



OPTI510R: Photonics

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Announcements

- HW #5 is assigned (due April 9)
- April 9th class will be in 305 instead of 307
- April 11th class will be at 2PM instead of 11AM, still in 307
- Final exam May 2



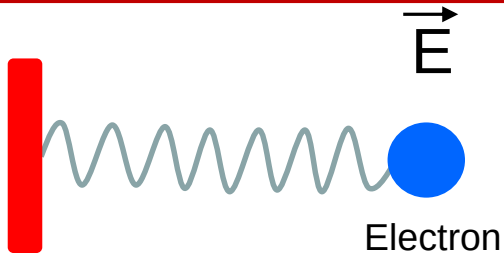
Nonlinear optical effects in fibers

- Introduction to nonlinear optics
- Stimulated Brillouin scattering
- Stimulated Raman scattering
- Self-phase modulation
- Cross phase modulation
- Soliton propagation
- Four-Wave-Mixing (FWM), *may not have time for this*





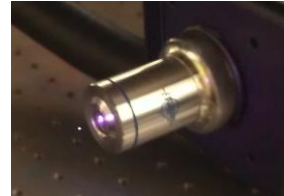
Nonlinear optics



$$P = \chi^{(1)} E + \chi^{(2)} E^2 + \chi^{(3)} E^3 + \dots$$

$\chi^{(2)}, \chi^{(3)} \dots$ are very small

Credit: Alex Erstad



Linear Optics



Nonlinear Optics



Extreme Nonlinear Optics



Nonlinear optical effects

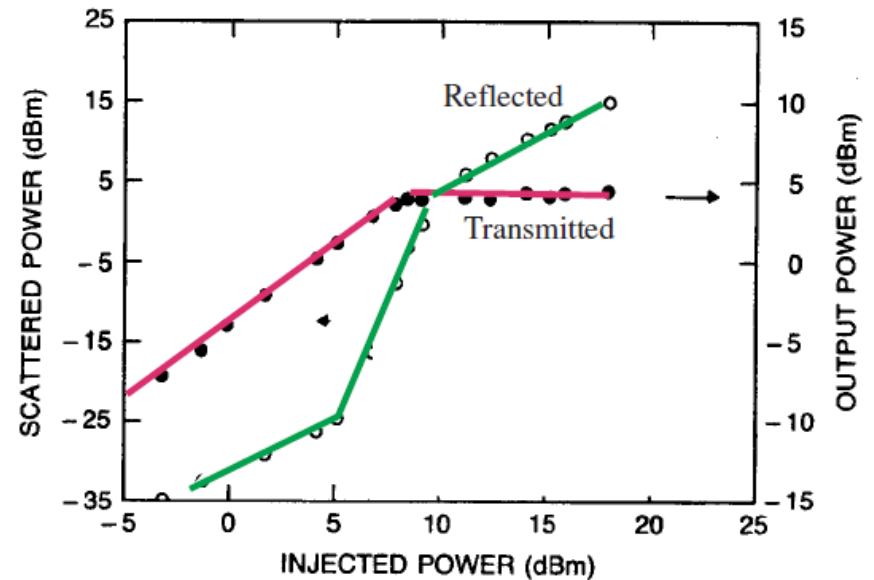
- Second harmonic generation (SHG)
- Third harmonic generation (THG)
- High harmonic generation (HHG)
- Sum/Difference frequency generation (SFG/DFG)
- Optical parametric processes (OPA, OPO, OPG)
- Kerr effect
- Self-focusing
- Self-phase modulation (SPM)
- Cross-phase modulation (XPM)
- Four-wave mixing (FWM)
- Multiphoton absorption
- Photo-ionization
- Raman/Brillouin scattering
- and more...



Stimulated Brillouin scattering

- Predicted by Leon Brillouin in 1922
- Scattering of light from acoustic waves
- Becomes a stimulated process when input power exceeds a threshold level
- Low threshold power for long fibers (5 mW)

Most of the power is reflected backward after SBS threshold is reached!





Stimulated Brillouin scattering

- Pump produces density variations through electrostriction, resulting in an index grating which generates Stokes wave through Bragg diffraction
- Energy and momentum conservation require:

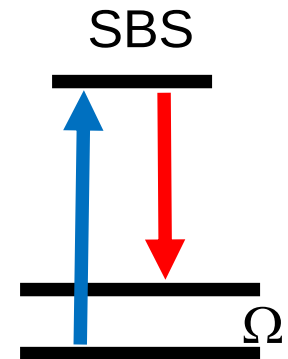
$$\Omega_B = \omega_p - \omega_s; \mathbf{k}_A = \mathbf{k}_p - \mathbf{k}_s$$

- Acoustic waves satisfy the dispersion relation:

$$\Omega_B = v_A * |\mathbf{k}_A| = v_A * |(\mathbf{k}_p - \mathbf{k}_s)| \sim 2 * v_A * |\mathbf{k}_p| = 2 * v_A * 2\pi * n_p / \lambda_p$$

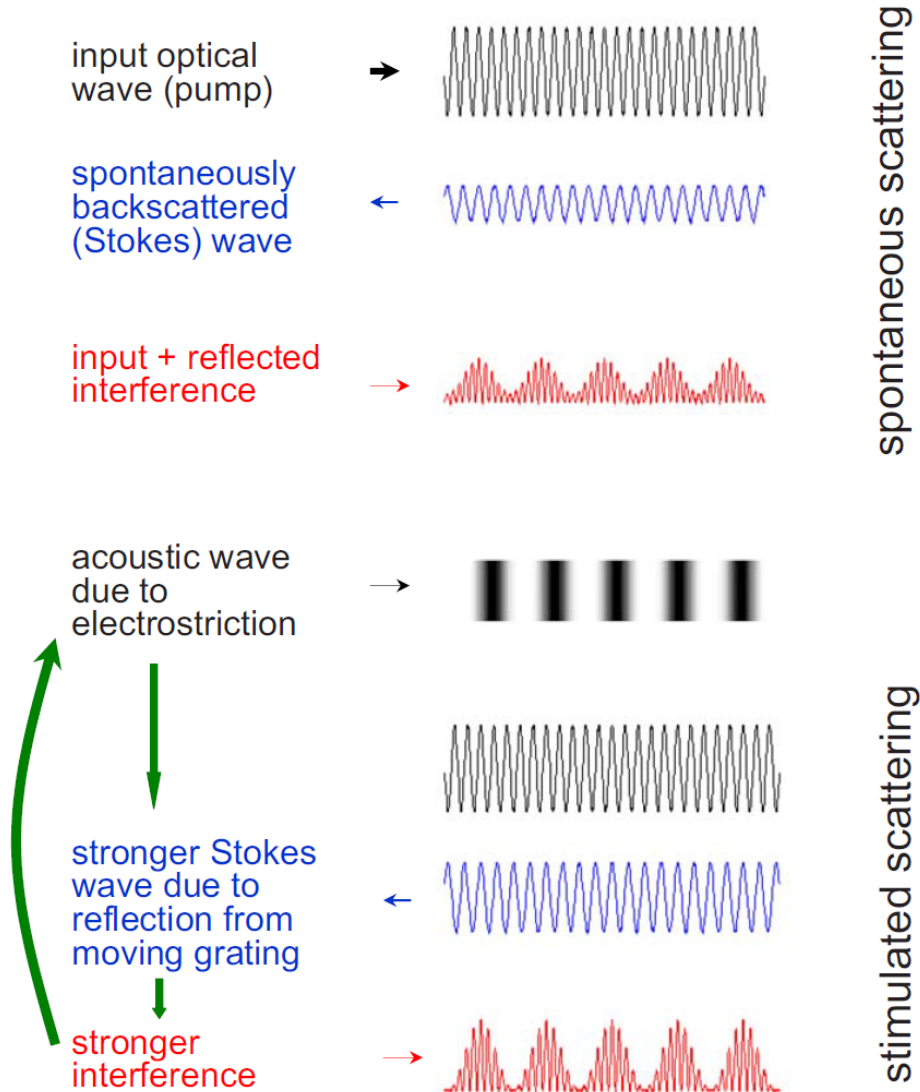
$$f_A = \Omega_B / 2\pi = 2 * v_A * 2\pi * n_p / \lambda_p \sim 11 \text{GHz} \quad (\text{Brillouin frequency shift})$$

if we use $v_A = 5.96 \text{ km/s}$, $n_p = 1.45$, and $\lambda_p = 1550 \text{ nm}$



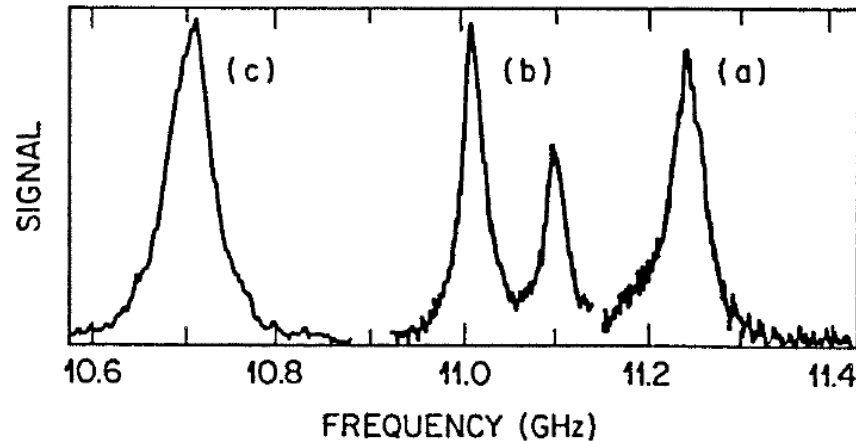


Stimulated Brillouin scattering





Brillouin gain spectrum in optical fibers



- Measured spectra for (a) silica-core (b) depressed-cladding, and (c) dispersion-shifted fibers
- Brillouin gain spectrum is quite narrow (50 MHz)
- Brillouin shift depends on GeO_2 doping within the core
- Multiple peaks are due to the excitation of different acoustic modes



Brillouin threshold in optical fibers

$$\begin{cases} \frac{dI_p}{dz} = -g_B I_p I_s - \alpha I_p, \\ \frac{dI_s}{dz} = -g_B I_p I_s + \alpha I_s, \end{cases}$$

α is the fiber loss

g_B is the Brillouin gain coefficient

$$P_o L_e = \int_{z=0}^L P(z) dz$$

L_e is the effective length

$$L_e = \frac{1 - e^{-\alpha L}}{\alpha}$$

$$\boxed{\frac{P_{th} g_B L_e}{A_{eff}} = 21}$$

P_{th} is the Brillouin threshold



Brillouin threshold in optical fibers

Telecommunication Fibers:

For long fibers, $L_e \sim 50$ km for $\alpha = 0.2$ dB at 1550 nm

For telecom fibers, $A_{eff} = 50 - 75 \mu\text{m}^2$

$$g_B = 5 \cdot 10^{-11} \text{ m/W}$$

Threshold power: $P_{th} \sim 1$ mW is relatively small!



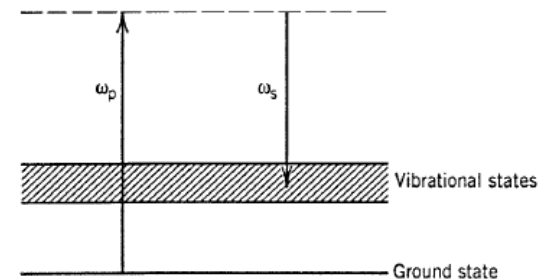
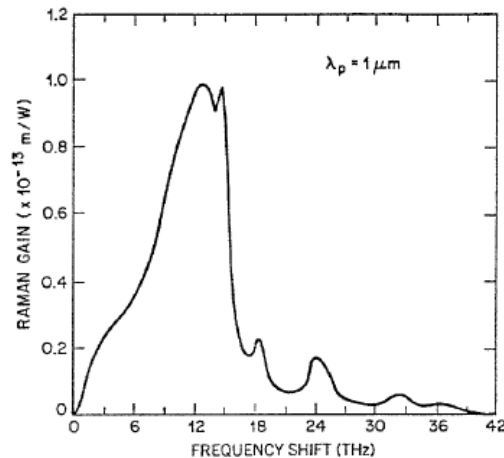
Control SBS

- Phase modulation: Sinusoidal modulation at several frequencies >0.1 GHz or with a pseudorandom bit pattern
- Cross-phase modulation by launching a pseudorandom pulse train at a different wavelength
- Temperature gradient along the fiber: Changes in $\nu_B = 2 * \nu_A * n_p / \lambda_p$ through temperature dependence of n_p
- Built-in strain along the fiber: Changes in ν_B through n_p
- Non-uniform core radius and dopant density: mode index n_p also depends on fiber design parameters (a and Δ)
- Control of overlap between the optical and acoustic modes
- Use of large-core fibers: A larger core reduces SBS threshold by enhancing A_{eff}



Stimulated Raman scattering

- Discovered by C. V. Raman in 1928
- Scattering of light from vibrating silica molecules
- Amorphous nature of silica turns vibrational state into a band
- Raman gain is maximum near 13 THz
- Scattered light red-shifted by 100 nm in the 1.5 μm region





SRS threshold

$$\begin{cases} \frac{dI_s}{dz} = g_R I_p I_s - \alpha_s I_s, \\ \frac{dI_p}{dz} = -\frac{\omega_p}{\omega_s} g_R I_p I_s - \alpha_p I_p, \end{cases} \quad \frac{g_R P_0^{cr} L_{\text{eff}}}{A_{\text{eff}}} \approx 16.$$

For telecom fibers, $A_{\text{eff}} = 50 - 75 \mu\text{m}^2$

$$g_B = 10^{-13} \text{ m/W}$$

- Threshold power $P_{th} \sim 100\text{mW}$ is too large to be of concern
- Inter-channel crosstalk in WDM systems because of Raman gain



SRS: Good or Bad?

- Inter-channel crosstalk in WDM systems because of Raman gain

But...

- Raman amplifiers are a boon for WDM systems (easy to implement)
- Can be used in the entire 1300-1650nm range
- EDFA bandwidth limited to 40 nm near 1550nm
- Distributed nature of Raman amplification lowers noise
- Needed for opening new transmission bands in telecom systems



Self-phase modulation (SPM)

- First observation:
F. Demartini et al., Phys. Rev. 164, 312 (1967)
F. Shimizu, PRL 19, 1097 (1967)
- Pulse compression though SPM was suggested by 1969:
R. A. Fisher and P. L. Kelley, APL 24, 140 (1969)
- First observation of optical Kerr effect inside optical fibers:
R. H. Stolen and A. Ashkin, APL 22, 294 (1973)
- SPM-induced spectral broadening in optical fibers:
R. H. Stolen and C. Lin Phys. Rev. A 17, 1448 (1978)
- Prediction and observation of solitons in optical fibers:
A. Hasegawa and F. Tappert, APL 23, 142 (1973)
Mollenauer, Stolen, and Gordon, PRL 45, 1095 (1980)



Self-phase modulation (SPM)

For an ultrashort pulse with a Gaussian shape and constant phase, the intensity at time t is given by $I(t)$:

$$I(t) = I_0 \exp\left(-\frac{t^2}{\tau^2}\right)$$

Optical Kerr effect:

$$n(I) = n_0 + n_2 \cdot I$$

This variation in refractive index produces a shift in the instantaneous phase of the pulse:

$$\phi(t) = \omega_0 t - kx = \omega_0 t - \frac{2\pi}{\lambda_0} \cdot n(I)L$$

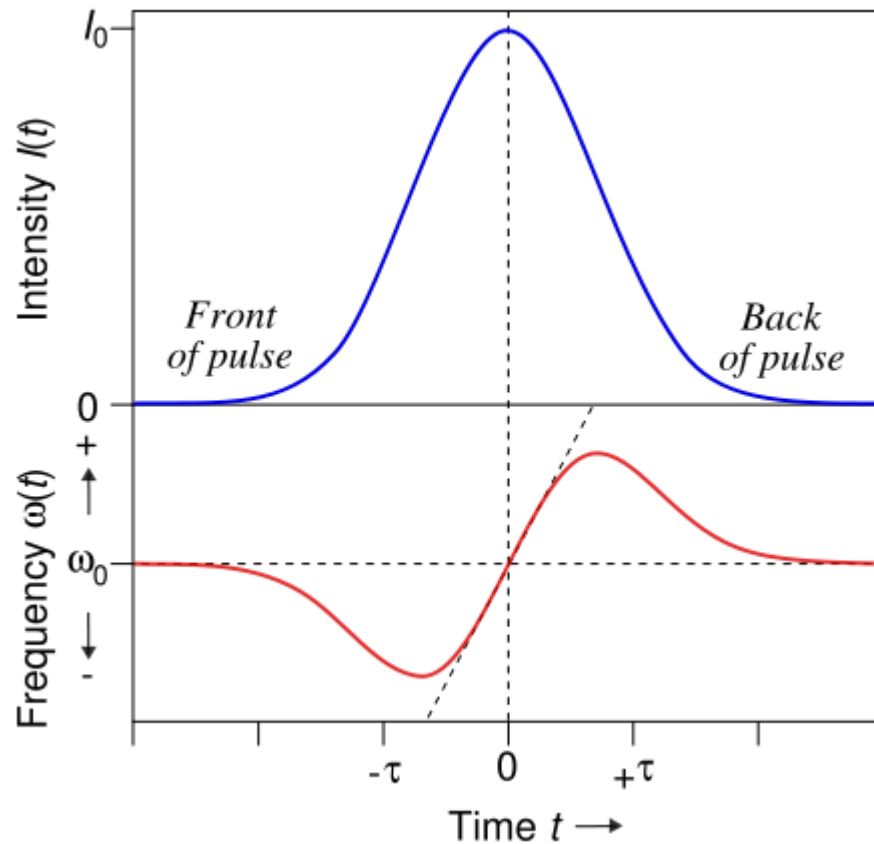
The phase shift results in a frequency shift of the pulse. The instantaneous frequency $\omega(t)$ is given by:

$$\omega(t) = \frac{d\phi(t)}{dt} = \omega_0 - \frac{2\pi L}{\lambda_0} \frac{dn(I)}{dt},$$

$$\omega(t) = \omega_0 + \frac{4\pi L n_2 I_0}{\lambda_0 \tau^2} \cdot t \cdot \exp\left(-\frac{t^2}{\tau^2}\right).$$



Self-phase modulation (SPM)





Self-phase modulation (SPM)

- An optical field modifies its own phase (SPM)
- Phase shift varies with time for pulses
- Each optical pulse becomes chirped
- As a pulse propagates along the fiber, its spectrum changes because of SPM

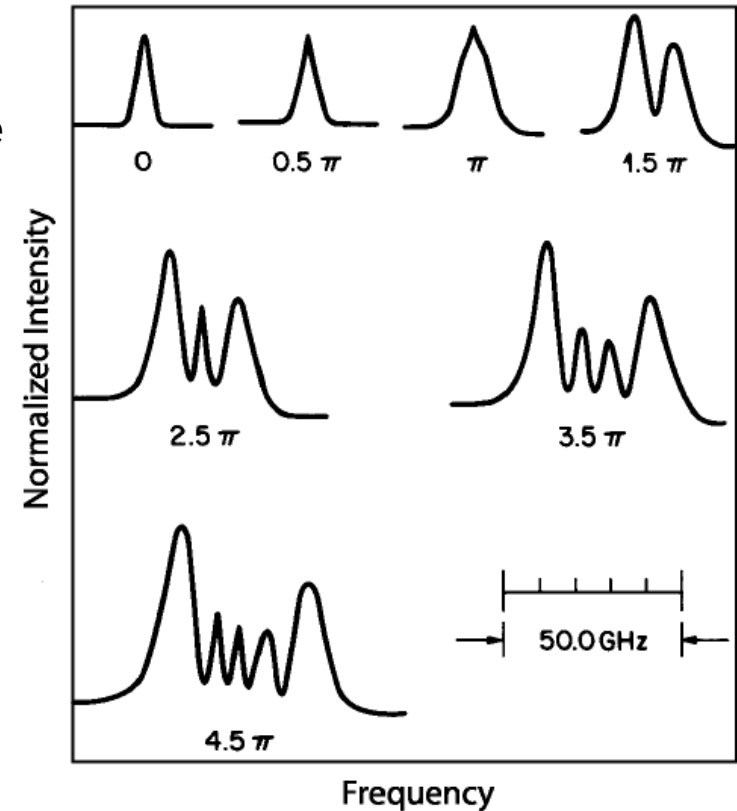


Self-phase modulation (SPM)

$$\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}}$$

$$\varphi_{NL} = \gamma \cdot P_0 \cdot L$$

Output spectrum
as the function of
the nonlinear phase
shift



- First observed inside optical fiber by Stolen and Lin (1978)
- 90-ps pulses transmitted through a 100-m-long fiber
- Output spectrum depends on shape and chirp of input pulses.
- Even spectral compression can occur for suitably chirped pulses



SPM: Good or Bad?

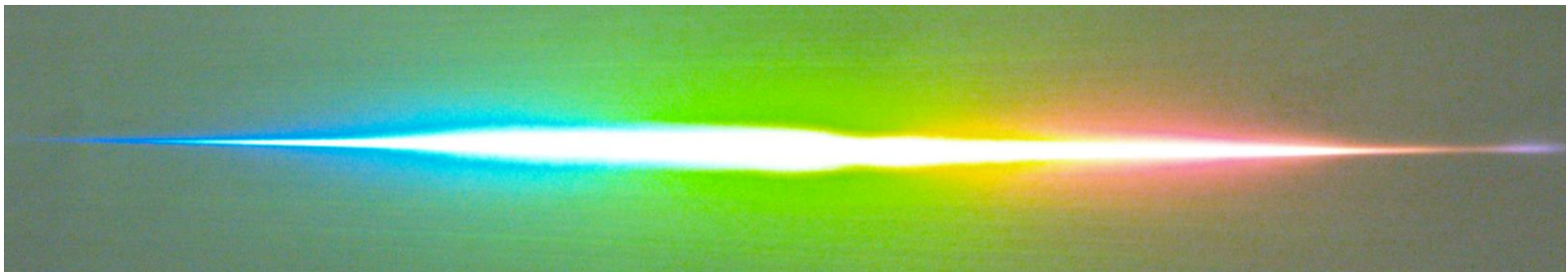
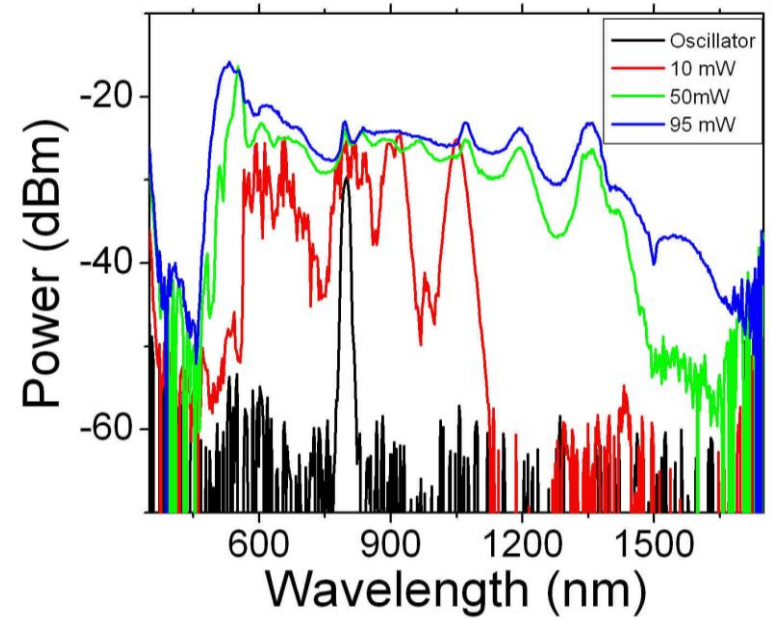
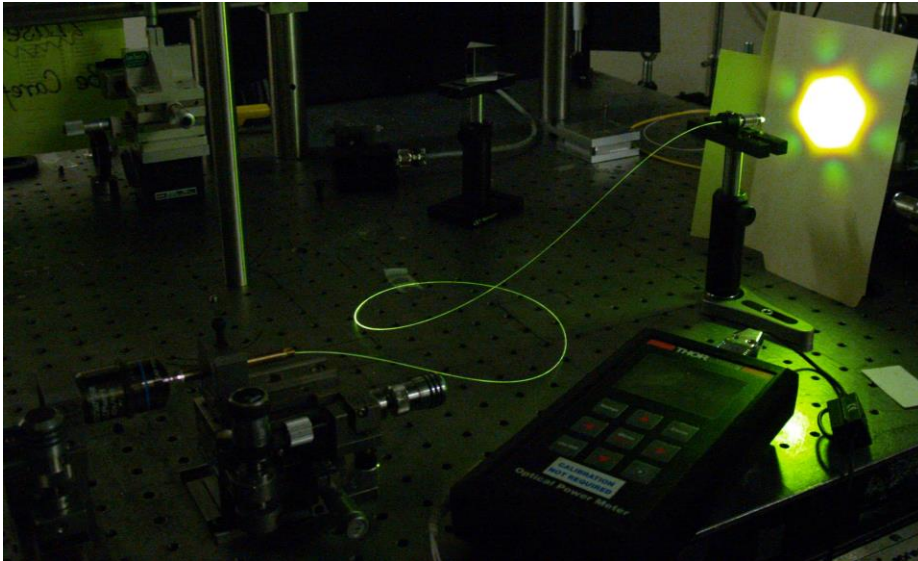
- SPM-induced spectral broadening can degrade performance of a lightwave system

But...

- SPM often used for fast optical switching (NOLM or MZI)
- Formation of standard and dispersion-managed optical solitons
- Useful for all-optical regeneration of WDM channels
- Other applications (pulse compression, supercontinuum generation chirped-pulse amplification, passive mode-locking, etc.)



Supercontinuum generation





Cross-phase modulation

Consider two optical fields propagating simultaneously:

- The nonlinear refractive index seen by one wave depends on the intensity of the other wave as:

$$\Delta n_{NL} = n_2(|A_1|^2 + b|A_2|^2)$$

- Total nonlinear phase shift in a fiber of length L:

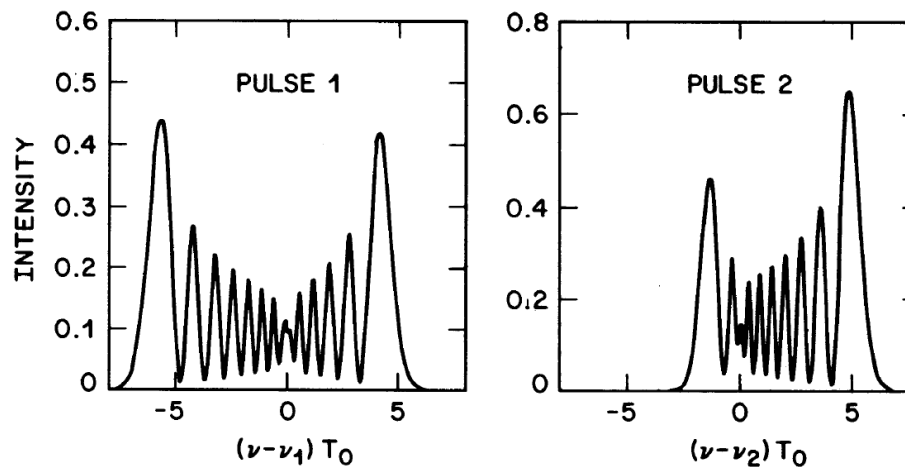
$$\phi_{NL} = (2\pi L/\lambda) * n_2 [I_1(t) + bI_2(t)]$$

- An optical beam modifies not only its own phase but also of other co-propagating beams (XPM)
- XPM induces nonlinear coupling among overlapping optical pulses.



Cross-phase modulation

- Fiber dispersion affects the XPM considerably
- Pulses belonging to different WDM channels travel at different speeds
- XPM occurs only when pulses overlap





XPM-Good or Bad?

- XPM leads to inter-channel crosstalk in WDM systems
- It can produce amplitude and timing jitter

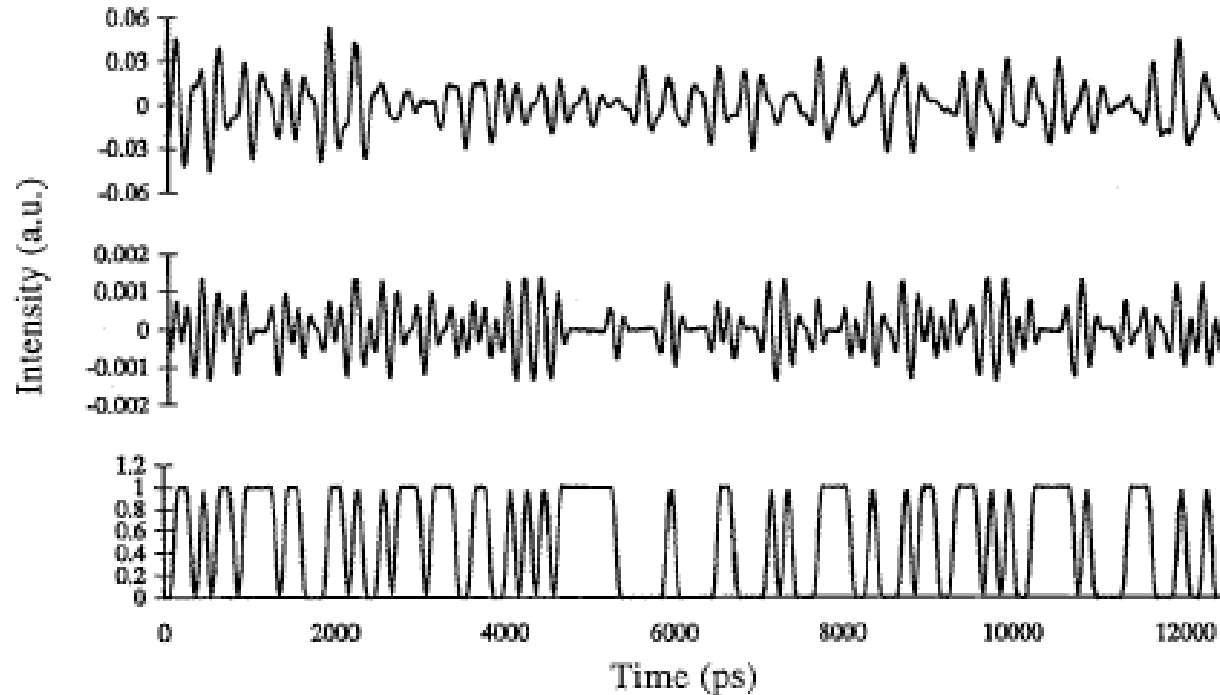
But...

XPM can be used for:

- Nonlinear Pulse Compression
- Passive mode locking
- Ultrafast optical switching
- De-multiplexing of OTDM channels
- Wavelength conversion of WDM channels



XPM-induced crosstalk



- A CW probe propagated with 10-Gb/s pump channel.
- Probe phase is modulated through XPM.
- Dispersion converts phase modulation into amplitude modulation.
- Probe power after 130 (middle) and 320 km (top) exhibits large fluctuations (Hui et al., JLT, 1999).



Soliton propagation in fibers

- The word *soliton* refers to special kinds of wave packets that can propagate undistorted over long distances: Ideal for long distance communication!
- The discovery of Optical Solitons dates back to 1971 when Zakharov and Shabat solved in 1971 the nonlinear Schrodinger (NLS) equation with the inverse scattering method.
- Hasegawa and Tappert realized in 1973 that the same NLS equation governs pulse propagation inside optical fibers. They predicted the formation of both bright and dark solitons.
- Bright solitons were first observed in 1980 by Mollenauer et al.



Nonlinear Schrodinger equation

From the Maxwell's equations it can be shown that an optical field propagating inside an optical fiber is governed by following equation:

$$i \frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0$$

Nonlinear Schrodinger equation

β_2 is the GVD of the optical fiber

γ is the nonlinear coefficient of the fiber, $\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}}$



Dispersion and nonlinear length

$$i \frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \cancel{\gamma |A|^2 A} = 0 \quad (\text{no nonlinear term})$$

$$\tau_{out} = \tau_{in} (1 + (\beta_2 \cdot L / \tau^2)^2)^{1/2} \quad (\text{assuming Gaussian pulse shape})$$

$$\tau_{out} = \tau_{in} (1 + (L/L_D)^2)^{1/2} \quad \text{Where, } L_D = \tau^2 / |\beta_2|, \text{ is the dispersion length}$$



Dispersion and nonlinear length

$$i \frac{\partial A}{\partial z} - \cancel{\frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2}} + \gamma |A|^2 A = 0 \quad (\text{no dispersion term})$$

→ $A(L, t) = A(0, t) \cdot \exp(i\varphi_{NL})$; where, $\varphi_{NL} = \gamma \cdot L \cdot |A(0, t)|^2$

Maximum nonlinear phase shift: $\varphi_{max} = \gamma P_0 L = L/L_{NL}$

Nonlinear length: $L_{NL} = (\gamma P_0)^{-1}$



Soliton propagation

$$i \frac{\partial A}{\partial z} - \frac{1}{2} \beta_2 \frac{\partial^2 A}{\partial T^2} + \gamma |A|^2 A = 0$$

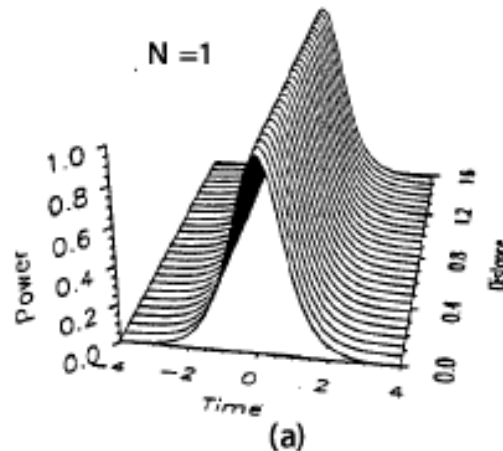
Solution depends on a single parameter:

$$N^2 = \frac{\gamma P_0 T_0^2}{|\beta_2|}$$

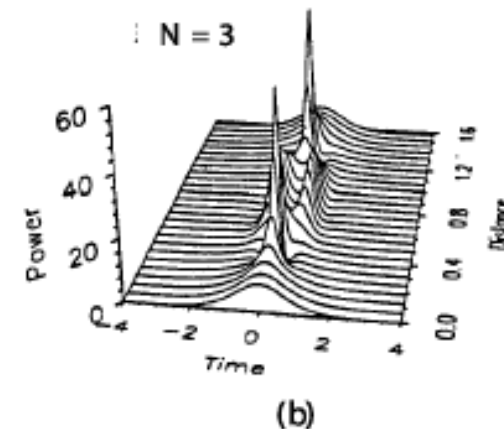
N is the soliton number

Since n_2 is positive
need β_2 to be negative

Fundamental soliton



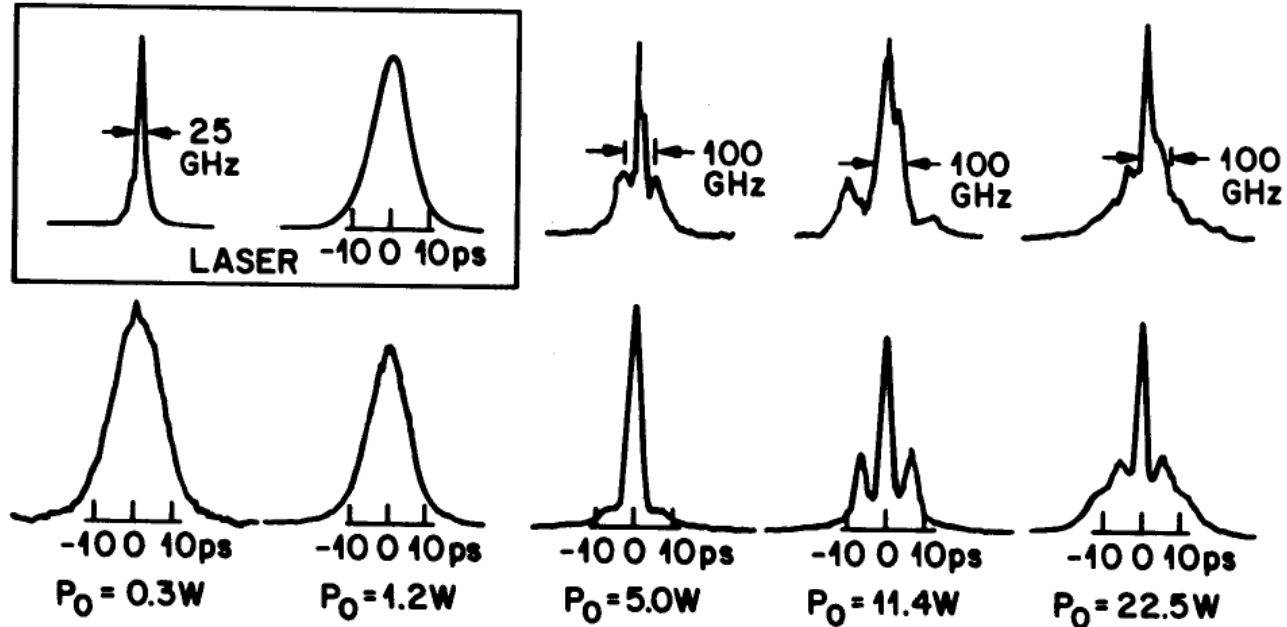
Third order soliton



$$L_{NL} = (\gamma P_0)^{-1} = L_D = T_0^2 / |\beta_2|$$



Soliton propagation

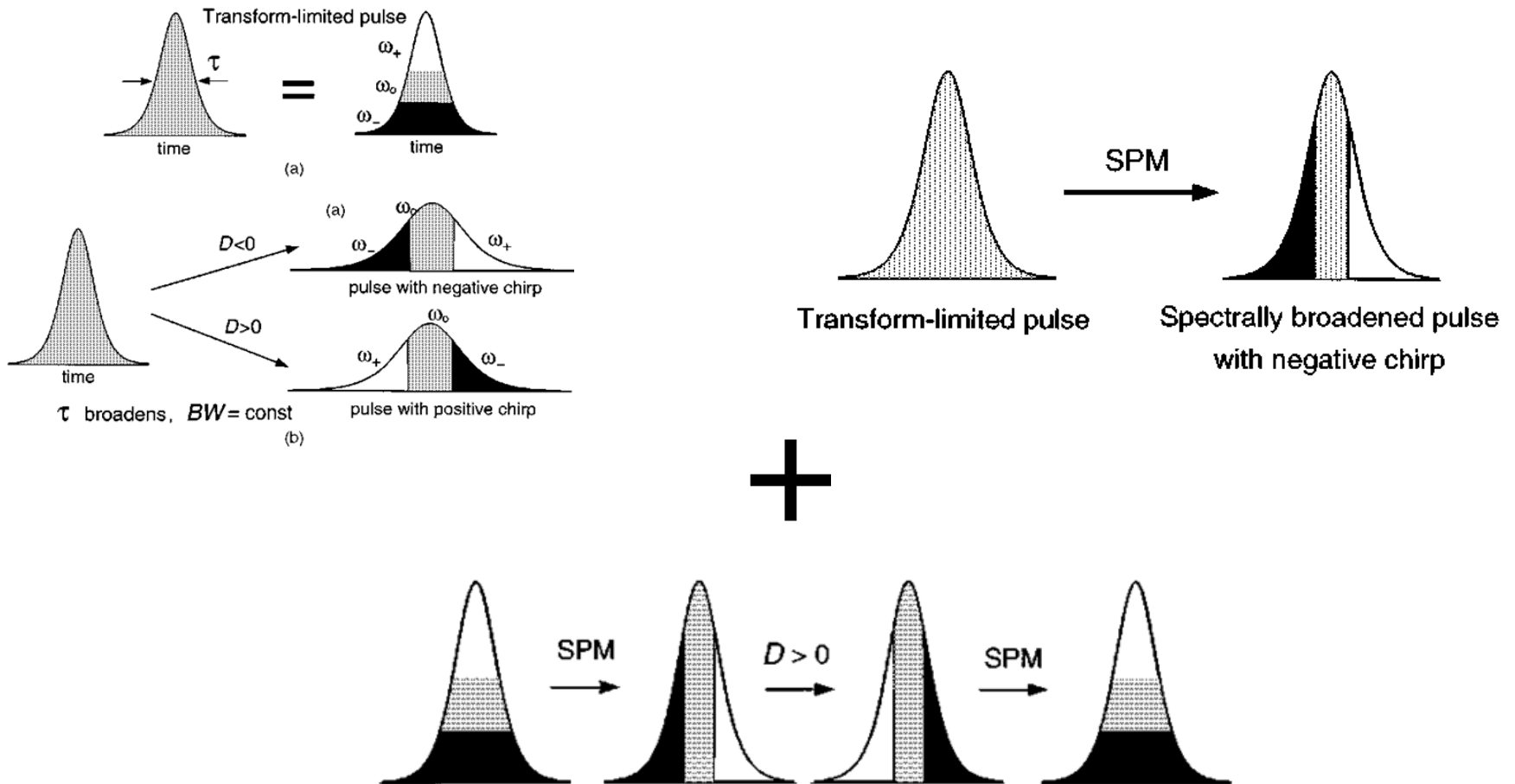


L. F. Mollenauer, R. H. Stolen, and J. P. Gordon, *Phys. Rev. Lett.* **45**, 1095 (1980)

700m fiber, $\lambda=1550\text{nm}$, 9.3 μm core diameter



Explain soliton





Four-wave-mixing

$$\mathbf{P} = \varepsilon_0 \left(\chi^{(1)} \cdot \mathbf{E} + \chi^{(2)} : \mathbf{E}\mathbf{E} + \chi^{(3)} : \mathbf{E}\mathbf{E}\mathbf{E} + \dots \right) \quad (\text{Induced polarization})$$

$$\mathbf{P}_{\text{NL}} = \varepsilon_0 \chi^{(3)} : \mathbf{E}\mathbf{E}\mathbf{E}, \quad (\text{third order nonlinear polarization term})$$

Consider four optical waves oscillating at frequencies ω_1 , ω_2 , ω_3 , and ω_4 and linearly polarized along the same axis x . *The total electric field can be written as:*

$$\mathbf{E} = \frac{1}{2} \hat{x} \sum_{j=1}^4 E_j \exp[i(k_j z - \omega_j t)] + \text{c.c.},$$

$$\rightarrow \mathbf{P}_{\text{NL}} = \frac{1}{2} \hat{x} \sum_{j=1}^4 P_j \exp[i(k_j z - \omega_j t)] + \text{c.c.},$$



Four-wave-mixing

We find that P_j ($j=1$ to 4) consists of a large number of terms involving the products of three electric fields.

For example, P_4 can be expressed as:

$$P_4 = \frac{3\epsilon_0}{4} \chi_{xxxx}^{(3)} [|E_4|^2 E_4 + 2(|E_1|^2 + |E_2|^2 + |E_3|^2) E_4 + 2E_1 E_2 E_3 \exp(i\theta_+) + 2E_1 E_2 E_3^* \exp(i\theta_-) + \dots],$$

where θ_+ and θ_- are defined as

$$\begin{aligned} \theta_+ &= (k_1 + k_2 + k_3 - k_4)z - (\omega_1 + \omega_2 + \omega_3 - \omega_4)t, \\ \theta_- &= (k_1 + k_2 - k_3 - k_4)z - (\omega_1 + \omega_2 - \omega_3 - \omega_4)t. \end{aligned}$$



Four-wave-mixing

There are two types of FWM. The term containing θ_+ corresponds to the case in which three photons transfer their energy to a single photon at the frequency $\omega_4 = \omega_1 + \omega_2 + \omega_3$. This term is responsible for the phenomena such as third-harmonic generation ($\omega_1 = \omega_2 = \omega_3$). In general, it is difficult to satisfy the phase-matching condition for such processes to occur in optical fibers with high efficiencies.

The term containing θ_- corresponds to the case in which two photons at frequencies ω_1 and ω_2 are annihilated with simultaneous creation of two photons at frequencies ω_3 and ω_4 such that:

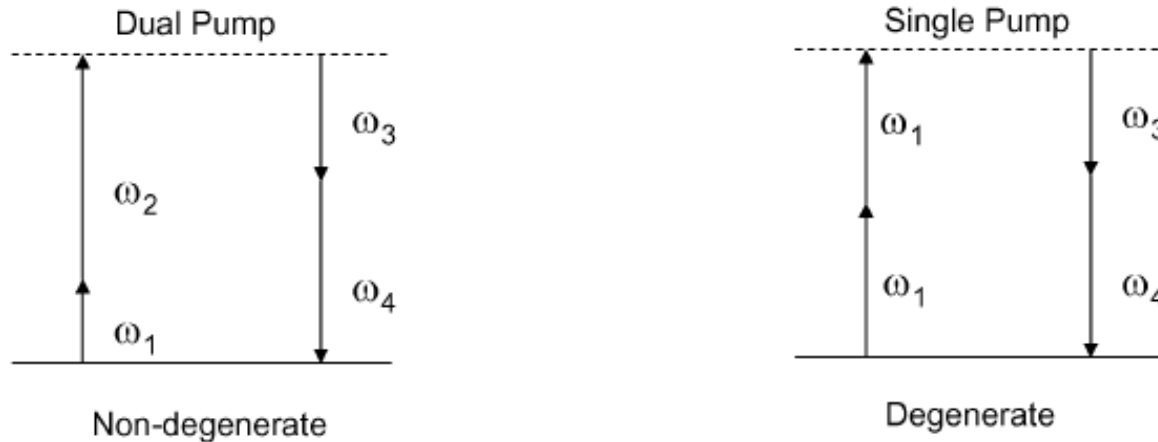
$$\omega_3 + \omega_4 = \omega_1 + \omega_2$$

The phase-matching requirement for this process to occur is:

$$\begin{aligned}\Delta k &= k_3 + k_4 - k_1 - k_2 \\ &= (n_3 \omega_3 + n_4 \omega_4 - n_1 \omega_1 - n_2 \omega_2)/c = 0.\end{aligned}$$



Four-wave-mixing



FWM efficiency governed by phase mismatch (in a waveguide):

$$\Delta = \beta(\omega_3) + \beta(\omega_4) - \beta(\omega_1) - \beta(\omega_2)$$

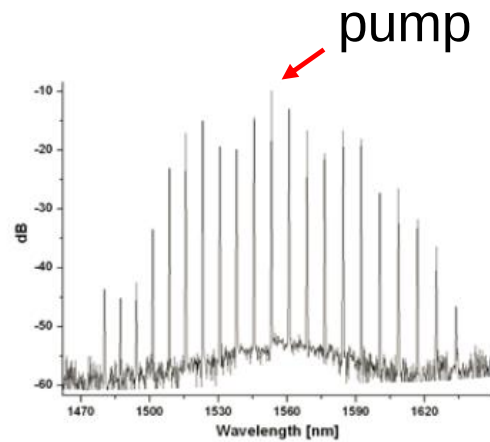
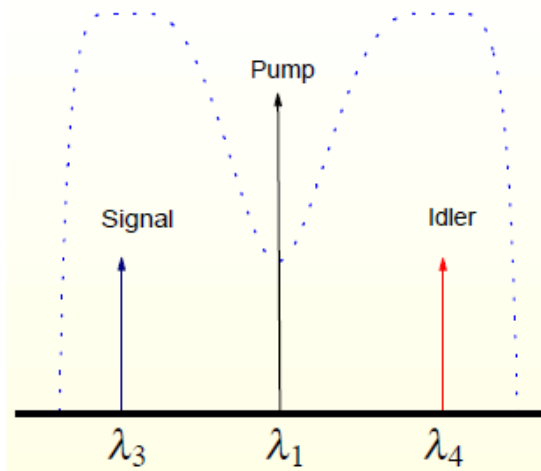
In the degenerate case ($\omega_1 = \omega_2$), $\omega_3 = \omega_1 + \Omega$, and $\omega_4 = \omega_1 - \Omega$

Expanding β in a Taylor series, $\Delta = \beta_2 \Omega^2$

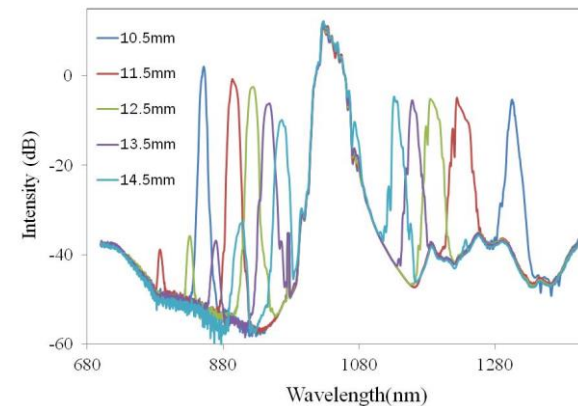
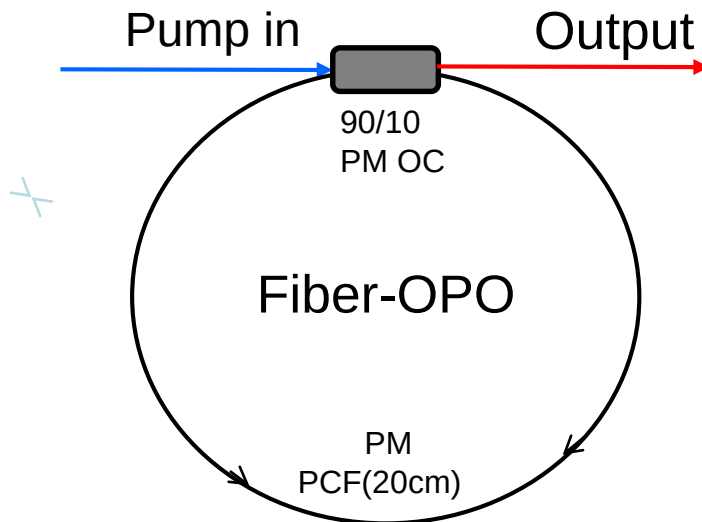
FWM becomes important for WDM systems designed with low dispersion fibers!



Four-wave-mixing



Cascaded FWM





FWM-good or bad?

- FWM leads to inter-channel crosstalk in WDM systems
- It can be avoided through dispersion management

On the other hand...

FWM can be used beneficially for:

- Parametric amplification and lasing
- Optical phase conjugation
- Wavelength conversion of WDM channels
- Supercontinuum generation



Summary

Major Nonlinear Effects:

- Self-Phase Modulation (SPM)
- Cross-Phase Modulation (XPM)
- Four-Wave Mixing (FWM)
- Stimulated Raman Scattering (SRS)
- Stimulated Brillouin Scattering (SBS)

Origin of Nonlinear Effects in Optical Fibers:

- Ultrafast third-order susceptibility χ_3



Literature

M. E. Marhic, *Fiber Optical Parametric Amplifiers, Oscillators and Related Devices* (Cambridge University, 2007)

G. P. Agrawal, *Nonlinear Fiber Optics*, (Academic Press, 2007)

R. W. Boyd, *Nonlinear Optics*, (Academic Press, 2008)