Announcements

- HW#3 is assigned due Feb. 20th
- Mid-term exam Feb 27, 2PM (open book/notes)
Planar waveguides

- Review waveguide modes by Maxwell’s equations
- Rectangular waveguides
- Slab directional coupler
- Waveguide materials and fabrication methods
Planar waveguides - next time

- Coupling light into planar waveguides
- Numerical simulation tools
- Waveguide loss
- Example of waveguides and performance
Planar dielectric waveguide

- Core film sandwiched between two layers of lower refractive index
- Bottom layer is often a substrate with $n = n_s$
- Top layer is called the cover layer ($n_c \neq n_s$)
- Air can also acts as a cover ($n_c = 1$)
- $n_c = n_s$ in symmetric waveguides
Using Maxwell’s equations

- An optical mode is a solution of the Maxwell's equations satisfying all boundary conditions.
- Its spatial distribution does not change with propagation.
- Modes are obtained by solving the curl equations.

\[ \nabla \times \mathbf{E} = i\omega \mu_0 \mathbf{H}, \quad \nabla \times \mathbf{H} = -i\omega \varepsilon_0 n^2 \mathbf{E} \]

- These six equations are solved in each layer of the waveguide.
- Boundary condition: Tangential component of \( \mathbf{E} \) and \( \mathbf{H} \) be continuous across both interfaces.
- Waveguide modes are obtained by imposing the boundary conditions.
Using Maxwell’s equations

\[
\begin{align*}
\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= i\omega \mu_0 H_x, \\
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} &= i\omega \mu_0 H_y, \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} &= i\omega \mu_0 H_z, \\
\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= i\omega \varepsilon_0 n^2 E_x, \\
\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} &= i\omega \varepsilon_0 n^2 E_y, \\
\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} &= i\omega \varepsilon_0 n^2 E_z.
\end{align*}
\]

- Assume waveguide is infinitely wide along the x axis
- E and H are then x-independent
- For any mode, all components vary with z as \( \exp(i\beta z) \). Thus,

\[
\begin{align*}
\frac{\partial E}{\partial x} &= 0, & \frac{\partial H}{\partial x} &= 0, & \frac{\partial E}{\partial z} &= i\beta E, & \frac{\partial H}{\partial z} &= i\beta H.
\end{align*}
\]
These equations have two distinct sets of linearly polarized solutions

For Transverse-Electric (TE) modes, $E_z = 0$ and $E_y = 0$

TE modes are obtained by solving:

$$\frac{d^2 E_x}{dy^2} + (n^2 k_0^2 - \beta^2) E_x = 0, \quad k_0 = \omega \sqrt{\varepsilon_0 \mu_0} = \omega / c$$

Magnetic field components are related to $E_x$ as:

$$H_y = -\frac{\beta}{\omega \mu_0} E_x, \quad H_x = 0, \quad H_z = -\frac{i}{\omega \mu_0} \frac{d E_x}{dy}$$
Using Maxwell’s equations

\[
\frac{d^2 E_X}{dy^2} + (n^2 k_0^2 - \beta^2) E_X = 0, \quad k_0 = \omega \sqrt{\varepsilon_0 \mu_0} = \omega / c
\]

- We solve this equation in each layer separately using \( n = n_c, n_1, \) and \( n_s. \)

\[
E_X(y) = \begin{cases} 
B_c \exp[-q_1(y - d)]; & y > d, \\
A \cos(p y - \phi); & |y| \leq d \\
B_s \exp[q_2(y + d)]; & y < -d, 
\end{cases}
\]

- Constants \( p, q_1, \) and \( q_2 \) are defined as

\[
p^2 = n_1^2 k_0^2 - \beta^2, \quad q_1^2 = \beta^2 - n_c^2 k_0^2, \quad q_2^2 = \beta^2 - n_s^2 k_0^2.
\]

- Constants \( B_c, B_s, A, \) and \( \phi \) are determined from the boundary conditions at the two interfaces.
Using Maxwell’s equations

- Tangential components of \( \mathbf{E} \) and \( \mathbf{H} \) continuous across any interface with index discontinuity.

- Mathematically, \( E_x \) and \( H_z \) should be continuous at \( y = \pm d \).

- \( E_x \) is continuous at \( y = \pm d \) if

  \[
  B_c = A \cos(pd - \phi); \quad B_s = A \cos(pd + \phi). 
  \]

- Since \( H_z \propto dE_x/dy, \) \( dE_x/dy \) should also be continuous at \( y = \pm d \):

  \[
  pA \sin(pd - \phi) = q_1B_c, \quad pA \sin(pd + \phi) = q_2B_s. 
  \]

- Eliminating \( A, B_c, B_s \) from these equations, \( \phi \) must satisfy

  \[
  \tan(pd - \phi) = q_1/p, \quad \tan(pd + \phi) = q_2/p 
  \]
Using Maxwell’s equations

- Boundary conditions are satisfied when

\[ pd - \phi = \tan^{-1}(q_1/p) + m_1\pi, \quad pd + \phi = \tan^{-1}(q_2/p) + m_2\pi \]

- Adding and subtracting these equations, we obtain

\[ 2\phi = m\pi - \tan^{-1}(q_1/p) + \tan^{-1}(q_2/p) \]
\[ 2pd = m\pi + \tan^{-1}(q_1/p) + \tan^{-1}(q_2/p) \]

- The last equation is called the eigenvalue equation.

- Multiple solutions for \( m = 0, 1, 2, \ldots \) are denoted by \( \text{TE}_m \).

- Effective index of each TE mode is \( \bar{n} = \beta / k_0 \).
For symmetric waveguides $n_c = n_s$.

Using $q_1 = q_2 \equiv q$, TE modes satisfy

$$q = p \tan(pd - m\pi/2).$$

Define a dimensionless parameter

$$V = d \sqrt{p^2 + q^2} = k_0 d \sqrt{n_1^2 - n_s^2},$$

If we use $u = pd$, the eigenvalue equation can be written as

$$\sqrt{V^2 - u^2} = u \tan(u - m\pi/2).$$

For given values of $V$ and $m$, this equation is solved to find $p = u/d$. 

G. Agrawal
Effective index $\tilde{n} = \beta / k_0 = (n_1^2 - p^2 / k_0^2)^{1/2}$.

Using $2\phi = m\pi - \tan^{-1}(q_1 / p) + \tan^{-1}(q_2 / p)$
with $q_1 = q_2$, phase $\phi = m\pi / 2$.

Spatial distribution of modes is found to be

$E_X(y) = \begin{cases} 
B_\pm \exp[-q(|y| - d)]; & |y| > d, \\
A \cos(p_y - m\pi / 2); & |y| \leq d, 
\end{cases}$

where $B_\pm = A \cos(pd \pm m\pi / 2)$ and the lower sign is chosen for $y < 0$.

Modes with even values of $m$ are symmetric around $y = 0$ (even modes).

Modes with odd values of $m$ are antisymmetric around $y = 0$ (odd modes).
Modes of asymmetric waveguide

- We can follow the same procedure for \( n_c \neq n_s \).

- Eigenvalue equation for TE modes:
  \[
  2pd = m\pi + \tan^{-1}(q_1/p) + \tan^{-1}(q_2/p)
  \]

- Eigenvalue equation for TM modes:
  \[
  2pd = m\pi + \tan^{-1}\left(\frac{n_1^2 q_1}{n_c^2 p}\right) + \tan^{-1}\left(\frac{n_1^2 q_2}{n_s^2 p}\right)
  \]

- Constants \( p, q_1, \) and \( q_2 \) are defined as
  \[
  p^2 = n_1^2 k_0^2 - \beta^2, \quad q_1^2 = \beta^2 - n_c^2 k_0^2, \quad q_2^2 = \beta^2 - n_s^2 k_0^2.
  \]

- Each solution for \( \beta \) corresponds to a mode with effective index \( \tilde{n} = \beta/k_0 \).

- If \( n_1 > n_s > n_c \), guided modes exist as long as \( n_1 > \tilde{n} > n_s \).
Modes of asymmetric waveguide

- Useful to introduce two normalized parameters as
  
  \[ b = \frac{n^2 - n_s^2}{n_1^2 - n_s^2}, \quad \delta = \frac{n_s^2 - n_c^2}{n_1^2 - n_s^2}. \]

- \( b \) is a normalized propagation constant \((0 < b < 1)\).
- Parameter \( \delta \) provides a measure of waveguide asymmetry.
- Eigenvalue equation for TE modes in terms \( V, b, \delta \):
  
  \[ 2V \sqrt{1-b} = m\pi + \tan^{-1} \sqrt{\frac{b}{1-b}} + \tan^{-1} \sqrt{\frac{b+\delta}{1-b}}. \]

- Its solutions provide **universal** dispersion curves.
Universal dispersion curve

Solid lines ($\delta = 5$); dashed lines ($\delta = 0$)
Springer Series in Electronics and Photonics 26

Theodor Tamir (Ed.)

Guided-Wave Optoelectronics

T. Tamir
Introduction
H. Kogelnik
Theory of Optical Waveguides
W.K. Burns and A.F. Milton
Waveguide Transitions and Junctions
R.C. Alferness
Titanium-Diffused Lithium Niobate Waveguide Devices
I.P. Kaminow and R.S. Tucker
Mode-Controlled Semiconductor Lasers
F.J. Leonberger and J.F. Donnelly
Semiconductor Integrated Optic Devices

Springer-Verlag
Rectangular mirror waveguide

\[ 2k_x d = 2 \quad m_x \quad m_x = 1, 2, \ldots \]
\[ 2k_y d = 2 \quad m_y \quad m_y = 1, 2, \ldots \]

\[ k_{ym} = n k_0 \sin \theta \quad m = m / d \]
Number of modes

\[ 2k_x d = 2 \ m_x \quad m_x = 1, 2, \ldots \]
\[ 2k_y d = 2 \ m_y \quad m_y = 1, 2, \ldots \]

\[ M = \frac{\text{Quadrant area}}{\text{Unit cell area}} = \frac{n^2 k_0^2}{4} = \frac{2d^2}{4} = \frac{d^2}{2} \]
Rectangular dielectric waveguide

\[ k_x^2 + k_y^2 = n_1^2 k_0^2 \sin^2 \frac{\phi}{c} \]

\[ c = \cos^{-1} \left( \frac{n_2}{n_1} \right) \]
Rectangular dielectric waveguide

\[ k_x^2 + k_y^2 \leq n_1^2 k_0^2 \sin^2 \theta_c \]

\[ \theta_c = \cos^{-1} \left( \frac{n_2}{n_1} \cdot \frac{1}{\lambda_o} \right) \]

Number of TE modes:

\[ M \approx \frac{\pi}{4} \left( \frac{2d}{\lambda_o} \right)^2 (NA)^2 \]
Slab directional coupler

\[
\frac{da_1}{dz} = -jC_{21} \exp(j\Delta\beta z) a_2(z)
\]

\[
\frac{da_2}{dz} = -jC_{12} \exp(-j\Delta\beta z) a_1(z)
\]

\[
\Delta\beta = \beta_1 - \beta_2 \quad \text{phase mismatch per unit length}
\]

\[
C_{21} = \frac{k_0^2}{2\beta_1} (n_2^2 - n^2) \int_a^{a+d} dy \ u_1(y) u_2(y)
\]

\[
C_{12} = \frac{k_0^2}{2\beta_2} (n_1^2 - n^2) \int_{-a-d}^{-a} dy \ u_2(y) u_1(y)
\]
Slab directional coupler

\[
P_1(z) = P_1(0) \left[ \cos^2(\gamma z) + \left( \frac{\Delta \beta}{2\gamma} \right)^2 \sin^2(\gamma z) \right]
\]

\[
P_2(z) = P_1(0) \left| \frac{C_{21}}{\gamma^2} \right| \sin^2(\gamma z)
\]

Coupling length

\[
L_0 = \frac{\pi}{2\sqrt{C_{12}C_{21}}}
\]

3dB coupler
Phase-mismatched vs. phase-matched

\[ P_1(z) = P_1(0) \left[ \cos^2 \gamma z + \left( \frac{\Delta \beta}{2 \gamma} \right)^2 \sin^2 \gamma z \right] \]

\[ P_2(z) = P_1(0) \frac{|c_{21}|^2}{\gamma^2} \sin^2 \gamma z. \]

\[ P_1(z) = P_1(0) \cos^2 c z \]
\[ P_2(z) = P_1(0) \sin^2 c z. \]

Phase mismatched \( \Delta \beta \neq 0 \)

Phase matched \( \Delta \beta = 0 \)

\[ \gamma^2 = \left( \frac{\Delta \beta}{2} \right)^2 + C_{12} C_{21} \]

(Homework)
Switching with directional coupler

Power transfer ratio

\[
T = \frac{P_2(L_0)}{P_1(0)} = \frac{\pi^2}{4} \sin^2 c \left[ \frac{1}{2} \sqrt{1 + \left( \frac{\Delta \beta L_0}{\pi} \right)^2} \right]
\]

\[
\sin c(x) = \frac{\sin(\pi x)}{\pi x}
\]

Phase mismatch can be tuned electrically in directional couplers. In tuning the phase mismatch from 0 to \(\sqrt{3}\pi\), light is switched from WG 2 to 1. Tuning can be done electro-optically or thermally, for example.
In (g) core layer is covered with two metal stripes.

Losses can be reduced by using a thin buffer layer (h).
Design of rectangular waveguide

Lasing in direct-bandgap GeSn alloy grown on Si

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Materials for waveguides

- **Active (light generating)**
  - Indium phosphide
  - Gallium arsenide
  - Aluminum gallium arsenide
  - Germanium Tin (new)

- **Passive (not light generating)**
  - Silica
  - Lithium niobate
  - Polymer
  - Silicon
  - Silicon nitride

Silica AWG wafers

Arrays of ten 10Gb/s transmitters and receivers
Semiconductor waveguides

- Semiconductors allow fabrication of electrically active devices
- Semiconductors belonging to III-V group often used
- Two semiconductors with different refractive indices needed
- They must have different bandgaps but same lattice constant

Figure 9-20. Lattice constant versus band gap for III-V semiconductors in the ultraviolet, visible, and near-infrared. Note discontinuity in lattice-spacing scale. Dashed lines are indirect band gap compounds.

By Jeff Hecht
A fraction of the lattice sites in a binary semiconductor (GaAs, InP, etc.) is replaced by other elements.

Ternary compound $Al_xGa_{1-x}As$ is made by replacing a fraction $x$ of Ga atoms by Al atoms.

Band-gap varies with $x$ as: $E_g(x) = 1.424 + 1.247x$ \hspace{1cm} (0 < x < 0.45)

Quaternary compound $In_{1-x}Ga_xAs_yP_{1-y}$ useful in the wavelength range 1.1 to 1.6 µm.

For matching lattice constant to InP substrate, $x/y = 0.45$

Band-gap varies with $y$ as: $E_g(y) = 1.35 - 0.72y + 0.12y^2$
Ternary and quaternary compounds

Epitaxial growth of multiple layers on a base substrate (GaAs or InP)

Three primary techniques:

• Liquid-phase epitaxy (LPE)
• Vapor-phase epitaxy (VPE)
• Molecular-beam epitaxy (MBE)

VPE is also called chemical-vapor deposition (CVD)
Quantum-well technology

- Thickness of the core layer plays a central role
- If it is small enough, electrons and holes act as if they are confined to a quantum well
- Confinement leads to quantization of energy bands into subbands
- Useful for making modern quantum-well, quantum wire, and quantum-dot lasers
- In MQW lasers, multiple core layers (thickness 5-10 nm) are separated by transparent barrier layers
- Use of intentional but controlled strain improves performance in strained quantum wells.
To build a laser, one needs to inject current into the core layer of the semiconductor waveguide.

This is accomplished through a p-n junction formed by making cladding layers p- and n-types.

n-type material requires a dopant with an extra electron.

p-type material requires a dopant with one less electron.

Doping creates free electrons or holes within a semiconductor. Fermi level lies in the middle of bandgap for undoped (intrinsic) semiconductors.

In a heavily doped semiconductor, Fermi level lies inside the conduction or valence band.
Electro-optics waveguide

- Use Pockels effect to change refractive index of the core layer with an external voltage
- Common electro-optic materials: LiNbO3, LiTaO3, BaTiO3
- LiNbO3 used commonly for making optical modulators
- For any anisotropic material: \( D_i = \varepsilon_0 \sum_{j=1}^{3} \varepsilon_{ij} E_j \)
Pockels Effect

\[ n(E) = n + a_1 E + \frac{1}{2} a_2 E^2 + \ldots \]

\[ \eta = \frac{\varepsilon_0}{\varepsilon} = \frac{1}{\varepsilon_r} = \frac{1}{n^2} \quad \text{(Impermeability)} \]

\[ \Delta \eta = \frac{d\eta}{dn} \Delta n = \frac{-2}{n^3} \left[ -\frac{1}{2} r n^3 E - \frac{1}{2} s n^3 E^2 \right] \]

\[ \Delta \eta = rE + sE^2 \]

\[ \eta = \eta(0) + rE + sE^2 \]

\[ r = \text{Pockels coefficient ("linear E-O coefficient")}, \]
\[ s = \text{Kerr coefficient ("quadratic E-O coefficient")}, \]

\[ n(E) = n - \frac{1}{2} r n^3 E - \frac{1}{2} s E^2 - \ldots \]

Named for Friedrich Pockels (1865-1913), who discovered this effect in 1893.
Pockel’s Cell
LiNbO$_3$ waveguides do not require an epitaxial growth.

A popular technique employs diffusion of metals into a LiNbO$_3$ substrate, resulting in a low-loss waveguide.

The most commonly used element: Titanium (Ti).

Diffusion of Ti atoms within LiNbO$_3$ crystal increases refractive index and forms the core region.

Surface flatness critical to ensure a uniform waveguide.
A proton-exchange technique is also used for LiNbO$_3$ waveguides

- A low-temperature process (200°C) in which Li ions are replaced with protons when the substrate is placed in an acid bath
- Proton exchange increases the extraordinary part of refractive index but leaves the ordinary part unchanged
- Such a waveguide supports only TM modes and is useful for some applications because of its polarization selectivity
- High-temperature annealing used to stabilizes the index difference
- Accelerated aging tests predict a lifetime of over 25 years at a temperature as high as 95°C.
LiNbO$_3$ waveguide

- Electrodes fabricated directly on the surface of wafer (or on an optically transparent buffer layer)
- An adhesion layer (typically Ti) first deposited to ensure that metal sticks to LiNbO$_3$
- Photolithography used to define the electrode pattern
Silica glass waveguides

- Silica layers deposited on top of a Si substrate
- Employs the technology developed for integrated circuits
- Fabricated using flame hydrolysis with reactive ion etching
  - Two silica layers are first deposited using flame hydrolysis
  - Top layer converted to core by doping it with germania
  - Both layers solidified by heating at 1300C (consolidation process)
  - Photolithography used to etch patterns on the core layer
  - Entire structure covered with a cladding formed using flame hydrolysis
Silica glass waveguides

Steps used to form silica waveguides on top of a Si Substrate
Silicon-on-insulator waveguides

- Core waveguide layer is made of Si ($n_1 = 3.45$)
- A silica layer under the core layer is used for lower cladding
- Air on top acts as the top cladding layer
- Tightly confined waveguide mode because of large index difference
- Silica layer formed by implanting oxygen, followed with annealing

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Polymer waveguides

Sartomer
n=1.478

PMMA/DIP
n=1.485

SiO₂
n=1.44

15 µm

8 µm
6 µm
4 µm

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Direct laser writing

- Maskless
- Waveguide symmetry is uniform and highly reproducible
- Fast waveguide fabrication
- No clean-room environment required
- Design flexibility

Top View

Side View

Femtosecond laser writing

R. Norwood lab
A new, high-speed technique to fabricate low loss EO polymer waveguide using all-reflective multiphoton imaging and direct laser writing system has been developed – provides for positioning EO polymer waveguides at arbitrary positions on nanophotonic chip with sub-micron resolution.