(p\rho)\exp(im\phi)\exp(i\beta z); & \rho \le a, \\ CK_m(q\rho)\exp(im\phi)\exp(i\beta z); & \rho > a. \end{cases}$$

$$H_z = \begin{cases} BJ_m(p\rho)\exp(im\phi)\exp(i\beta z); & \rho \le a, \\ DK_m(q\rho)\exp(im\phi)\exp(i\beta z); & \rho > a. \end{cases}$$

$$p^2 = n_1^2 k_0^2 - \beta^2, \quad q^2 = \beta^2 - n_2^2 k_0^2.$$

- Other components can be written in terms of E_z and H_z:

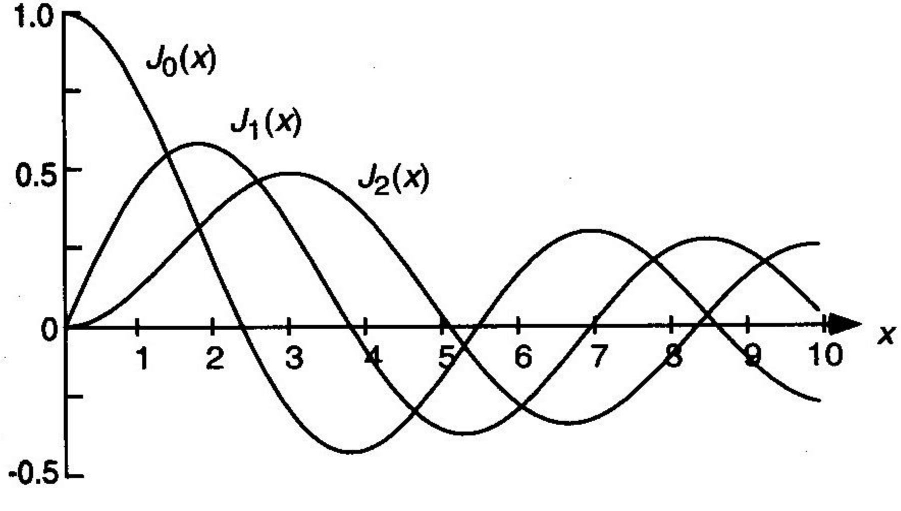
$$E_\rho = \frac{i}{p^2}\left(\beta\frac{\partial E_z}{\partial \rho} + \mu_0\frac{\omega}{\rho}\frac{\partial H_z}{\partial \phi}\right), \qquad E_\phi = \frac{i}{p^2}\left(\frac{\beta}{\rho}\frac{\partial E_z}{\partial \phi} - \mu_0\omega\frac{\partial H_z}{\partial \rho}\right),$$

$$H_\rho = \frac{i}{p^2}\left(\beta\frac{\partial H_z}{\partial \rho} - \varepsilon_0 n^2\frac{\omega}{\rho}\frac{\partial E_z}{\partial \phi}\right), \qquad H_\phi = \frac{i}{p^2}\left(\frac{\beta}{\rho}\frac{\partial H_z}{\partial \phi} + \varepsilon_0 n^2\omega\frac{\partial E_z}{\partial \rho}\right).$$

(credit: G. Agrawal)

Bessel function basics

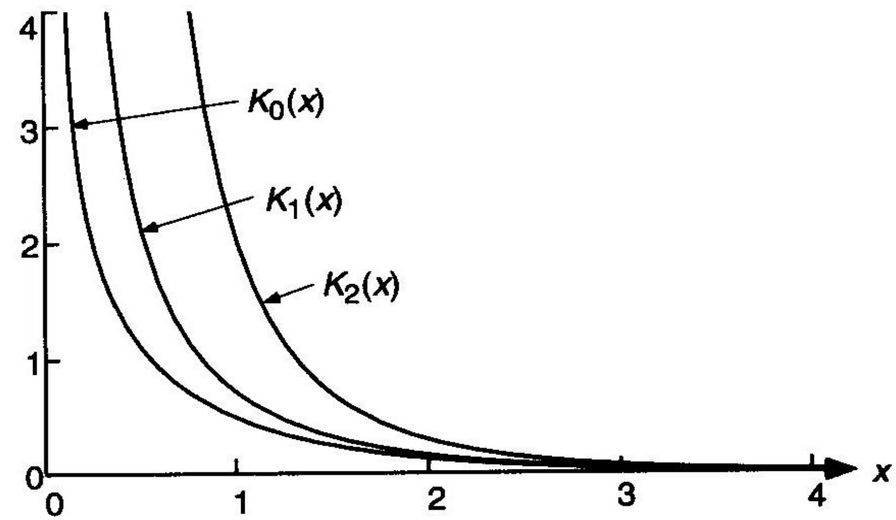
Bessel functions of the first kind



$$u(r) \propto J_l(k_T r)$$

(core)

Modified Bessel functions of the second kind



$$u(r) = K_l(\gamma r)$$

(cladding)

Eigen-value equation

- Boundary conditions: E_z, H_z, E_ϕ, and H_ϕ should be continuous across the *core–cladding interface*.

- Continuity of E_z and H_z at $\rho = a$ leads to
  $AJ_m(pa) = CK_m(qa), \quad BJ_m(pa) = DK_m(qa).$

  Continuity of E_ϕ and H_ϕ provides two more equations.

- Four equations lead to the eigenvalue equation

$$\left[\frac{J_m'(pa)}{pJ_m(pa)} + \frac{K_m'(qa)}{qK_m(qa)}\right]\left[\frac{J_m'(pa)}{pJ_m(pa)} + \frac{n_2^2}{n_1^2}\frac{K_m'(qa)}{qK_m(qa)}\right]$$

$$= \frac{m^2}{a^2}\left(\frac{1}{p^2}+\frac{1}{q^2}\right)\left(\frac{1}{p^2}+\frac{n_2^2}{n_1^2}\frac{1}{q^2}\right)$$

$$p^2 = n_1^2k_0^2 - \beta^2, \quad q^2 = \beta^2 - n_2^2k_0^2.$$

(credit: G. Agrawal)

Eigen-value equation

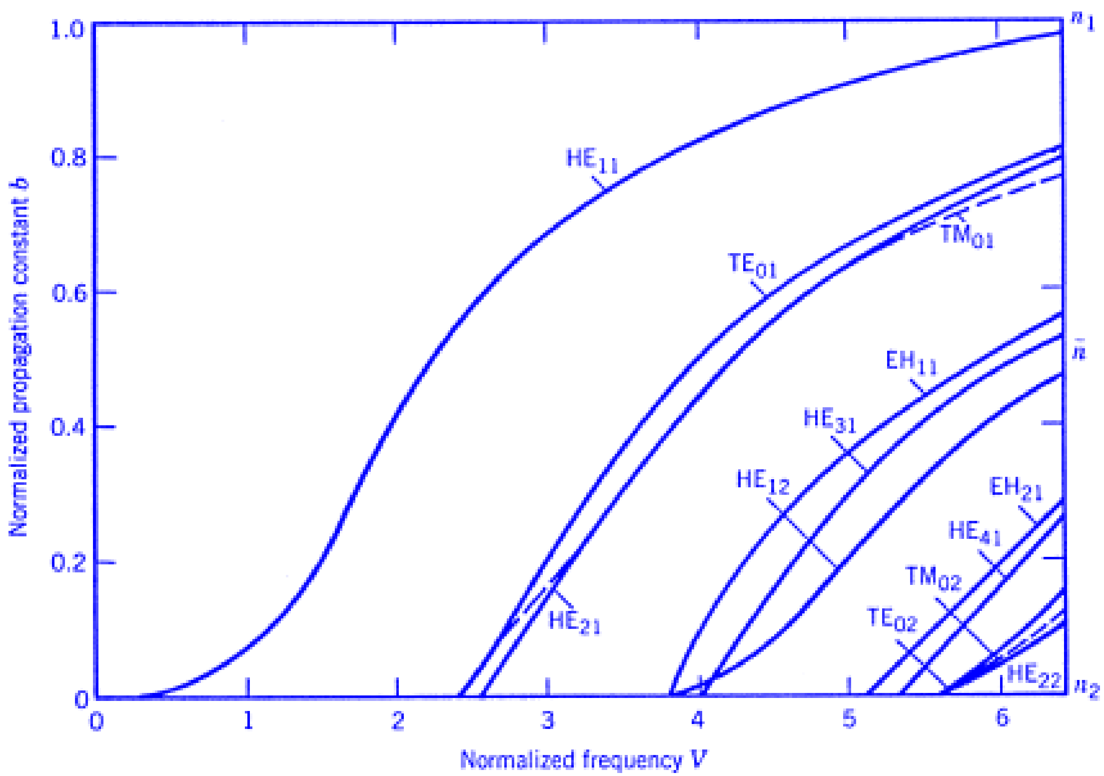
- Eigenvalue equation involves Bessel functions and their derivatives. It needs to be solved numerically.
- Noting that $p^2 + q^2 = (n_1^2 - n_2^2)k_0^2$, we introduce the dimensionless V parameter as

$$V = k_0 a \sqrt{n_1^2 - n_2^2}.$$

- Multiple solutions for β for a given value of V.
- Each solution represents an optical mode.
- Number of modes increases rapidly with V parameter.
- Effective mode index $\bar{n} = \beta / k_0$ lies between n_1 and n_2 for all bound modes.

(credit: G. Agrawal)

Eigen-value equation



- Useful to introduce a normalized quantity

$$b = (\bar{n} - n_2)/(n_1 - n_2), \quad (0 < b < 1)$$

- Modes quantified through $\beta(\omega)$ or $b(V)$.

(credit: G. Agrawal)

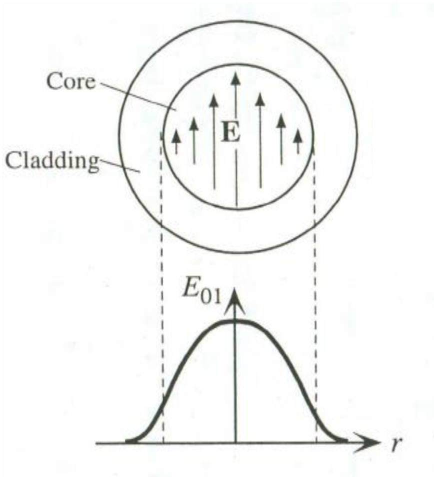
Fundamental modes

- Single-mode fibers require $V < 2.405$ (first zero of J_0).

  They transport light through the fundamental HE_{11} mode.

- This mode is almost linearly polarized $(|E_z|^2 \ll |E_x|^2)$

$$E_x(\rho,\phi,z) = \begin{cases} A[J_0(p\rho)/J_0(pa)]e^{i\beta z}\,; & \rho \le a, \\ A[K_0(q\rho)/K_0(qa)]e^{i\beta z}; & \rho > a. \end{cases}$$



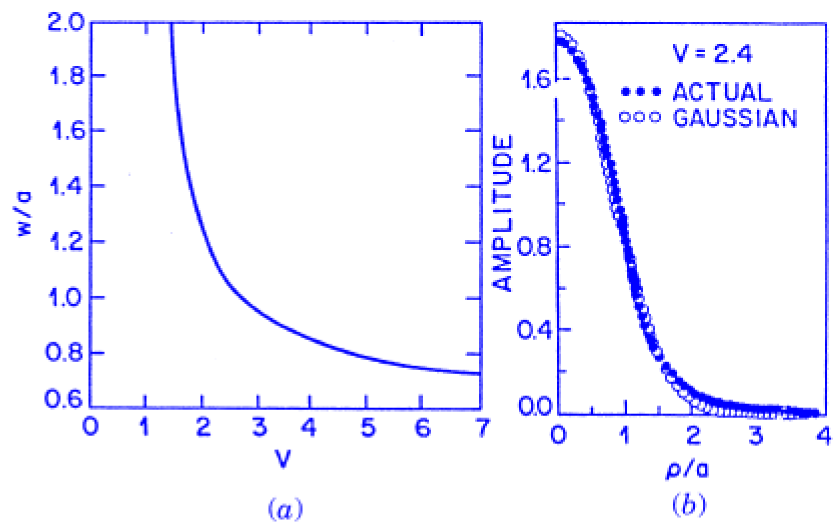
Optical Fiber Communications, Gerd Keiser, third edition, McGraw Hill, 2000.

Fundamental modes

- Use of Bessel functions is not always practical.
- It is possible to approximate spatial distribution of HE_{11} mode with a Gaussian for V in the range 1 to 2.5.

  $$E_x(\rho,\phi,z) \approx A\exp(-\rho^2/w^2)e^{i\beta z}.$$

- Spot size w depends on V parameter.



Fundamental modes

- Spot size: $w/a \approx 0.65 + 1.619V^{-3/2} + 2.879V^{-6}$.
- Mode index:

$$\bar{n} = n_2 + b(n_1 - n_2) \approx n_2(1 + b\Delta),$$

$$b(V) \approx (1.1428 - 0.9960/V)^2.$$

- Confinement factor:

$$\Gamma = \frac{P_{\text{core}}}{P_{\text{total}}} = \frac{\int_0^a |E_x|^2 \rho \, d\rho}{\int_0^\infty |E_x|^2 \rho \, d\rho} = 1 - \exp\left(-\frac{2a^2}{w^2}\right).$$

$\Gamma \approx 0.8$ for $V = 2$ but drops to 0.2 for $V = 1$.

- Mode properties completely specified if V parameter is known

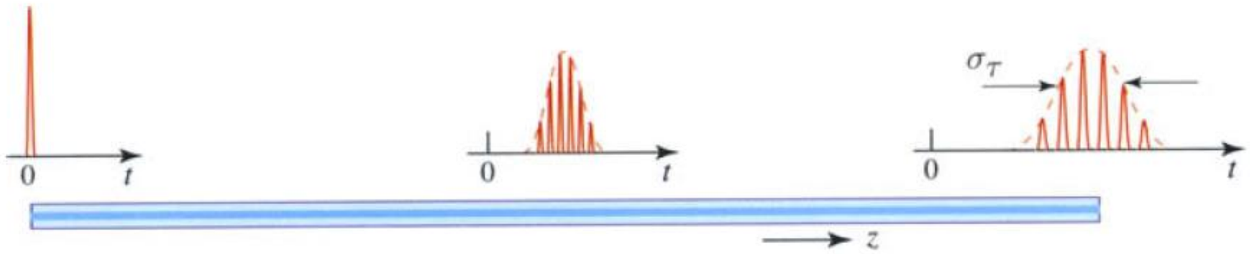
(credit: G. Agrawal)

Sources of dispersion in optical fiber

- Modal dispersion
  - Occurs in multimode fibers coming from differences in group velocity for different modes
- Material dispersion
  - Results from the wavelength dependence of the bulk refractive index
- Waveguide dispersion
  - Results from the wavelength dependence of the effective index in a waveguide
  - Material + waveguide dispersion is termed chromatic dispersion
- Polarization mode dispersion
  - Results from the fact that different polarizations travel at different speeds due to small birefringence that is present
- Nonlinear dispersion – an example is self-phase modulation

Modal dispersion

- Modal dispersion occurs in multimode fibers as a result of differences in the group velocities of the various modes.
- A single pulse of light entering an M-mode fiber spreads into M pulses.



- Estimate of pulse spread $$\sigma_\tau = \frac{1}{2}\left(\frac{L}{v_{\min}} - \frac{L}{v_{\max}}\right),$$

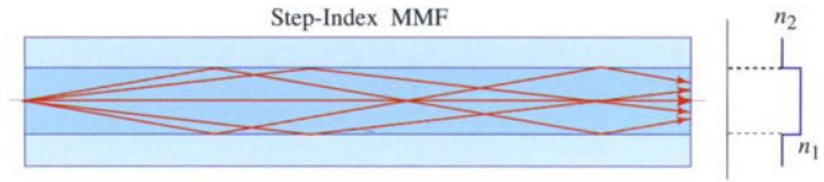
  – Where v_{min} and v_{max} are the smallest and largest group velocity of the modes.

Modal dispersion

- For multimode step-index fiber,

$$v_{\min} \approx c_1(1-\Delta), \quad v_{\max} \approx c_1, \quad \Delta = (n_1^2 - n_2^2)/2n_1^2$$ (see chapter 9 S&T)

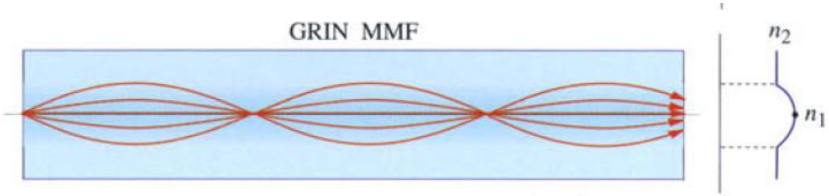
$$\sigma_\tau \approx \frac{L\Delta}{2c_1}$$



- For multimode graded-index fiber,

$$v_{\min} \approx c_1(1-\Delta^2/2), \quad v_{\max} \approx c_1, \quad \Delta = (n_1^2 - n_2^2)/2n_1^2$$ (see chapter 9 S&T)

$$\sigma_\tau \approx \frac{L\Delta^2}{4c_1}$$



Material dispersion

- Spread of wave packet after traveling a distance L through a dispersive material

$$\Delta\tau = \frac{L}{c}\left(N_g(\lambda_1) - N_g(\lambda_2)\right) = \frac{L}{c}\Delta N_g = \frac{L}{c}\frac{dN_g}{d\lambda}\Delta\lambda$$

Since $\frac{dN_g}{d\lambda} = \frac{d}{d\lambda}(n - \lambda\frac{dn}{d\lambda}) = -\lambda\frac{d^2n}{d\lambda^2}$

We get $\Delta\tau = -\lambda\frac{d^2n}{d\lambda^2}\frac{L}{c}\Delta\lambda$ where

$D_M = -\frac{\lambda}{c}\frac{\partial^2 n}{\partial\lambda^2}$ is the material dispersion parameter

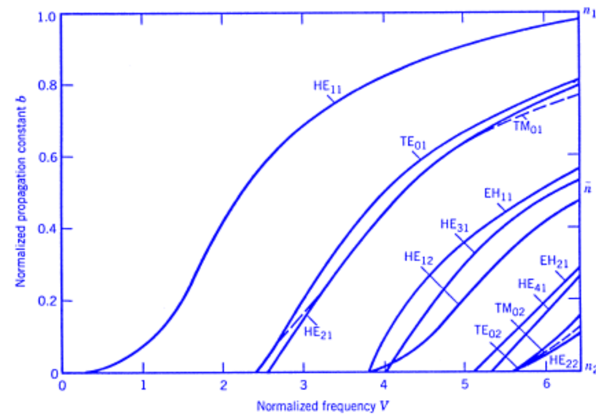
$\Delta\tau = D_m * L * \Delta\lambda$

Waveguide dispersion

In single-mode fibers the group delay, τ_g, determines the transit time of a pulse traveling through a unit length of fiber. To get the waveguide dispersion we want to express the group delay in terms of normalized parameters, b and V:

Normalized propagation constant:
$$b = \frac{\beta / k_0 - n_{clad}}{n_{core} - n_{clad}}.$$

Normalized frequency:
$$V = \frac{2\pi a}{\lambda_0}\sqrt{n_{core}^2 - n_{clad}^2}$$



$$\tau_g = \frac{d\beta}{d\omega} = \frac{d\beta}{dk_0}\frac{dk_0}{d\omega} = \frac{1}{c}\frac{d\beta}{dk_0}.$$

Using normalized parameters :

$$\frac{d\beta}{dk_0} = \frac{d\beta}{dV}\frac{dV}{dk_0} = \frac{d\beta}{dV}\frac{V}{k_0}.$$

We get : $\tau_g = \frac{1}{c}\frac{d\beta}{dk_0} = \frac{1}{c}\frac{V}{k_0}\frac{d\beta}{dV}.$

Waveguide dispersion

Defining $\Delta = \frac{n_{core} - n_{clad}}{n_{core}}$ and with $\beta = [b(n_{core} - n_{clad}) + n_{clad}]k_0 \approx k_0 n_{clad}(1 + b\Delta)$ (when Δ is small).

$$\rightarrow \quad \tau_g = \frac{1}{c}\frac{V}{k_0}\frac{d}{dV}\left[k_0 n_{clad}(1 + b\Delta)\right]$$

$$= \frac{1}{c}\frac{V}{k_0}\frac{d(k_0 n_{clad})}{dV} + \frac{1}{c}\frac{V}{k_0}\frac{d(b\Delta k_0 n_{clad})}{dV}$$

τ_W : waveguide delay

τ_m : material delay

$\tau_W = \frac{1}{c}\frac{V}{k_0}\frac{d(b\Delta k_0 n_{clad})}{dV}$. Noting that $V = \sqrt{2\Delta}\, n_{clad} k_0 a$, we get :

$\tau_W = \frac{1}{c} n_{clad}\Delta \frac{d(bV)}{dV}$, from which we get get the waveguide dispersion

$\frac{d\tau_W}{dV} = \frac{1}{c} n_{clad}\Delta \frac{d^2(bV)}{dV^2}$. With $dV = a\sqrt{n_{core}^2 - n_{clad}^2}\, dk_0$ and $d\lambda = -\lambda \frac{dk_0}{k_0}$,

we finally get :

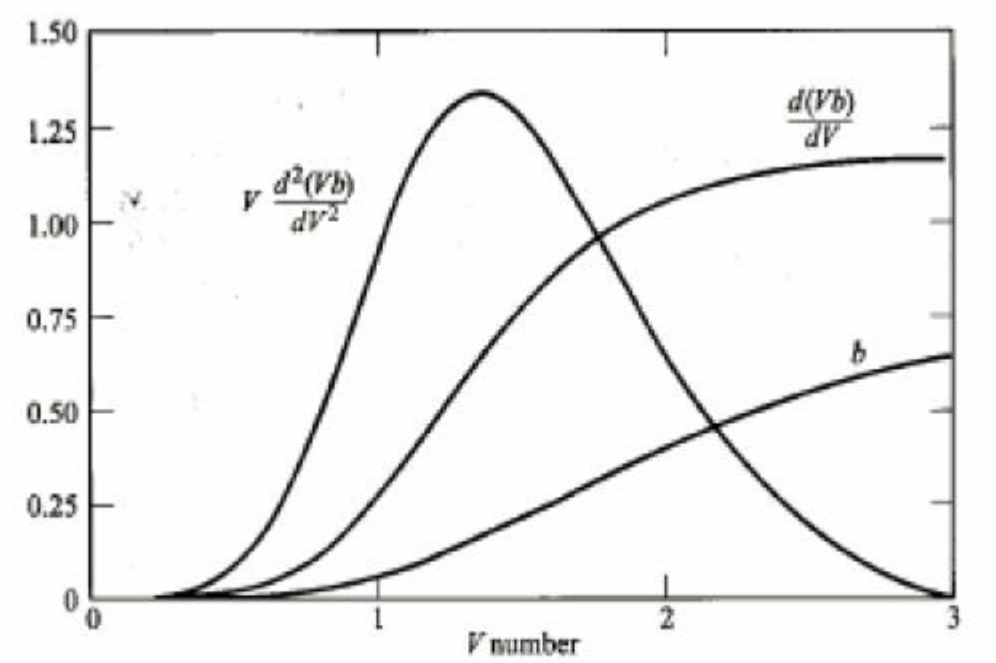
$$\frac{d\tau_W}{d\lambda} = -\frac{n_{clad}\Delta}{c\lambda} V \frac{d^2(bV)}{dV^2}.$$

Waveguide dispersion!

Note: Here we have neglected the dependence of Δ on k_0, which is negligibly small.

Waveguide dispersion

$b,\ d(Vb)/dV$ and $V\dfrac{d^2(Vb)}{dV^2}$ as a function of the V number:



Page 113, in *Optical Fiber Communications*, Gerd Keiser, third edition, McGraw Hill, 2000.

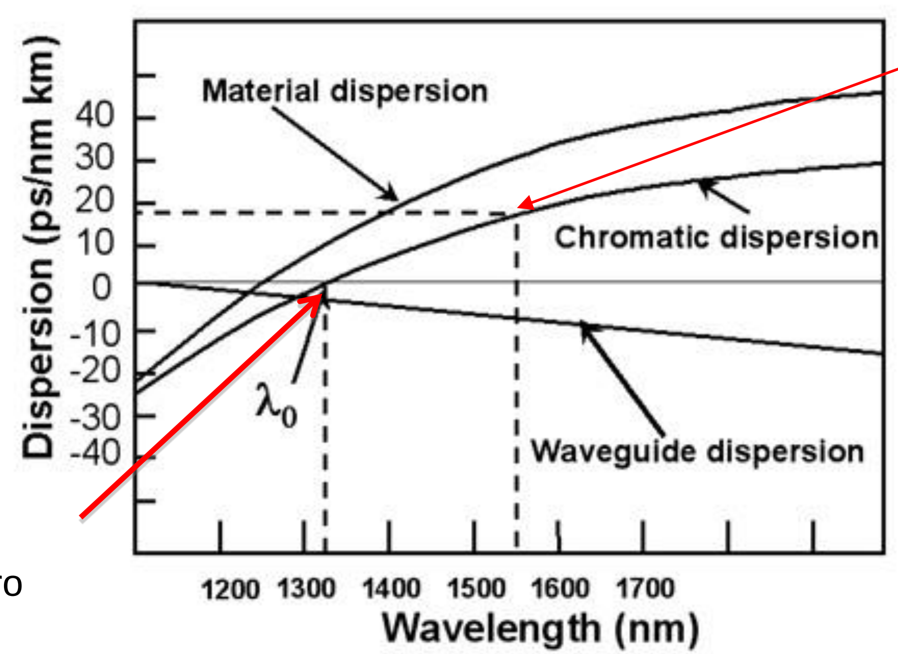
Chromatic dispersion of SMF

- Chromatic dispersion is the combination of material and waveguide dispersion in single-mode fiber (made of fused silica)

$$D = -\frac{\lambda}{c}\frac{d^2 n_{core}}{d\lambda^2} - \frac{n_{core}\Delta}{c\lambda} V \frac{d^2 (Vb)}{dV^2}$$

$$(D_{Material}) \qquad (D_{Waveguide})$$

at 1.55 μm,
D=+17ps/km-nm



at 1.312 μm,
dispersion is zero

D. Gloge, "Dispersion in weakly guiding fibers," *Appl. Opt.* **10**, pp. 2442-2445, 1971

Dispersive pulse broadening

Group velocity: $v_g = (d\beta / d\omega)^{-1}$

Pulse broadening ΔT when spectral width of the pulse is $\Delta\omega$:

$$\Delta T = \frac{dT}{d\omega}\Delta\omega = \frac{d}{d\omega}\left(\frac{L}{v_g}\right)\Delta\omega = L\frac{d^2\beta}{d\omega^2}\Delta\omega$$

$\frac{d^2\beta}{d\omega^2}$ is the Group Velocity Dispersion (GVD) parameter

With $\omega = 2\pi c/\lambda$ and $\Delta\omega = \left(-2\pi c/\lambda^2\right)\Delta\lambda$

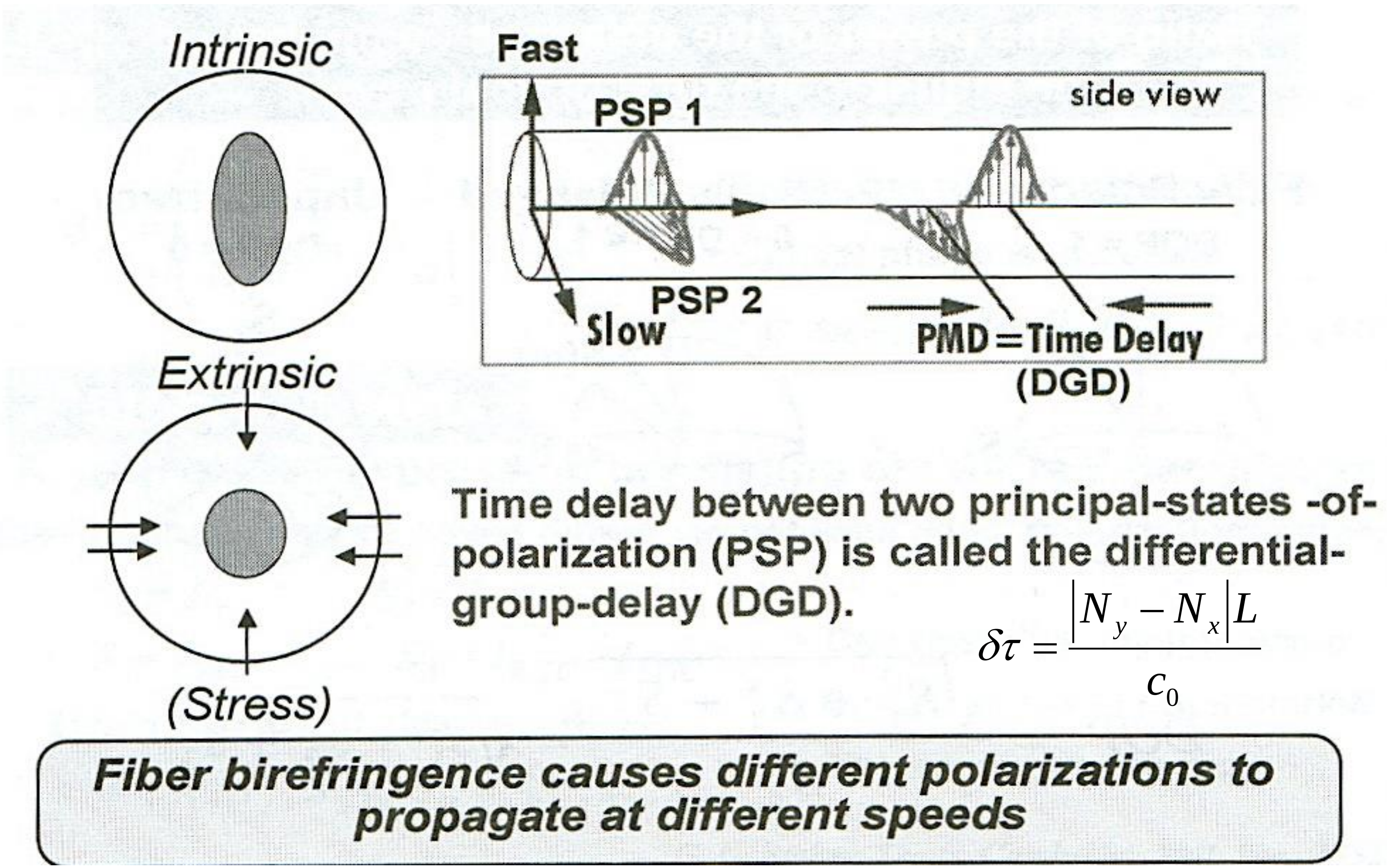
We get: $\Delta T = \frac{d}{d\lambda}\left(\frac{L}{v_g}\right)\Delta\lambda = DL\Delta\lambda$

Here D is the *dispersion parameter*:
[expressed in units of ps/(km-nm)]

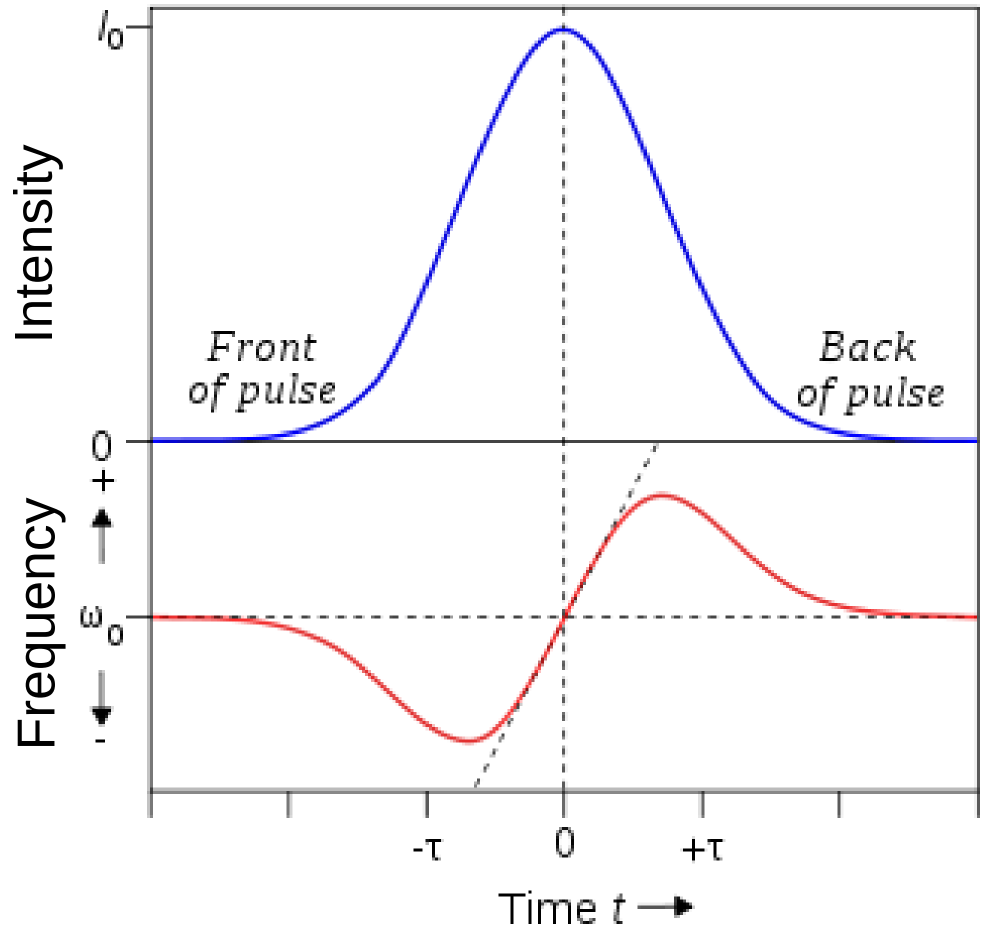
$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2}\frac{d^2\beta}{d\omega^2}$$

Polarization mode dispersion (PMD)

LP_{01} has two polarization modes - Degeneracy is removed by birefringence.



Nonlinear dispersion: Self phase modulation



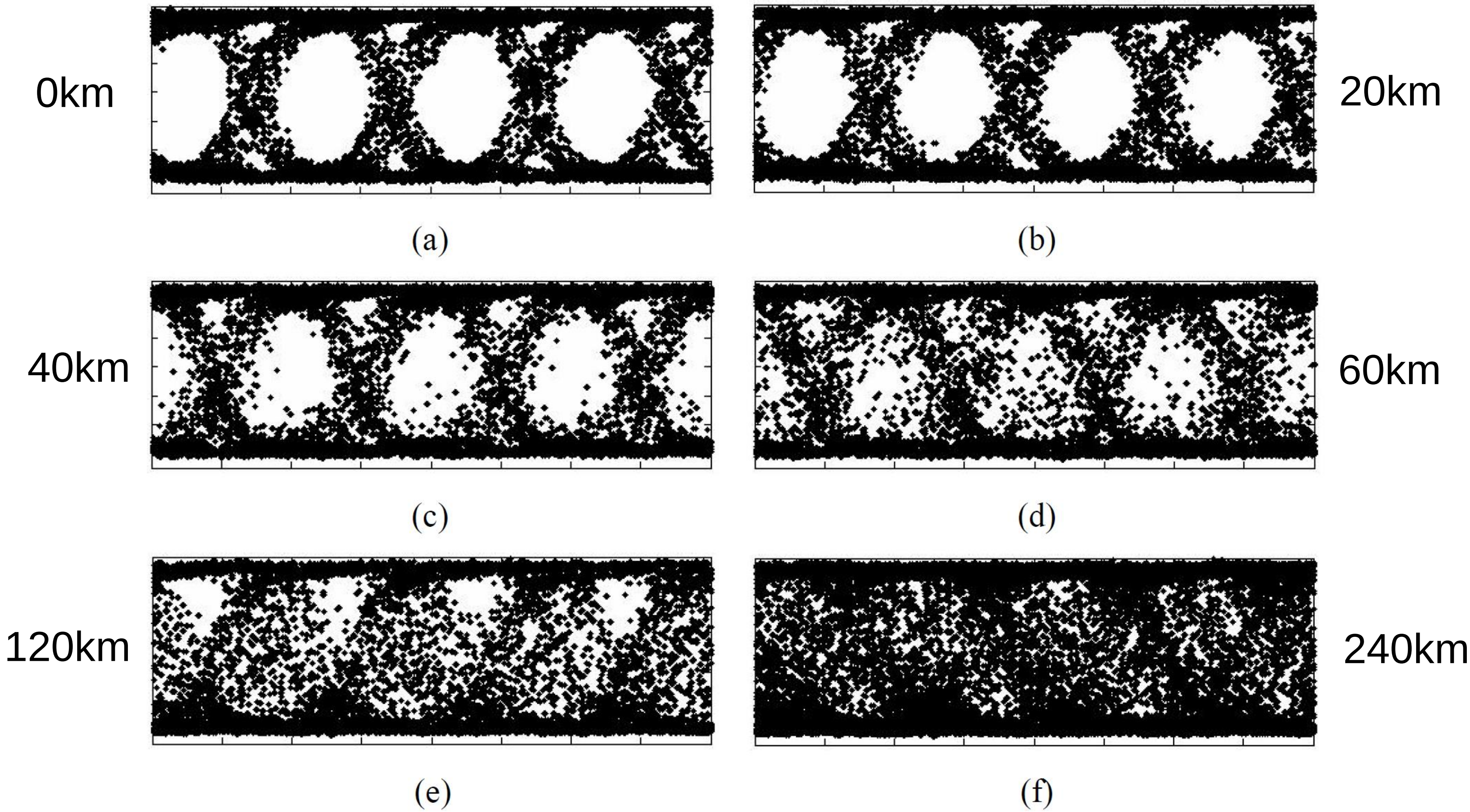
$$n(t) = n_o + n_2 I(t)$$

$$\omega(t) = \omega_0 - \frac{2\pi L}{\lambda_0}\frac{dn(I)}{dt}$$

Nonlinearity results in creating a frequency chirp in the pulse. For a Gaussian pulse we have

$$\omega(t) = \omega_0 + \frac{4\pi L n_2 I_0}{\lambda_0 \tau^2}\, t \exp\left(\frac{t^2}{\tau^2}\right.$$

Dispersive pulse broadening



Lee’s PhD thesis

Dispersion compensation

- Pre-chirp technique
- Dispersion engineered fibers
- Dispersion compensating fibers
- Chirped fiber Bragg grating
- Tunable dispersion compensation

Pre-chirp technique

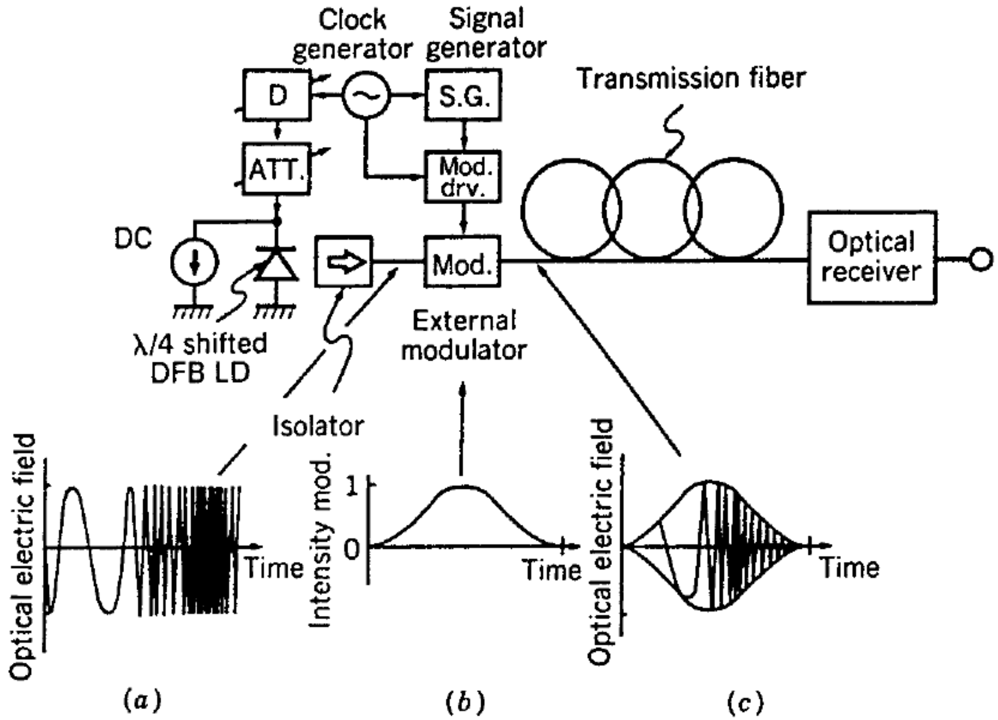
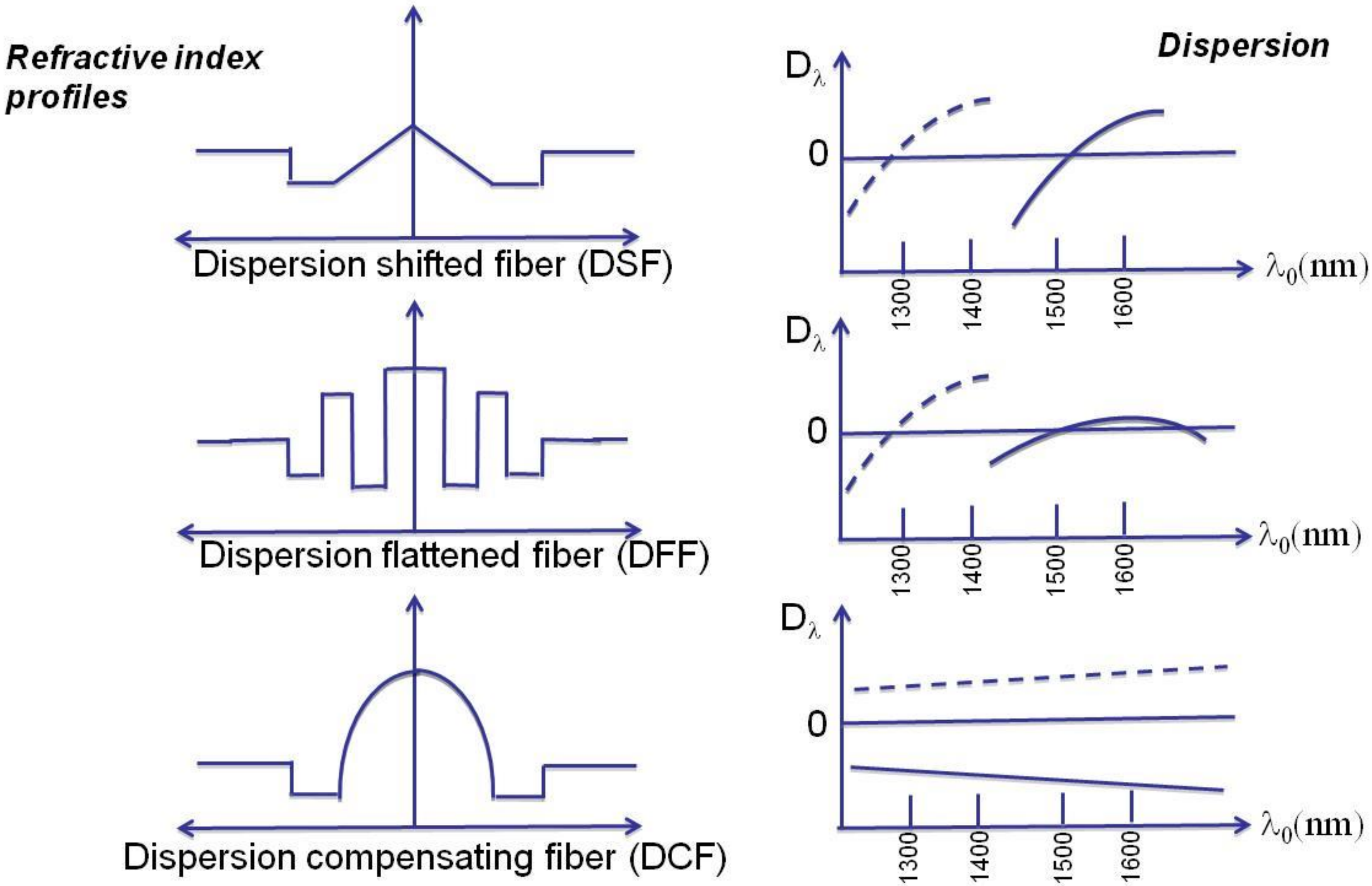
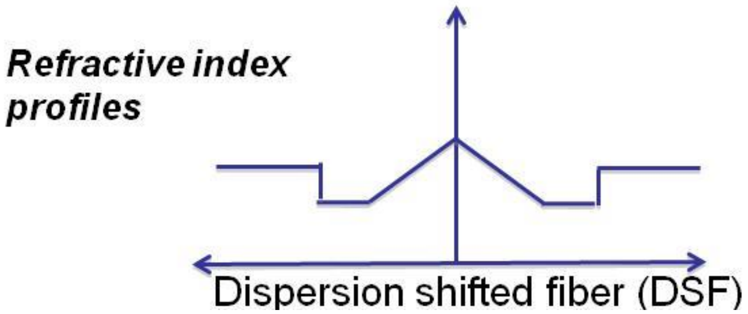


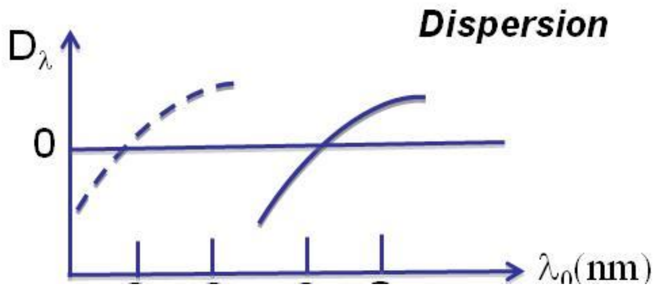
Figure 7.1: Schematic of the prechirp technique used for dispersion compensation: (a) FM output of the DFB laser; (b) pulse shape produced by external modulator; and (c) prechirped pulse used for signal transmission. (After Ref. [9]; ©1994 IEEE; reprinted with permission.)

Dispersion engineered fiber

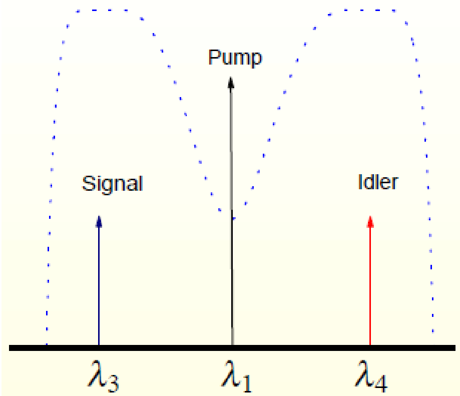


Dispersion engineered fiber

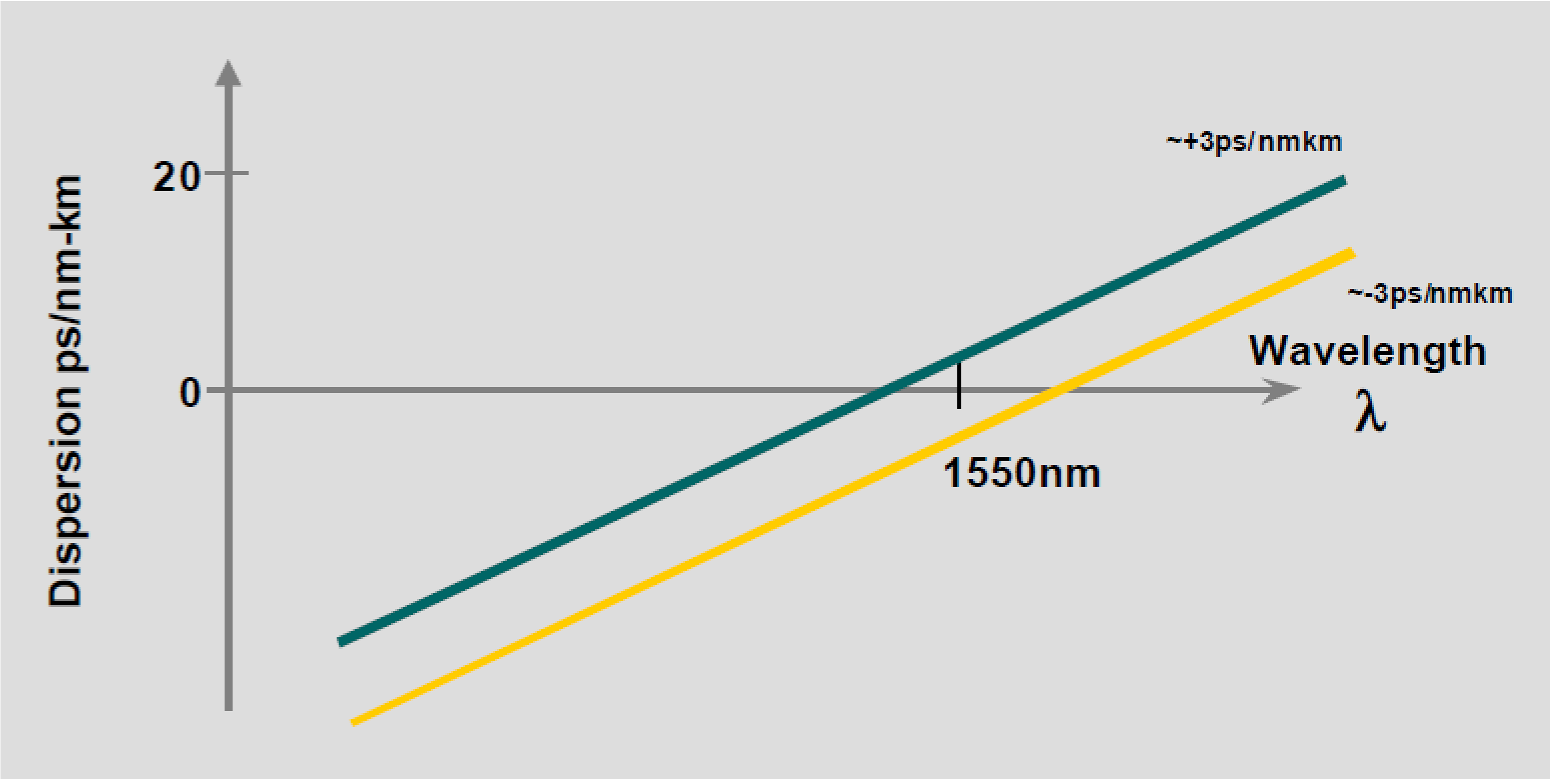




- No dispersion at 1550nm
- But strong Four-Wave-Mixing cross-talk
- Need non-zero dispersion shifted fibers

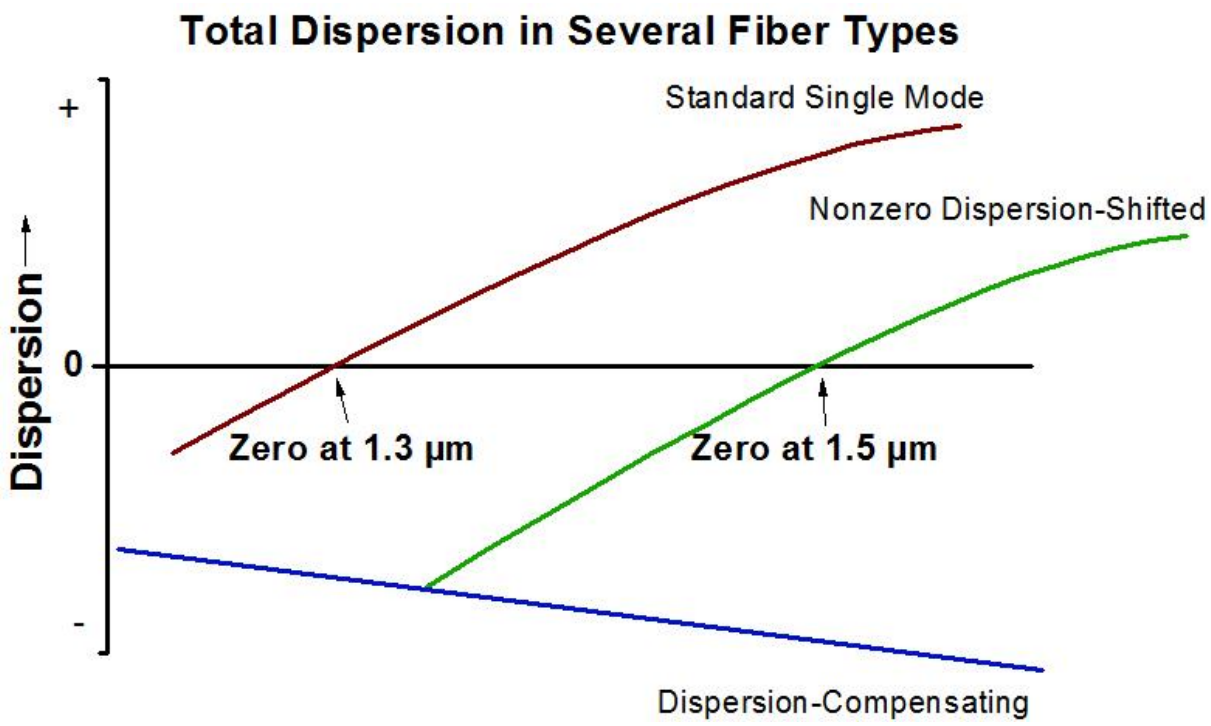


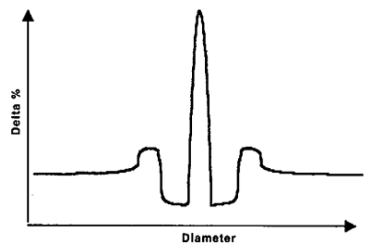
Dispersion engineered fiber



Non-zero dispersion shifted fibers

Dispersion compensating fibers





Typical index profile
of a DCF

Dispersion compensating fibers

$$\beta_{21}L_1 + \beta_{22}L_2 = 0, \text{ or}$$

$$D_1L_1 + D_2L_2 = 0.$$


DCF modules

Advantage:

- Fiber format
- Low cost
- Broadband

Disadvantage:

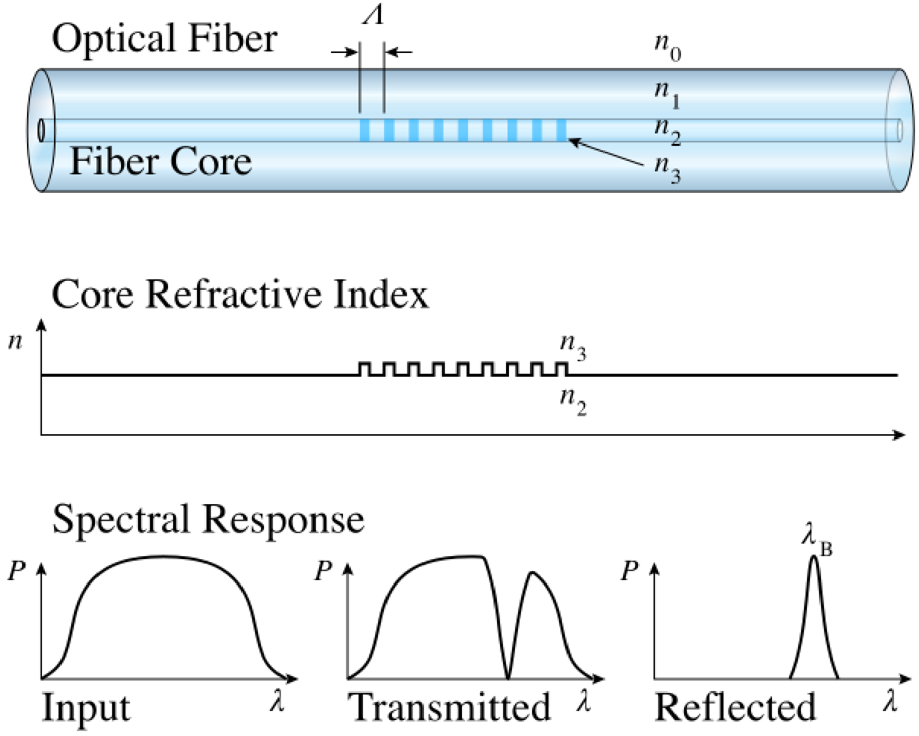
- Small mode field diameter
- Higher loss

Chirped fiber Bragg grating

Bragg condition:

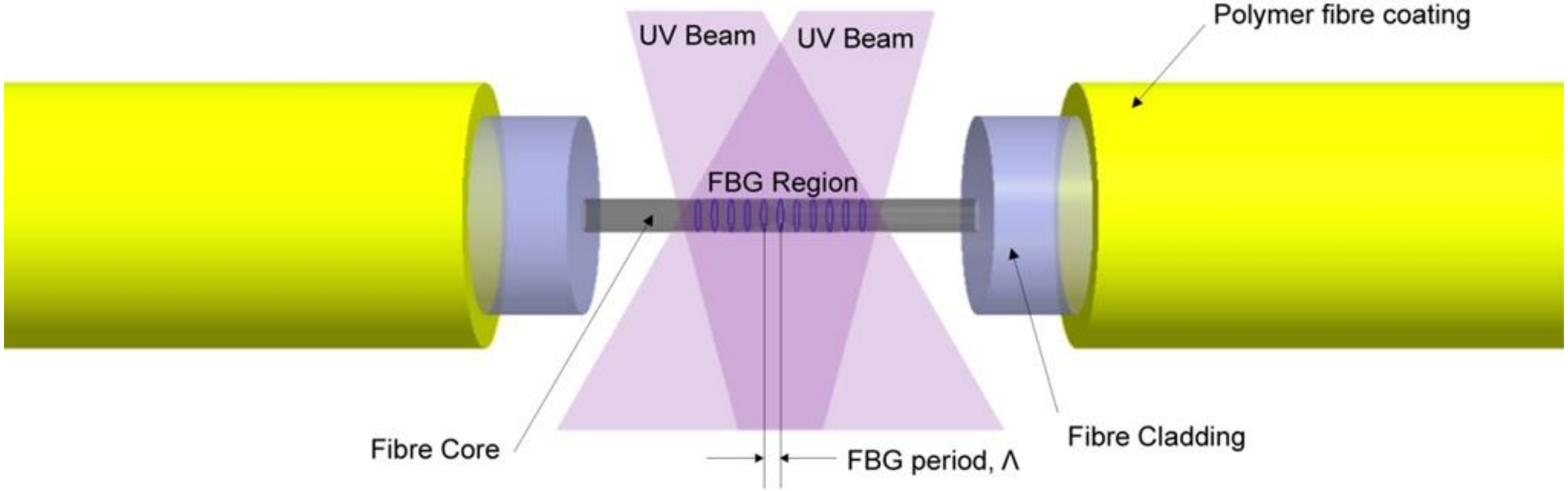
$$\lambda_B = 2n_e\Lambda$$

Fiber Bragg Grating

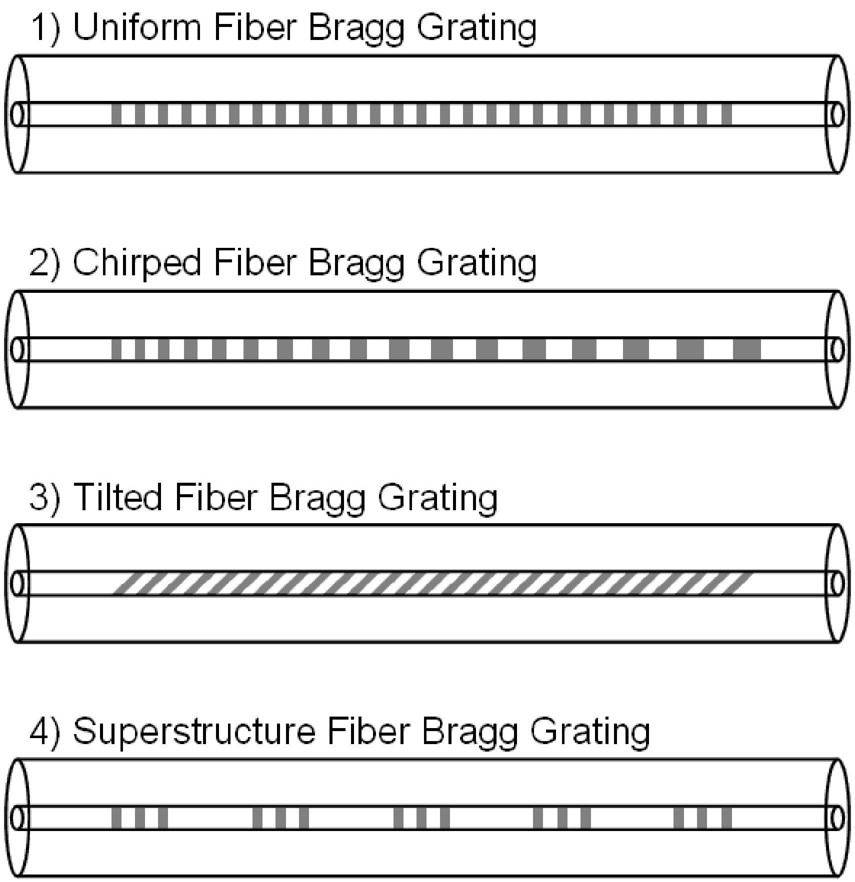


Chirped fiber Bragg grating

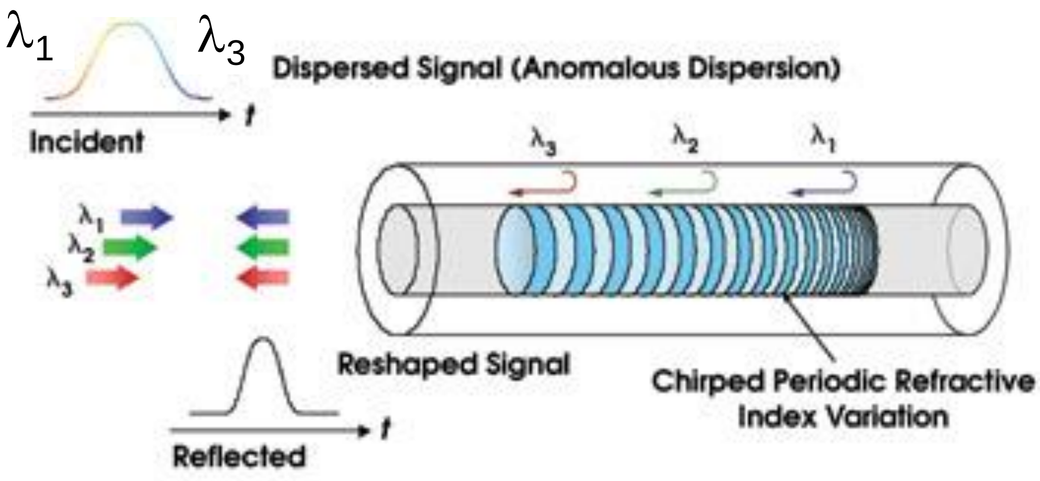
How to make fiber Bragg gratings?



Types of fiber Bragg grating



Chirped fiber Bragg grating



Advantage:

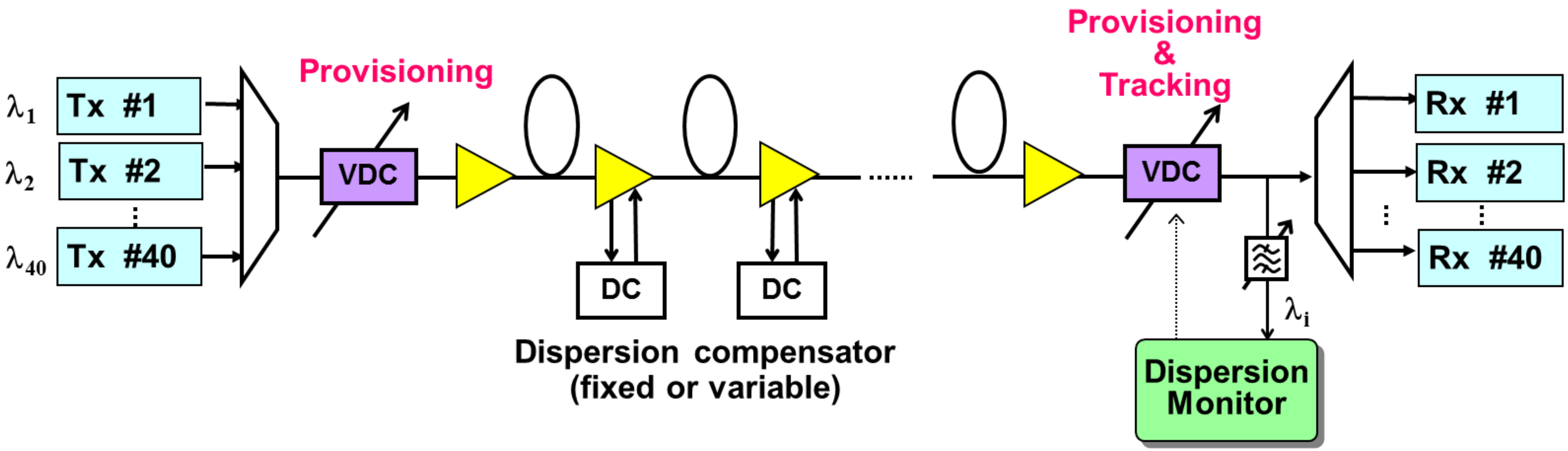
- Low cost
- Low loss

Disadvantage:

- Narrow band

Tunable dispersion compensation

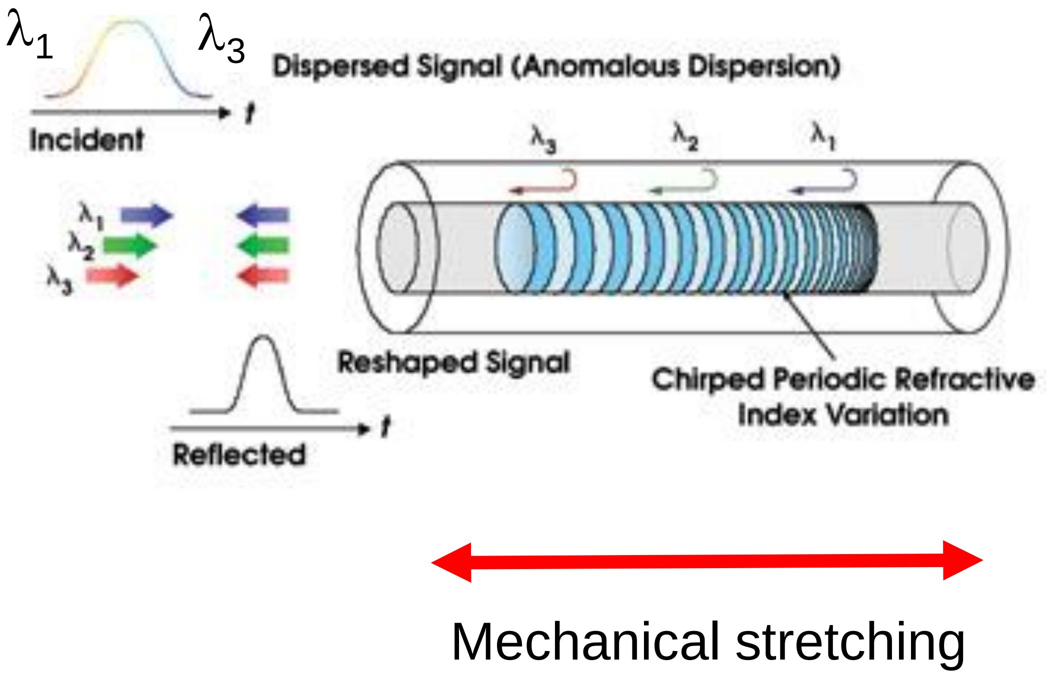
- Provide adjustable dispersion
- Compensate residual dispersion



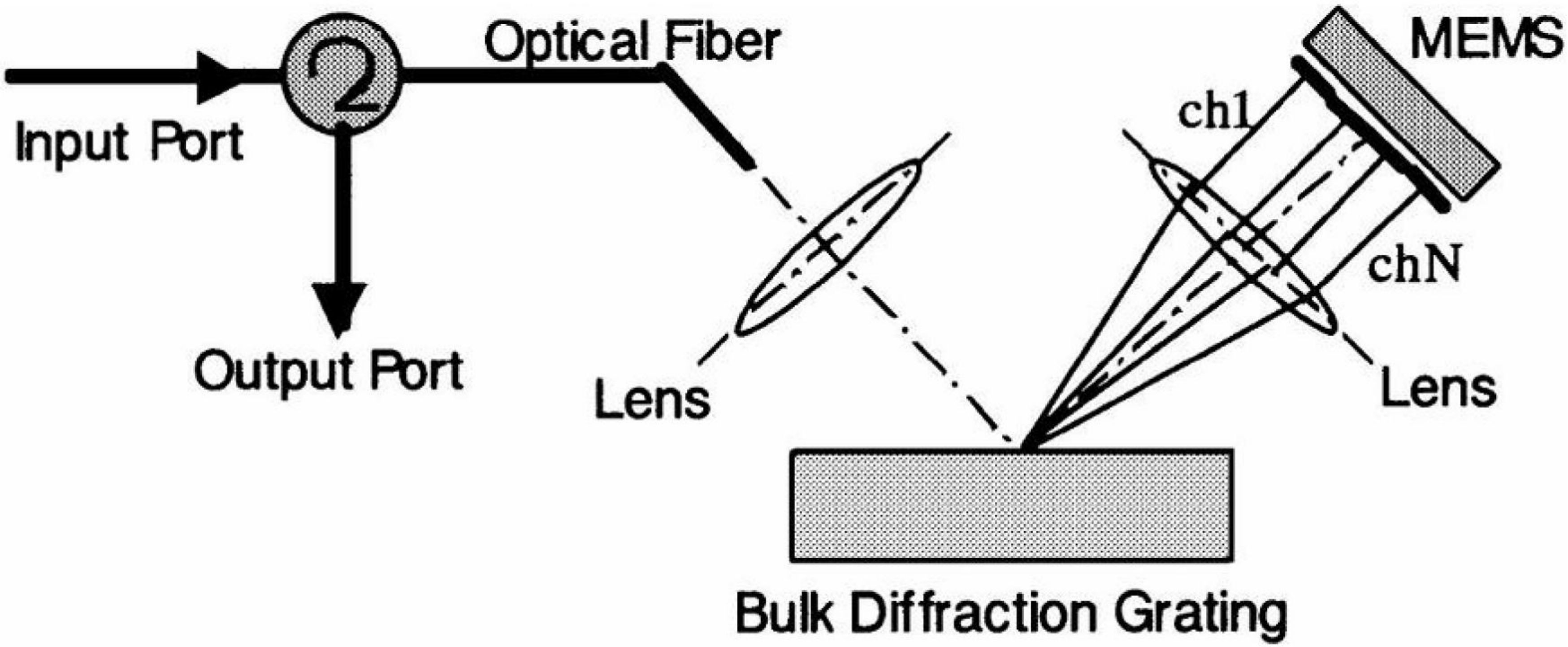
Fujitsu

Tunable dispersion compensation

- Provide adjustable dispersion
- Compensate residual dispersion



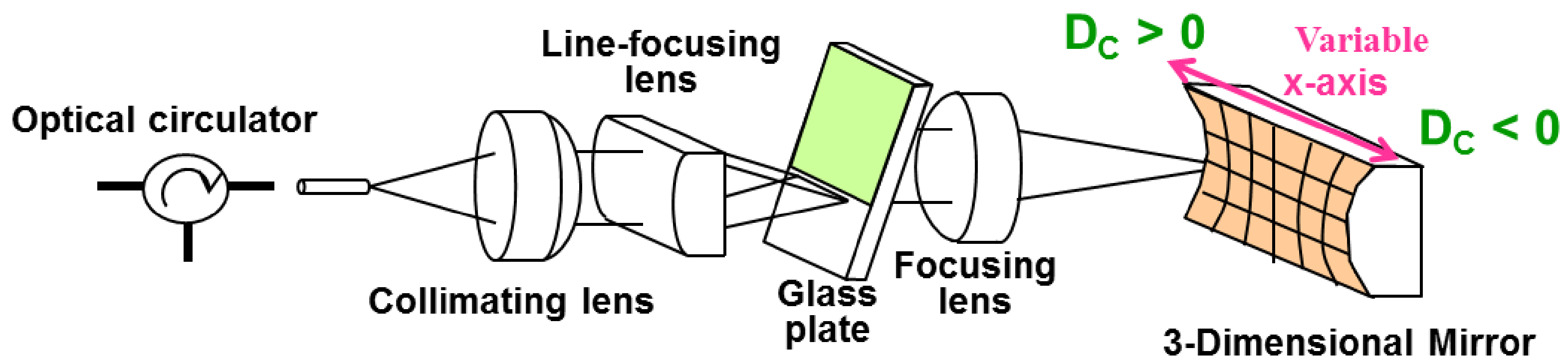
Tunable dispersion compensation



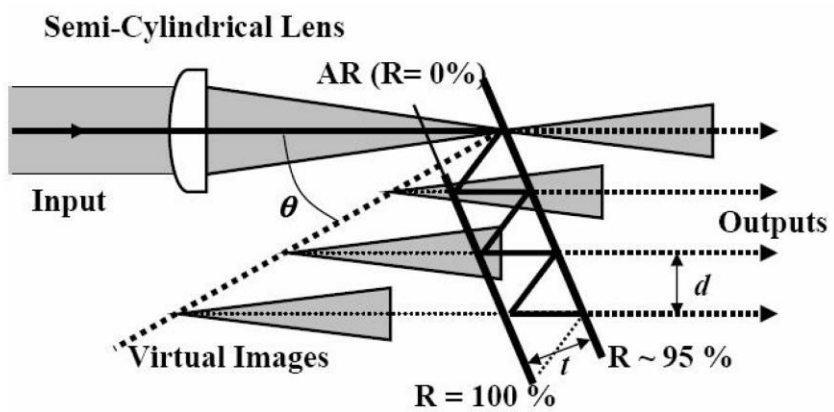
Lee's PhD thesis

Tunable dispersion compensation

VIPA variable dispersion compensator



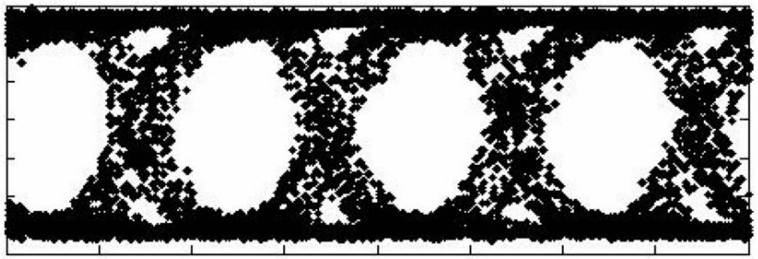
VIPA : Virtually Imaged Phased Array



Fujitsu

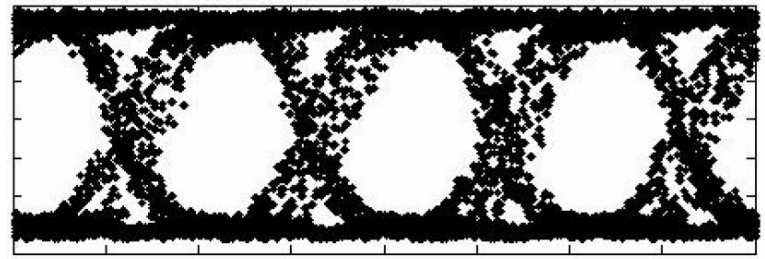
Tunable dispersion compensation

20km



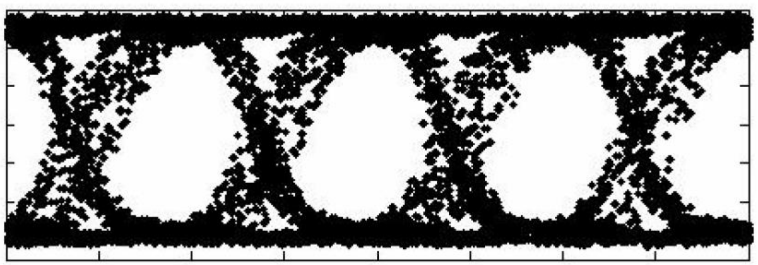
(a)

40km



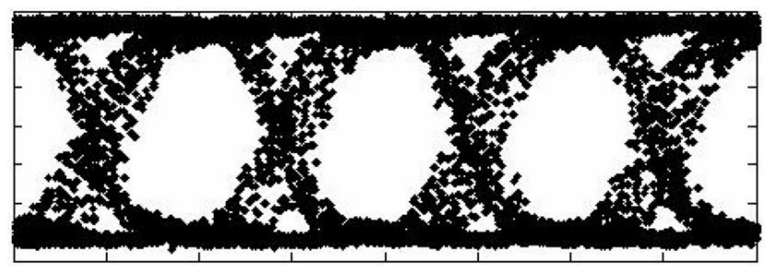
(b)

60km



(c)

120km



(d)

240km

(f)

Lee’s PhD thesis