OPTI510R: Photonics

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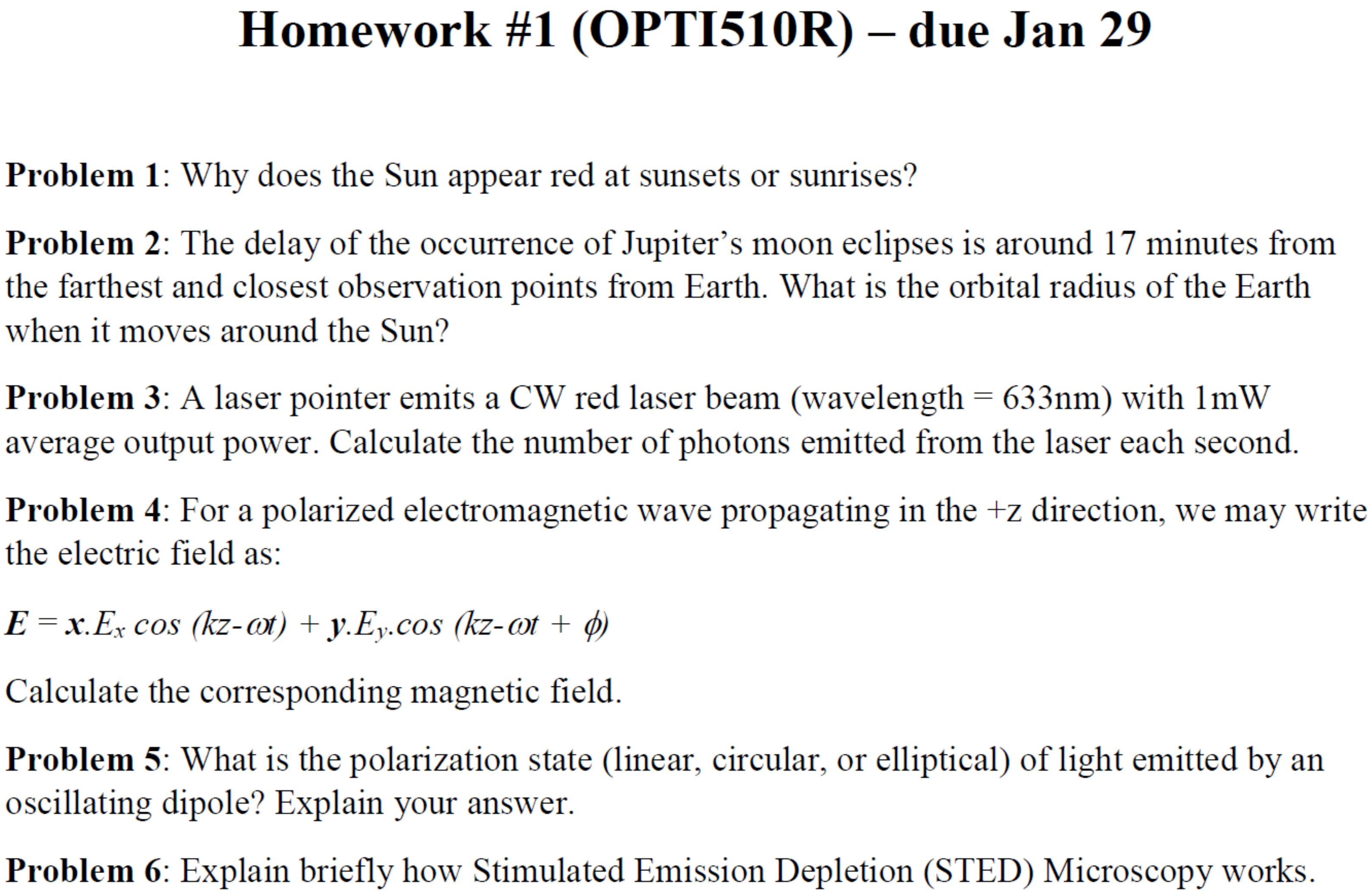
Announcements

- No class Monday, Feb 26
- Mid-term exam will be on Feb 28th (open books/notes)

Homework solutions



Homework #1 (OPTI510R) – due Jan 29

Problem 1: Why does the Sun appear red at sunsets or sunrises?

Problem 2: The delay of the occurrence of Jupiter's moon eclipses is around 17 minutes from the farthest and closest observation points from Earth. What is the orbital radius of the Earth when it moves around the Sun?

Problem 3: A laser pointer emits a CW red laser beam (wavelength = 633nm) with 1mW average output power. Calculate the number of photons emitted from the laser each second.

Problem 4: For a polarized electromagnetic wave propagating in the +z direction, we may write the electric field as:

$$\mathbf{E} = \mathbf{x}.E_x \cos(kz-\omega t) + \mathbf{y}.E_y.\cos(kz-\omega t + \phi)$$

Calculate the corresponding magnetic field.

Problem 5: What is the polarization state (linear, circular, or elliptical) of light emitted by an oscillating dipole? Explain your answer.

Problem 6: Explain briefly how Stimulated Emission Depletion (STED) Microscopy works.

Homework 2 (OPTI510R) – due Feb. 12

Problem 1: A single frequency fiber laser has a full width at half maximum (FWHM) spectral linewidth of 1 kHz. What is the spectral linewidth of the laser in nanometer. What is the coherent length of the laser beam? You can assume that the laser center wavelength is at 1550nm.

Problem 2: We use laser light in the C-band (with wavelength around 1550nm) for long distance optical communication. What is the energy of a single photon with wavelength of 1550nm? Due to limitation in detector's sensitivity we need a minimum of 1000 photon in a '1' bit of information. What would be the total light energy we need to detect for watching a movie which has a size of 1GB? You can assume that we don't need any photon for the '0' bit of information and the probabilities of having '1' or '0' bits are equal.

Problem 3: Obtain the unit vector along the direction of propagation for a wave, the displacement of which is given by:

$$\psi(x, y, z, t) = a * \cos(2x + 3y + 4z - 5t)$$

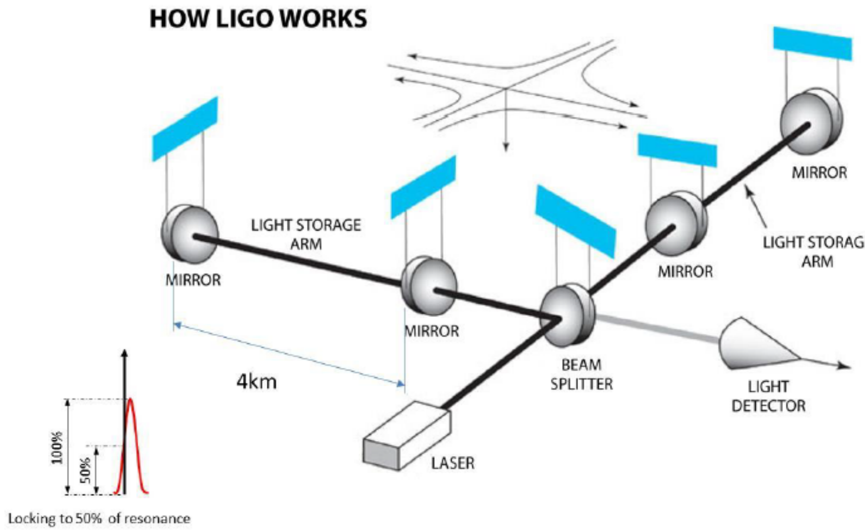
Where x, y, and z are measured in microns and t in femtoseconds. What is the frequency and wavelength of the wave?

Problem 4: Derive the complex susceptibility of a medium using the classical theory of permittivity. Calculate the real and imaginary part of the complex susceptibility. What is the behavior of the real and imaginary part of the complex susceptibility when the driving field frequency (ω) approaches zero, infinity, and the system resonant frequency Ω_0?

Problem 5: Calculate the time bandwidth product of the pulse shapes shown in the first column of the table below.

Pulse Shape	Fourier Transform	Pulse Width
$\underline{A}(t)$	$\underline{\tilde{A}}(\omega) = \int_{-\infty}^{\infty} a(t) e^{-j\omega t} dt$	Δt
Gaussian: $e^{-\frac{t^2}{2\tau^2}}$	$\sqrt{2\pi}\tau e^{-\frac{1}{2}\tau^2\omega^2}$	$2\sqrt{\ln 2}\tau$
Hyperbolic Secant: $\mathrm{sech}(\frac{t}{\tau})$	$\frac{\tau}{2}\,\mathrm{sech}\left(\frac{\pi}{2}\tau\omega\right)$	$1.7627\ \tau$
Rect-function: $= \begin{cases} 1, & \lvert t\rvert \le \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	$\tau \frac{\sin(\tau\omega/2)}{\tau\omega/2}$	τ
Lorentzian: $\frac{1}{1+(t/\tau)^2}$	$2\pi\tau e^{-\lvert\tau\omega\rvert}$	$1.287\ \tau$
Double-Exp.: $e^{-\left\lvert\frac{t}{\tau}\right\rvert}$	$\frac{\tau}{1+(\omega\tau)^2}$	$\ln 2\ \tau$

Problem 6:



To increase its sensitivity each arm of the LIGO interferometer works as a Fabry-Perot resonator. The reflectivity of each mirror is 99.99%. The distance between them is 4km.

a) What are the free spectral range, finesse, and resonant linewidth of the Fabry-Perot resonator?

b) Sketch the transmission spectrum as a function of frequency for light at normal incident. Label the vertical axis from 0 to 100%.

c) A powerful CW laser beam (wavelength 1064nm, 20W average power) is locked to a resonance line of the Fabry-Perot resonators (see the inset of the figure above). The locking point is set to be at half the transmission resonance line (50%) to get the maximum sensitivity with regard to change of the distance between the two mirrors.

- What is average power of light that the light detector detects? (Assuming identical path length for both arms and no loss in the system).
- A short burst of gravitational wave arrived to the LIGO interferometer and changed the distance between the two mirrors by about 1/100 the size of a proton (the size of a proton is roughly 1 femtometer). What is the power change on the light detector? You can assume that the distance between the two Fabry-Perot mirrors in one arm is reduced and in the other arm is increased.

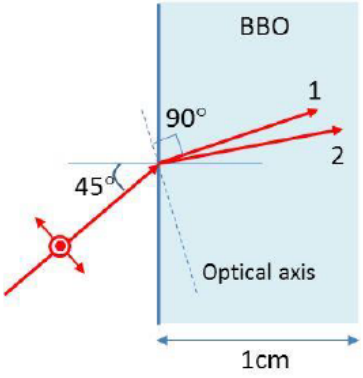
Homework #3 (OPTI510R) – due Feb. 21

Problem 1:

Beta Barium Borate (β-BaB_2O_4), or BBO, is one of the most versatile nonlinear optical crystal (uniaxial) materials available. BBO is most commonly used for second or higher order harmonic generation of Nd:YAG, Ti:Sapphire, Argon Ion and Alexandrite lasers.

Optical Homogeneity (cm^{-1})	$\delta n < 10^{-6}$
Transparency Range (nm)	190 - 3500
Sellmeier Equations (λ in μm)	$n_o^2 = 2.7405 + [0.0184/(\lambda^2 - 0.0179)] - 0.0155\lambda^2$ $n_e^2 = 2.3730 + [0.0128/(\lambda^2 - 0.0156)] - 0.0044\lambda^2$

An un-polarized Gaussian pulse with FWHM pulse duration of 100fs propagates through a 1cm thick **BBO** crystal as shown. The speed of light in vacuum is 2.9979×10^8m/s. The center wavelength of the pulse in vacuum is 1.55μm.



Give numerical answers to the following questions:

a) What are the phase velocities of the ordinary and extraordinary waves?
b) What is the group velocity of the extraordinary waves?
c) The pulse will be divided into two pulses as shown. Indicate the polarization for ray 1 and 2. Which ray is the extraordinary ray?
d) What is the time delay between the two pulses as they exit the crystal?
e) What is the duration of the extraordinary (use the Sellmeier equation for extraordinary wave) pulse at the output of the crystal?

Modal Dispersion. Light of wavelength $\lambda_o = 0.633$ μm is transmitted through a mirror waveguide of mirror separation $d = 10$ μm and $n = 1$. Determine the number of TE and TM modes. Determine the group velocities of the fastest and the slowest mode. If a narrow pulse of light is carried by all modes for a distance 1 m in the waveguide, how much does the pulse spread as a result of the differences of the group velocities?

Problem 3: 8.2-5 (From S&T textbook)

Field Distribution. The transverse distribution $u_m(y)$ of the electric-field complex amplitude of a TE mode in a slab waveguide is given by (8.2-10) and (8.2-13). Derive an expression for the ratio of the proportionality constants. Plot the distribution of the $m = 0$ TE mode for a slab waveguide with parameters $n_1 = 1.48$, $n_2 = 1.46$, $d = 0.5$ μm, and $\lambda_o = 0.85$ μm, and determine its confinement factor (percentage of power in the slab).

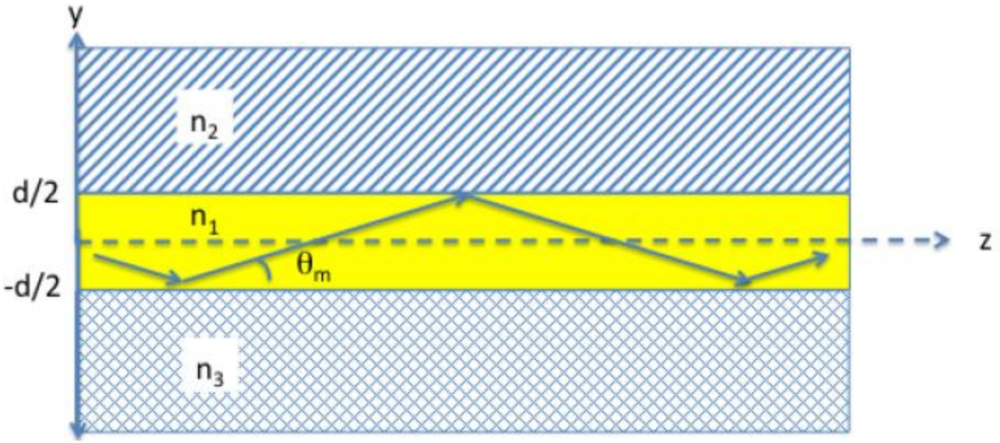
Problem 4:

Light of free-space wavelength $\lambda_0 = 0.87$ µm is guided by a thin planar film of width $d = 2$ µm and refractive index $n_1 = 1.6$ surrounded by a medium of refractive index $n_2 = 1.4$.

a) Determine the critical angle θ_c and its complement $\bar{\theta}_c$, the numerical aperture NA, and the maximum acceptance angle for light originating in air (n = 1).
b) Determine the number of TE modes.
c) Determine the bounce angle θ_0 and the group velocity v_0 of the $m = 0$ TE mode (5pts)
d) At which wavelengths the waveguide is singlemode?

Problem 5:

This problem concerns the waveguide structure shown below



A slab dielectric waveguide is made as shown. The thickness of the light guiding layer, with refractive index n_1, is d. The top cladding has refractive index n_2, and the bottom cladding has refractive index n_3, with $n_2 \neq n_3$. Light is propagated at wavelength λ (material wavelength) along the z direction.

a) Write down the self-consistency condition for the TE modes of this waveguide.

b) What is the transcendental equation that will be satisfied for the TE mode angle, θ_m? You may make use of the fact that

$$\tan(a+b) = \frac{\tan a + \tan b}{1 \square \tan a \tan b}$$

and you need not simplify the expression beyond the first step that shows clear dependence on known quantities only (other than θ_m).

c) If $n_1 - n_3 << n_1 - n_2$, roughly sketch how you expect the transverse profile of the lowest order TE waveguide mode to look.

d) In general, what is different about the lowest order TE mode of this waveguide and its symmetric equivalent (i.e. $n_2 = n_3$) as $d \rightarrow 0$?

e) What is the propagation constant β for the fundamental mode (m = 0) if $n_1 = 1.46$, $n_2 = n_3 = 1.44$, $d = 10$ µm, $\lambda = 1.04$ µm?

Review

- Properties of light
- Wave optics
- Interference and devices
- Diffraction and devices
- Polarization optics
- Guided-wave optics

Properties of light

Corpuscular theory of light: light consists of corpuscles or very small particles flying at finite velocity (Isaac Newton).

The wave theory of light: At first, the nature of light is wave propagating in a medium called *luminiferous aether.* But all attempts to detect the aether have failed so far. So the theory based on the aether was more or less abandoned. The **electromagnetic theory of light** was developed culminating in the Maxwell’s equations.

The quantum theory of light: In an attempt to explain blackbody radiation, Planck postulated that electromagnetic energy could be emitted only in quantized form, in other words, the energy could only be a multiple of an elementary unit, $E = h\nu$, where h is Planck's constant.

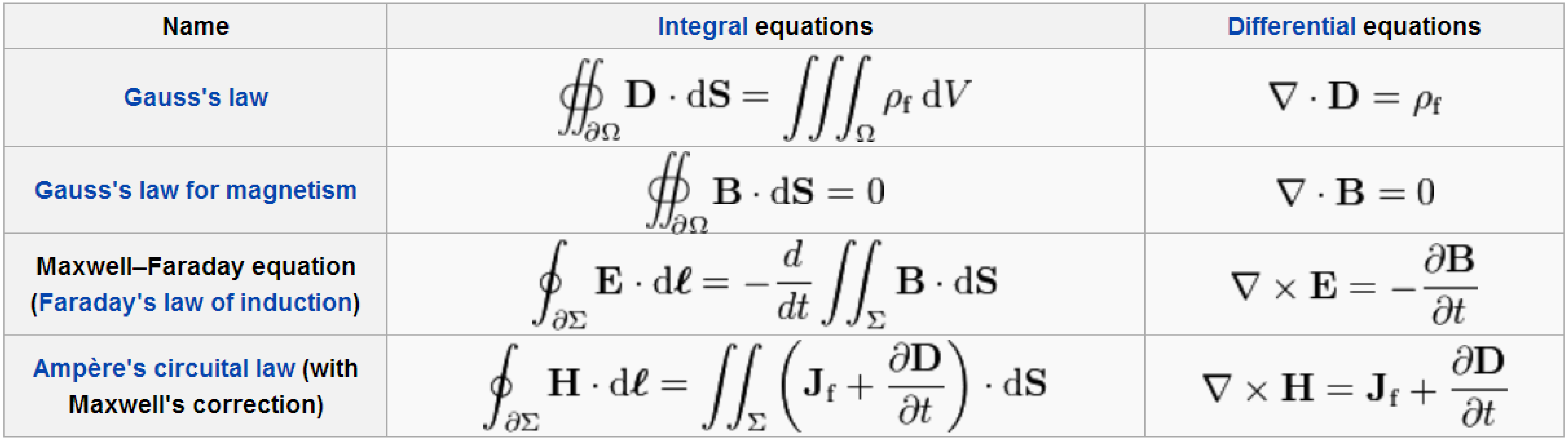
Properties of light

- Velocity of light
- Frequency and wavelength
- Polarization
- Coherence
- Other light characteristics

Wave optics

- Maxwell’s equations
- Wave equation
- Plane wave solution
- Classical theory of permittivity
- Sellmeier Equation and Kramers-Kroenig Relations

Maxwell’s equations



Name	Integral equations	Differential equations
Gauss's law	$\oiint_{\partial\Omega} \mathbf{D}\cdot \mathrm{d}\mathbf{S} = \iiint_{\Omega} \rho_\text{f}\,\mathrm{d}V$	$\nabla\cdot\mathbf{D} = \rho_\text{f}$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot \mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot \mathrm{d}\boldsymbol{\ell} = -\frac{d}{dt}\iint_{\Sigma} \mathbf{B}\cdot \mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's correction)	$\oint_{\partial\Sigma} \mathbf{H}\cdot \mathrm{d}\boldsymbol{\ell} = \iint_{\Sigma} \left(\mathbf{J}_\text{f} + \frac{\partial\mathbf{D}}{\partial t}\right)\cdot \mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{H} = \mathbf{J}_\text{f} + \frac{\partial\mathbf{D}}{\partial t}$

Here, **E** and **H** are the electric and magnetic field, **D** the dielectric flux, **B** the magnetic flux, $\mathbf{J}_\mathbf{f}$ the current density of free charges, $\boldsymbol{\rho}_\mathbf{f}$ is the free charge density.

Maxwell’s equations

Material equations:

$$\begin{aligned} \vec{D} &= \epsilon_0 \vec{E} + \vec{P}, \\ \vec{B} &= \mu_0 \vec{H} + \vec{M} \end{aligned}$$

P is the polarization, ***M*** is the magnetization

Uniform optical medium

$$\left\{\begin{aligned} \nabla \cdot \vec{E} &= 0 \\ \nabla \cdot \vec{B} &= 0 \\ \nabla \times \vec{E} &= -\frac{\partial \vec{B}}{\partial t} \\ \nabla \times \vec{B} &= \epsilon\mu \frac{\partial \vec{E}}{\partial t} \end{aligned}\right.$$

$$\begin{aligned} \nabla \times (\nabla \times \vec{E}) &= \nabla \times \left(-\frac{\partial \vec{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \vec{B}) \\ &= -\frac{\partial}{\partial t}\left(\epsilon\mu \frac{\partial \vec{E}}{\partial t}\right) \end{aligned}$$

$$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2 \vec{E}$$

$$\longrightarrow \quad \boxed{\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}}$$

Wave equation

Plane-wave solution

$$\vec{E}(\vec{r},t) = \vec{E}_0 e^{i(\omega t - \vec{k}\cdot\vec{r})}$$

Simple harmonic plane wave

$$\begin{aligned}\nabla^2 \vec{E} &= \vec{E}_0 \nabla^2 e^{i(\omega t - \vec{k}\cdot\vec{r})} \\ &= \vec{E}_0 \nabla \cdot \left[-i\vec{k} e^{i(\omega t - \vec{k}\cdot\vec{r})}\right] \\ &= -k^2 \vec{E}_0 e^{i(\omega t - \vec{k}\cdot\vec{r})} \\ &= -k^2 \vec{E}\end{aligned}$$

$$\mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = -\omega^2 \mu\epsilon \vec{E}$$

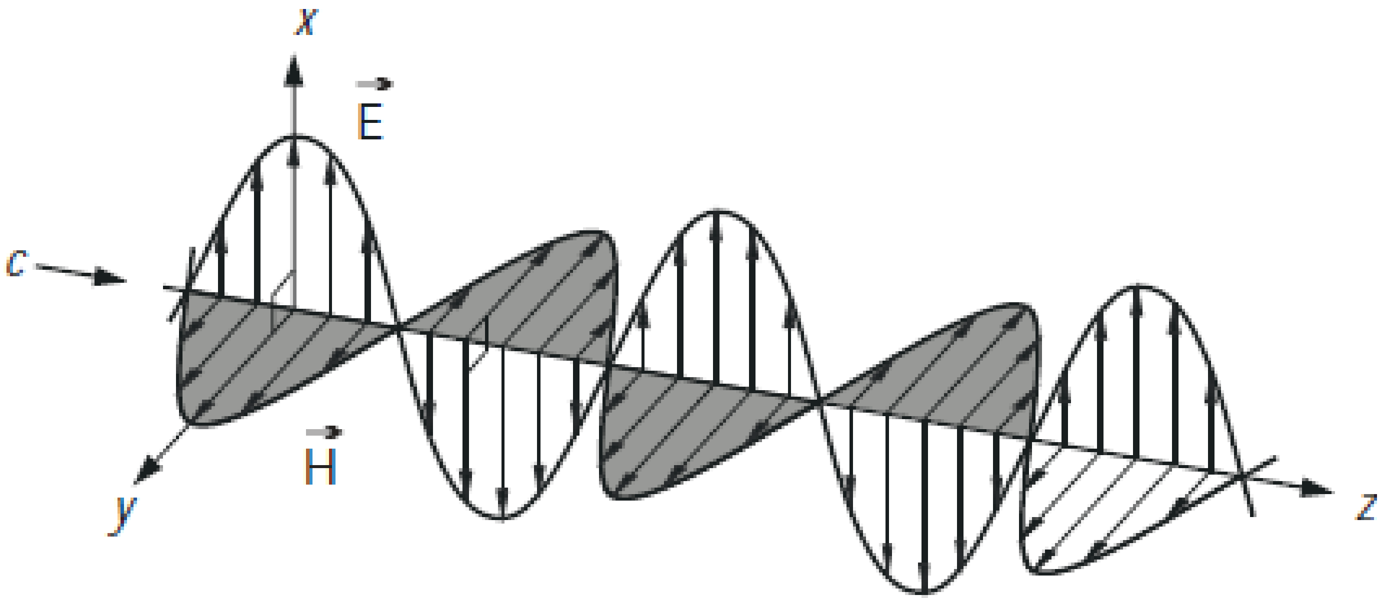
$$\boxed{\vec{k}\cdot\vec{E}_0 = 0}$$

$$\boxed{\vec{k}\times\vec{E} = \omega\vec{B}}$$

$$\longrightarrow \boxed{k = \omega\sqrt{\mu\epsilon}}$$

Dispersion relation

Plane-wave solution



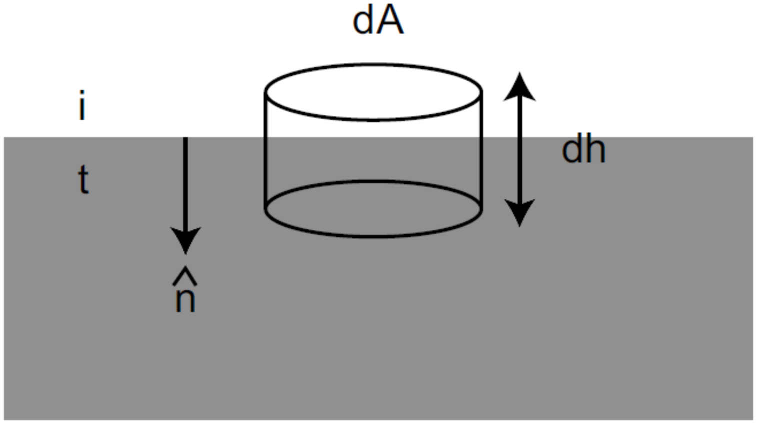
$$\vec{e} \perp \vec{h}, \quad \vec{k} \perp \vec{e}, \quad \vec{k} \perp \vec{h}.$$

Transverse electromagnetic wave

Boundary conditions at the interface

Maxwell's equations in integral form

$$\left\{\begin{array}{l} \oint_s \vec{D}\cdot d\vec{s} = 0 \\ \oint_s \vec{B}\cdot d\vec{s} = 0 \\ \oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int_s \vec{B}\cdot d\vec{s} \\ \oint \vec{H}\cdot d\vec{l} = \frac{\partial}{\partial t}\int_s \vec{D}\cdot d\vec{s} \end{array}\right.$$



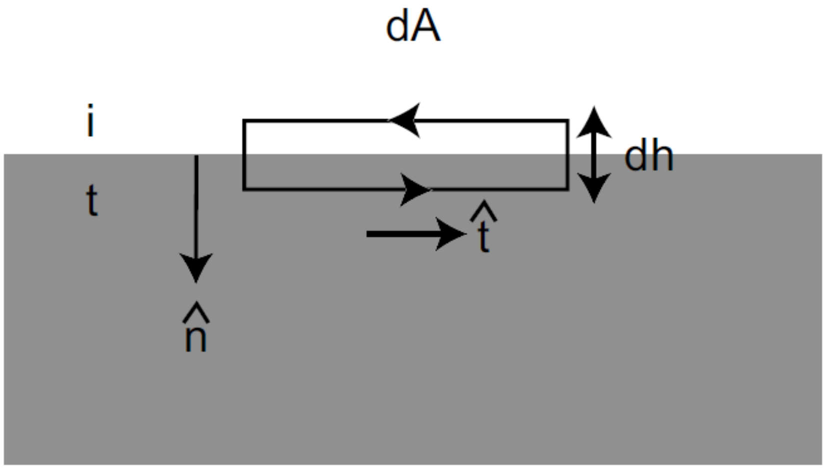
$$\vec{D}_i\cdot\hat{n} = \vec{D}_t\cdot\hat{n}$$
$$\vec{B}_i\cdot\hat{n} = \vec{B}_t\cdot\hat{n}$$

The normal component of D and B are continuous across the dielectric interface

Boundary conditions at the interface

Maxwell's equations in integral form

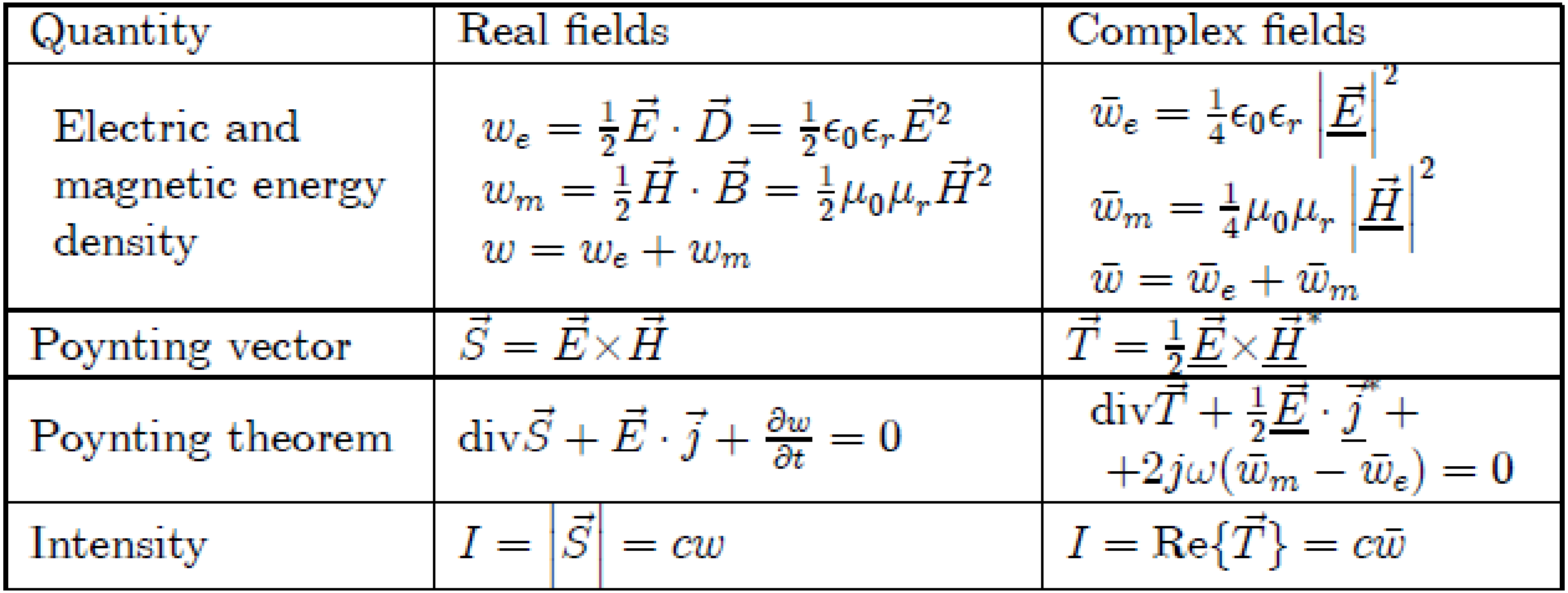
$$\begin{cases} \oint_s \vec{D} \cdot d\vec{s} = 0 \\ \oint_s \vec{B} \cdot d\vec{s} = 0 \\ \oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_s \vec{B} \cdot d\vec{s} \\ \oint \vec{H} \cdot d\vec{l} = \frac{\partial}{\partial t} \int_s \vec{D} \cdot d\vec{s} \end{cases}$$



$$\vec{E}_i \cdot \hat{t} = \vec{E}_t \cdot \hat{t}$$
$$\vec{H}_i \cdot \hat{t} = \vec{H}_t \cdot \hat{t}$$

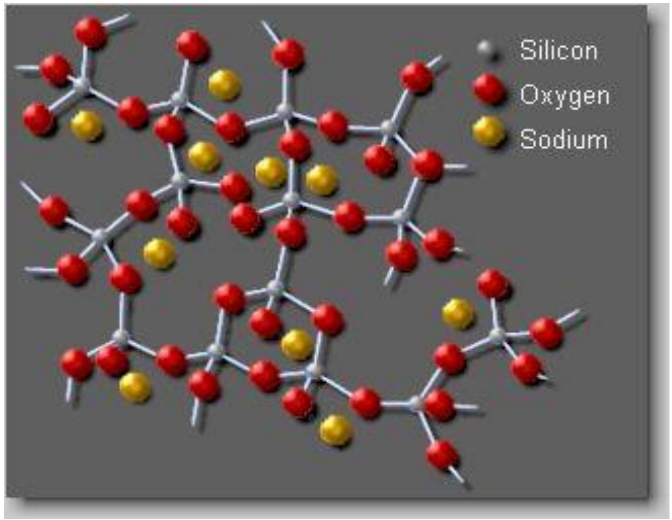
The tangential component of E and H are continuous across the dielectric interface

Pointing vector, energy and Intensity

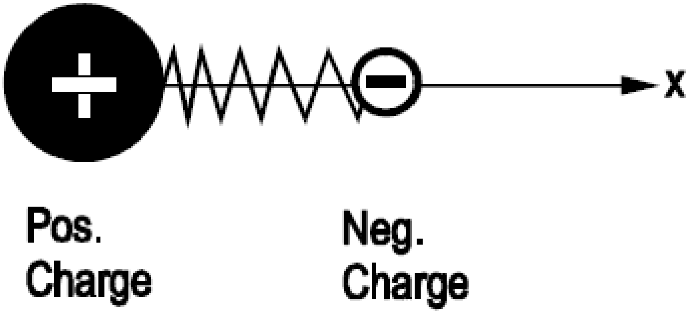


Quantity	Real fields	Complex fields
Electric and magnetic energy density	$w_e = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}\epsilon_0\epsilon_r\vec{E}^2$ $w_m = \frac{1}{2}\vec{H}\cdot\vec{B} = \frac{1}{2}\mu_0\mu_r\vec{H}^2$ $w = w_e + w_m$	$\bar{w}_e = \frac{1}{4}\epsilon_0\epsilon_r\left\lvert\underline{\vec{E}}\right\rvert^2$ $\bar{w}_m = \frac{1}{4}\mu_0\mu_r\left\lvert\underline{\vec{H}}\right\rvert^2$ $\bar{w} = \bar{w}_e + \bar{w}_m$
Poynting vector	$\vec{S} = \vec{E}\times\vec{H}$	$\vec{T} = \frac{1}{2}\underline{\vec{E}}\times\underline{\vec{H}}^*$
Poynting theorem	$\mathrm{div}\vec{S} + \vec{E}\cdot\vec{j} + \frac{\partial w}{\partial t} = 0$	$\mathrm{div}\vec{T} + \frac{1}{2}\underline{\vec{E}}\cdot\underline{\vec{j}}^* +$ $+2j\omega(\bar{w}_m - \bar{w}_e) = 0$
Intensity	$I = \left\lvert\vec{S}\right\rvert = cw$	$I = \mathrm{Re}\{\vec{T}\} = c\bar{w}$

Classical theory of permittivity



Glass



A dipole

$$\underline{\tilde{\vec{P}}}(\omega) = \frac{\text{dipole moment}}{\text{volume}} = N \cdot \langle \underline{\tilde{\vec{p}}}(\omega) \rangle = \epsilon_0 \tilde{\chi}(\omega) \underline{\tilde{\vec{E}}}(\omega),$$

(Induced polarization)

Classical theory of permittivity

The electron motion equation:

$$m\frac{d^2x}{dt^2} + 2\frac{\Omega_0}{Q}m\frac{dx}{dt} + m\Omega_0^2 x = e_0 E(t),$$

Where: $\underline{E}(t) = \underline{\tilde{E}}e^{j\omega t}$

Trial solution: $x\left(t\right) = \tilde{x}e^{j\omega t}$

Induced dipole moment: $p(t) = e_0 x(t) = \underline{\tilde{p}}e^{j\omega t}$

Classical theory of permittivity

$$\longrightarrow \qquad \underline{\tilde{p}} = \frac{\frac{e_0^2}{m}}{(\Omega_0^2 - \omega^2) + 2\mathrm{j}\frac{\Omega_0}{Q}\omega}\underline{\tilde{E}}.$$

,for 1 dipole

$$\longrightarrow \qquad \underline{\chi}(\omega) = \frac{N\frac{e_0^2}{m}\frac{1}{\epsilon_0}}{(\Omega_0^2 - \omega^2) + 2\mathrm{j}\omega\frac{\Omega_0}{Q}}$$

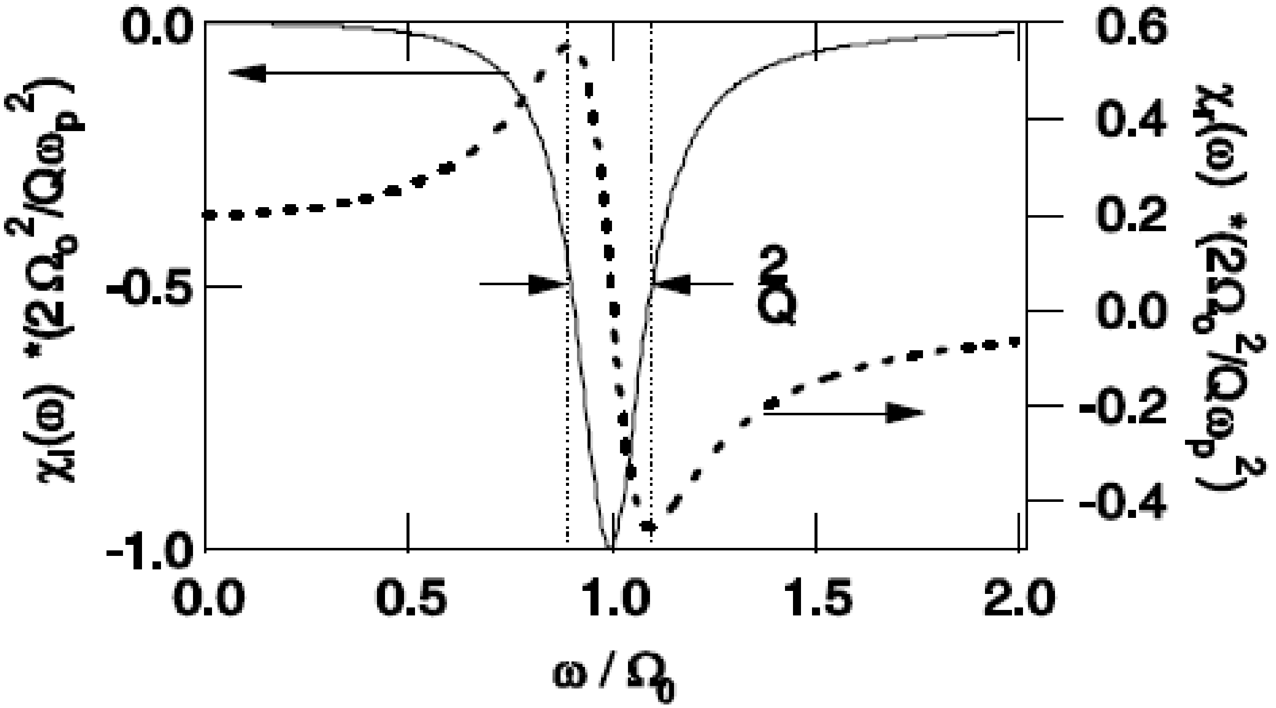
or:

$$\underline{\tilde{\chi}}(\omega) = \frac{\omega_p^2}{(\Omega_0^2 - \omega^2) + 2\mathrm{j}\omega\frac{\Omega_0}{Q}}$$

,where: $\omega_p^2 = Ne_0^2/m\epsilon_0$

(Plasma frequency)

Classical theory of permittivity



Real part (dashed line) and imaginary part (solid line) of the susceptibility of the classical oscillator model for the dielectric polarizability

Sellmeier Equation and Kramers-Kroenig Relations

$$\chi_r(\Omega) = \frac{2}{\pi}\int\limits_0^\infty \frac{\omega\chi_i(\omega)}{\omega^2-\Omega^2}d\omega = n_r^2(\Omega)-1,$$

$$\chi_i(\Omega) = -\frac{2}{\pi}\int\limits_0^\infty \frac{\Omega\chi_r(\omega)}{\omega^2-\Omega^2}d\omega.$$

The refractive index and absorption of a medium are not independent

Sellmeier Equation and Kramers-Kroenig Relations

If the media are used in a frequency range far away from resonances. Then the imaginary part of the susceptibility related to absorption can be approximated by:

$$\chi_i(\Omega) = \sum_i A_i \delta\left(\omega - \omega_i\right)$$

The Kramers-Kroenig relation results in the Sellmeier Equation for the refractive index:

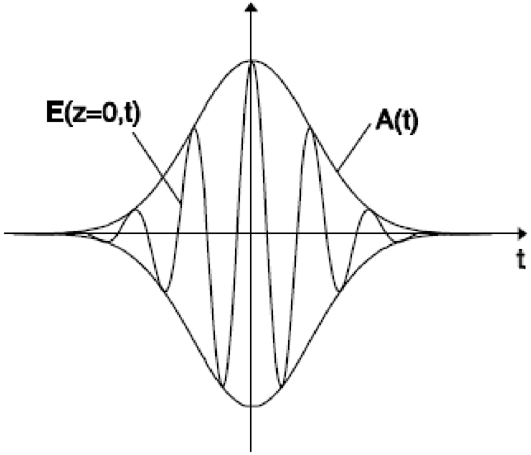
$$n^2(\Omega) = 1 + \sum_i A_i \frac{\omega_i}{\omega_i^2 - \Omega^2} = 1 + \sum_i a_i \frac{\lambda^2}{\lambda^2 - \lambda_i^2}.$$

Pulse propagation

Optical pulses often have relatively small spectral width compared to the center frequency of the pulse ω_0. In such cases it is useful to separate the complex electric field into a carrier frequency ω_0 and an envelope $A(t)$ and represent the absolute frequency as $\Omega = \omega_0 + \omega$. We can then rewrite:

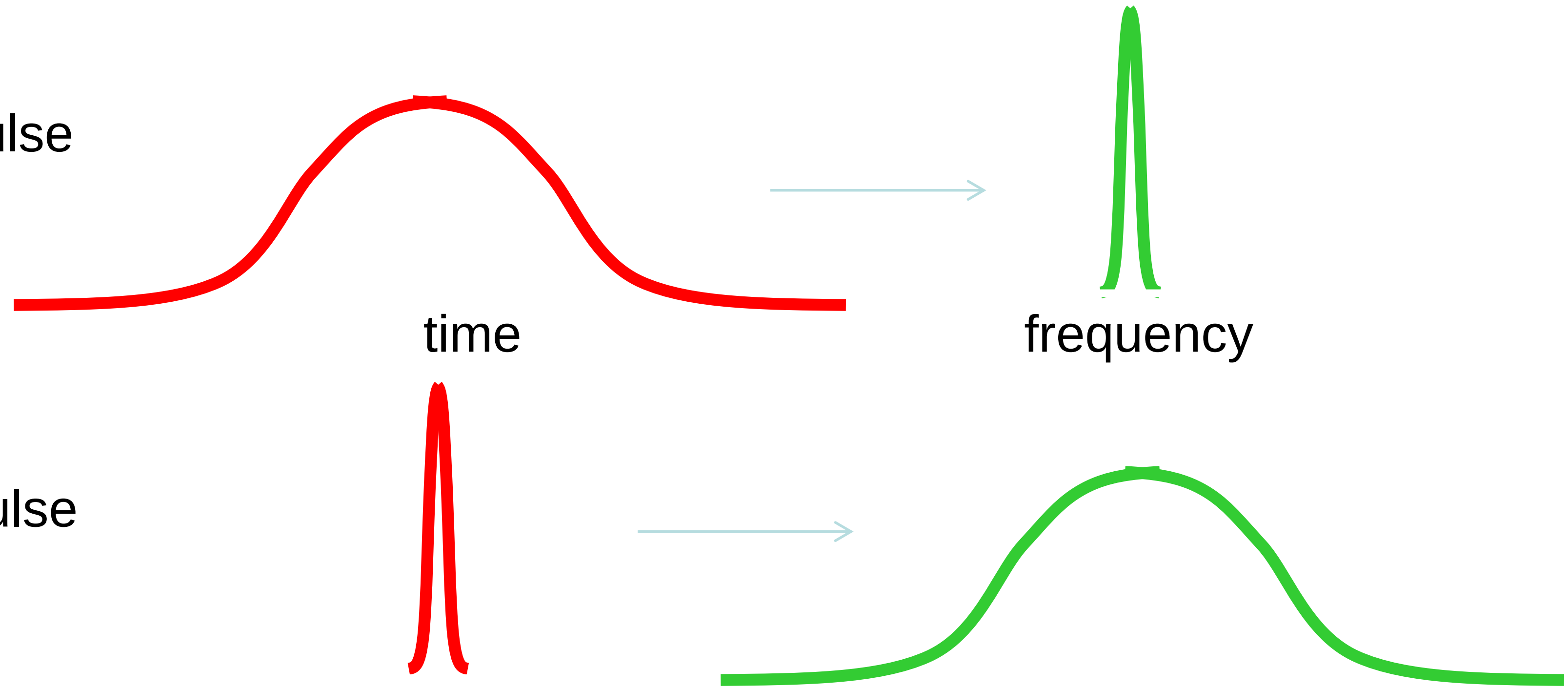
$$\begin{aligned} \underline{E}(z &= 0,t) = \frac{1}{2\pi}\int_{-\omega_0}^{\infty} \underline{\tilde{E}}(\omega_0+\omega)e^{j(\omega_0+\omega)t}d\omega \\ &= A(t)e^{j\omega_0 t}. \end{aligned}$$

The optical spectrum of a pulse can be calculated through the Fourier transform of the envelop.



Pulse propagation

Time domain and frequency domain:



Spectrum

Long pulse

time

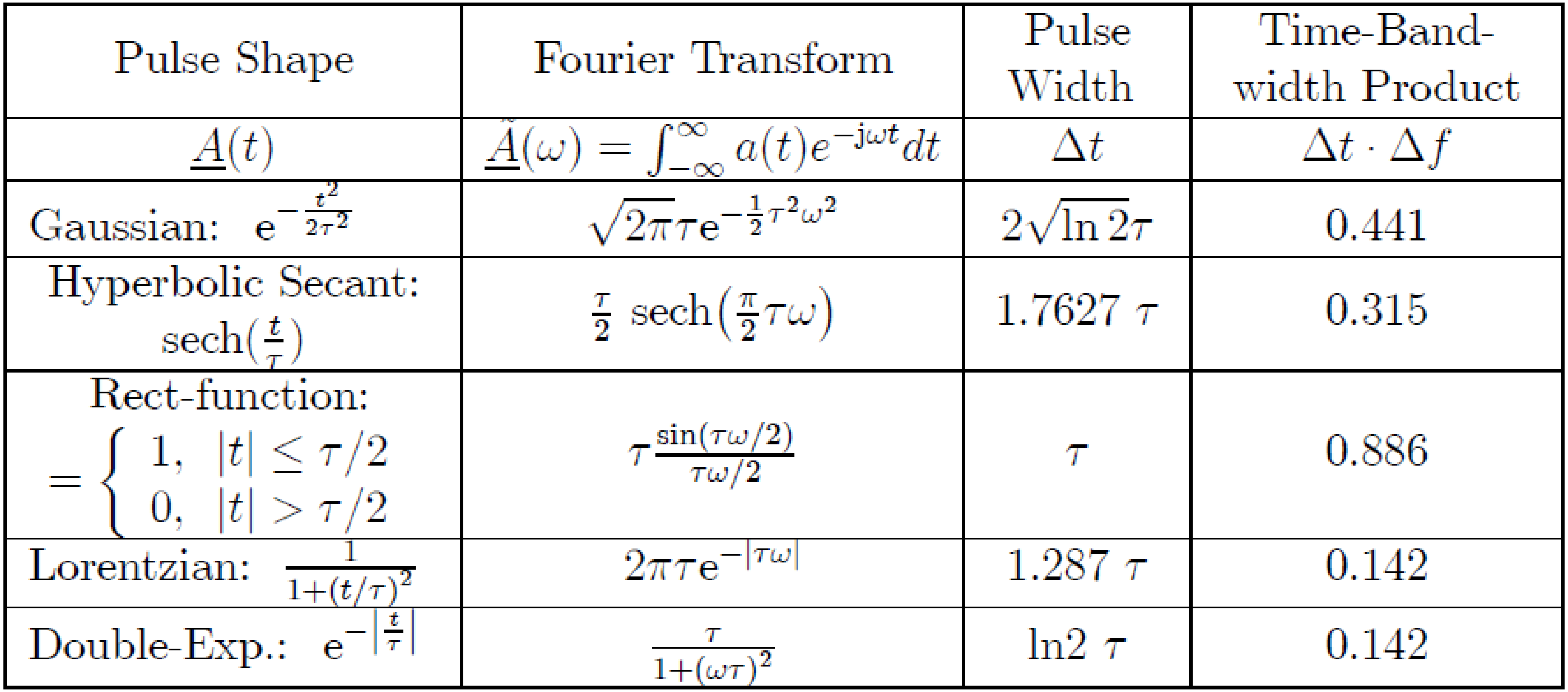
frequency

Short pulse

time

frequency

Pulse propagation



Pulse Shape	Fourier Transform	Pulse Width	Time-Band-width Product
$\underline{A}(t)$	$\underline{\tilde{A}}(\omega) = \int_{-\infty}^{\infty} a(t) e^{-j\omega t} dt$	Δt	$\Delta t \cdot \Delta f$
Gaussian: $e^{-\frac{t^2}{2\tau^2}}$	$\sqrt{2\pi}\tau e^{-\frac{1}{2}\tau^2\omega^2}$	$2\sqrt{\ln 2}\tau$	0.441
Hyperbolic Secant: $\mathrm{sech}(\frac{t}{\tau})$	$\frac{\tau}{2}\ \mathrm{sech}\left(\frac{\pi}{2}\tau\omega\right)$	$1.7627\ \tau$	0.315
Rect-function: $= \begin{cases} 1, & \lvert t\rvert \le \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	$\tau \frac{\sin(\tau\omega/2)}{\tau\omega/2}$	τ	0.886
Lorentzian: $\frac{1}{1+(t/\tau)^2}$	$2\pi\tau e^{-\lvert\tau\omega\rvert}$	$1.287\ \tau$	0.142
Double-Exp.: $e^{-\left\lvert\frac{t}{\tau}\right\rvert}$	$\frac{\tau}{1+(\omega\tau)^2}$	$\ln 2\ \tau$	0.142

Pulse shapes, corresponding spectra and time bandwidth products

Pulse propagation

We can separate an optical pulse into a carrier wave at frequency ω_0 and a complex envelope:

$$\underline{E}(z,t) = \underline{A}(z,t)e^{\mathrm{j}(\omega_0 t - K(\omega_0)z)}$$

By introducing the offset frequency ω, the offset wave-number k(ω) and spectrum of the envelope $\underline{\tilde{A}}(\omega)$:

$$\begin{aligned} \omega &= \Omega - \omega_0, \\ k(\omega) &= K(\omega_0 + \omega) - K(\omega_0), \\ \underline{\tilde{A}}(\omega) &= \underline{\tilde{E}}(\Omega = \omega_0 + \omega). \end{aligned}$$

We have:

$$\underline{A}(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \underline{\tilde{A}}(\omega)e^{\mathrm{j}(\omega t - k(\omega)z)}d\omega,$$

Dispersion

In general dispersion is a function of frequency:

$$k(\omega) = k'\omega + \frac{k''}{2}\omega^2 + \frac{k^{(3)}}{6}\omega^3 + O(\omega^4)$$

$$\frac{\partial \underline{\tilde{A}}(z,\omega)}{\partial z} = -\mathrm{j}k(\omega)\underline{\tilde{A}}(z,\omega)$$,in the frequency domain

$$\frac{\partial \underline{A}(z,t)}{\partial z} = -\mathrm{j}\sum_{n=1}^{\infty}\frac{k^{(n)}}{n!}\left(-\mathrm{j}\frac{\partial}{\partial t}\right)^n \underline{A}(z,t)$$,in the time domain

Group velocity

If we keep only the first term:

$$\frac{\partial \underline{A}(z,t)}{\partial z} + \frac{1}{v_{g0}}\frac{\partial \underline{A}(z,t)}{\partial t} = 0$$

$$v_{g0} = 1/k' = \left(\left. \frac{dk(\omega)}{d\omega} \right|_{\omega=0} \right)^{-1}$$

This is the equation describing a wave-package moving at the speed of v_{g0}

Group velocity

$$\begin{aligned} z' &= z, \\ t' &= t - z/v_{g0}, \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial z} &= \frac{\partial}{\partial z'} - \frac{1}{v_{g0}}\frac{\partial}{\partial t'}, \\ \frac{\partial}{\partial t} &= \frac{\partial}{\partial t'} \end{aligned}$$

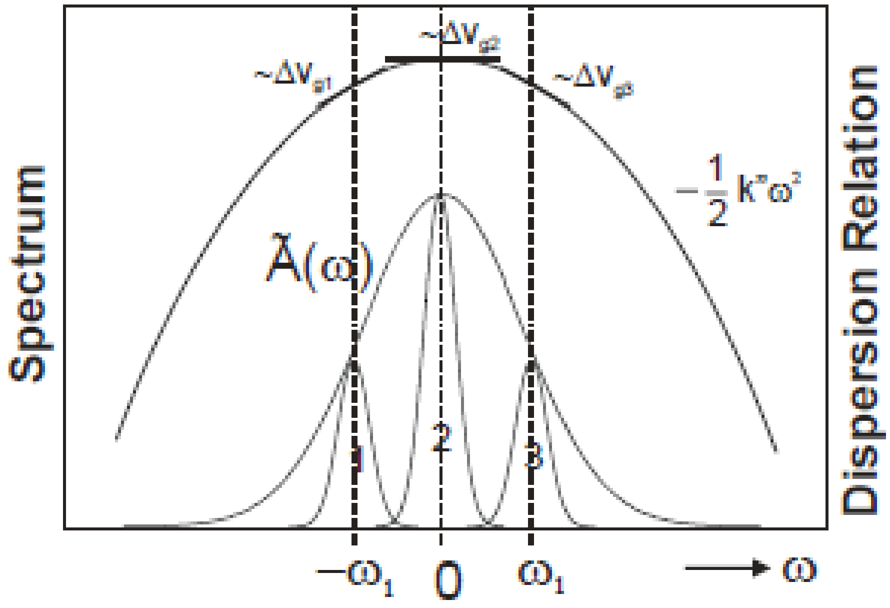
$$\frac{\partial \underline{A}(z', t')}{\partial z'} = 0$$

The pulse travels with the group velocity without changing its shape!

Pulse spreading due to second order dispersion

$$\frac{\partial \underline{A}(z,t')}{\partial z} = j\frac{k''}{2}\frac{\partial^2 \underline{A}(z,t')}{\partial t'^2}$$

This equation does not have an analytical solution, generally.



Decomposition of a pulse into wave packets with different center frequencies. In a medium with dispersion the wave-packets move at different relative group velocities.

Pulse spreading due to second order dispersion

$$\frac{\partial \underline{A}(z,t')}{\partial z} = \mathrm{j}\frac{k''}{2}\frac{\partial^2 \underline{A}(z,t')}{\partial t'^2}$$

k" is the group velocity dispersion (GVD)

Fortunately, for a Gaussian pulse, the pulse propagation equation can be solved analytically. The initial pulse is then of the form:

$$\begin{aligned} \underline{E}(z &= 0,t) = \underline{A}(z=0,t)e^{\mathrm{j}\omega_0 t} \\ \underline{A}(z &= 0, t=t') = \underline{A}_0 \exp\left[-\frac{1}{2}\frac{t'^2}{\tau^2}\right] \end{aligned}$$

$$\frac{\partial \underline{\tilde{A}}(z,\omega)}{\partial z} = -\mathrm{j}\frac{k''\omega^2}{2}\underline{\tilde{A}}(z,\omega)$$

(We have this from the Fourier transform)

$$\longrightarrow \quad \underline{\tilde{A}}(z,\omega) = \underline{\tilde{A}}(z=0,\omega)\exp\left[-\mathrm{j}\frac{k''\omega^2}{2}z\right]$$

Pulse spreading due to second order dispersion

Since:

$$\begin{aligned}\underline{E}(z &= 0,t) = \underline{A}(z=0,t)e^{\mathrm{j}\omega_0 t}\\ \underline{A}(z &= 0,t=t') = \underline{A}_0 \exp\left[-\frac{1}{2}\frac{t'^2}{\tau^2}\right]\end{aligned}$$

Fourier transform of the Gaussian function is a Gaussian function:

$$\underline{\tilde{A}}(z=0,\omega) = A_0\sqrt{2\pi}\tau \exp\left[-\frac{1}{2}\tau^2\omega^2\right]$$

Therefore, in the spectral domain the solution at an arbitrary propagation distance z is:

$$\underline{\tilde{A}}(z,\omega) = A_0\sqrt{2\pi}\tau \exp\left[-\frac{1}{2}\left(\tau^2 + \mathrm{j}k''z\right)\omega^2\right]$$

Pulse spreading due to second order dispersion

Now we go back to the time domain (again using the Fourier transform):

$$\underline{A}(z,t') = A_0 \left(\frac{\tau^2}{(\tau^2 + \mathrm{j}k''z)}\right)^{1/2} \exp\left[-\frac{1}{2}\frac{t'^2}{(\tau^2 + \mathrm{j}k''z)}\right]$$

After splitting the real and imaginary part we get:

$$\underline{A}(z,t') = A_0 \left(\frac{\tau^2}{(\tau^2 + \mathrm{j}k''z)}\right)^{1/2} \exp\left[-\frac{1}{2}\frac{\tau^2 t'^2}{(\tau^4 + (k''z)^2)} + \mathrm{j}\frac{1}{2}k''z\frac{t'^2}{(\tau^4 + (k''z)^2)}\right]$$

Here is the starting point:

$$\underline{A}(z \;=\; 0, t = t') = \underline{A}_0 \exp\left[-\frac{1}{2}\frac{t'^2}{\tau^2}\right]$$

Pulse spreading due to second order dispersion

Initial pulse duration: $\tau_{FWHM} = 2\sqrt{\ln 2}\,\tau$

$$\exp\left[-\frac{|\tau(\tau'_{FWHM}/2)|^2}{(\tau^4+(k''z)^2)}\right] = 0.5$$

FWHM pulse duration after propagating distance *L*:

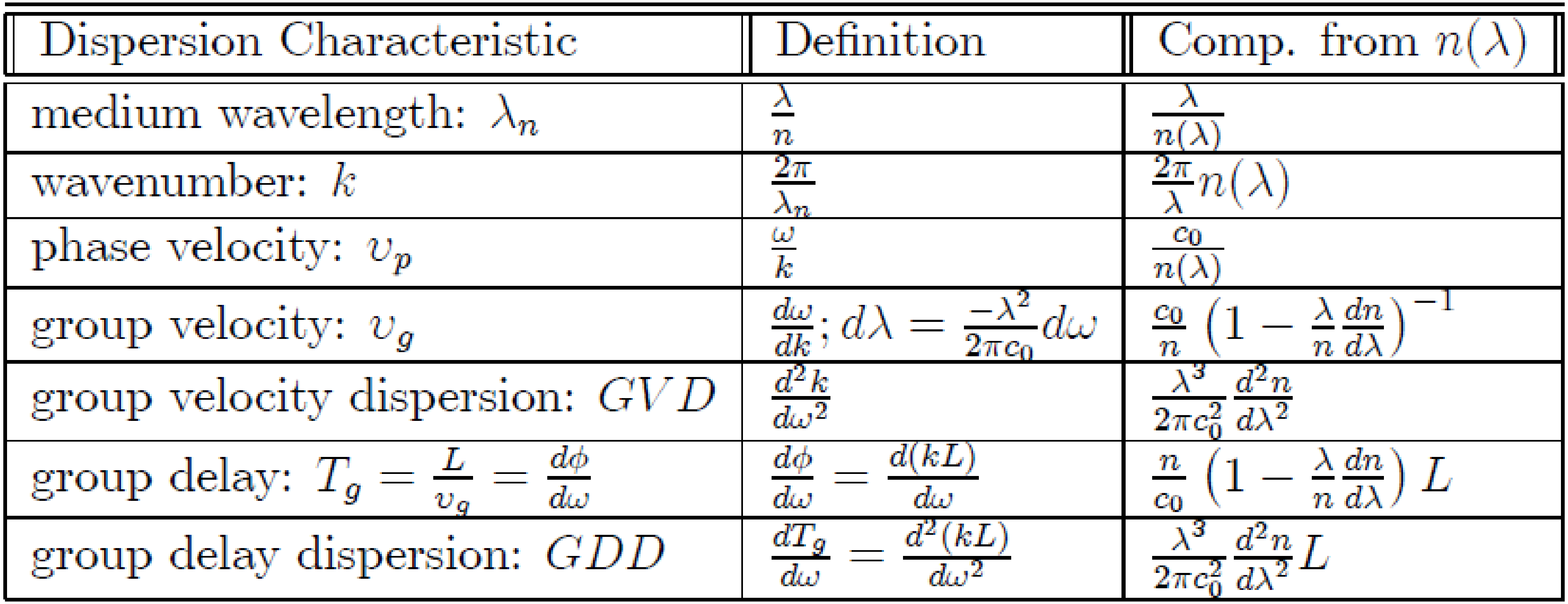
$$\tau'_{FWHM} = 2\sqrt{\ln 2}\,\tau\sqrt{1+\left(\frac{k''L}{\tau^2}\right)^2}$$

Pulse spreading due to second order dispersion

$$\frac{\tau'_{FWHM}}{\tau_{FWHM}} = \sqrt{1 + \left(\frac{k''L}{\tau^2}\right)^2} \sim \frac{k''L}{\tau^2}$$

- Pulse spreading is proportional to propagation distance
- Pulse spreading is proportional to the GVD
- Inversely proportional to the (initial pulse duration)^2

Pulse spreading due to second order dispersion



Dispersion Characteristic	Definition	Comp. from $n(\lambda)$
medium wavelength: λ_n	$\frac{\lambda}{n}$	$\frac{\lambda}{n(\lambda)}$
wavenumber: k	$\frac{2\pi}{\lambda_n}$	$\frac{2\pi}{\lambda}n(\lambda)$
phase velocity: v_p	$\frac{\omega}{k}$	$\frac{c_0}{n(\lambda)}$
group velocity: v_g	$\frac{d\omega}{dk}; d\lambda = \frac{-\lambda^2}{2\pi c_0}d\omega$	$\frac{c_0}{n}\left(1 - \frac{\lambda}{n}\frac{dn}{d\lambda}\right)^{-1}$
group velocity dispersion: GVD	$\frac{d^2k}{d\omega^2}$	$\frac{\lambda^3}{2\pi c_0^2}\frac{d^2n}{d\lambda^2}$
group delay: $T_g = \frac{L}{v_g} = \frac{d\phi}{d\omega}$	$\frac{d\phi}{d\omega} = \frac{d(kL)}{d\omega}$	$\frac{n}{c_0}\left(1 - \frac{\lambda}{n}\frac{dn}{d\lambda}\right)L$
group delay dispersion: GDD	$\frac{dT_g}{d\omega} = \frac{d^2(kL)}{d\omega^2}$	$\frac{\lambda^3}{2\pi c_0^2}\frac{d^2n}{d\lambda^2}L$

Important parameters

Dispersion example: Fused silica

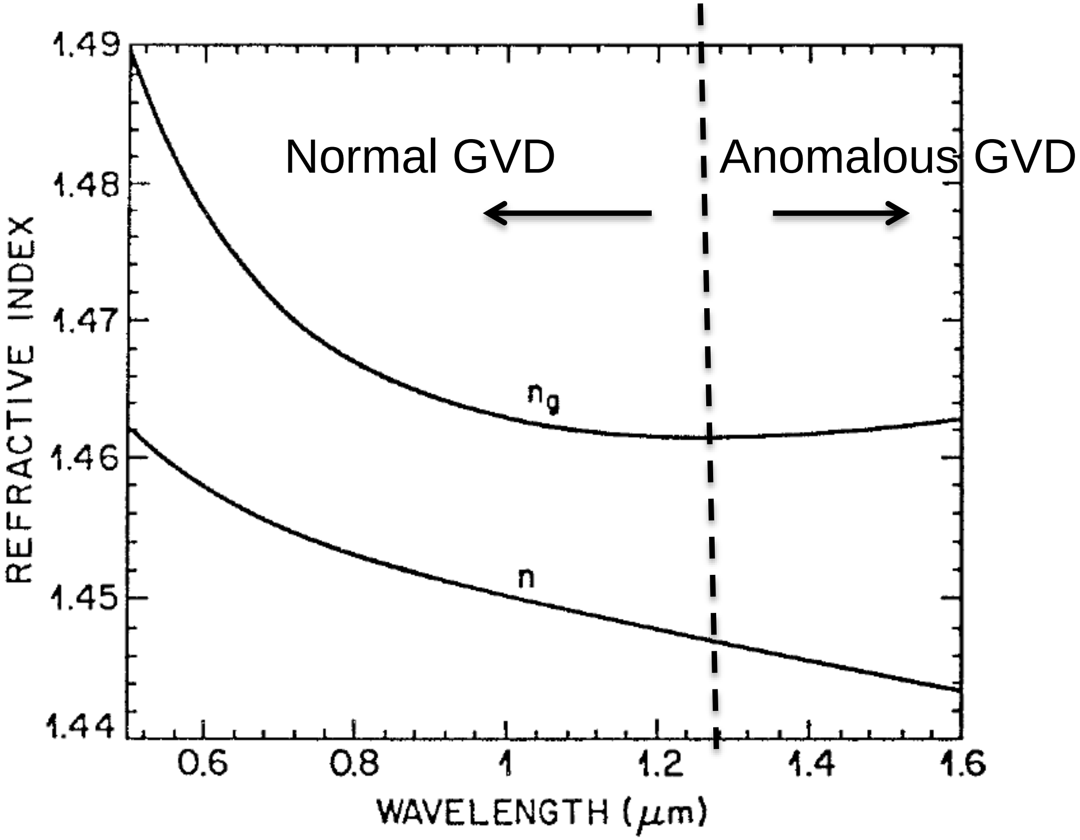
Sellmeier equation:

$$n^2(\lambda) = 1 + \sum_{i=1}^{3} \frac{A_i \lambda^2}{\lambda^2 - \lambda_i^2}$$

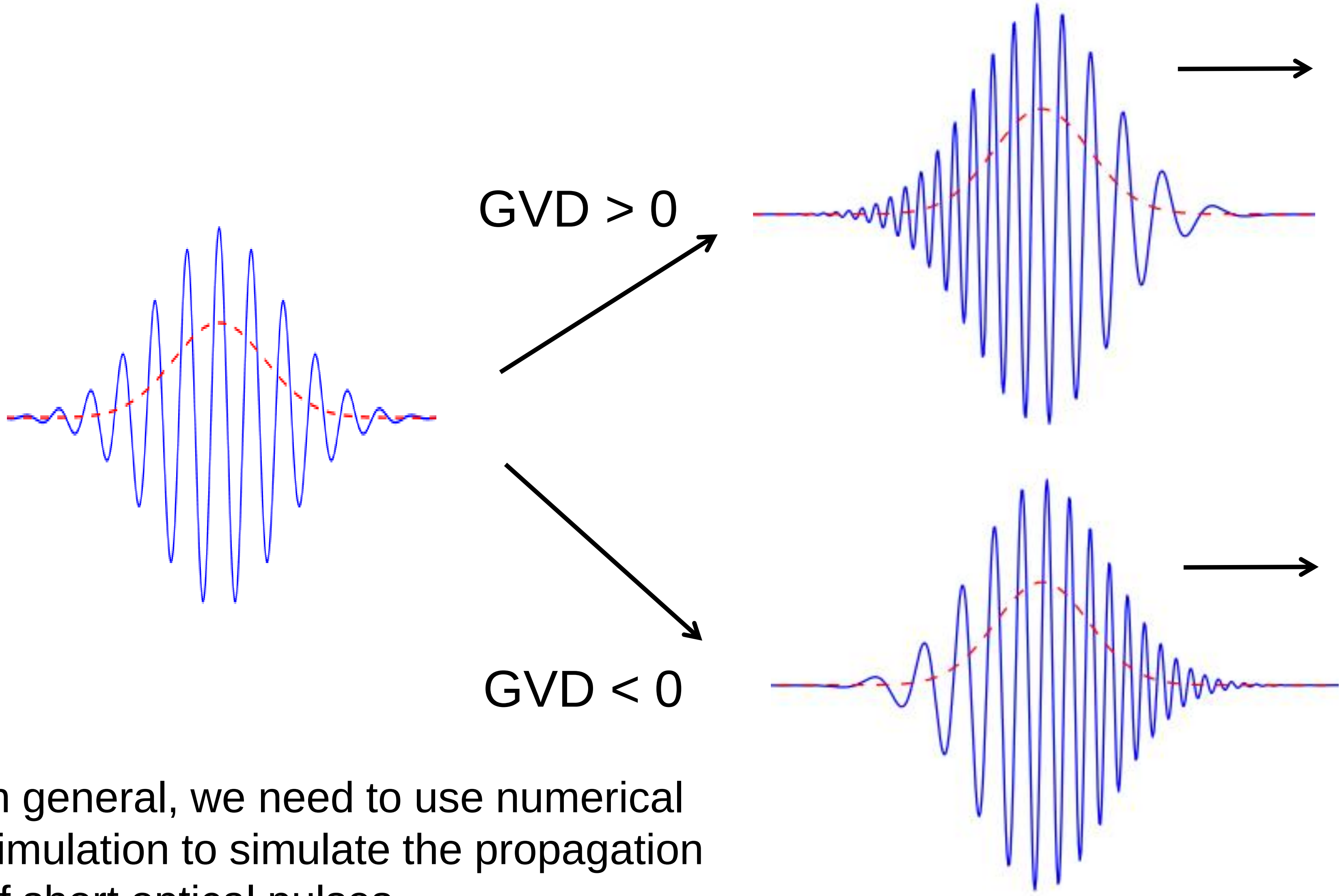
$n(\lambda)$	is the refractive index
A_i	is the i:th Sellmeier coefficient related to the i:th oscillator wavelength ($i = 1 \ldots 3$)
λ_i	is the i:th oscillator wavelength
λ	is the signal wavelength

$$\lambda_{1s} = 0.0684\ \mu\text{m}, \quad A_{1s} = 0.69617$$
$$\lambda_{2s} = 0.1162\ \mu\text{m}, \quad A_{2s} = 0.40794$$
$$\lambda_{3s} = 9.8962\ \mu\text{m}, \quad A_{3s} = 0.89748$$

Dispersion example-fused silica



Chirped pulses



In general, we need to use numerical simulation to simulate the propagation of short optical pulses.

Other important topics

- Fresnel reflection
- Brewster angle
- Total internal reflection (phase shift, evanescent field)
- Goos-Haenchen phase shift
- Frustrated TIR
- Metamaterials

Interference and Devices

- Interference
- Interferometers (Michelson, Mach-Zehnder, Fabry-Perot, Sagnac, etc.)
- Autocorrelator
- Mach-Zehnder Modulators
- Sagnac interferometer (Rotation sensor)

Interference

Two plane waves (solutions of the wave equation) interference:

$$\begin{aligned}\vec{E}_1(\vec{r},t) &= E_1\cos(\omega_1 t - \vec{k}_1\cdot\vec{r} + \varphi_1)\ \vec{e}_1,\\ \vec{E}_2(\vec{r},t) &= E_2\cos(\omega_2 t - \vec{k}_1\cdot\vec{r} + \varphi_2)\ \vec{e}_2,\end{aligned}$$

$$\vec{E}(\vec{r},t) = \vec{E}_1(\vec{r},t) + \vec{E}_2(\vec{r},t)$$

(still a solution of the wave equation)

We detect the intensity instead of the amplitude:

$$\vec{E}(\vec{r},t)^2 = \left(\vec{E}_1(\vec{r},t) + \vec{E}_2(\vec{r},t)\right)^2$$

$$\vec{E}(\vec{r},t)^2 = \vec{E}_1(\vec{r},t)^2 + \vec{E}_2(\vec{r},t)^2 + 2\vec{E}_1(\vec{r},t)\cdot\vec{E}_2(\vec{r},t)$$

Interference

$$\vec{E}_1(\vec{r},t)^2 = \frac{E_1^2}{2}\left(1+\cos 2(\omega_1 t - \vec{k}_1\cdot\vec{r}+\varphi_1)\right),$$

$$\vec{E}_2(\vec{r},t)^2 = \frac{E_2^2}{2}\left(1+\cos 2(\omega_2 t - \vec{k}_2\cdot\vec{r}+\varphi_2)\right),$$

$$\begin{aligned}\vec{E}_1(\vec{r},t)\cdot\vec{E}_2(\vec{r},t) \quad &= \quad (\vec{e}_1\cdot\vec{e}_2)\,E_1E_2\cos(\omega_1 t-\vec{k}_1\cdot\vec{r}+\varphi_1)\\ &\qquad \cdot\cos(\omega_2 t-\vec{k}_2\cdot\vec{r}+\varphi_2)\end{aligned}$$

$$\longrightarrow \quad \vec{E}_1(\vec{r},t)\cdot\vec{E}_2(\vec{r},t) = \frac{1}{2}(\vec{e}_1\cdot\vec{e}_2)\,E_1E_2\,\cdot$$

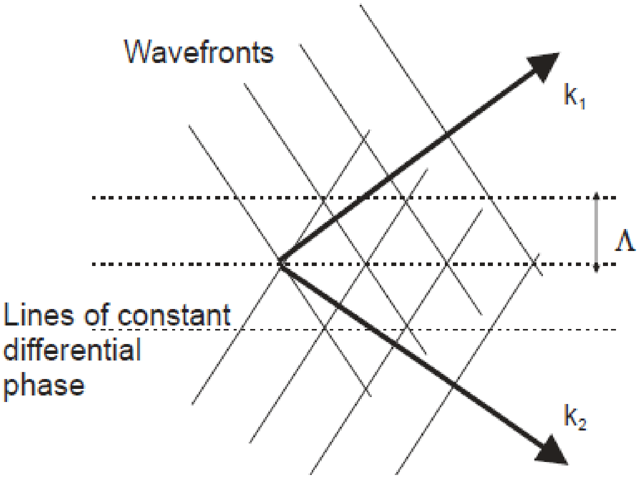
$$\cdot\left[\begin{array}{c}\cos\left((\omega_1-\omega_2)\,t-\left(\vec{k}_1-\vec{k}_2\right)\cdot\vec{r}+(\varphi_1-\varphi_2)\right)\\ +\cos\left((\omega_1+\omega_2)\,t-\left(\vec{k}_1+\vec{k}_2\right)\cdot\vec{r}+(\varphi_1+\varphi_2)\right)\end{array}\right]$$

Interference

Since the oscillation is very fast, the time average becomes:

$$\overline{\vec{E}(\vec{r},t)^2} = \frac{E_1^2}{2} + \frac{E_2^2}{2} + (\vec{e}_1 \cdot \vec{e}_2)\, E_1 E_2 \cdot$$

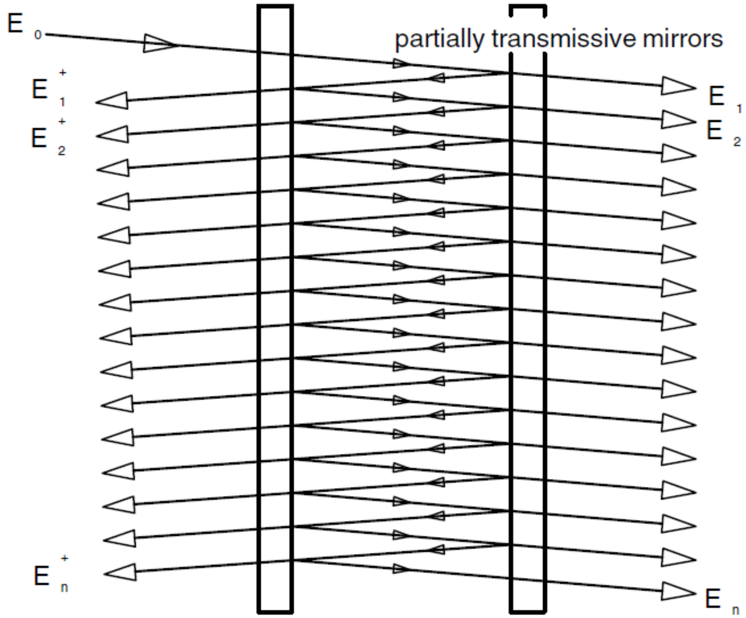
$$\cdot \cos\left((\omega_1 - \omega_2)\, t - \left(\vec{k}_1 - \vec{k}_2\right) \cdot \vec{r} + (\varphi_1 - \varphi_2)\right)$$

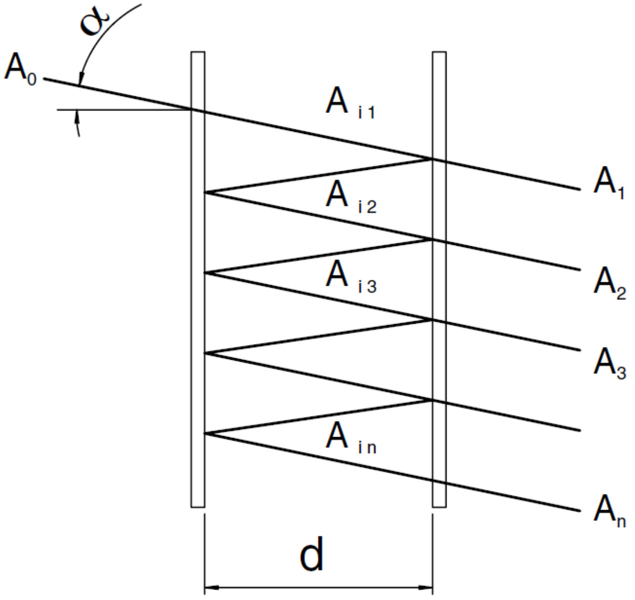


$$\Lambda = \frac{2\pi}{\left|\vec{k}_1 - \vec{k}_2\right|}$$

Interference pattern generated by two monochromatic plane waves

Fabry-Perot Resonator





R = 1-T (T = Transmission)

$$E = A \cdot \cos(\omega t + kx + \delta)$$

$$I_1 = (1 - R) \cdot I_0 = T \cdot I_0$$

since $I = E^2$

we have $A_{i1} = \sqrt{1 - R} \cdot A_0$

Fabry-Perot Resonator

$$A_{i2} = R \cdot A_{i1} = \sqrt{1-R} \cdot R^{1} \cdot A_0$$

$$A_{i3} = R \cdot A_{i2} = \sqrt{1-R} \cdot R^{2} \cdot A_0$$

$$A_{i4} = R \cdot A_{i3} = \sqrt{1-R} \cdot R^{3} \cdot A_0$$

$$A_{in} = R \cdot A_{i(n-1)} = \sqrt{1-R} \cdot R^{n-1} \cdot A_0$$

$$A_1 = \sqrt{1-R} \cdot A_{i1} = (1-R) \cdot A_0$$

$$A_2 = \sqrt{1-R} \cdot A_{i2} = (1-R) \cdot R^{2} A_0$$

$$A_3 = \sqrt{1-R} \cdot A_{i3} = (1-R) \cdot R^{3} A_0$$

$$A_n = \sqrt{1-R} \cdot A_{in} = (1-R) \cdot R^{n} A_0$$

Fabry-Perot Resonator

$$\delta = \frac{2kd}{\cos\alpha}$$

$$E_1 = A_1 \cdot \cos(\omega t + kx)$$

$$E_2 = A_2 \cdot \cos(\omega t + kx + \delta)$$

$$E_3 = A_3 \cdot \cos(\omega t + kx + 2 \cdot \delta)$$

$$E_n = A_n \cdot \cos(\omega t + kx + (n-1) \cdot \delta)$$

$$E_n = (1-R) \cdot R^n \cdot A_0 \cdot \cos(\omega t + kx + (n-1) \cdot \delta)$$

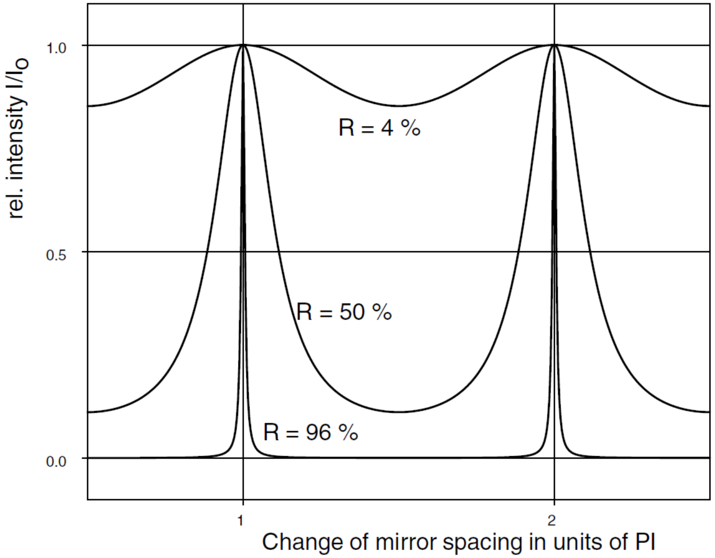
$$E = \sum_0^\infty E_n$$

Fabry-Perot Resonator

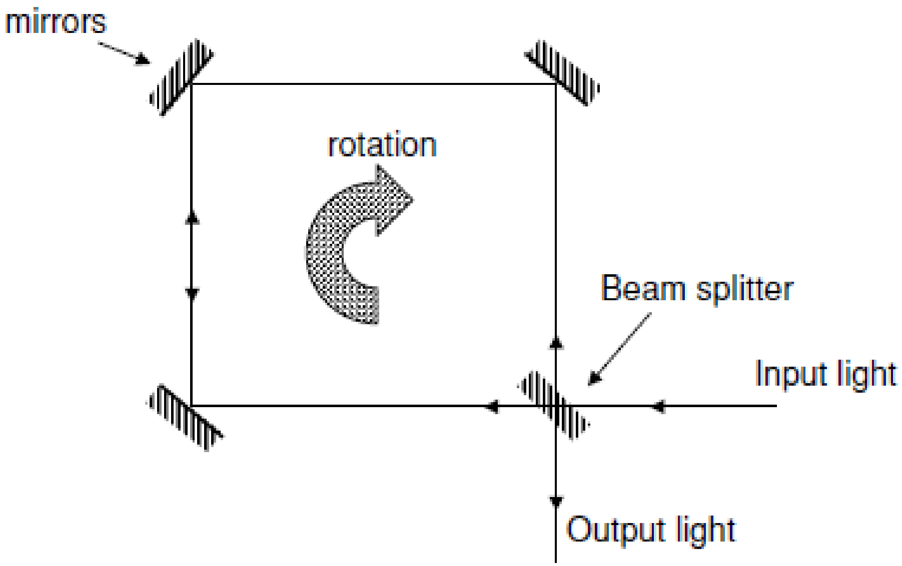
$$I = I_0 \frac{(1-R)^2}{(1-R)^2 + 4\cdot R\cdot \sin^2(\delta/2)}$$

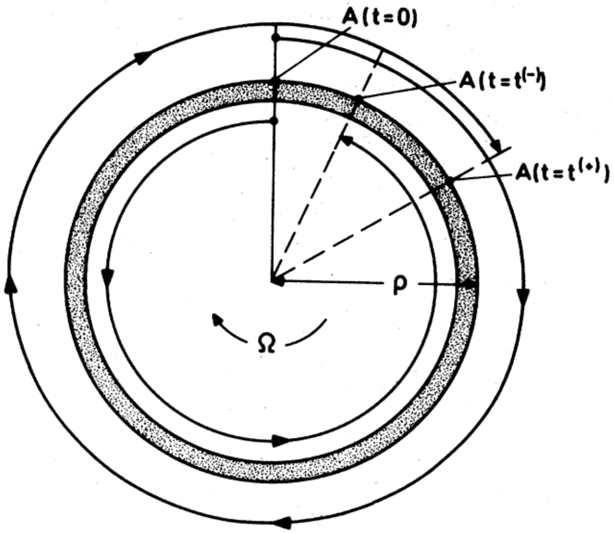
$$I = I_0 \frac{(1-R)^2}{(1-R)^2 + 4\cdot R\cdot \sin^2\left(\frac{2\pi d}{\lambda}\right)}$$

(α = 0, normal incident)



Sagnac Interferometer





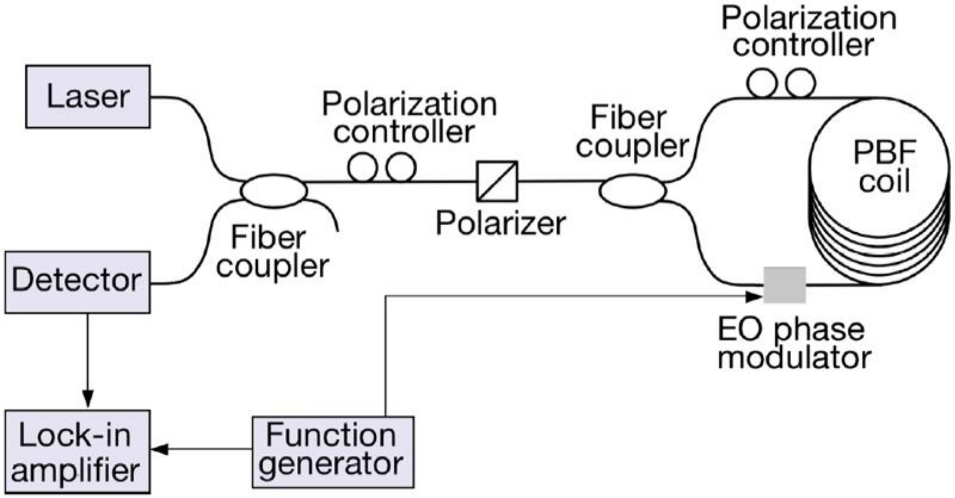
$$\Delta t = t^{+} - t^{-} = \frac{4\pi\rho^2\Omega}{c^2 - \rho^2\Omega^2}$$

$$\Delta t \cong \frac{4\pi\rho^2\Omega}{c^2}.$$

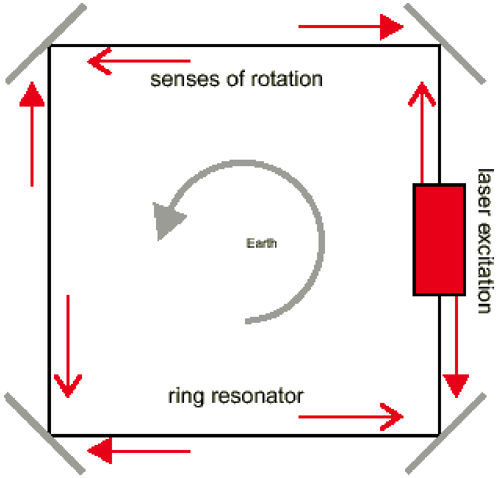
$$\Delta L = c\Delta t = \frac{4\pi\rho^2\Omega}{c}$$

The sagnac sensor has the best sensitivity compared to other type of sensors.

Fiber Optics Gyroscope



Laser Gyroscope

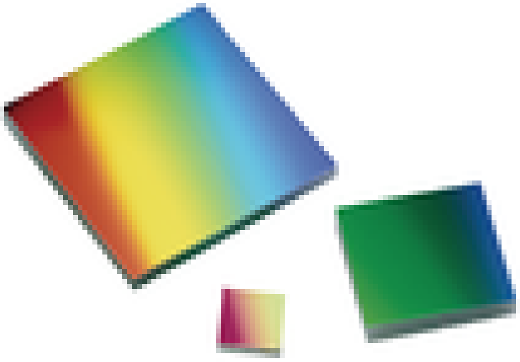


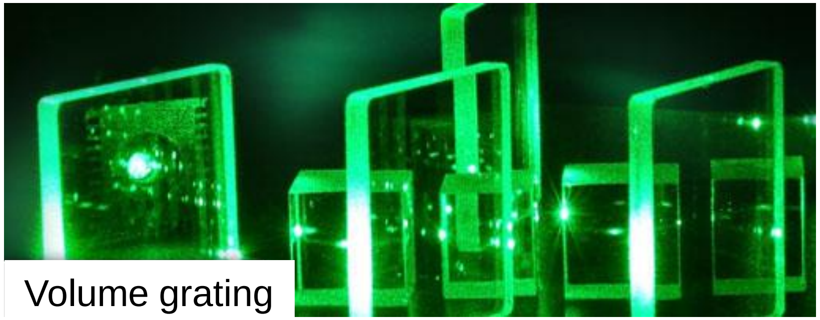
$$f_{beat} = 2\Delta v = \frac{4\hat{\Omega}.\hat{A}}{\lambda P}$$

We can easily measure f_{beat} with <1Hz precision. What would be the smallest rotation rate that we can measure using a ring resonator with 1m radius?

Diffraction and Devices

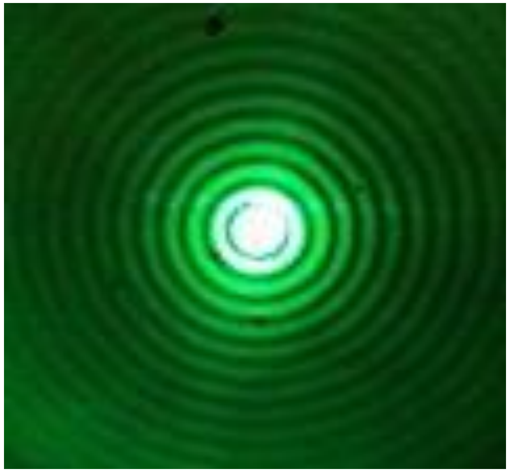
- Diffraction
- Diffraction gratings
  - ✓ Ruled grating
  - ✓ Holographic grating
  - ✓ Volume grating
- Applications
  - ✓ Tunable laser
  - ✓ Spectroscopy
  - ✓ Laser stabilization
  - ✓ Pulse compression



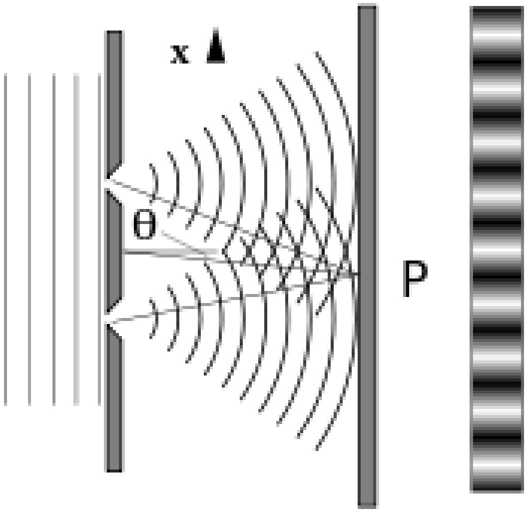

Volume grating

Diffraction

Diffraction relies on the interference of waves emanating from the same source taking different paths to the same point on a screen → Diffraction can be explained by interference

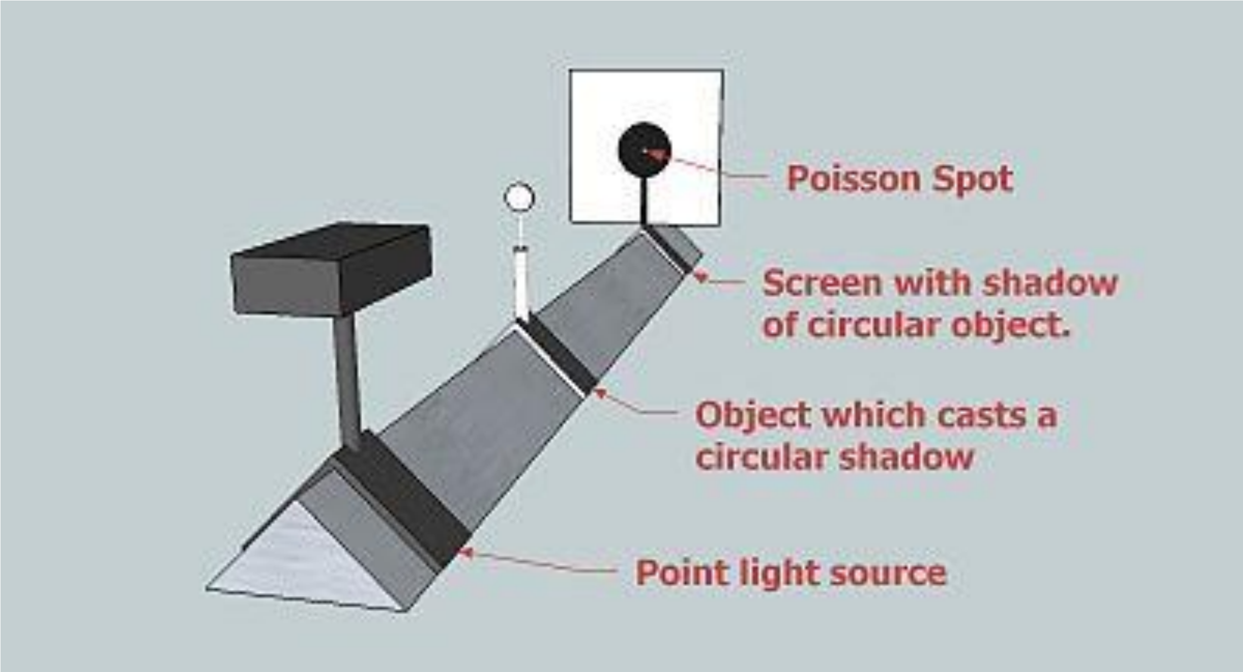


Diffraction of a laser beam through a small circular hole (Airy disk)



Young's double-slit interferometer (Homework)

Diffraction and nature of light



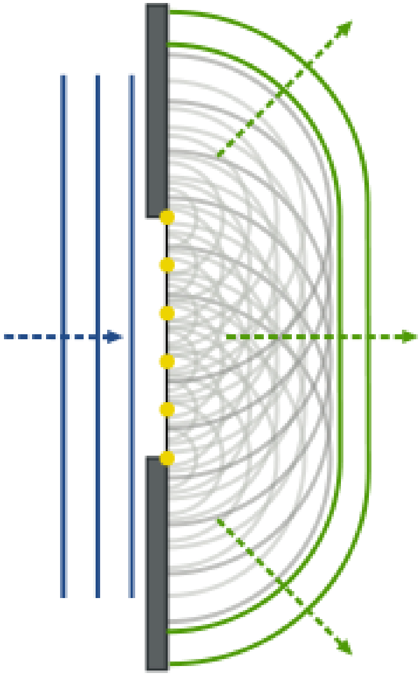
Need to be in the near field:

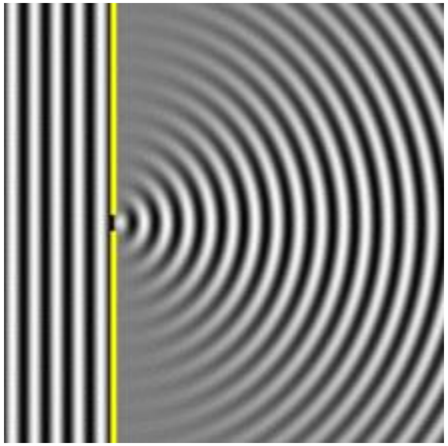
$$F = \frac{d^2}{l\lambda} \gtrsim 1$$

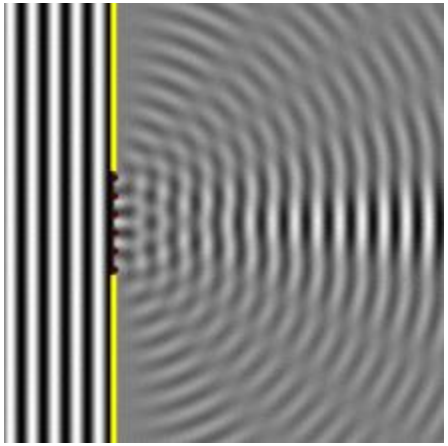
Arago spot, Fresnel bright spot, or Poisson spot

This experiment confirmed the wave nature of light!

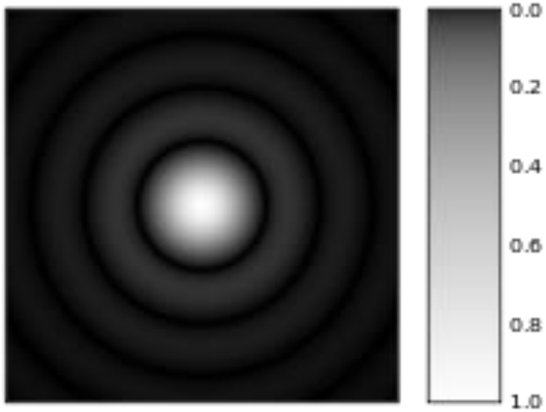
Huygens–Fresnel principle

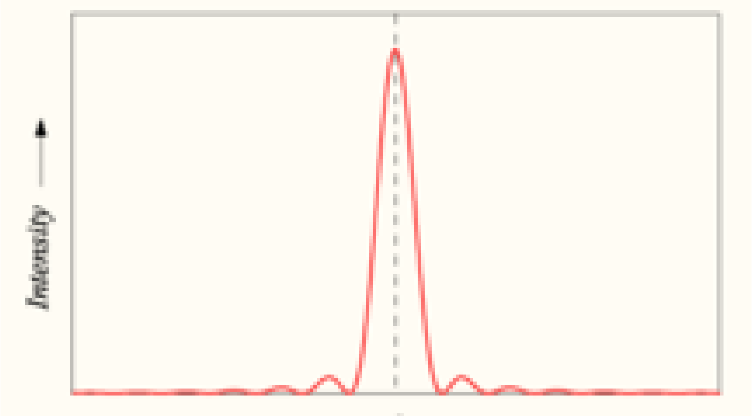


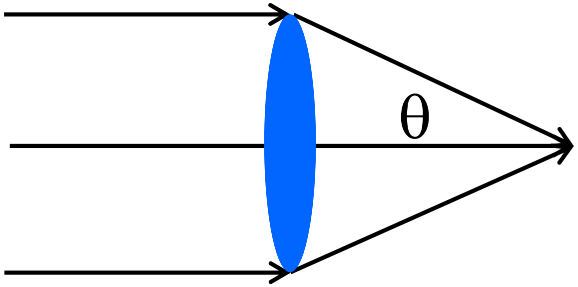




Diffraction limit





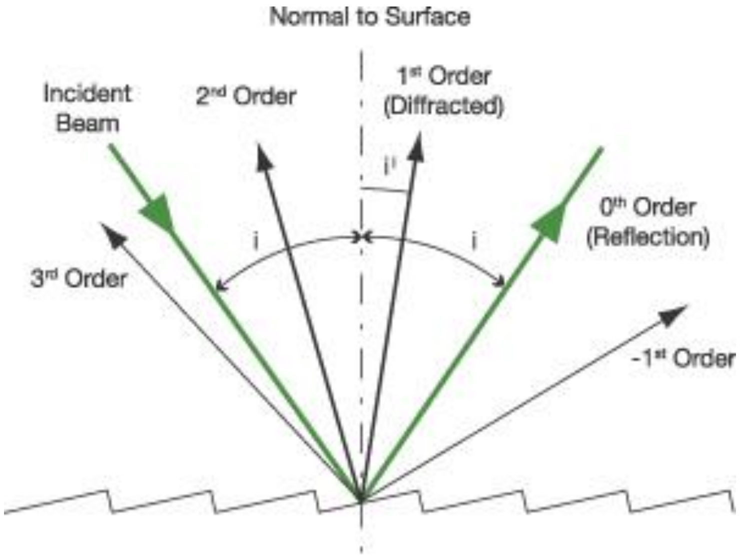


$$d = \frac{\lambda}{2(n \sin \theta)}$$

How to overcome the diffraction limit?

Diffraction Grating

A periodic structure that diffracts light into different directions. Grating can be flat, concave, convex and arbitrary shape



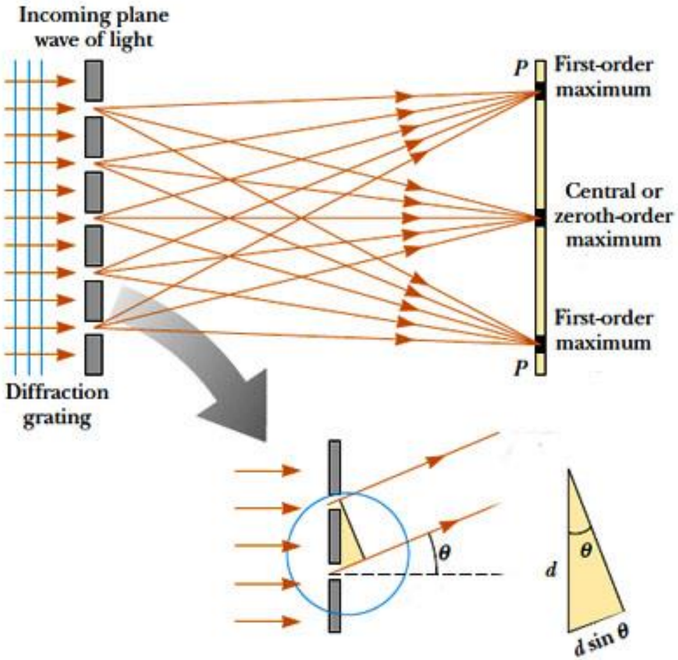
HeNe laser incident on a diffraction grating showing zero, first and second order beams

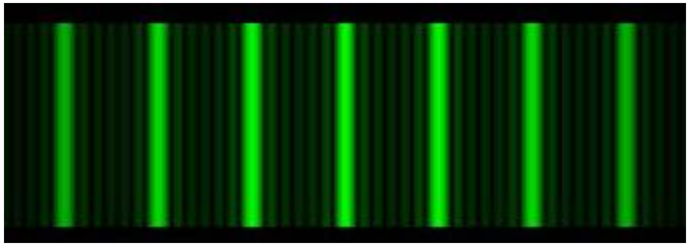
Basic equations

$$d \sin\theta_m = m\lambda$$

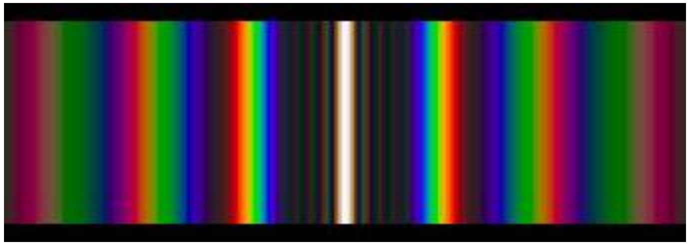
$$d\,(\sin\theta_i + \sin\theta_m) = m\lambda$$

$$\theta_m = \arcsin\left(\frac{m\lambda}{d} - \sin\theta_i\right)$$





Monochromatic source



White light

Polarization optics

- ➢ Polarization optics
- ➢ Anisotropic media
  - ✓ Index ellipsoid
  - ✓ Uniaxial crystal
  - ✓ Double refraction
- ➢ Polarization devices
  - ✓ Polarizer
  - ✓ Waveplates
  - ✓ Isolators
  - ✓ Polarization microscope

Calcite CaCO3

Potassium Niobate

Polarization optics

The polarization of light is determined by the trajectory of the end of the electric vector in time at a given position.

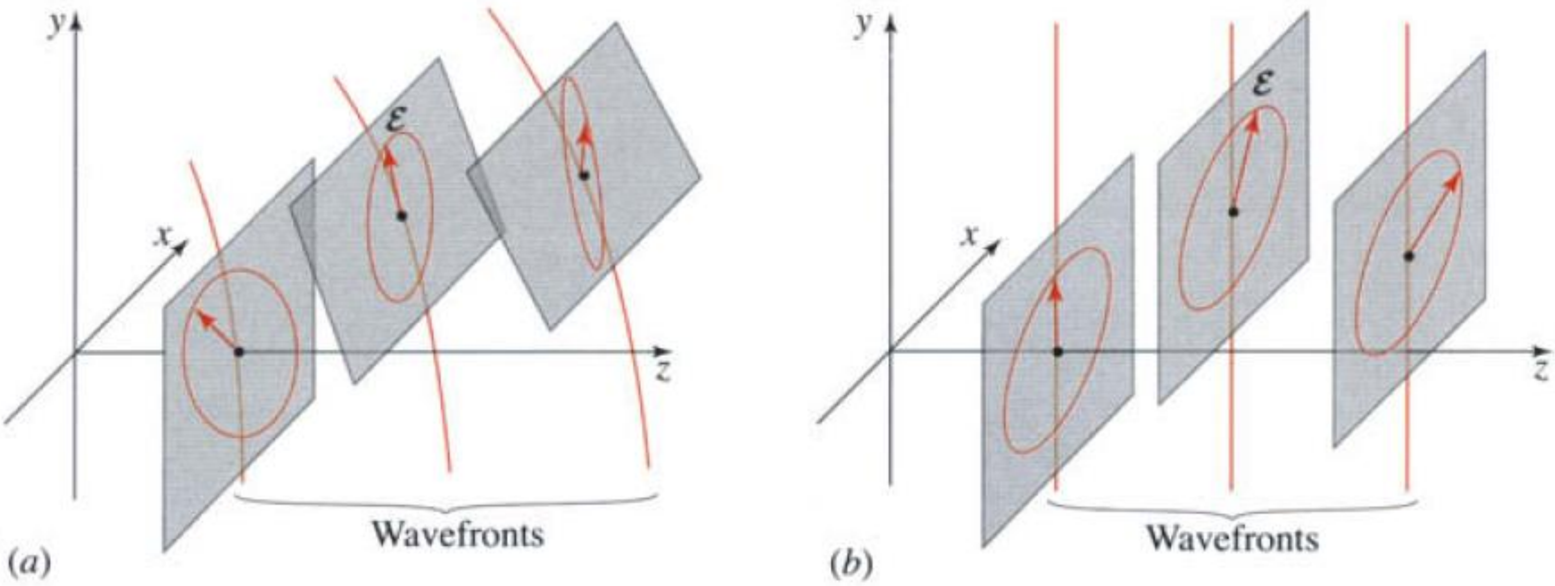


Figure 6.0-1 Time course of the electric field vector of monochromatic light at several positions: (*a*) arbitrary wave; (*b*) plane wave or paraxial wave traveling in the *z* direction.

Polarization optics

Plane wave propagating in the z direction:

$$\mathcal{E}(z,t) = \mathrm{Re}\left\{\mathbf{A}\exp\left[j\omega\left(t-\frac{z}{c}\right)\right]\right\}$$

$$\mathcal{E}(z,t) = \mathcal{E}_x\hat{\mathbf{x}} + \mathcal{E}_y\hat{\mathbf{y}},$$

$$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$

$$A_x = \mathrm{a}_x\exp(j\varphi_x)$$

$$A_y = \mathrm{a}_y\exp(j\varphi_y)$$

Where:

$$\begin{cases} \mathcal{E}_x = \mathrm{a}_x\cos\left[\omega\left(t-\frac{z}{c}\right)+\varphi_x\right] \\ \mathcal{E}_y = \mathrm{a}_y\cos\left[\omega\left(t-\frac{z}{c}\right)+\varphi_y\right] \end{cases}$$

(A: complex envelope)

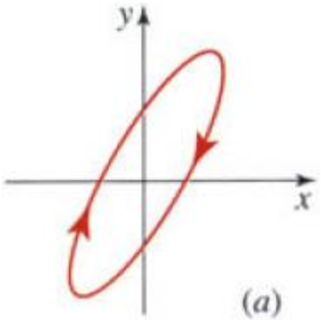
$$\frac{\mathcal{E}_x^2}{\mathrm{a}_x^2} + \frac{\mathcal{E}_y^2}{\mathrm{a}_y^2} - 2\cos\varphi\frac{\mathcal{E}_x\mathcal{E}_y}{\mathrm{a}_x\mathrm{a}_y} = \sin^2\varphi$$

describes an ellipse (z = const)

where $\varphi = \varphi_y - \varphi_x$

Polarization optics

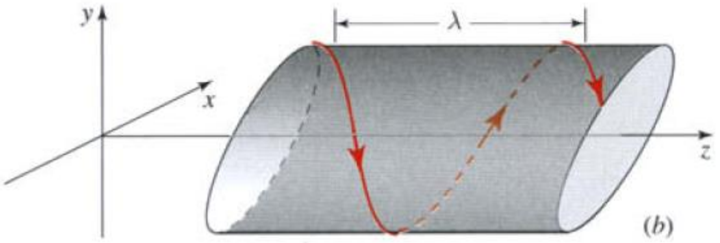
Plane wave propagating in the z direction:



$$\mathcal{E}(z,t) = \mathrm{Re}\left\{\mathbf{A}\exp\left[j\omega\left(t-\frac{z}{c}\right)\right]\right\}$$

$$\mathcal{E}(z,t) = \mathcal{E}_x\hat{\mathbf{x}} + \mathcal{E}_y\hat{\mathbf{y}},$$

Where:



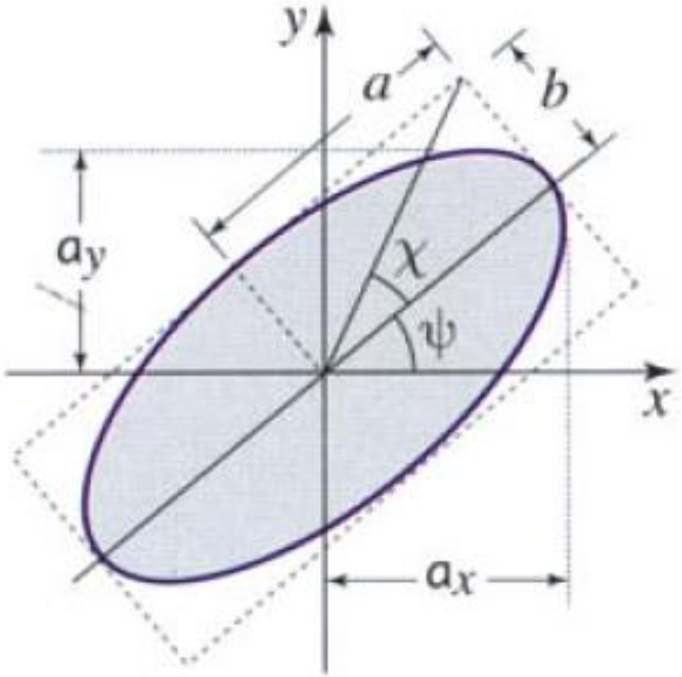
$$\begin{cases} \mathcal{E}_x = \mathrm{a}_x\cos\left[\omega\left(t-\dfrac{z}{c}\right)+\varphi_x\right] \\ \mathcal{E}_y = \mathrm{a}_y\cos\left[\omega\left(t-\dfrac{z}{c}\right)+\varphi_y\right] \end{cases}$$

$$\frac{\mathcal{E}_x^2}{\mathrm{a}_x^2} + \frac{\mathcal{E}_y^2}{\mathrm{a}_y^2} - 2\cos\varphi\,\frac{\mathcal{E}_x\mathcal{E}_y}{\mathrm{a}_x\mathrm{a}_y} = \sin^2\varphi$$

describes an ellipse (z = const)

where $\varphi = \varphi_y - \varphi_x$

Polarization optics

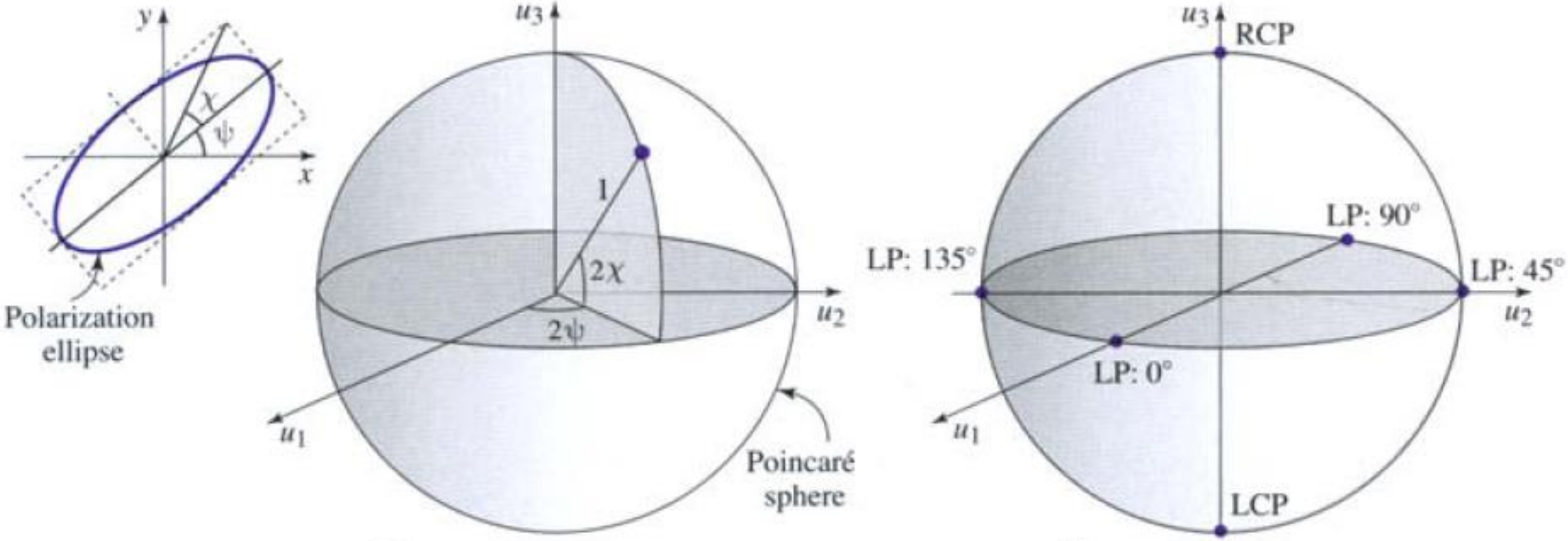


$$\tan 2\psi = \frac{2R}{1-R^2}\cos\varphi\,, \qquad R = \frac{a_y}{a_x}\,,$$

$$\sin 2\chi = \frac{2R}{1+R^2}\sin\varphi\,, \qquad \varphi = \varphi_y - \varphi_x$$

(Homework)

Poincaré sphere



(*a*)

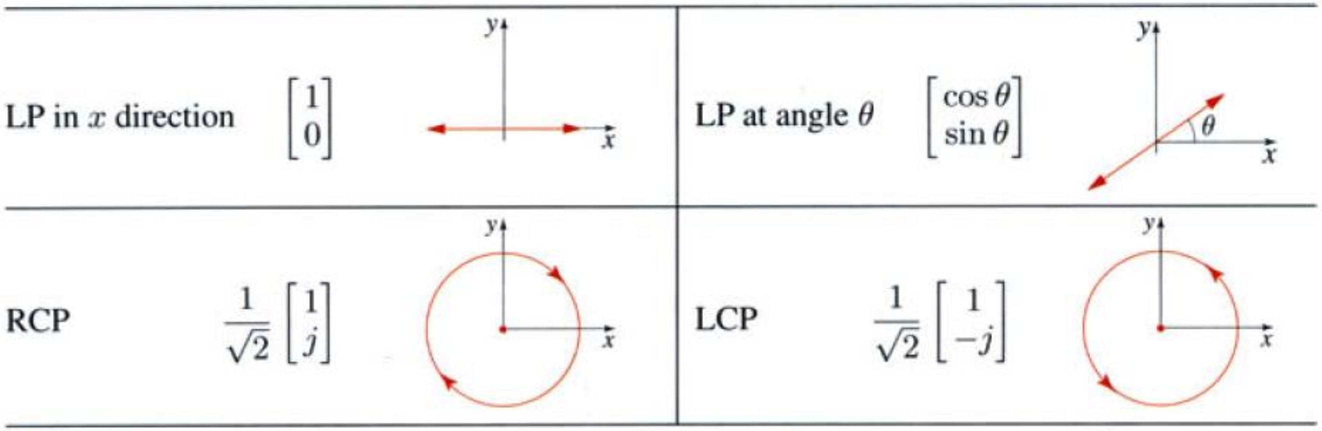
(*b*)

Other representations

Stokes parameters:

$$
\begin{aligned}
S_0 &= a_x^2 + a_y^2 &&= |A_x|^2 + |A_y|^2 \\
S_1 &= a_x^2 - a_y^2 &&= |A_x|^2 - |A_y|^2 \\
S_2 &= 2a_x a_y \cos\varphi &&= 2\,\mathrm{Re}\{A_x^* A_y\} \\
S_3 &= 2a_x a_y \sin\varphi &&= 2\,\mathrm{Im}\{A_x^* A_y\}\,.
\end{aligned}
$$

Jones vector:



LP in x direction	$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$		LP at angle θ	$\begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$	
RCP	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$		LCP	$\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -j \end{bmatrix}$	

Anisotropic media

$$D_i = \sum_j \epsilon_{ij} E_j$$

The electric permittivity is a matrix (tensor)

D and **E** may point to different directions

By choice of the coordinate system we can simplify the math:

$$D_1 = \epsilon_1 E_1, \qquad D_2 = \epsilon_2 E_2, \qquad D_3 = \epsilon_3 E_3,$$

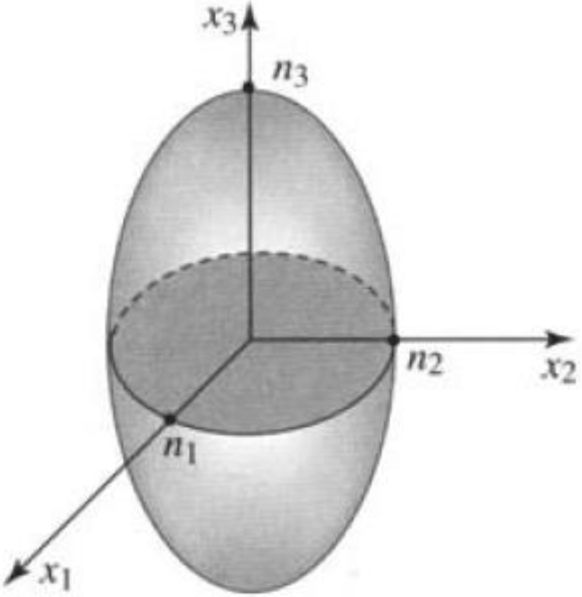
$$n_1 = \sqrt{\epsilon_1/\epsilon_o}, \qquad n_2 = \sqrt{\epsilon_2/\epsilon_o}, \qquad n_3 = \sqrt{\epsilon_3/\epsilon_o},$$

(Principle refractive index)

Anisotropic media

Index ellipsoid (representation of the tensor):

$$\frac{x_1^2}{n_1^2}+\frac{x_2^2}{n_2^2}+\frac{x_3^2}{n_3^2}=1$$

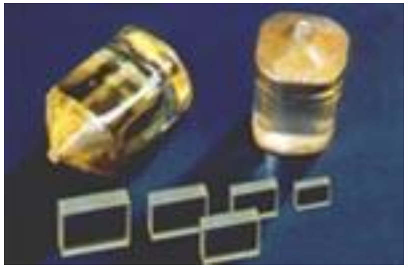


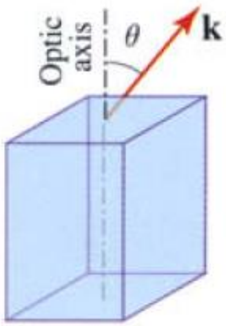
Uniaxial crystals

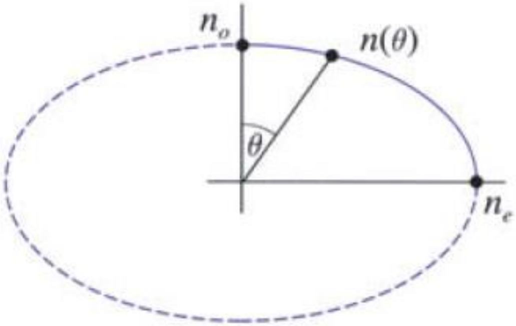
$n_1 = n_2 = n_o$ and $n_3 = n_e$

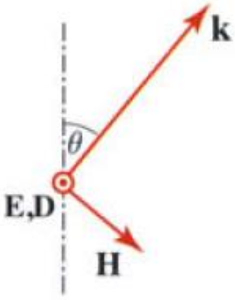
$$\frac{1}{n^2(\theta)} = \frac{\cos^2\theta}{n_o^2} + \frac{\sin^2\theta}{n_e^2}$$

Calcite, Rutile TiO2, Yttrium Vanadate

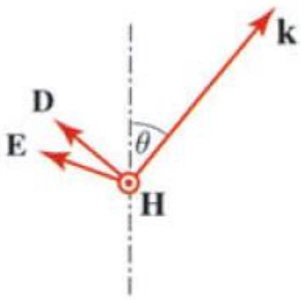






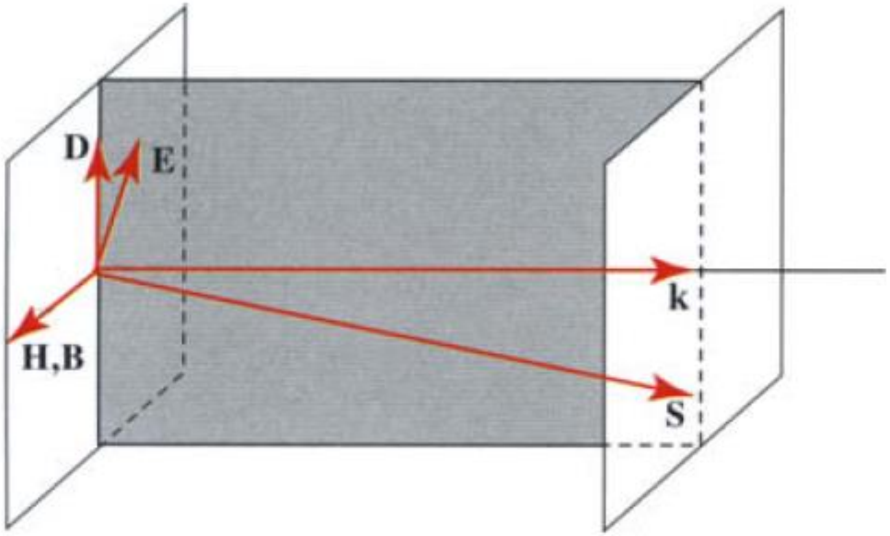


o wave

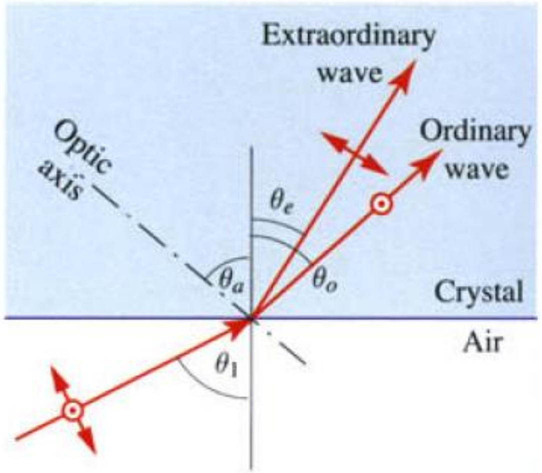


e wave

Uniaxial crystals



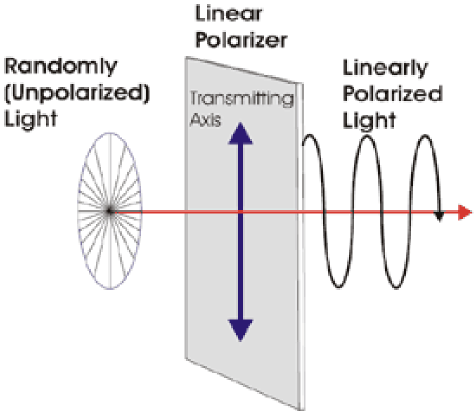
The direction of energy flow is not the same as the wave front propagation direction



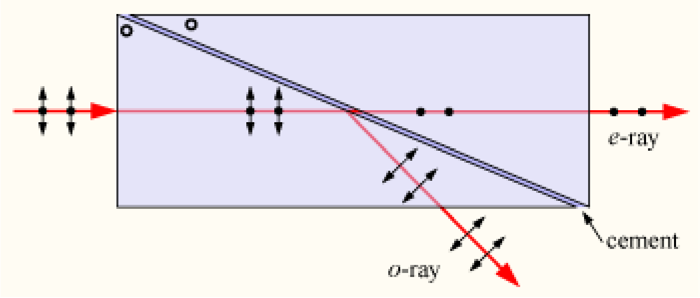
Double refraction

Polarization devices

Polarizer:



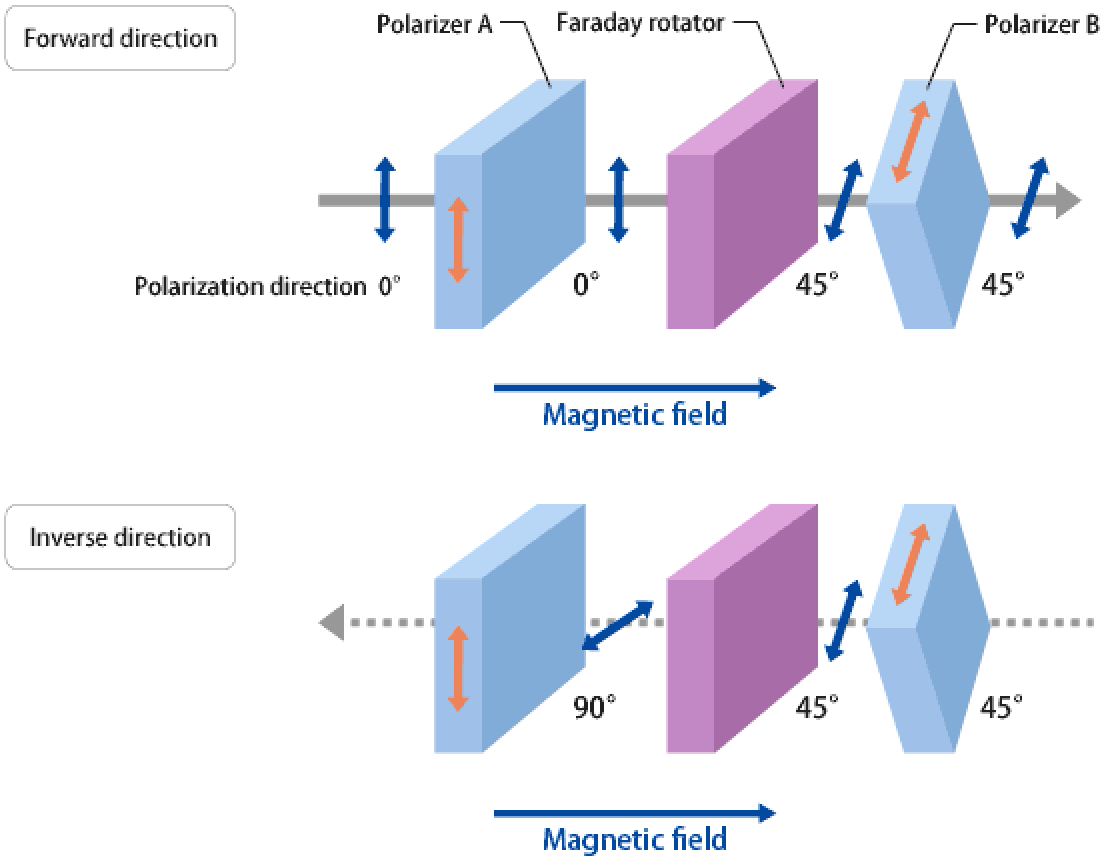
Glan-Thompson polarizer



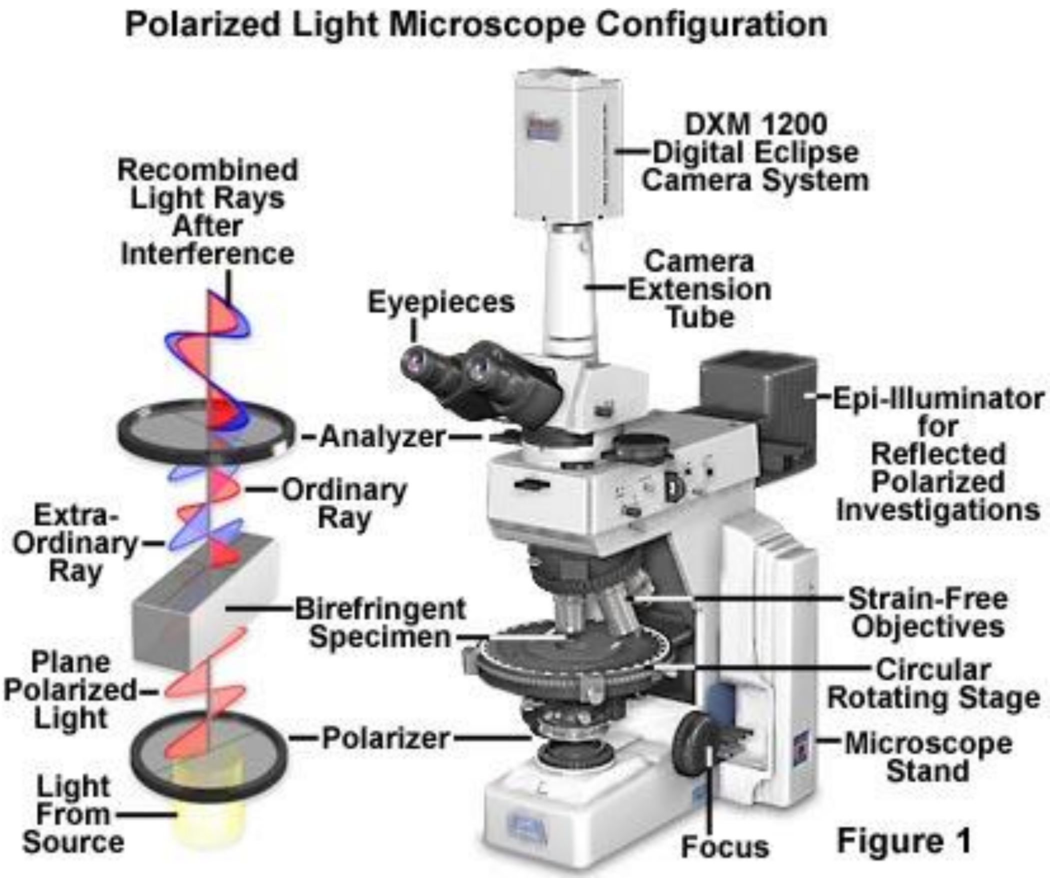
- Dichroic polarizer (Stretched Polyvinyl Alcohol (PVA)
- Double refraction polarizer
- Glan-Thompson polarizer

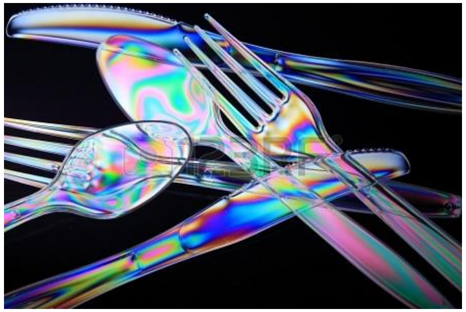
(Polarization extinction ratio: PER)

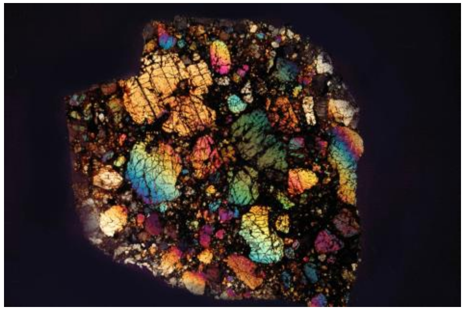
Isolators


Forward direction
Polarizer A
Faraday rotator
Polarizer B
Polarization direction 0°
0°
45°
45°
Magnetic field
Inverse direction
90°
45°
45°
Magnetic field

Polarization microscope

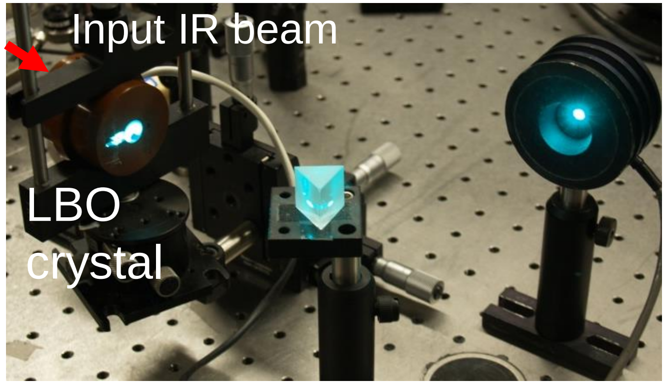




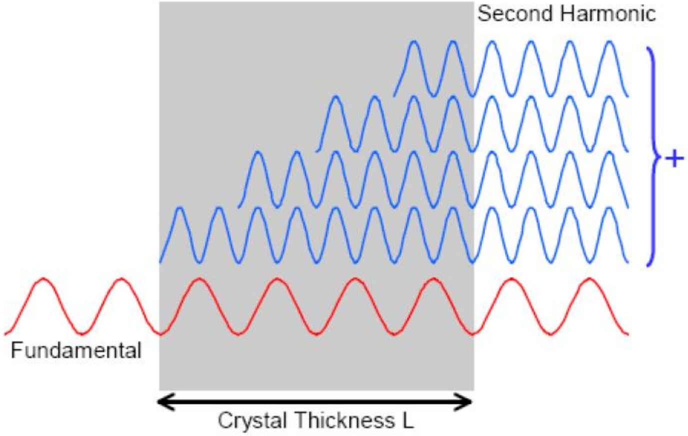


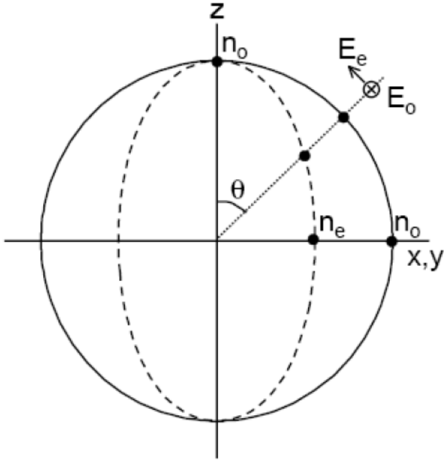
Phase matching

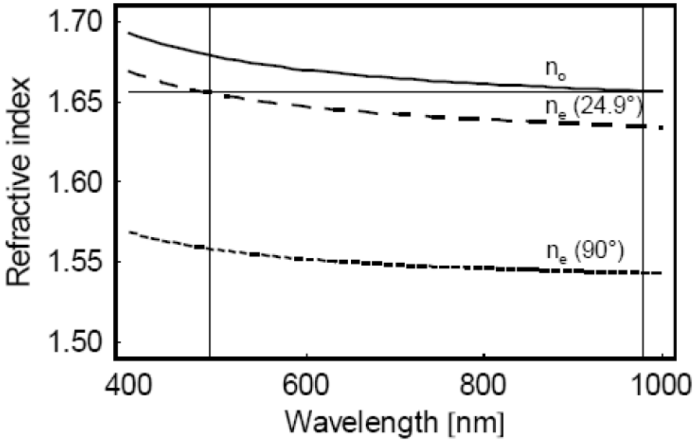
Second harmonic generation



Phase matching concept







Angle and Wavelength Dependence of Refractive index of BBO

Guided-wave optics

- Introduction
- Overview of guided-wave devices
  - ✓ Optical fibers
  - ✓ Planar waveguides
  - ✓ Integrated optical devices
  - ✓ Compact lasers
- Planar mirror waveguides
  - ✓ Waveguide modes
  - ✓ Dispersion relation

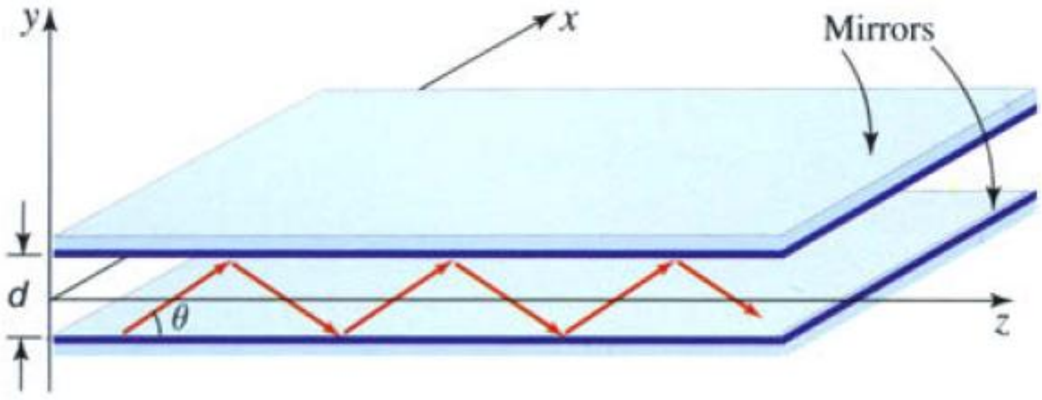
Why guided wave optics?

- Propagate light over long distances without the need for lenses
- Escape the slavery of diffraction
- Take advantage of semiconductor manufacturing processes to determine dimensions and designs
- Enable unique devices impossible to make in other ways (arrayed waveguide grating)
- Drive the size of photonics down dramatically (compact and silicon photonics)

Important trends

- Integrated optical circuits: combination of multiple optical functions on a single substrate
  - Functions include splitting, filtering, switching, modulating, isolating, coupling (general passive functions), generation (lasers) and detection
  - Monolithic integration – a single material is used
  - Hybrid integration – multiple materials are used that perform different functions
- Optoelectronic integrated circuits (OEIC)
  - Includes both integrated optical circuits and conventional electronic circuits on the same substrate
  - Generally limited to semiconductor substrate materials
  - High refractive indices create the potential for ultracompact circuitry
  - Increasing need for optical interconnections between and within computers

Planar mirror waveguides



$$\lambda = \lambda_0 / n$$

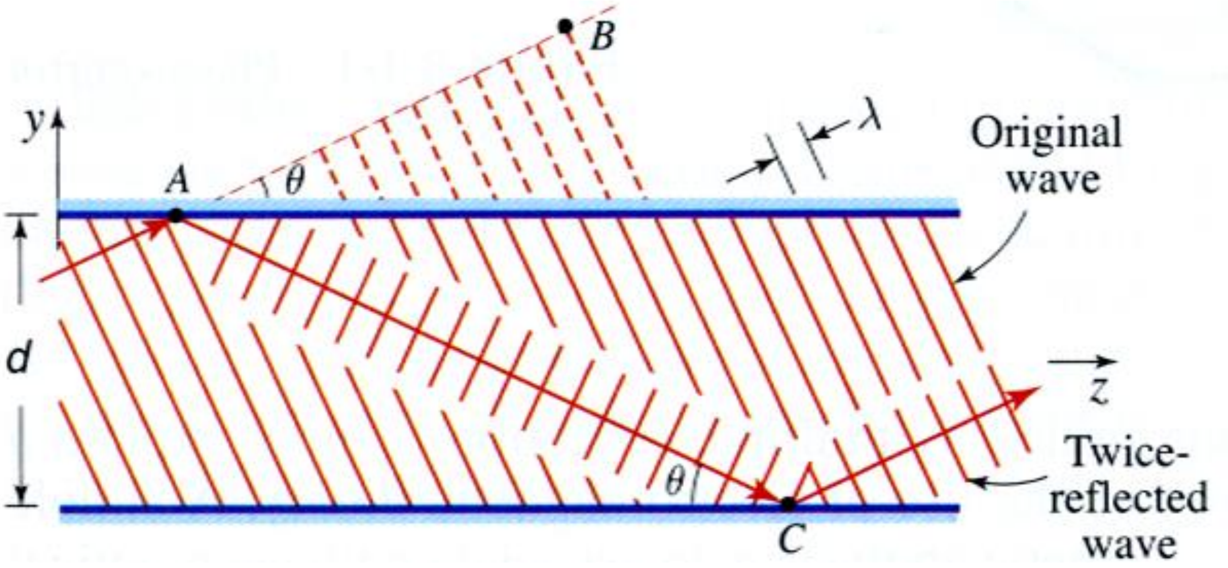
$$k = nk_0$$

$$c = c_0 / n$$

TEM plane wave
TE: E polarized in x-direction
TM: H polarized in x-direction

- π phase shift for each reflection (boundary conditions)
- Amplitude and polarization do not change (perfect mirror).
- Not practical due to the fact that there is no perfect metal mirror

Planar mirror waveguides



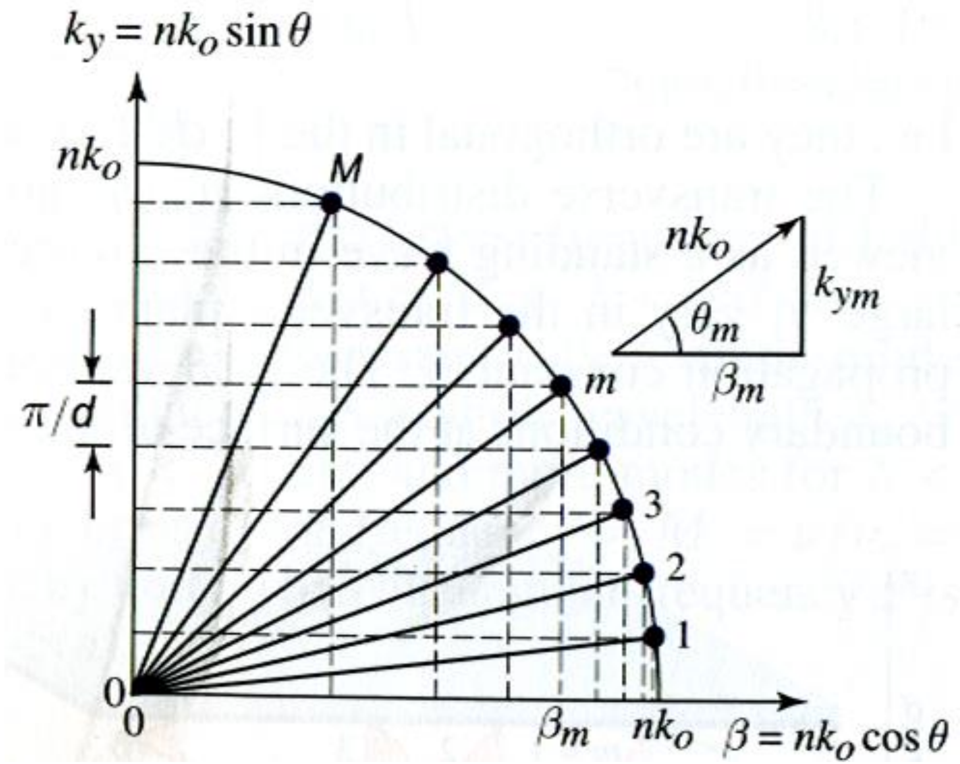
- Self-consistency: The wave reflects twice and reproduces itself
- Therefore the phase shift in travelling from A to B must be equal to or differ by an integer multiple of 2π from the phase shift from A to C
- Modes are fields that maintain the same transverse distribution and polarization at all locations along the waveguide axis.

$$\boxed{\sin\theta_m = (q+1)\lambda/2d = m\lambda/2d}$$

Planar mirror waveguides

A guided wave consists of the superposition of two plane waves in the y-z plane at angle ±θ with respect to the z axis. The components of the mode wave vector are

$$k_{ym} = nk_0 \sin\theta_m = m\pi / d \qquad \beta_m^2 = k_{zm}^2 = k^2 - \frac{m^2\pi^2}{d^2}$$



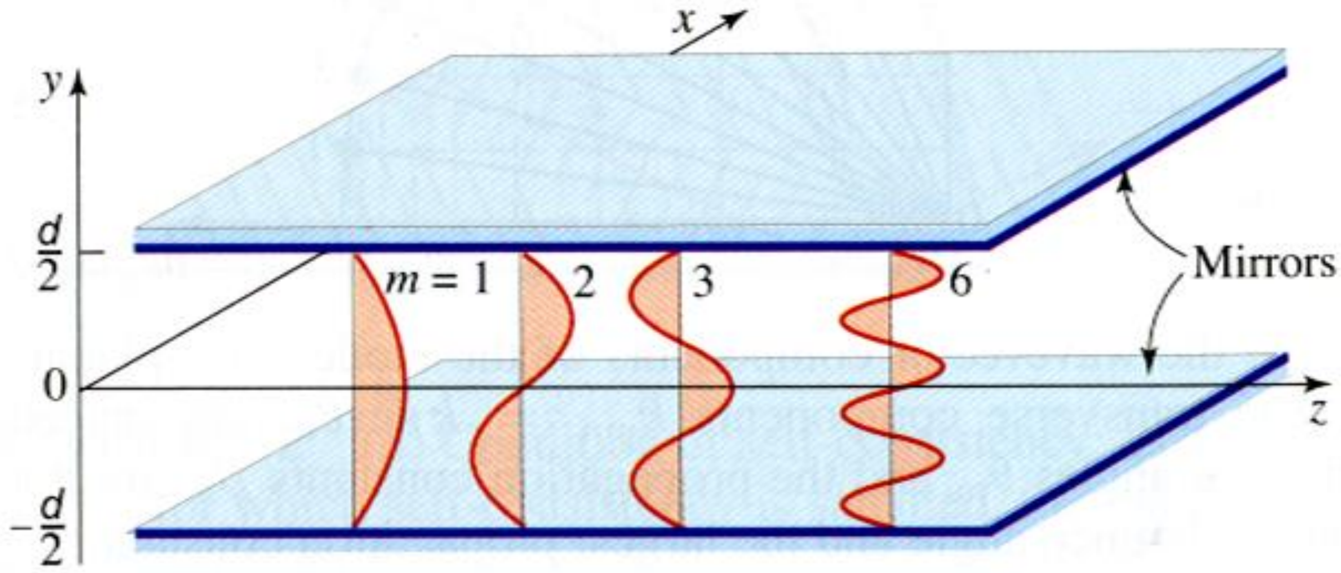
Mode field profile

TE modes

$$E_x(y,z) = a_m u_m(y)\exp(-j\beta_m z)$$

$$a_m = \sqrt{2d}\,A_m \quad \text{or} \quad j\sqrt{2d}\,A_m$$

$$u_{m(y)} = \begin{cases} \sqrt{\dfrac{2}{d}}\cos\left(\dfrac{m\pi y}{d}\right), & m = 1,3,5\ldots \\ \sqrt{\dfrac{2}{d}}\sin\left(\dfrac{m\pi y}{d}\right), & m = 2,4,6\ldots \end{cases}$$



Modes are orthogonal and normalized

Mode properties

TE modes

$$E_x(y,z) = a_m u_m(y)\exp(-j\beta_m z)$$

$$a_m = \sqrt{2d}\,A_m \quad \text{or} \quad j\sqrt{2d}\,A_m$$

$$u_{m(y)} = \begin{cases} \sqrt{\dfrac{2}{d}}\cos\left(\dfrac{m\pi y}{d}\right), & m = 1,3,5\ldots \\ \sqrt{\dfrac{2}{d}}\sin\left(\dfrac{m\pi y}{d}\right), & m = 2,4,6\ldots \end{cases}$$

Modes are orthogonal and normalized

$$\int_{-d/2}^{d/2} u_m(y)\,u_l(y)\,dy = 0, \qquad l \neq m,$$

$$\int_{-d/2}^{d/2} u_m^2(y)\,dy = 1$$

Orthogonal condition

Normalized condition

$\longrightarrow$ Any field distribution can be discomposed into a sum of modes

Number of modes, Cutoff

Number of modes

$$\sin\theta_m = m\lambda/2d < 1, \quad M = 2d/\lambda$$

Reduce to nearest integer

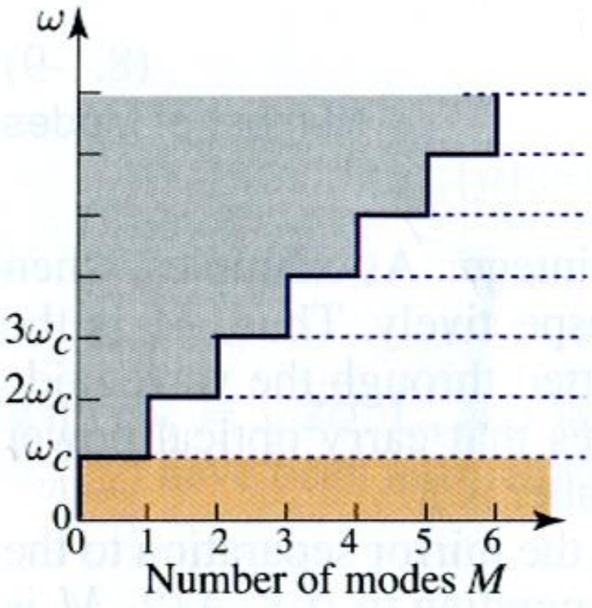
Dispersion relation

$$\beta_m^2 = (\omega/c)^2 - m^2\pi^2/d^2$$

Cutoff wavelength and frequency

$$\lambda_c = 2d, \quad \nu_c = c/2d$$

$$\omega_c = 2\pi\nu_c = \pi c/d$$

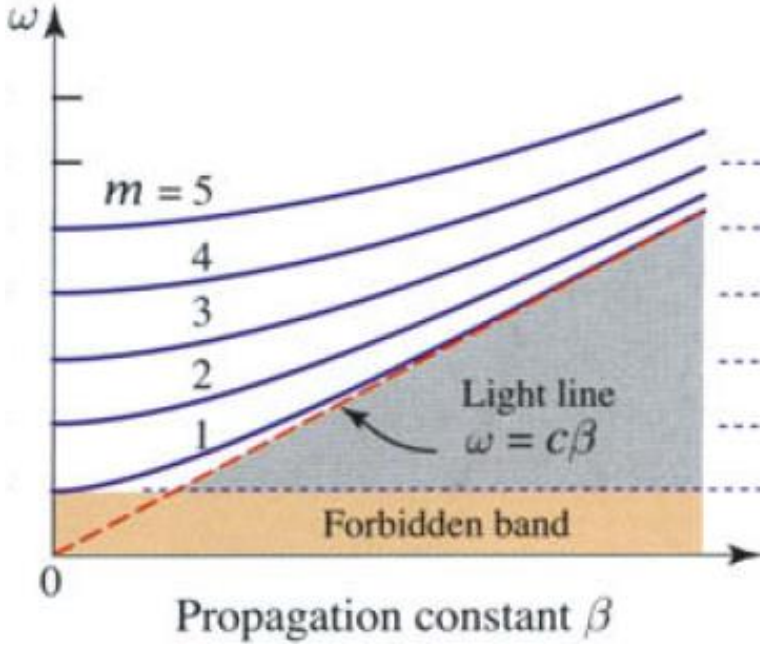


For $\lambda > \lambda_c$ or $\nu < \nu_c$ there is no guided mode

Dispersion relation

Dispersion relation .

$$\beta_m^2 = (\omega/c)^2 - m^2\pi^2/d^2$$



This leads to waveguide dispersion

Group velocity

Dispersion relation

$$\beta_m^2 = (\omega/c)^2 - m^2\pi^2/d^2$$

Cutoff frequency

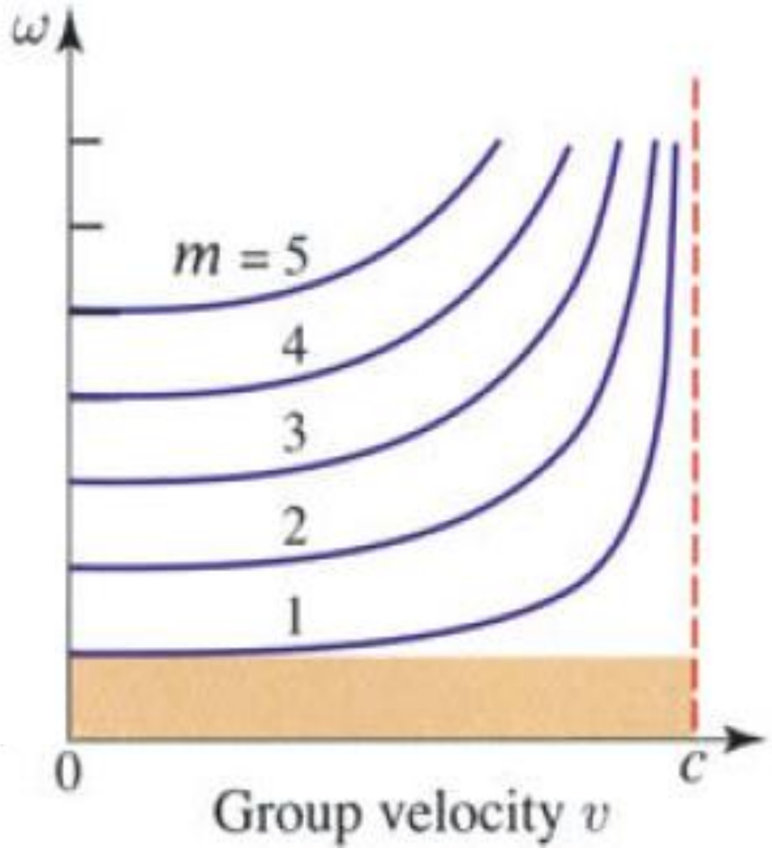
$$\omega_c = 2\pi\nu_c = \pi c/d$$

$$\longrightarrow \quad \boxed{\beta_m = \frac{\omega}{c}\sqrt{1 - m^2\frac{\omega_c^2}{\omega^2}}\,.} \quad \longrightarrow \quad 2\beta_m\, d\beta_m/d\omega = 2\omega/c^2$$

$$v = d\omega/d\beta = c^2\beta_m/\omega = c^2 k\cos\theta_m/\omega = c\cos\theta_m$$

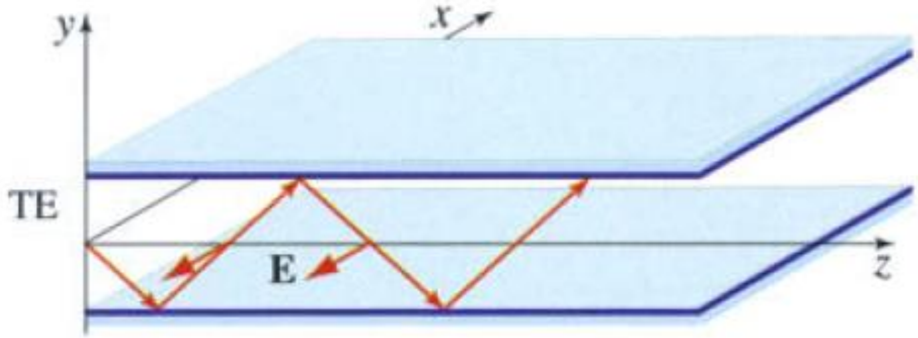
$$\longrightarrow \quad v_m = c\cos\theta_m = c\sqrt{1 - m^2\frac{\omega_c^2}{\omega^2}}$$

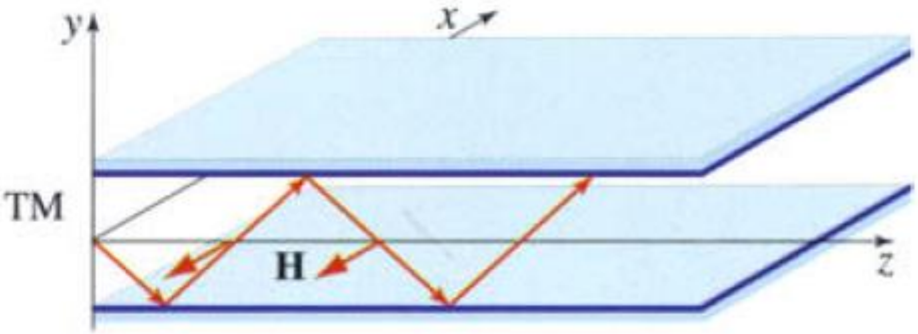
Group velocity



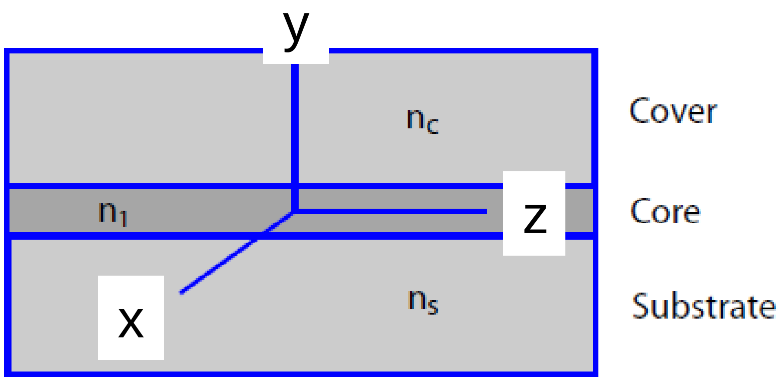
Is this normal or anomalous dispersion?

TE versus TM





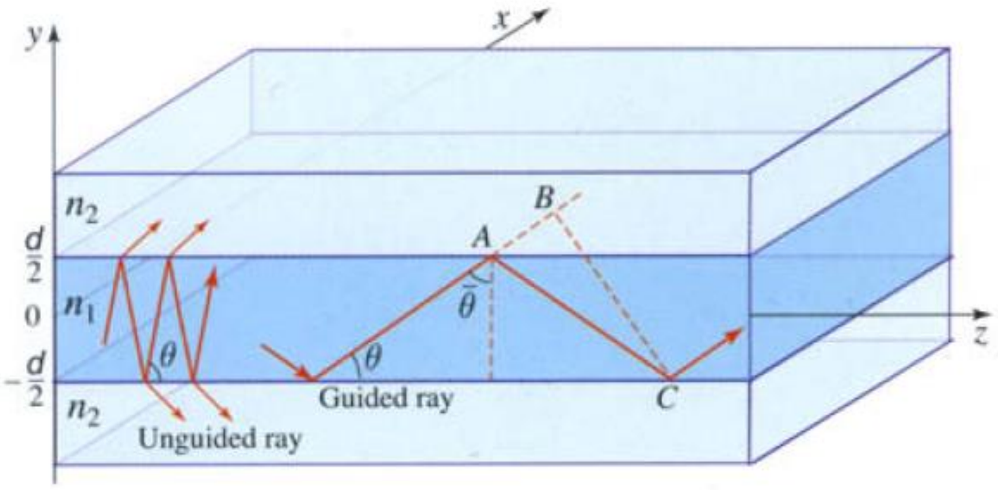
Planar dielectric waveguide



- Core film sandwiched between two layers of lower refractive index
- Bottom layer is often a substrate with $n = n_s$
- Top layer is called the cover layer ($n_c \neq n_s$)
- Air can also acts as a cover ($n_c = 1$)
- $n_c = n_s$ in symmetric waveguides

Planar dielectric waveguide

Symmetric waveguide



Reflection due to TIR

(similar to planar mirror waveguide)

$$\sin\bar{\theta}_c = n_2 / n_1$$

$$\theta \le \frac{\pi}{2} - \bar{\theta}_c = \frac{\pi}{2} - \sin^{-1}\left(\frac{n_2}{n_1}\right) = \cos^{-1}\left(\frac{n_2}{n_1}\right)$$

Self Consistency

$$\frac{2\pi}{\lambda} 2d\sin\theta - 2\varphi_r = 2\pi m$$

$$2k_y d - 2\varphi_r = 2\pi m$$

Phase shift for TIR

TE wave

$$\tan\frac{\varphi_r}{2} = \sqrt{\frac{\sin^2\bar{\theta}_c}{\sin^2\theta} - 1}$$

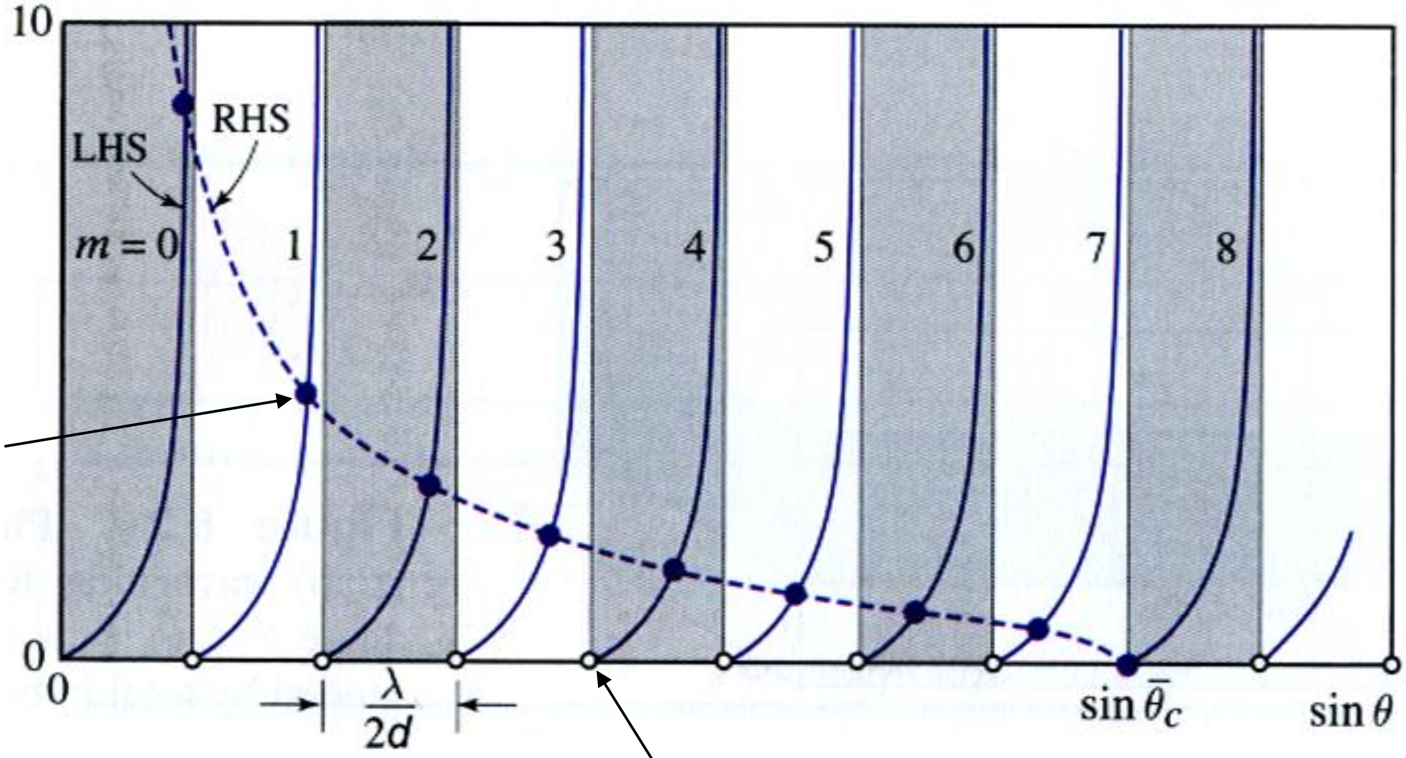
TM wave

$$\tan\frac{\varphi_r}{2} = \frac{-n_1^2}{n_2^2}\sqrt{\frac{\sin^2\bar{\theta}_c}{\sin^2\theta} - 1}$$

We can now arrive at an equation for the mode angles

Transcendental equation for modes

$$\tan\left(\frac{\pi d}{\lambda}\sin\theta - \frac{m\pi}{2}\right) = \sqrt{\frac{\sin^2\bar{\theta}_c}{\sin^2\theta} - 1}$$



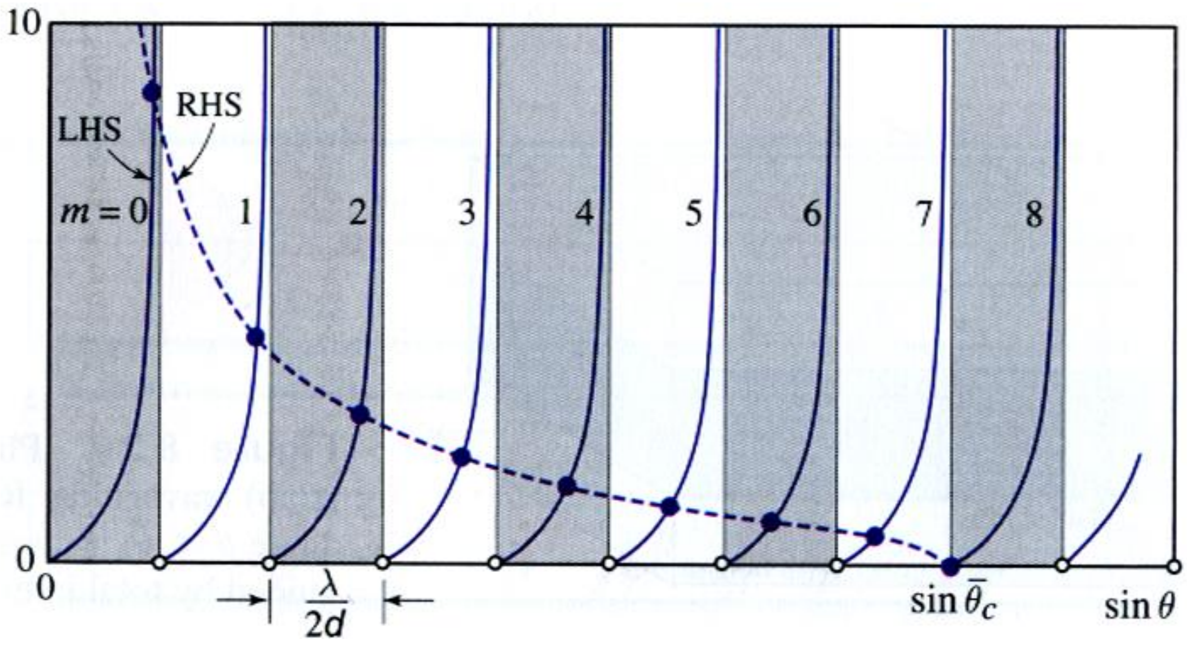
dielectric waveguide

mirror waveguide $\varphi_r = \pi$, or $\tan(\varphi_r / 2) = \infty$

Number of modes

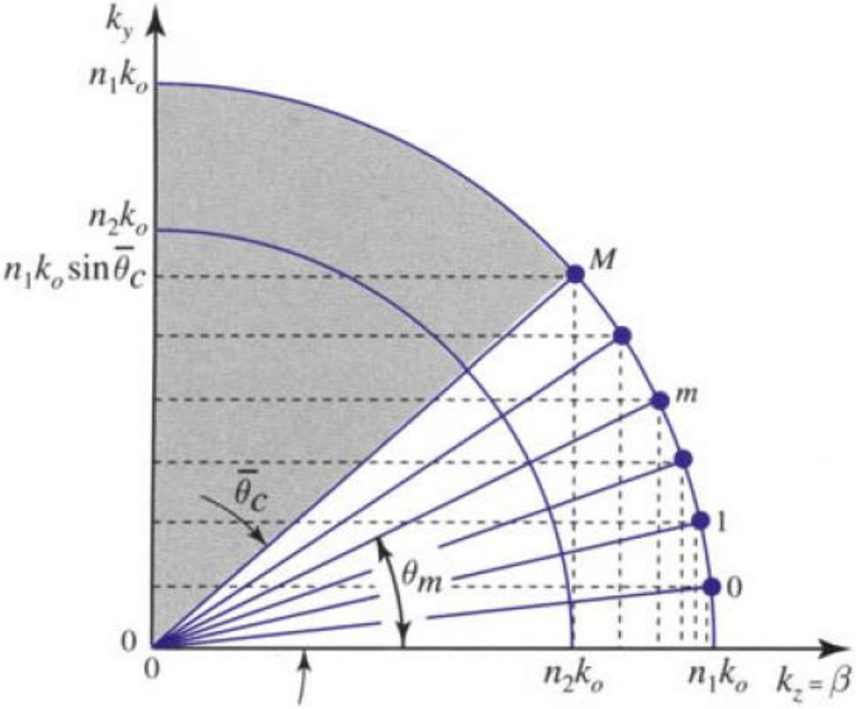
$M \doteq \frac{\sin\bar{\theta}_c}{\lambda/2d}$,or $M \doteq \frac{2d}{\lambda_o}\mathrm{NA}$,where $\mathrm{NA} = \sqrt{n_1^2 - n_2^2}$

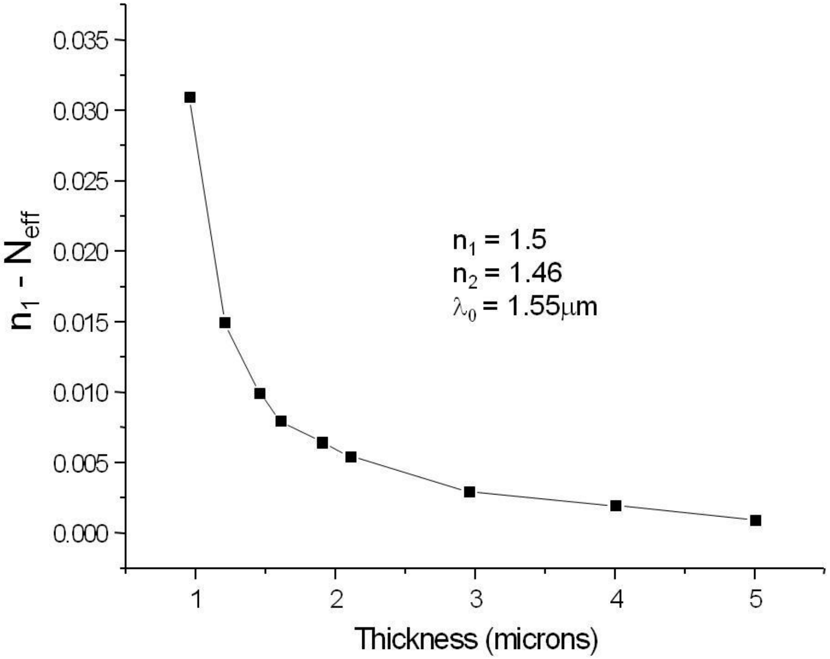
Single mode $\frac{2d}{\lambda_0}NA < 1$



Transcendental equation for modes

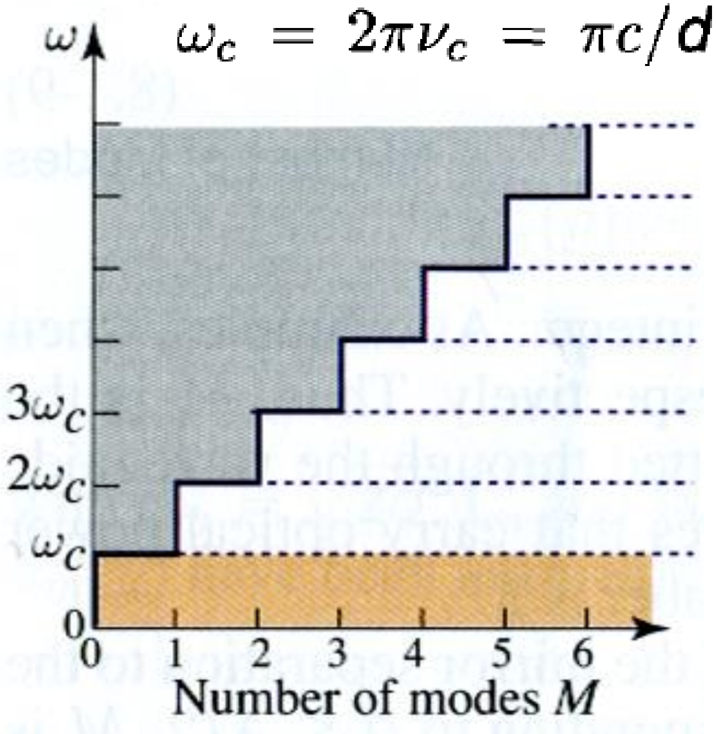
$$\beta_m = n_1 k_0 \cos\theta_m \qquad b_m = N_{eff}^{m} w / c_0$$





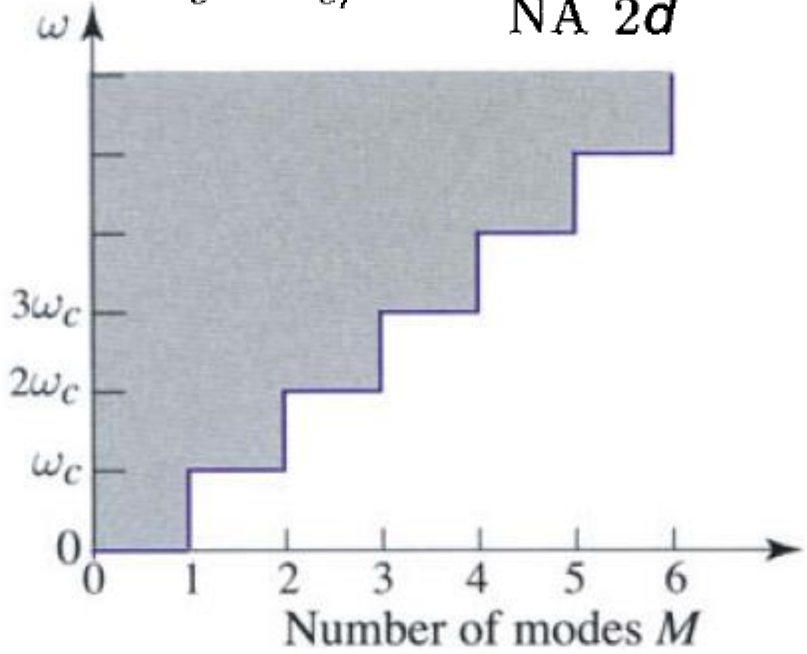
Cut-off frequency

$$\omega_c = 2\pi\nu_c = \pi c/d$$



Mirror waveguide

$$\nu_c = \omega_c/2\pi = \frac{1}{\text{NA}}\frac{c_o}{2d}$$



Dielectric waveguide

There is no gap for dielectric waveguide – always one guided mode for a symmetric slab (not so for asymmetric)

Oscillating field component

The electric field in a symmetric dielectric waveguide is harmonic within the slab and exponentially decaying outside the slab.

$$E_x(y,z) = a_m u_m(y)\exp(-j\beta_m z)$$

In the core

$$u_m(y) \propto \begin{cases} \cos\left(\frac{2\pi}{\lambda}\sin\theta_m y\right), & m = 0,2,4,\ldots \\ \sin\left(\frac{2\pi}{\lambda}\sin\theta_m y\right), & m = 1,3,5,\ldots \end{cases}, \quad -\frac{d}{2} \le y \le \frac{d}{2}$$

Evanescent field component

$$u_m(y) \propto \begin{cases} \exp(-\gamma_m y), & y > d/2 \\ \exp(\gamma_m y), & y < -d/2 \end{cases}$$

The z dependence must be identical in order to satisfy continuity at ± d/2. Signs are chosen to obtain a decaying field

$$E_x(y,z) = \sum_{m=0}^{M} a_m u_m(y) \exp(-j\beta_m z)$$

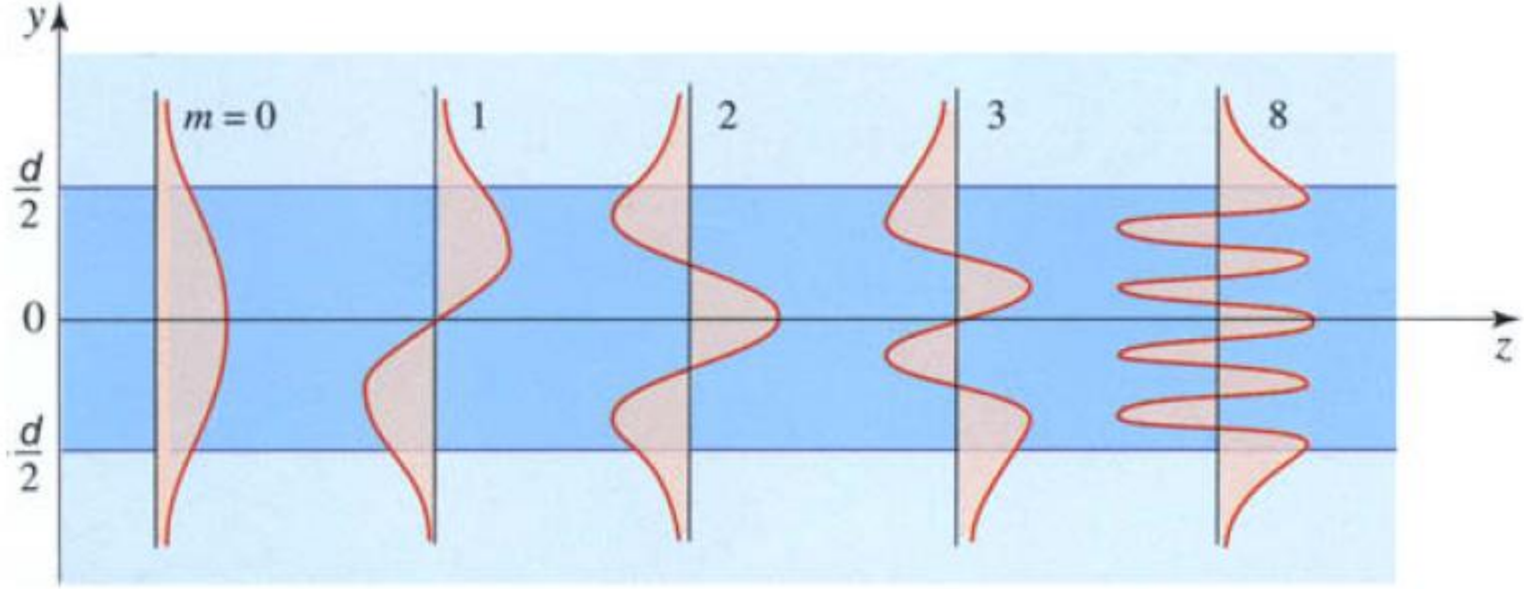
$$(\nabla^2 + n^2 k_0^2) E_x(y,z) = 0$$

Extinction coefficient

$$\gamma_m^2 = \beta_m^2 - n_2^2 k_0^2$$

$$\gamma_m = n_2 k_0 \sqrt{\frac{\cos^2\theta_m}{\cos^2\overline{\theta}_c} - 1}$$

TE field distribution



$$E_x(y,z) = \sum_{m=0}^{M} a_m u_m(y) \exp(-j\beta_m z)$$

Dispersion relation

Self Consistency

$$\frac{2\pi}{\lambda}2d\sin\theta-2\varphi_r=2\pi m \longrightarrow 2d\sqrt{\frac{\omega^2}{c_1^2}-\beta^2}=2\varphi_r+2\pi m$$

$$2k_y d-2\varphi_r=2\pi m$$

TE wave

$$\tan\frac{j_r}{2}=\sqrt{\frac{\sin^2\bar{q}_c}{\sin^2 q}-1} \longrightarrow \tan^2\frac{\varphi_r}{2}=\frac{\beta^2-\omega^2/c_2^2}{\omega^2/c_1^2-\beta^2}$$

Dispersion relation

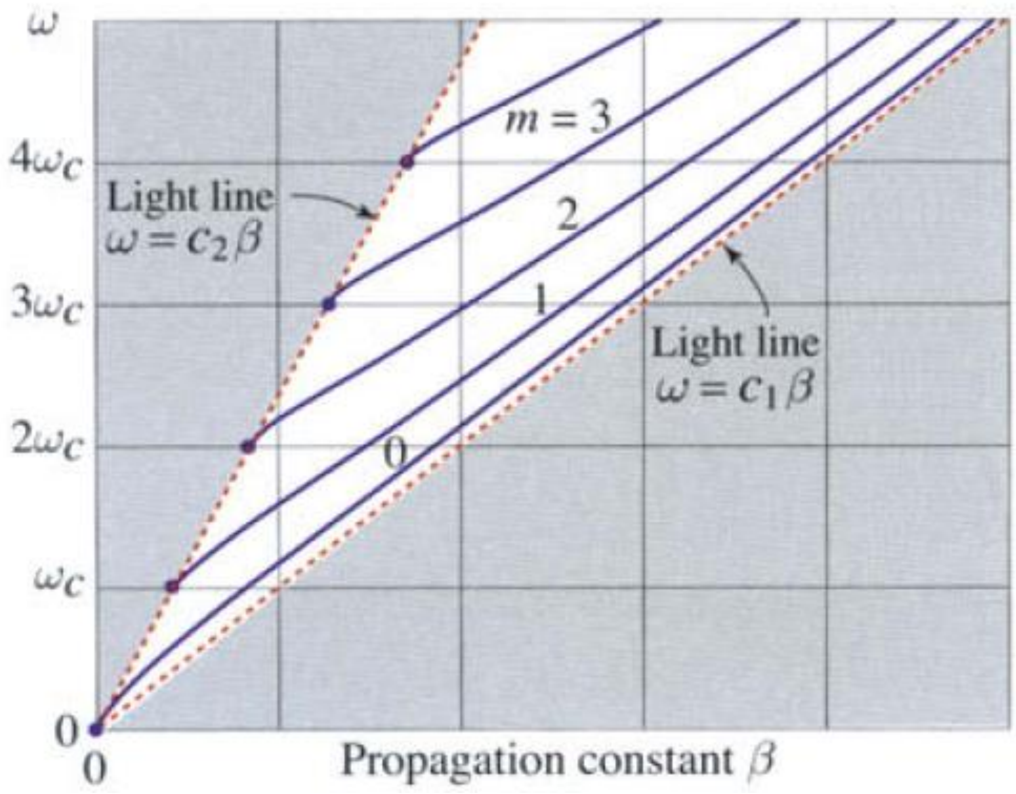
$$\longrightarrow \qquad \tan^2\left(\frac{d}{2}\sqrt{\frac{\omega^2}{c_1^2}-\beta^2}-m\frac{\pi}{2}\right)=\frac{\beta^2-\omega^2/c_2^2}{\omega^2/c_1^2-\beta^2}$$

Dispersion relation in parametric form:

$$\frac{\omega}{\omega_c}=\frac{\sqrt{n_1^2-n_2^2}}{\sqrt{n_1^2-n^2}}\left(m+\frac{2}{\pi}\tan^{-1}\sqrt{\frac{n^2-n_2^2}{n_1^2-n^2}}\right), \qquad \beta=n\omega/c_o$$

(n: effective refractive index)

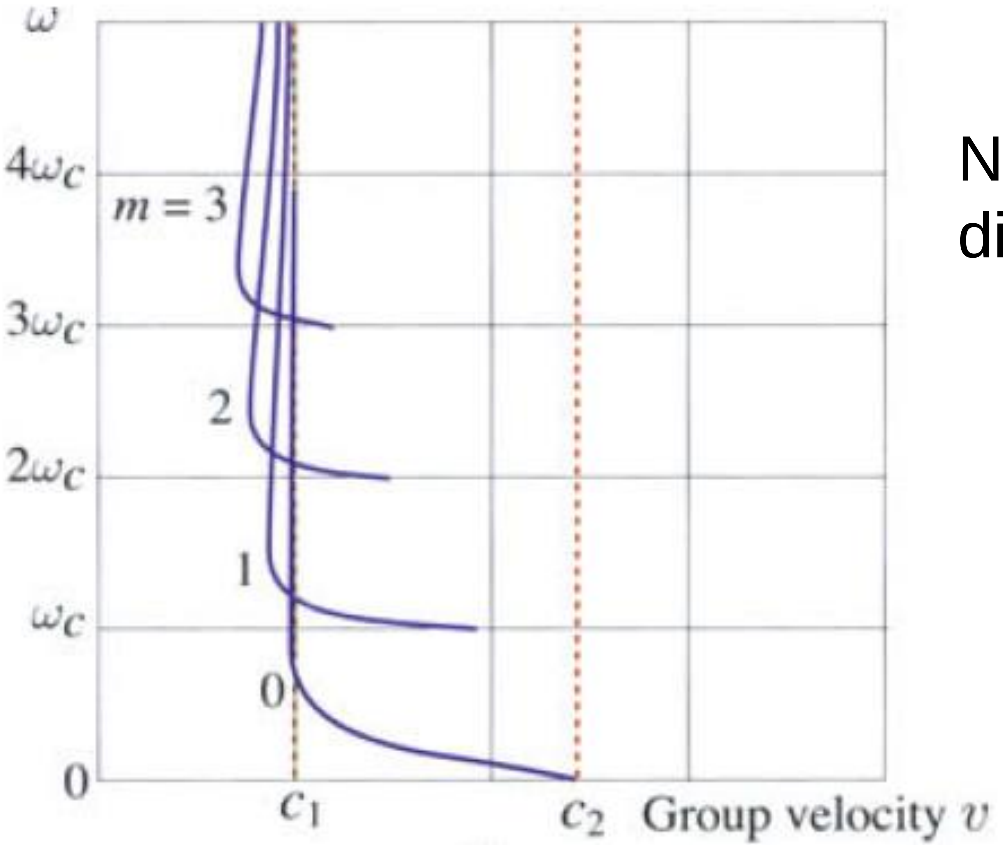
Dispersion relation



n_1, n_2 are constant

$$\frac{\omega}{\omega_c} = \frac{\sqrt{n_1^2 - n_2^2}}{\sqrt{n_1^2 - n^2}} \left(m + \frac{2}{\pi} \tan^{-1} \sqrt{\frac{n^2 - n_2^2}{n_1^2 - n^2}} \right), \quad \beta = n\omega / c_o$$

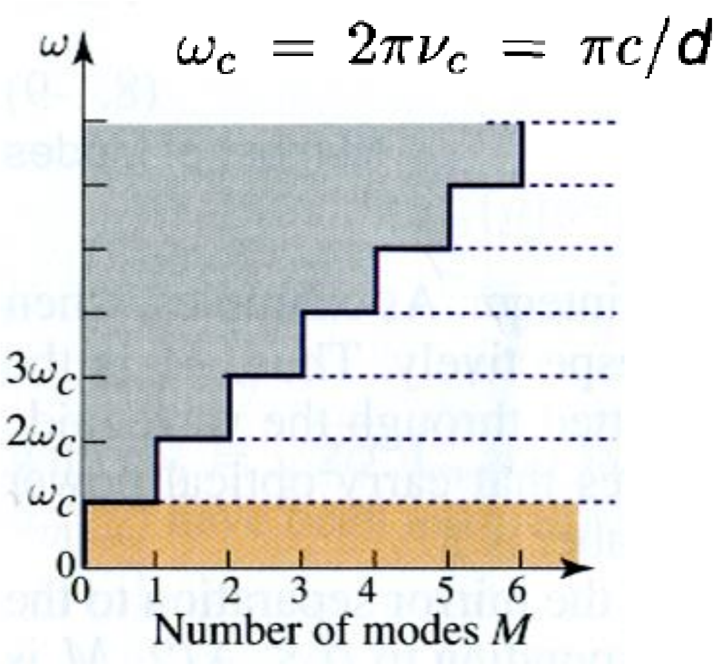
Dispersion relation



Normal or anomalous dispersion?

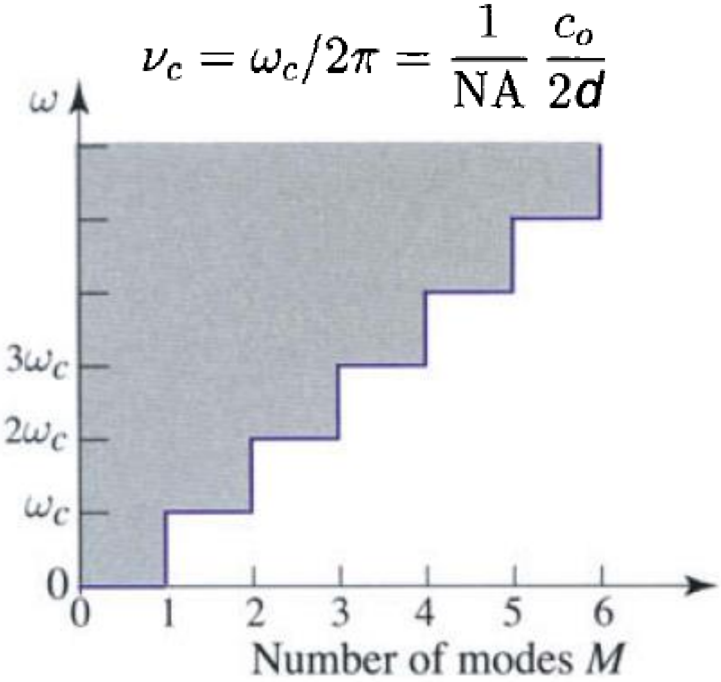
$$v = d\omega/d\beta$$

What is the smallest waveguide?



$$\omega_c = 2\pi\nu_c = \pi c/d$$

Mirror waveguide



$$\nu_c = \omega_c/2\pi = \frac{1}{\mathrm{NA}}\,\frac{c_o}{2d}$$

Dielectric waveguide

There is no gap for dielectric waveguide – always one guided mode for a symmetric slab (not so for asymmetric)

Can we then make an infinitely small dielectric waveguide?

Using Maxwell's equations

- An optical mode is solution of Maxwell's equations satisfying all boundary conditions

- Its spatial distribution does not change with propagation

- Modes are obtained by solving the curl equations

$$\nabla \times \mathbf{E} = i\omega\mu_0\mathbf{H}, \qquad \nabla \times \mathbf{H} = -i\omega\varepsilon_0 n^2\mathbf{E}$$

- These six equations are solved in each layer of the waveguide

- Boundary condition: Tangential component of **E** and **H** be continuous across both interfaces

- Waveguide modes are obtained by imposing the boundary conditions

Using Maxwell's equations

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = i\omega\mu_0 H_x, \qquad \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = i\omega\varepsilon_0 n^2 E_x$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = i\omega\mu_0 H_y, \qquad \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = i\omega\varepsilon_0 n^2 E_y$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = i\omega\mu_0 H_z, \qquad \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = i\omega\varepsilon_0 n^2 E_z$$

- Assume waveguide is infinitely wide along the x axis
- E and H are then x-independent
- For any mode, all components vary with *z* as *exp(iβz)*. Thus,

$$\frac{\partial \mathbf{E}}{\partial x} = 0, \quad \frac{\partial \mathbf{H}}{\partial x} = 0, \quad \frac{\partial \mathbf{E}}{\partial z} = i\beta\mathbf{E}, \quad \frac{\partial \mathbf{H}}{\partial z} = i\beta\mathbf{H}$$

Using Maxwell's equations

- These equations have two distinct sets of linearly polarized solutions
- For Transverse-Electric (TE) modes, $E_z = 0$ and $E_y = 0$
- TE modes are obtained by solving:

$$\frac{d^2 E_x}{dy^2} + (n^2 k_0^2 - \beta^2) E_x = 0, \qquad k_0 = \omega\sqrt{\varepsilon_0 \mu_0} = \omega / c$$

- Magnetic field components are related to E_x as:

$$H_y = -\frac{\beta}{\omega\mu_0} E_x, \qquad H_x = 0, \qquad H_z = -\frac{i}{\omega\mu_0}\frac{dE_x}{dy}$$

Using Maxwell's equations

$$\frac{d^2E_x}{dy^2}+(n^2k_0^2-\beta^2)E_x=0, \qquad k_0=\omega\sqrt{\varepsilon_0\mu_0}=\omega/c$$

- We solve this equation in each layer separately using $n=n_c,\ n_1$, and n_s.

$$E_x(y)=\begin{cases} B_c\exp[-q_1(y-d)]; & y>d, \\ A\cos(py-\phi) \quad ; & |y|\le d \\ B_s\exp[q_2(y+d)] \quad ; & y<-d, \end{cases}$$

- Constants $p,\ q_1$, and q_2 are defined as

$$p^2=n_1^2k_0^2-\beta^2, \quad q_1^2=\beta^2-n_c^2k_0^2, \quad q_2^2=\beta^2-n_s^2k_0^2.$$

- Constants $B_c,\ B_s,\ A$, and ϕ are determined from the boundary conditions at the two interfaces.

Using Maxwell's equations

- Tangential components of $\mathbf{E}$ and $\mathbf{H}$ continuous across any interface with index discontinuity.
- Mathematically, E_x and H_z should be continuous at $y = \pm d$.
- E_x is continuous at $y = \pm d$ if

$$B_c = A\cos(pd - \phi); \qquad B_s = A\cos(pd + \phi).$$

- Since $H_z \propto dE_x/dy$, dE_x/dy should also be continuous at $y = \pm d$:

$$pA\sin(pd - \phi) = q_1 B_c, \qquad pA\sin(pd + \phi) = q_2 B_s.$$

- Eliminating A, B_c, B_s from these equations, ϕ must satisfy

$$\tan(pd - \phi) = q_1/p, \qquad \tan(pd + \phi) = q_2/p$$

Using Maxwell's equations

- Boundary conditions are satisfied when

$$pd - \phi = \tan^{-1}(q_1/p) + m_1\pi, \qquad pd + \phi = \tan^{-1}(q_2/p) + m_2\pi$$

- Adding and subtracting these equations, we obtain

$$2\phi = m\pi - \tan^{-1}(q_1/p) + \tan^{-1}(q_2/p)$$

$$2pd = m\pi + \tan^{-1}(q_1/p) + \tan^{-1}(q_2/p)$$

- The last equation is called the eigenvalue equation.
- Multiple solutions for $m = 0, 1, 2, \ldots$ are denoted by TE$_m$.
- Effective index of each TE mode is $\bar{n} = \beta / k_0$.

TE mode for symmetric waveguide

- For symmetric waveguides $n_c = n_s$.
- Using $q_1 = q_2 \equiv q$, TE modes satisfy

$$q = p\tan(pd - m\pi/2).$$

- Define a dimensionless parameter

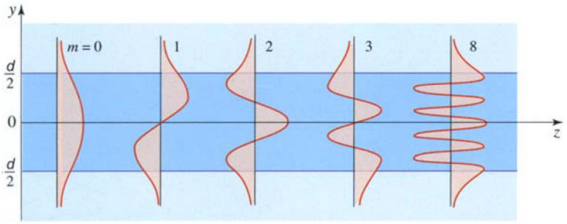
$$V = d\sqrt{p^2 + q^2} = k_0 d\sqrt{n_1^2 - n_s^2},$$

- If we use $u = pd$, the eigenvalue equation can be written as

$$\sqrt{V^2 - u^2} = u\tan(u - m\pi/2).$$

- For given values of V and m, this equation is solved to find $p = u/d$

TE mode for symmetric waveguide



- Effective index $\bar{n} = \beta/k_0 = (n_1^2 - p^2/k_0^2)^{1/2}$.
- Using $2\phi = m\pi - \tan^{-1}(q_1/p) + \tan^{-1}(q_2/p)$ with $q_1 = q_2$, phase $\phi = m\pi/2$.
- Spatial distribution of modes is found to be

$$E_y(x) = \begin{cases} B_\pm \exp[-q(|x|-d)]; & |x| > d, \\ A\cos(px - m\pi/2)\ ; & |x| \le d, \end{cases}$$

where $B_\pm = A\cos(pd \mp m\pi/2)$ and the lower sign is chosen for $x < 0$.

- Modes with even values of m are symmetric around $x = 0$ (even modes).
- Modes with odd values of m are antisymmetric around $x = 0$ (odd modes).

Modes of asymmetric waveguide

- We can follow the same procedure for $n_c \neq n_s$.
- Eigenvalue equation for TE modes:

$$2pd = m\pi + \tan^{-1}(q_1/p) + \tan^{-1}(q_2/p)$$

- Eigenvalue equation for TM modes:

$$2pd = m\pi + \tan^{-1}\left(\frac{n_1^2 q_1}{n_c^2 p}\right) + \tan^{-1}\left(\frac{n_1^2 q_2}{n_s^2 p}\right)$$

- Constants p, q_1, and q_2 are defined as

$$p^2 = n_1^2 k_0^2 - \beta^2, \quad q_1^2 = \beta^2 - n_c^2 k_0^2, \quad q_2^2 = \beta^2 - n_s^2 k_0^2.$$

- Each solution for β corresponds to a mode with effective index $\bar{n} = \beta/k_0$.
- If $n_1 > n_s > n_c$, guided modes exist as long as $n_1 > \bar{n} > n_s$.

Modes of asymmetric waveguide

- Useful to introduce two normalized parameters as

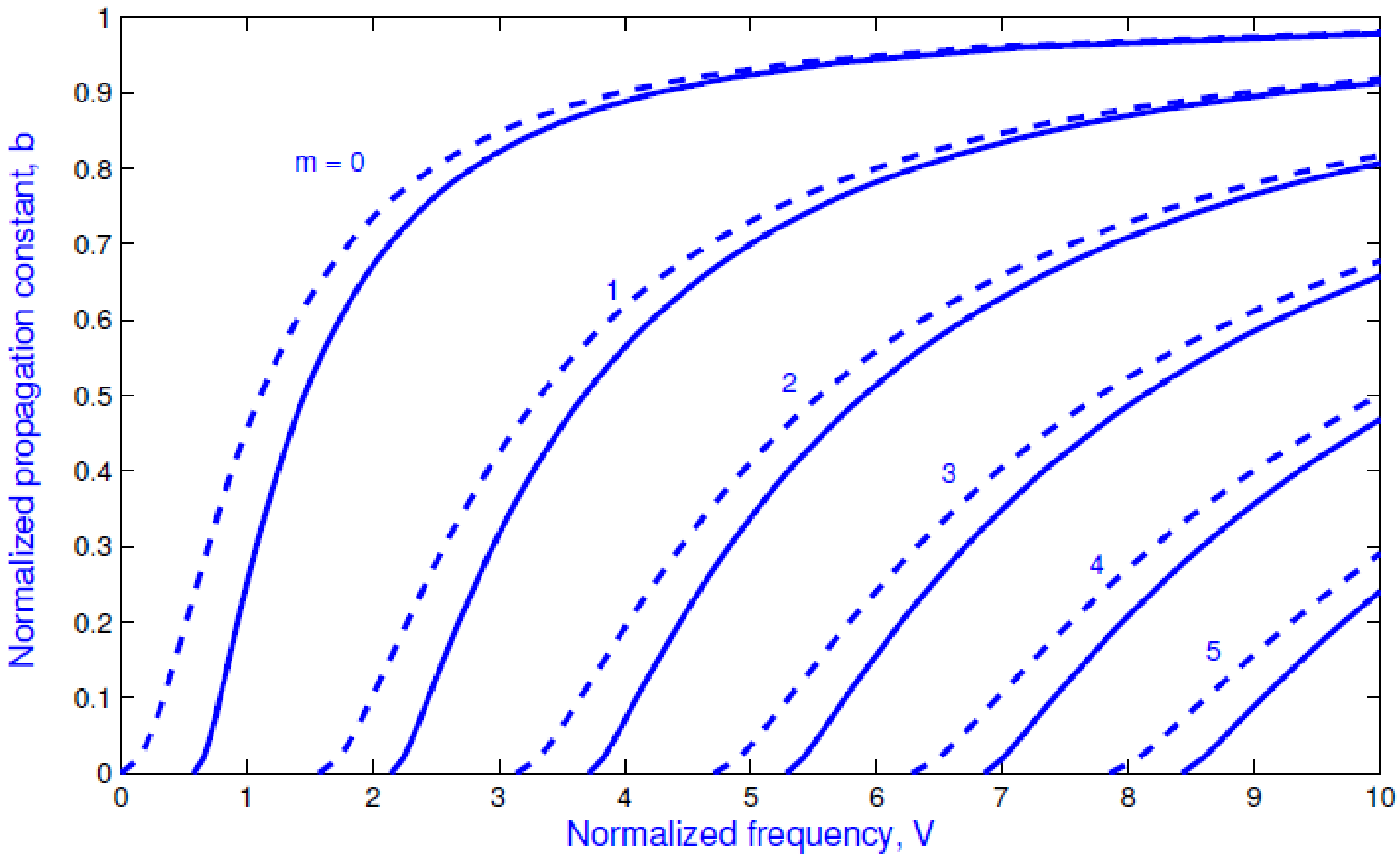
$$b = \frac{\bar{n}^2 - n_s^2}{n_1^2 - n_s^2}, \qquad \delta = \frac{n_s^2 - n_c^2}{n_1^2 - n_s^2}.$$

- b is a normalized propagation constant $(0 < b < 1)$.
- Parameter δ provides a measure of waveguide asymmetry.
- Eigenvalue equation for TE modes in terms V, b, δ:

$$2V\sqrt{1-b} = m\pi + \tan^{-1}\sqrt{\frac{b}{1-b}} + \tan^{-1}\sqrt{\frac{b+\delta}{1-b}}.$$

- Its solutions provide **universal** dispersion curves.

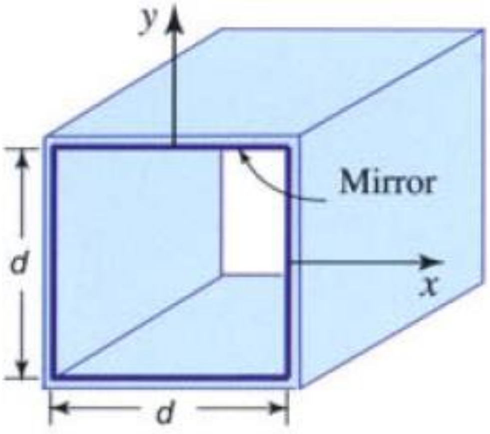
Universal dispersion curve



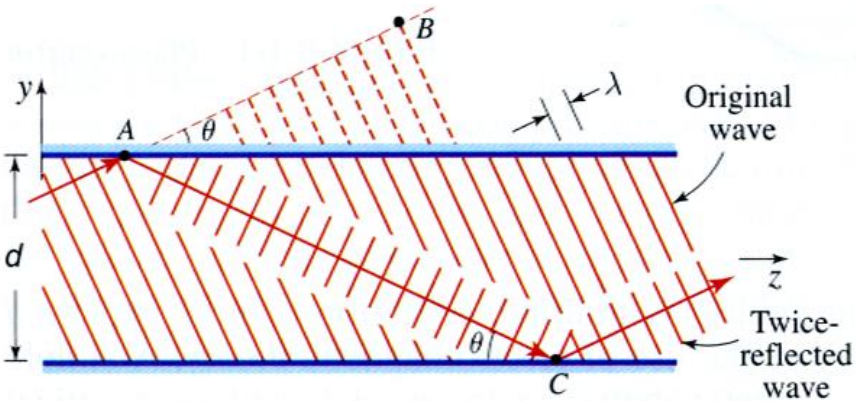
Solid lines ($\delta = 5$); dashed lines ($\delta = 0$)

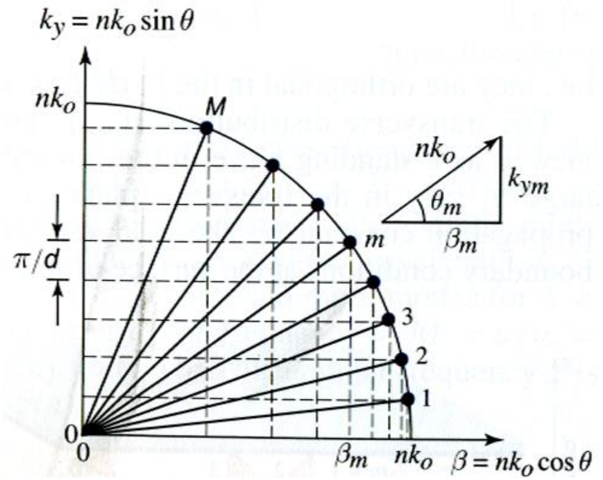
G. Agrawal

Rectangular mirror waveguide



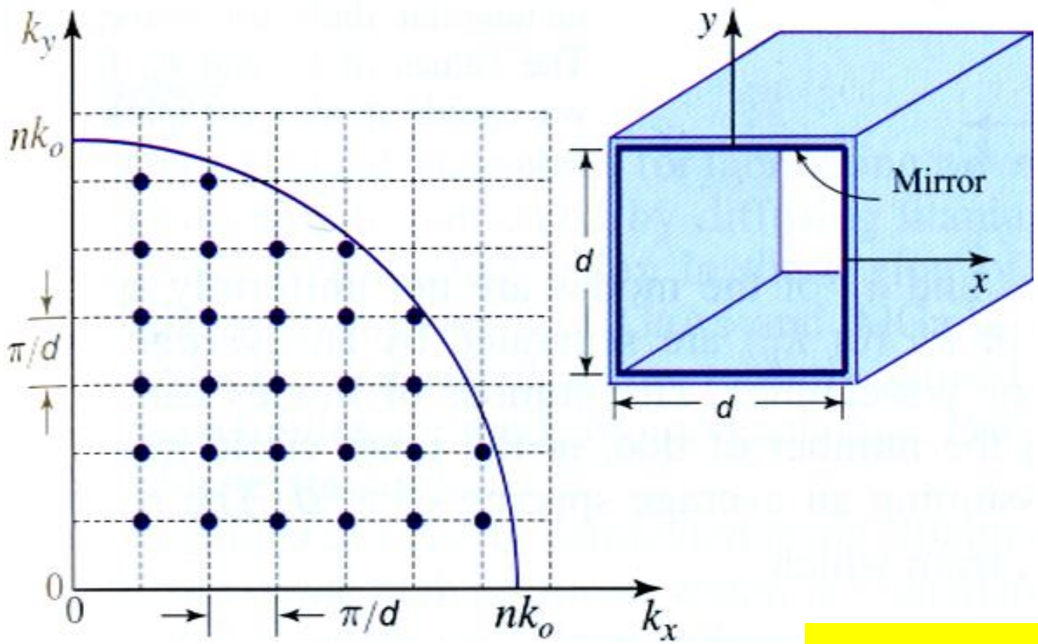
$$2k_x d = 2\pi m_x \qquad m_x = 1,2,\ldots$$
$$2k_y d = 2\pi m_y \qquad m_y = 1,2,\ldots$$





$$k_{ym} = nk_0 \sin\theta_m = m\pi / d$$

Number of modes



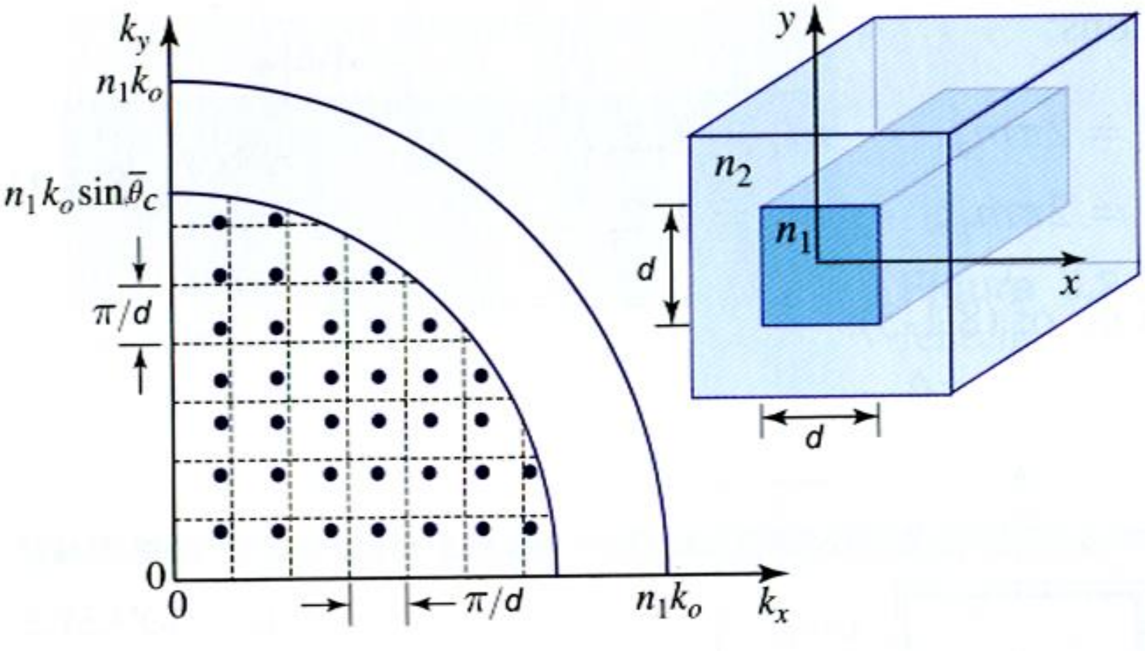
Number of modes

$$2k_x d = 2\pi m_x \qquad m_x = 1, 2, \ldots$$

$$2k_y d = 2\pi m_y \qquad m_y = 1, 2, \ldots$$

$$M = \frac{\text{Quadrant area}}{\text{Unit cell area}} = \frac{\frac{\pi n^2 k_0^2}{4}}{\frac{\pi^2}{d^2}} = \left(\frac{2d}{\lambda}\right)^2 \frac{\pi}{4}$$

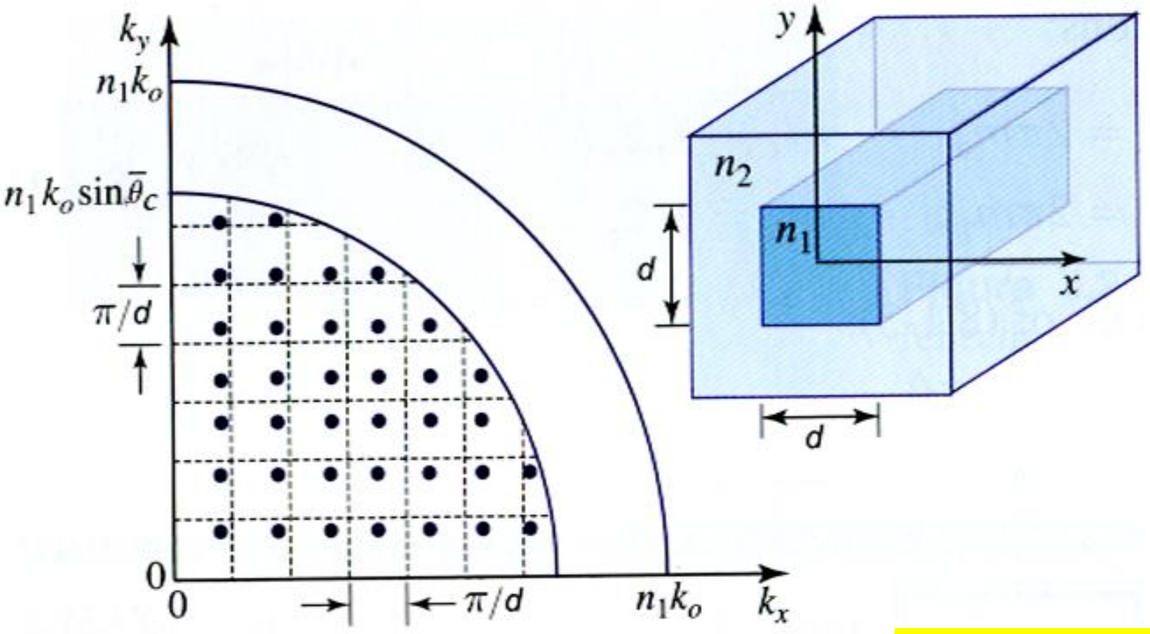
Rectangular dielectric waveguide



$$k_x^2 + k_y^2 \le n_1^2 k_0^2 \sin^2 \bar{\theta}_c$$

$$\bar{\theta}_c = \cos^{-1}\left(\frac{n_2}{n_1}\right)$$

Rectangular dielectric waveguide



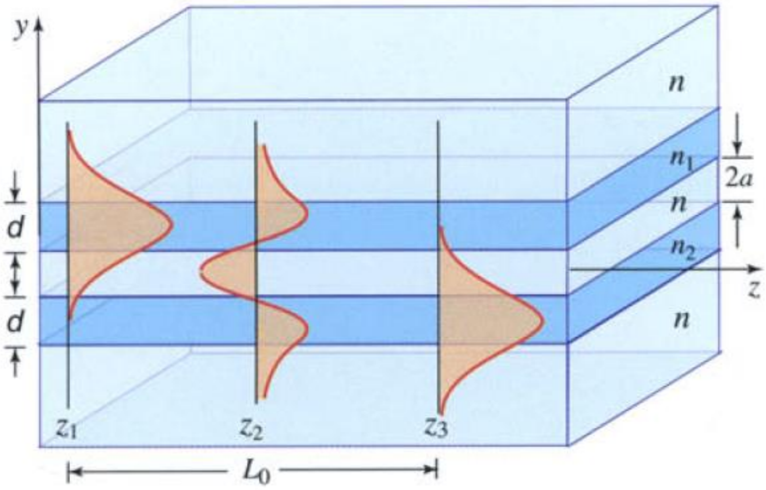
$$k_x^2 + k_y^2 \le n_1^2 k_0^2 \sin^2 \bar{\theta}_c$$

$$\bar{\theta}_c = \cos^{-1}\left(\frac{n_2}{n_1}\right)$$

Number of TE modes:

$$M \approx \frac{\pi}{4}\left(\frac{2d}{\lambda_o}\right)^2 (\mathrm{NA})^2,$$

Slab directional coupler



$$\frac{da_1}{dz} = -jC_{21}\exp(j\Delta\beta z)a_2(z)$$

$$\frac{da_2}{dz} = -jC_{12}\exp(-j\Delta\beta z)a_1(z)$$

$$\Delta\beta = \beta_1 - \beta_2$$ phase mismatch per unit length

$$C_{21} = \frac{k_0^2}{2\beta_1}(n_2^2 - n^2)\int_{a}^{a+d} dy\ u_1(y)u_2(y)$$

$$C_{12} = \frac{k_0^2}{2\beta_2}(n_1^2 - n^2)\int_{-a-d}^{-a} dy\ u_2(y)u_1(y)$$

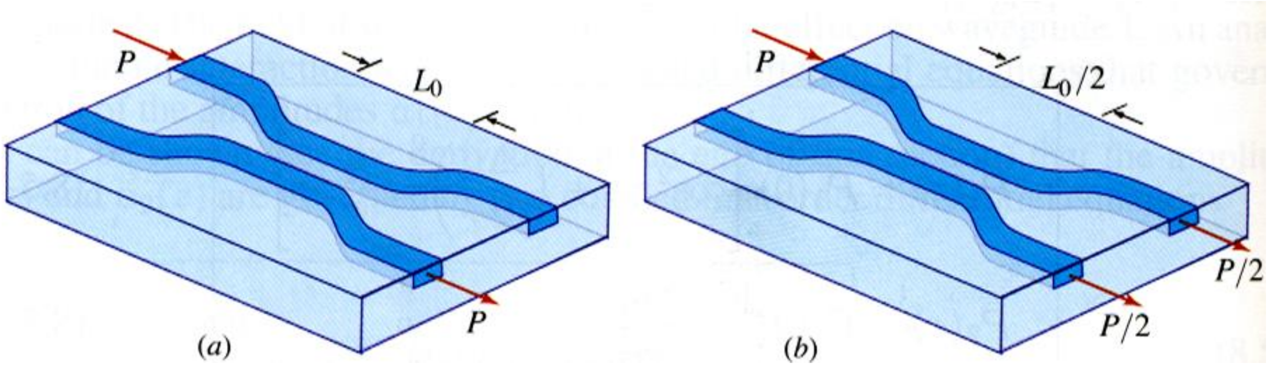
Slab directional coupler

$$P_1(z) = P_1(0)\left[\cos^2(\gamma z) + \left(\frac{\Delta\beta}{2\gamma}\right)^2 \sin^2(\gamma z)\right]$$

$$P_2(z) = P_1(0)\frac{|C_{21}|^2}{\gamma^2}\sin^2(\gamma z)$$

Coupling length

$$L_0 = \frac{\pi}{2\sqrt{C_{12}C_{21}}}$$



3dB coupler

Phase-mismatched vs. phase-matched

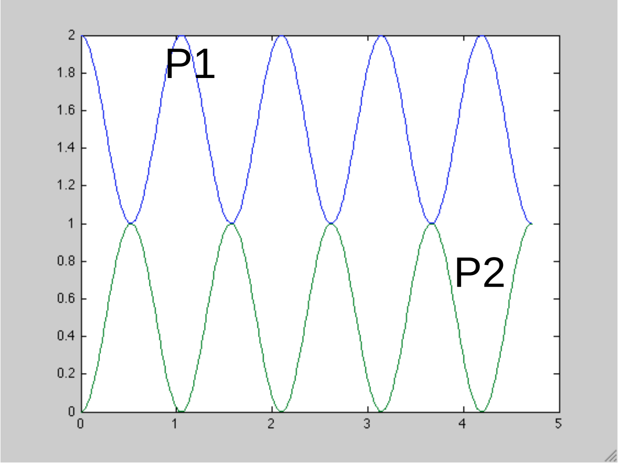
$$P_1(z) = P_1(0)\left[\cos^2 \gamma z + \left(\frac{\Delta\beta}{2\gamma}\right)^2 \sin^2 \gamma z\right]$$

$$P_2(z) = P_1(0)\frac{|\mathcal{C}_{21}|^2}{\gamma^2}\sin^2 \gamma z.$$

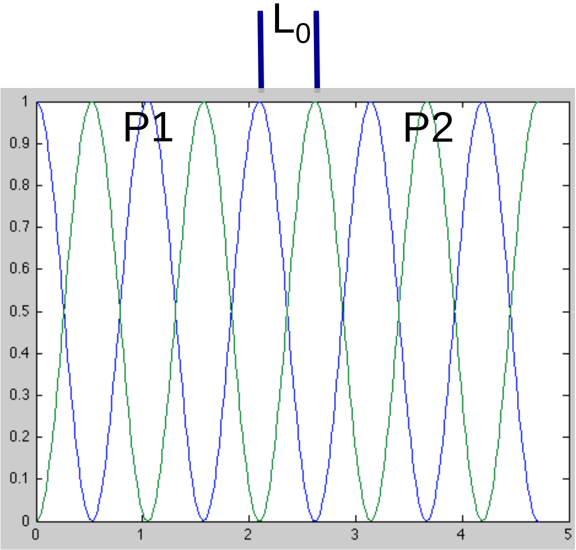
$$P_1(z) = P_1(0)\cos^2 \mathcal{C}z$$

$$P_2(z) = P_1(0)\sin^2 \mathcal{C}z.$$

Phase mismatched
$\Delta\beta \neq 0$

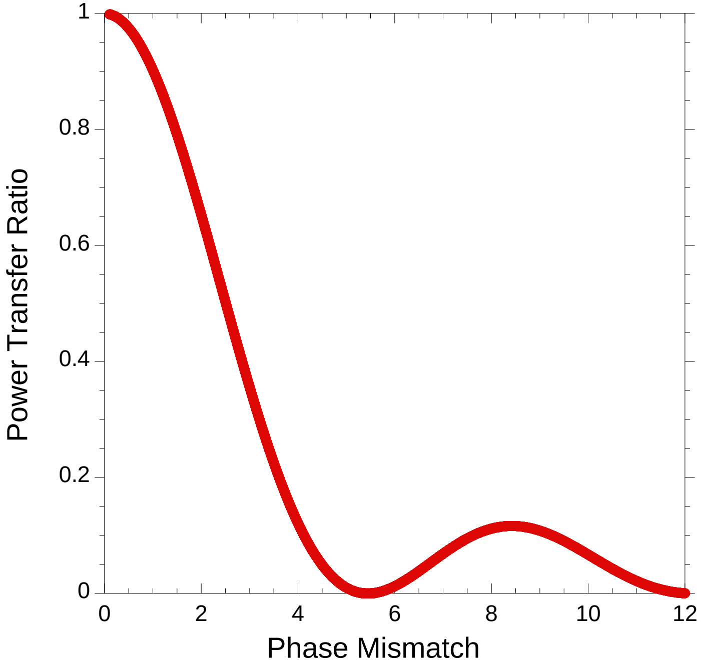


Phase matched
$\Delta\beta = 0$



$$\gamma^2 = \left(\frac{\Delta\beta}{2}\right)^2 + C_{12}C_{21}$$

Switching with directional coupler



Power transfer ratio

$$T = \frac{P_2(L_0)}{P_1(0)} = \frac{\pi^2}{4} \sin c^2 \left[\frac{1}{2} \sqrt{1 + \left(\frac{\Delta\beta L_0}{\pi} \right)^2} \right]$$

$$\sin c(x) = \frac{\sin(\pi x)}{\pi x}$$

Phase mismatch can be tuned electrically in directional couplers.
In tuning the phase mismatch from 0 to √3π, light is switch from WG 2 to 1.
Tuning can be done electro-optically or thermally, for example.

Homework #2 (OPTI510R) – due Feb. 15

Problem 1:

We use laser light in the C-band (with wavelength around 1550nm) for long distance optical communication. What is the energy of a single photon with wavelength of 1550nm? Due to limitation in detector's sensitivity we need a minimum of 1000 photon in a '1' bit of information. What would be the total light energy we need to detect for watching a movie which has a size of 1GB? You can assume that we don't need any photon for the '0' bit of information and the probabilities of having '1' or '0' bits are equal.

Problem 2:

Derive the complex susceptibility of a medium using the classical theory of permittivity. Calculate the real and imaginary part of the complex susceptibility. What is the behavior of the real and imaginary part of the complex susceptibility when the driving field frequency (ω) approaches zero, infinity, and the system resonant frequency Ω_0?

Problem 3:

Optical pulses are wave packets constructed by a continuous superposition of monochromatic plane waves:

$$\underline{\vec{E}}(\vec{r},t) = \int_0^\infty \frac{d\Omega}{2\pi} \underline{\widetilde{E}}(\Omega) e^{\mathrm{j}(\Omega t - K(\Omega) z)}\ \vec{e}_x.$$

Show that the corresponding magnetic field is given by:

$$\underline{\vec{H}}(\vec{r},t) = \int_0^\infty \frac{d\Omega}{2\pi Z_F(\Omega)} \underline{\widetilde{E}}(\Omega) e^{\mathrm{j}(\Omega t - K(\Omega) z)}\ \vec{e}_y$$

Where $Z_F\,(\Omega) = \sqrt{\frac{\mu}{\varepsilon}}$ is the impedance of the medium

Problem 4:

Calculate the time bandwidth product of the pulse shapes shown in the first column of the table below.

Pulse Shape	Fourier Transform	Pulse Width
$\underline{A}(t)$	$\underline{\tilde{A}}(\omega) = \int_{-\infty}^{\infty} a(t) e^{-j\omega t} dt$	Δt
Gaussian: $\mathrm{e}^{-\frac{t^2}{2\tau^2}}$	$\sqrt{2\pi}\tau \mathrm{e}^{-\frac{1}{2}\tau^2\omega^2}$	$2\sqrt{\ln 2}\tau$
Hyperbolic Secant: $\mathrm{sech}(\frac{t}{\tau})$	$\frac{\tau}{2}\ \mathrm{sech}\left(\frac{\pi}{2}\tau\omega\right)$	$1.7627\ \tau$
Rect-function: $= \begin{cases} 1, & \lvert t\rvert \leq \tau/2 \\ 0, & \lvert t\rvert > \tau/2 \end{cases}$	$\tau \frac{\sin(\tau\omega/2)}{\tau\omega/2}$	τ
Lorentzian: $\frac{1}{1+(t/\tau)^2}$	$2\pi\tau \mathrm{e}^{-\lvert\tau\omega\rvert}$	$1.287\ \tau$
Double-Exp.: $\mathrm{e}^{-\left\lvert\frac{t}{\tau}\right\rvert}$	$\frac{\tau}{1+(\omega\tau)^2}$	$\ln 2\ \tau$

Problem 5:

Calculate and plot the second and third order group velocity dispersion of fused silica for wavelengths from 1um to 2um. Estimate the duration of a transform limited 100fs Gaussian pulse (at 1550nm) after propagating through 1km of fused silica.

The Sellmeier coefficients for fused silica are:

$$\begin{aligned} \lambda_{1s} &= 0.0684\ \mu\mathrm{m}, & A_{1s} &= 0.69617 \\ \lambda_{2s} &= 0.1162\ \mu\mathrm{m}, & A_{2s} &= 0.40794 \\ \lambda_{3s} &= 9.8962\ \mu\mathrm{m}, & A_{3s} &= 0.89748 \end{aligned}$$

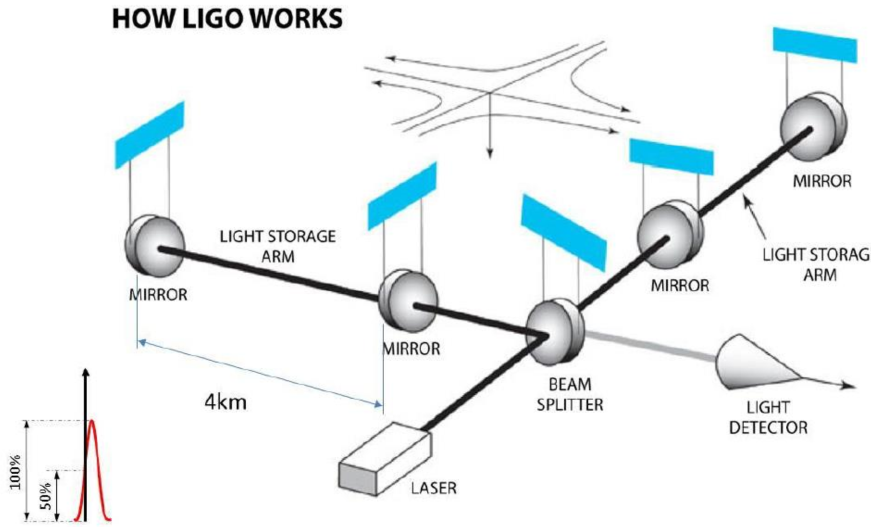
Problem 6:

This question deals with an active ring laser gyroscope. We can easily measure the beating frequency between the two clockwise and the counter-clockwise directions f_{beat} with ~1Hz precision. The laser wavelength is 632.3nm.

a) What would be the smallest rotation rate that we can measure using a ring resonator with 1m radius?

b) What is the difference in path length seen by light propagating in the two directions when the gyroscope is rotated at the smallest detectable rotation rate?

Problem 7:



Locking to 50% of resonance

To increase its sensitivity each arm of the LIGO interferometer works as a Fabry-Perot resonator. The reflectivity of each mirror is 99.99%. The distance between them is 4km.

a) What are the free spectral range, finesse, and resonant linewidth of the Fabry-Perot resonator?

b) Sketch the transmission spectrum as a function of frequency for light at normal incident. Label the vertical axis from 0 to 100%.

c) A powerful CW laser beam (wavelength 1064nm, 20W average power) is locked to a resonance line of the Fabry-Perot resonators (see the inset of the figure above). The locking point is set to be at half the transmission resonance line (50%) to get the maximum sensitivity with regard to change of the distance between the two mirrors.

   1. What is average power of light that the light detector detects? (Assuming identical path length for both arms and no loss in the system).
   2. A short burst of gravitational wave arrived to the LIGO interferometer and changed the distance between the two mirrors by about 1/100 the size of a proton (the size of a proton is roughly 1 femtometer). What is the power change on the light detector? You can assume that the distance between the two Fabry-Perot mirrors in one arm is reduced and in the other arm is increased.

Homework #3 (OPTI510R) – due Feb. 27

Problem 1:

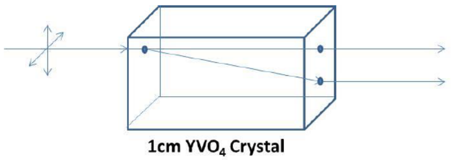
The electric field strength E of a plane electromagnetic wave varies following the expression: $E(z;\, t) = 20\cos(2\pi \cdot 4 \cdot 10^{14} \cdot t - 2\pi/(5 \cdot 10^{-7}) \cdot z + \pi/2)$. Determine the frequency ν, wavelength λ, and velocity V of the wave. Write an explicit expression for the magnitude of the Pointing vector $\boldsymbol{S}$ and calculate the value of the intensity I [W/m^2] of this wave in the uniform transparent medium. The velocity of light in vacuum is $3 \cdot 10^8$ m/s. (the unit of t is in second)

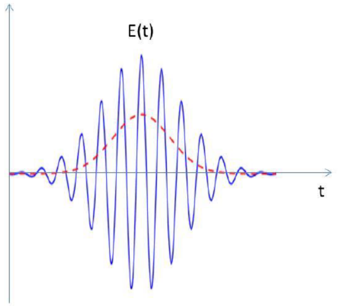
Problem 2:

Yttrium Orthovanadate (YVO_4) is a transparent uniaxial crystal that is used in many applications such as polarizing prisms, polarization independent isolators, or in divided-pulse amplification (DPA) of ultra-short optical pulses (which is a technique used to avoid nonlinear optical effects in high peak power fiber amplifiers). The Sellmeier equations for $\mathbf{YVO_4}$ (λ has unit in μm) are:

$$n_e^2 = 4.59905 + 0.110534/(\lambda^2 - 0.04813) - 0.012267612\,\lambda^2 \text{ (for extraordinary wave)}$$

$$n_o^2 = 3.77834 + 0.069736/(\lambda^2 - 0.04724) - 0.0108133\,\lambda^2 \text{ (for ordinary wave)}$$





An unpolarized ultrashort Gaussian pulse (the electric field E(t) of the pulse as the function of time is shown above on the right) propagates through a 10cm thick $\mathbf{YVO_4}$ crystal as shown. The speed of light in vacuum is 2.9979×10^8m/s. The center wavelength of the pulse is 1.56μm.

Give numerical answers to the following questions:

a) What is the FWHM duration of the input ultrashort Gaussian pulse?
b) What are the phase velocities of the ordinary and extraordinary waves?
c) What are the group velocities of the ordinary and extraordinary waves?
d) The pulse will be divided into two pulses as shown. Assuming a walk off angle of 5 degrees, what is the delay time between the two pulses?
e) Indicate the polarization of each pulse.

Problem 3: 8.1-4 (From S&T textbook)

Modal Dispersion. Light of wavelength $\lambda_o = 0.633\ \mu$m is transmitted through a mirror waveguide of mirror separation $d = 10\ \mu$m and $n = 1$. Determine the number of TE and TM modes. Determine the group velocities of the fastest and the slowest mode. If a narrow pulse of light is carried by all modes for a distance 1 m in the waveguide, how much does the pulse spread as a result of the differences of the group velocities?

Problem 4: 8.2-5 (From S&T textbook)

Field Distribution. The transverse distribution $u_m(y)$ of the electric-field complex amplitude of a TE mode in a slab waveguide is given by (8.2-10) and (8.2-13). Derive an expression for the ratio of the proportionality constants. Plot the distribution of the $m = 0$ TE mode for a slab waveguide with parameters $n_1 = 1.48$, $n_2 = 1.46$, $d = 0.5\ \mu$m, and $\lambda_o = 0.85\ \mu$m, and determine its confinement factor (percentage of power in the slab).

Problem 5:

Light of free-space wavelength $\lambda_0 = 0.87$ µm is guided by a thin planar film of width $d = 2$ µm and refractive index $n_1 = 1.6$ surrounded by a medium of refractive index $n_2 = 1.4$.

a) Determine the critical angle θ_c and its complement $\overline{\theta}_c$, the numerical aperture NA, and the maximum acceptance angle for light originating in air (n = 1).
b) Determine the number of TE modes.
c) Determine the bounce angle θ_0 and the group velocity v_0 of the $m = 0$ TE mode (5pts)
d) At which wavelengths the waveguide is singlemode?