OPTI510R: Photonics

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Announcements

- Homework #2 is due today, HW#3 is assigned due Feb. 21st
- No class Monday Feb. 26
- Pre-record lecture Friday Feb. 23 at 2PM
- Mid-term exam will be on Feb. 28
Electronics-Photonics

Electrons, Integrated circuit

Photonics integrated circuit

Optoelectronics chip
Planar waveguides

- Planar mirror waveguide
  - Waveguide modes
  - Number of modes
  - Cut-off condition
  - Dispersion

- Dielectric waveguide
  - Waveguide modes
  - Number of modes
  - Cut-off condition
  - Dispersion
Some useful math

\[ h = 6.625 \times 10^{-34} \text{ J} \cdot \text{s} = 4.134 \times 10^{-15} \text{ eV} \cdot \text{s} \]
\[ c = 1.6 \times 10^{-19} \text{ C} \]
\[ c = 3.0 \times 10^8 \text{ m/s} \]
\[ k_B = 1.38 \times 10^{-23} \text{ J/K} \]
\[ \sigma = 5.67 \times 10^{-8} \text{ W/K}^4 \cdot \text{m}^2 \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \]
\[ \mu_0 = 1.26 \times 10^{-6} \text{ H/m} \]
\[ \sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \]
\[ \cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \]
\[ 2 \cos A \cos B = \cos(A - B) + \cos(A + B) \]
\[ 2 \sin A \sin B = \cos(A - B) - \cos(A + B) \]
\[ 2 \sin A \cos B = \sin(A + B) + \sin(A - B) \]
\[ 2 \cos A \sin B = \sin(A + B) - \sin(A - B) \]
\[ \sin 2A = 2 \sin A \cos A \]
\[ \cos 2A = 2 \cos^2 A - 1 \]
\[ \cos 2A = 1 - 2 \sin^2 A \]
\[ \sinh x = \frac{1}{2} (e^x - e^{-x}) \]
\[ \cosh x = \frac{1}{2} (e^x + e^{-x}) \]
\[ \nabla(\phi + \psi) = \nabla\phi + \nabla\psi \]
\[ \nabla \phi \psi = \phi \nabla \psi + \psi \nabla \phi \]
\[ \nabla \cdot (F + G) = \nabla \cdot F + \nabla \cdot G \]
\[ \nabla \times (F + G) = \nabla \times F + \nabla \times G \]
\[ \nabla \cdot (F \cdot G) = (\nabla \cdot F)G + (G \cdot \nabla)F + F \times (\nabla \times G) + G \times (\nabla \times F) \]
\[ \nabla \cdot (\phi F) = \phi \nabla \cdot F + F \cdot \nabla \phi \]
\[ \nabla \cdot (F \times G) = G \cdot \nabla \times F - F \cdot \nabla \times G \]
\[ \nabla \cdot (\nabla \times F) = 0 \]
\[ \nabla \times (\phi F) = \phi (\nabla \times F) + \nabla \phi \times F \]
\[ \nabla \times (F \times G) = F(\nabla \cdot G) - G(\nabla \cdot F) + (G \cdot \nabla)F - (F \cdot \nabla)G \]
\[ \nabla \times (\nabla \times F) = \nabla(\nabla \cdot F) - \nabla^2 F \]
\[ \nabla \times \nabla \phi = 0 \]
\[ \oint_{\mathcal{S}} (F \cdot n) \, da = \int_{\mathcal{V}} (\nabla \cdot F) \, d^3 x \]
\[ \oint_{\mathcal{C}} F \cdot dl = \int_{\mathcal{S}} (\nabla \times F) \cdot n \, da \]
\[ \oint_{\mathcal{S}} \phi n \, da = \int_{\mathcal{V}} \nabla \phi \, d^3 x \]
\[ \oint_{\mathcal{S}} F(G \cdot n) \, da = \int_{\mathcal{V}} [F(\nabla \cdot G) + (G \cdot \nabla)F] \, d^3 x \]
\[ \oint_{\mathcal{S}} (n \times F) \, da = \int_{\mathcal{V}} (\nabla \times F) \, d^3 x \]
Planar mirror waveguides

\[ \lambda = \lambda_0 / n \]
\[ k = nk_0 \]
\[ c = c_0 / n \]

TEM plane wave
TE: E polarized in x-direction
TM: H polarized in x-direction

- \( \pi \) phase shift for each reflection (boundary conditions)
- Amplitude and polarization do not change (perfect mirror).
- Not practical due to the fact that there is no perfect metal mirror
Planar mirror waveguides

- Self-consistency: The wave reflects twice and reproduces itself
- Therefore the phase shift in travelling from A to B must be equal to or differ by an integer multiple of $2\pi$ from the phase shift from A to C
- Modes are fields that maintain the same transverse distribution and polarization at all locations along the waveguide axis.

\[ \sin \theta_m = \frac{(q + 1)\lambda}{2d} = \frac{m\lambda}{2d} \]
Planar mirror waveguides

A guided wave consists of the superposition of two plane waves in the y-z plane at angle $\pm \theta$ with respect to the z axis. The components of the mode wave vector are:

$$k_{ym} = nk_0 \sin \theta$$

$$m = m / d$$

$$k^2_m = k_{zm}^2 = k^2 \frac{m^2}{d^2}$$
Mode field profile

A guided wave consists of the superposition of two plane waves in the y-z plane at angle $\pm \theta$ with respect to the z axis.

\[
\begin{align*}
\left\{ \begin{array}{l}
A_m \exp(-jk_{ym} y - j\beta_m z) \\
+ e^{j(m-1)\pi} A_m \exp(+jk_{ym} y - j\beta_m z)
\end{array} \right. \\
\text{, upward wave}
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{l}
2A_m \cos(k_{ym} y) \exp(-j\beta_m z) \\
2jA_m \sin(k_{ym} y) \exp(-j\beta_m z)
\end{array} \right. \\
\text{, m is odd}
\end{align*}
\]

\[
\begin{align*}
\left\{ \begin{array}{l}
2A_m \cos(k_{ym} y) \exp(-j\beta_m z) \\
2jA_m \sin(k_{ym} y) \exp(-j\beta_m z)
\end{array} \right. \\
\text{, m is even}
\end{align*}
\]
Mode field profile

TE modes

\[ E_x(y, z) = a_m u_m(y) \exp(-j\beta_m z) \]

\[ a_m = \sqrt{2d} A_m \quad \text{or} \quad j\sqrt{2d} A_m \]

\[ u_{m(y)} = \begin{cases} 
\sqrt{\frac{2}{d}} \cos\left(\frac{m\pi y}{d}\right), & m = 1,3,5,... \\
\sqrt{\frac{2}{d}} \sin\left(\frac{m\pi y}{d}\right), & m = 2,4,6,... 
\end{cases} \]

Modes are orthogonal and normalized
Mode properties

TE modes

\[ E_x(y, z) = a_m u_m(y) \exp(-j \beta_m z) \]

\[ a_m = \sqrt{2d} A_m \quad \text{or} \quad j \sqrt{2d} A_m \]

\[ u_m(y) = \begin{cases} 
\sqrt{\frac{2}{d}} \cos\left(\frac{m \pi y}{d}\right), & m = 1, 3, 5, \ldots \\
\frac{2}{d} \sin\left(\frac{m \pi y}{d}\right), & m = 2, 4, 6, \ldots
\end{cases} \]

Modes are orthogonal and normalized

\[ \int_{-d/2}^{d/2} u_m(y) u_l(y) \, dy = 0, \quad l \neq m, \quad \int_{-d/2}^{d/2} u_m^2(y) \, dy = 1 \]

Orthogonal condition

Normalized condition

Any field distribution can be discomposed into a sum of modes
Number of modes, Cutoff

Number of modes

\[
\sin \theta_m = \frac{m\lambda}{2d} < 1, \quad M = \frac{2d}{\lambda}
\]

Reduce to nearest integer

Dispersion relation

\[
\beta_m^2 = \left(\frac{\omega}{c}\right)^2 - \frac{m^2 \pi^2}{d^2}
\]

Cutoff wavelength and frequency

\[
\lambda_c = 2d, \quad \nu_c = \frac{c}{2d}
\]

\[
\omega_c = 2\pi \nu_c = \frac{\pi c}{d}
\]

For \(\lambda > \lambda_c\) or \(\nu < \nu_c\) there is no guided mode
Dispersion relation

\[ \beta_m^2 = \left(\frac{\omega}{c}\right)^2 - m^2 \frac{\pi^2}{d^2} \]

This leads to waveguide dispersion
Group velocity

**Dispersion relation**

\[ \beta_m^2 = (\omega / c)^2 - m^2 \pi^2 / d^2 \]

\[ \beta_m = \frac{\omega}{c} \sqrt{1 - m^2 \frac{\omega_c^2}{\omega^2}}. \]

**Cutoff frequency**

\[ \omega_c = 2\pi \nu_c = \pi c / d \]

\[ 2\beta_m \frac{d\beta_m}{d\omega} = 2\omega / c^2 \]

\[ v = \frac{d\omega}{d\beta} = c^2 \beta_m / \omega = c^2 k \cos \theta_m / \omega = c \cos \theta_m \]

\[ v_m = c \cos \theta_m = c \sqrt{1 - m^2 \frac{\omega_c^2}{\omega^2}} \]
Group velocity

Is this normal or anomalous dispersion?
TE versus TM

(Homework)
Multimode fields

In general, a field with arbitrary distribution and vanishing amplitude at the mirror can be guided.

\[ E_x(y, z) = \sum_{m=0}^{M} a_m u_m(y) \exp(-j\beta_m z) \]
Planar dielectric waveguide

- Core film sandwiched between two layers of lower refractive index
- Bottom layer is often a substrate with $n = n_s$
- Top layer is called the cover layer ($n_c \neq n_s$)
- Air can also acts as a cover ($n_c = 1$)
- $n_c = n_s$ in symmetric waveguides
Planar dielectric waveguide

Symmetric waveguide

Reflection due to TIR

(similar to planar mirror waveguide)

\[
\sin c = \frac{n_2}{n_1}
\]

\[
\frac{c}{2} = \frac{1}{2} \sin \left( \frac{n_2}{n_1} \right) \cos \left( \frac{n_2}{n_1} \right)
\]

Self Consistency

\[
\frac{2\pi}{\lambda} \quad 2d \sin \theta - 2\varphi_r = 2\pi m
\]

\[
2k_y d - 2\varphi_r = 2\pi m
\]
Phase shift for TIR

TE wave

\[ \tan \frac{r}{2} = \sqrt{\frac{\sin^2 - c}{\sin^2}} \]

TM wave

\[ \tan \frac{r}{2} = \frac{n_1^2}{n_2^2} \sqrt{\frac{\sin^2 - c}{\sin^2}} \]

We can now arrive at an equation for the mode angles
Transcendental equation for modes

\[
\tan\left(\frac{\pi d}{\lambda} \sin \theta - \frac{m \pi}{2}\right) = \sqrt{\frac{\sin^2 \bar{\theta}_c}{\sin^2 \theta}} - 1
\]

Mirror waveguide \( \varphi_r = \pi \), or \( \tan(\varphi_r/2) = \infty \)
Number of modes

\[ M = \frac{\sin \bar{\theta}_c}{\lambda/2d} \]

, or

\[ M = \frac{2d}{\lambda_o} \] NA

, where

\[ NA = \sqrt{n_1^2 - n_2^2} \]

Single mode

\[ \frac{2d}{NA} < 1 \]
Transcendental equation for modes

\[ \beta_m = n_1 k_0 \cos \theta_m \quad m = N_{\text{eff}}^m / c_0 \]
Cut-off frequency

\[ \omega_c = 2\pi \nu_c = \pi c / d \]

\[ \nu_c = \omega_c / 2\pi = \frac{1}{\text{NA}} \frac{c_o}{2d} \]

Mirror waveguide

Dielectric waveguide

There is no gap for dielectric waveguide – always one guided mode for a symmetric slab (not so for asymmetric)
The electric field in a symmetric dielectric waveguide is harmonic within the slab and exponentially decaying outside the slab.

\[
E_x(y, z) = a_m u_m(y) \exp(-j\beta_m z)
\]

In the core

\[
\begin{align*}
&u_m(y) \mu \\
&\cos \frac{2}{d} \sin \frac{m y}{d}, \ m = 0, 2, 4, \ldots \\
&\sin \frac{2}{d} \sin \frac{m y}{d}, \ m = 1, 3, 5, \ldots
\end{align*}
\]
Evanescent field component

\[ u_m(y) \propto \begin{cases} 
\exp(-\gamma_m y), & y > d/2 \\
\exp(\gamma_m y), & y < -d/2 
\end{cases} \]

The z dependence must be identical in order to satisfy continuity at ± d/2. Signs are chosen to obtain a decaying field

\[ E_x(y, z) = \sum_{m=0}^{M} a_m u_m(y) \exp(-j\beta_m z) \]

\[ (\nabla^2 + n^2 k_0^2)E_x(y, z) = 0 \]

Extinction coefficient

\[ \gamma_m^2 = \beta_m^2 - n_2^2 k_0^2 \]

\[ \gamma_m = n_2 k_0 \sqrt{\frac{\cos^2 \theta_m}{\cos^2 \theta_c}} - 1 \]
\[ E_x (y, z) = \sum_{m=0}^{M} a_m u_m (y) \exp(-j \beta_m z) \]
Gaussian beam and waveguide mode

Gaussian beam in free space

Fundamental mode in dielectric waveguide (no diffraction!)
TE versus TM

(Homework)