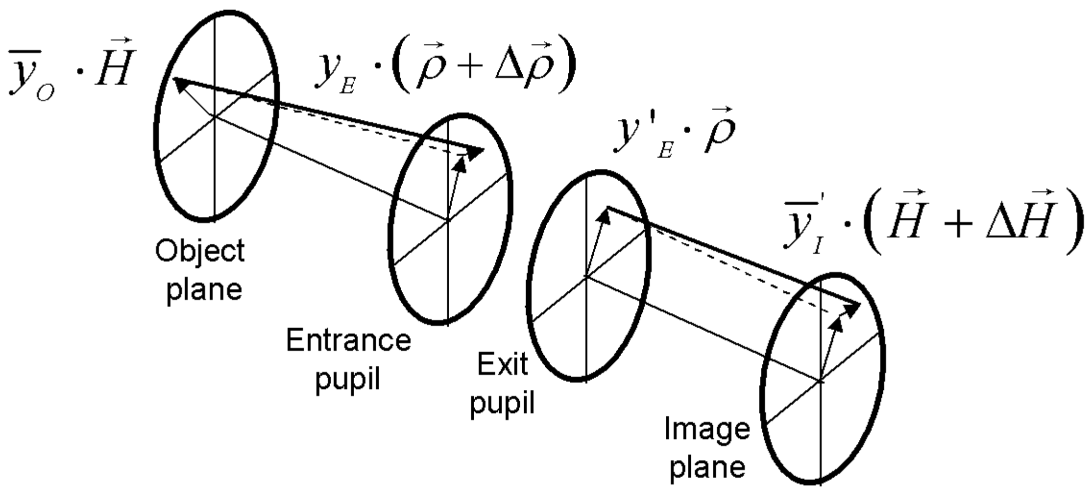
Introduction to aberrations

OPTI 518
Lecture 13



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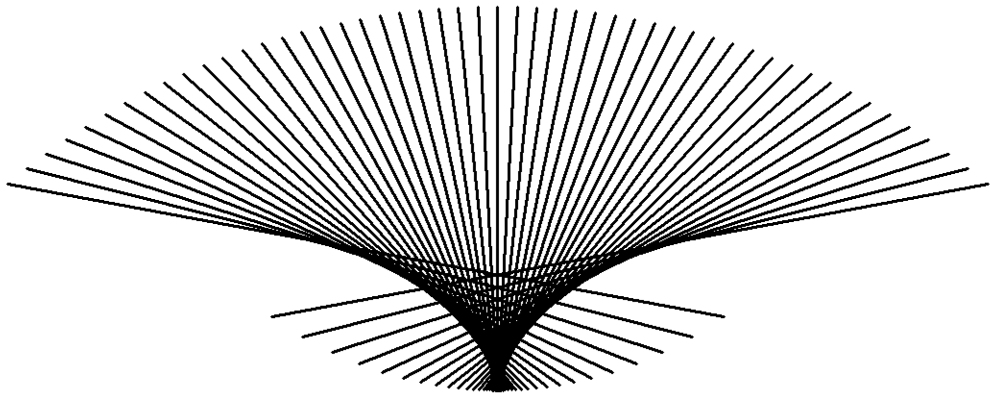
Caustics



Caustic ~ burning

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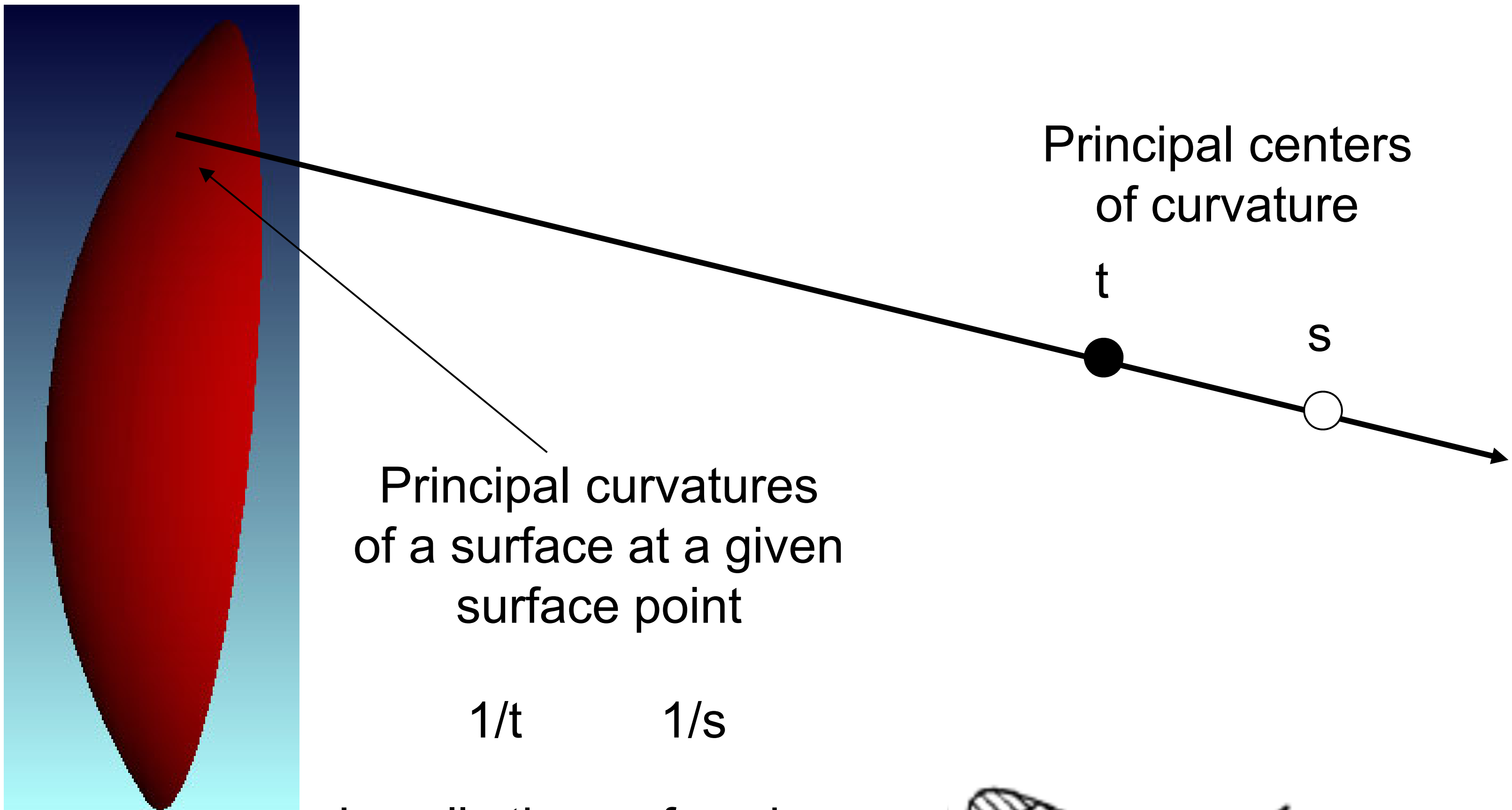
Caustic



Principal curvatures
of a surface at a given
surface point

1/t 1/s

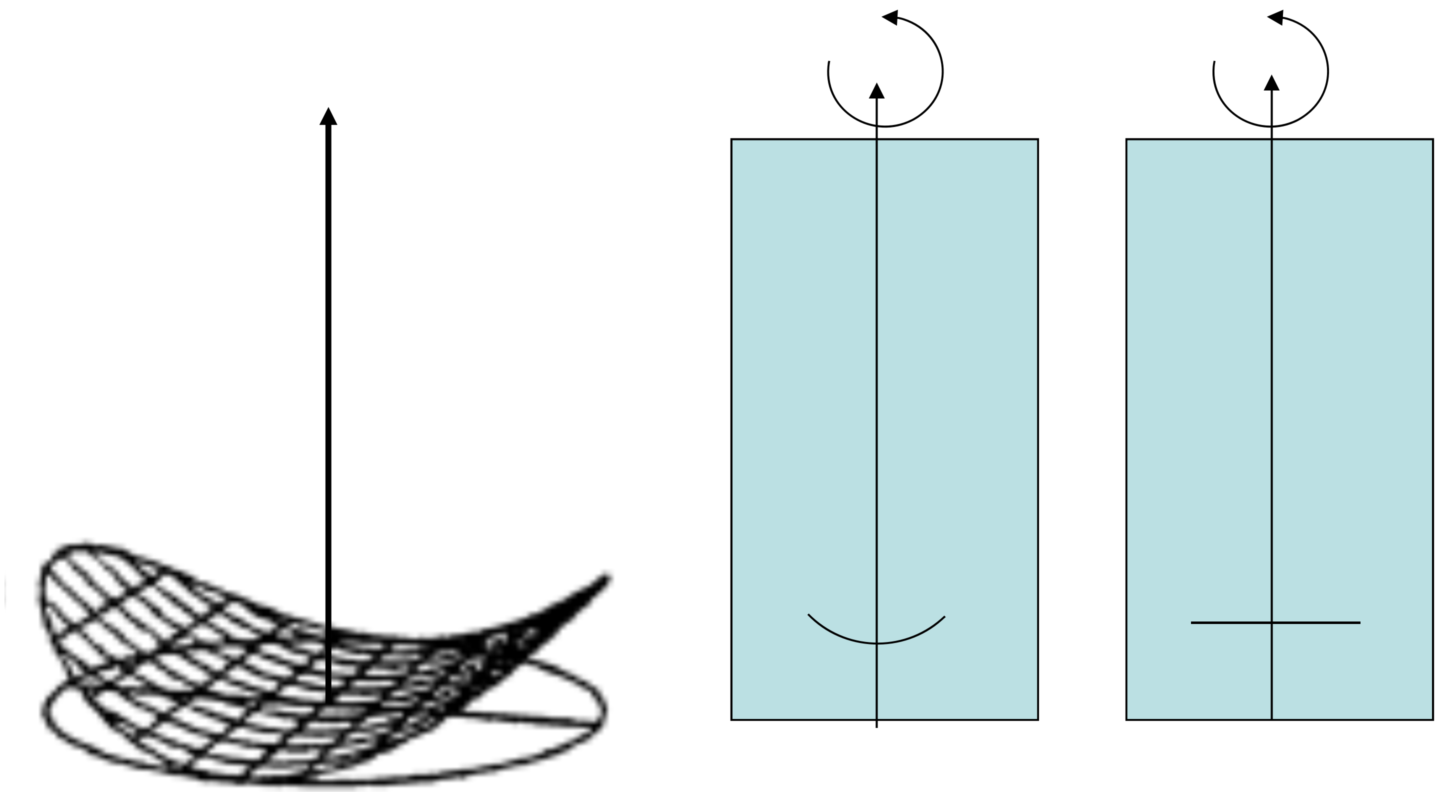
Locally the surface is
a cylindrical parabolic

Principal curvatures

- For any smoothly continuous surface, at each point in the surface there is a normal line. A plane containing the normal intersects the surface in a plane curve. With a finite curvature at the point in question. The center of this curvature lies on the normal line. If the plane is rotated about the normal, the resultant curvature in general fluctuates continuously between two extreme values. These extreme values are the *principal curvatures* of the surface at the point in question, and because of symmetry they lie in planes which are necessarily perpendicular to each other. Their centers of curvature are the *principal centers of curvature* and the intermediate centers of curvature lie between them.

Principal curvatures

$$Z=\frac{x^2}{2R_t}+\frac{y^2}{2R_s};\ \ Z=\frac{1}{2R_s}\left(\vec{I}_{\perp}\cdot\vec{\rho}\right)^2+\frac{1}{2R_t}\left(\vec{I}_{\parallel}\cdot\vec{\rho}\right)^2$$



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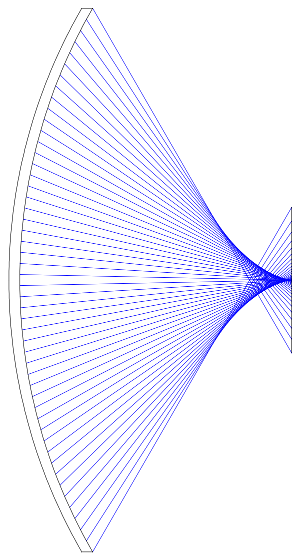
Caustic

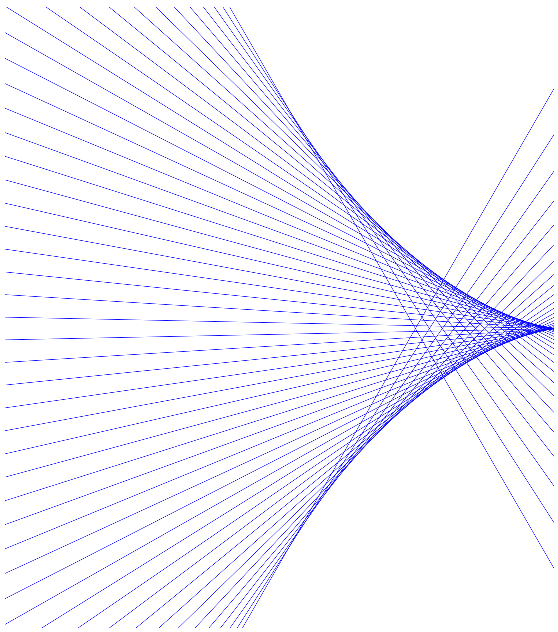
- For each point on the surface there are two principal centers of curvature, and for neighboring points on the surface, the principal centers of curvature are also pair-wise neighbors. Thus, for every given smoothly continuous surface there exits also in general a pair of surfaces containing the principal centers of curvature of the given surface. This pair is called the caustic (of two sheets) of the surface. Either or both sheets of the caustic may degenerate into a line, or, for a spherical surface, they degenerate to two coincident points.

Caustic

- If a sheet is not degenerate, then the normal to the given surface passing through the principal center of curvature lying in the caustic sheet is tangent to the sheet at that point, and all neighboring normal lines lie on the same side.

Ray caustic from a spherical mirror





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Caustic

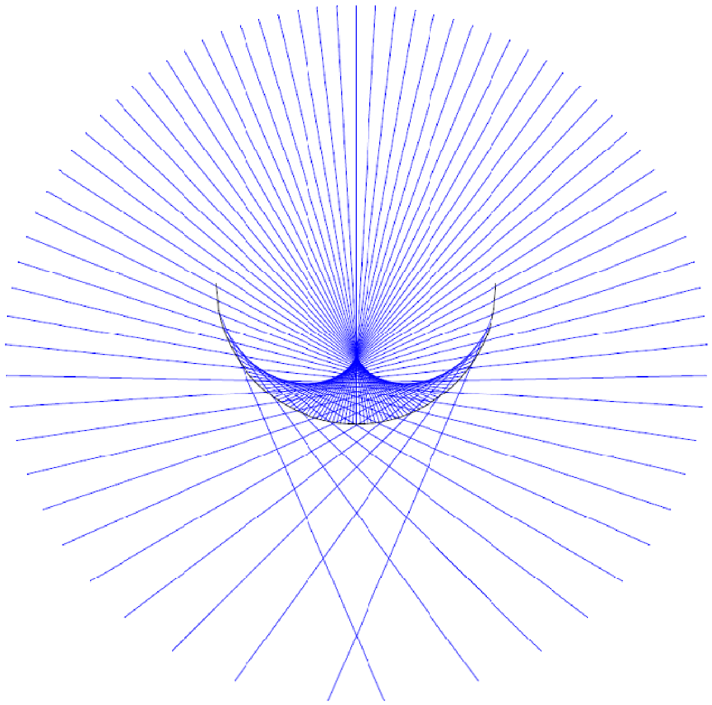
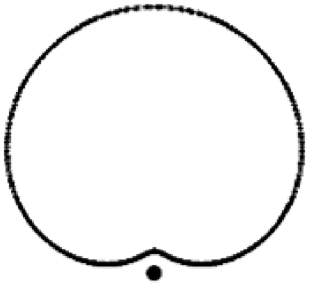


Photo and drawing by *Jose Sasian*



Cardiod, from the Greek "Heart"

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Principal curvatures

$$z(x, y) = ax + by + cx^2 + dxy + ey^2$$

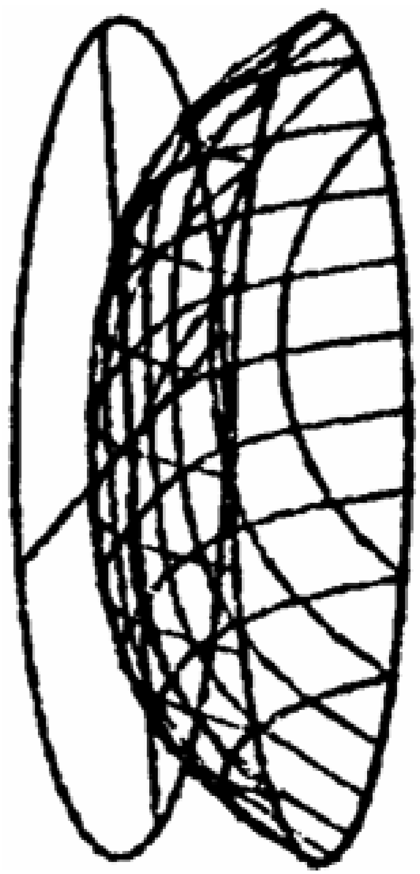
$$z(x, y) = cx^2 + ey^2$$

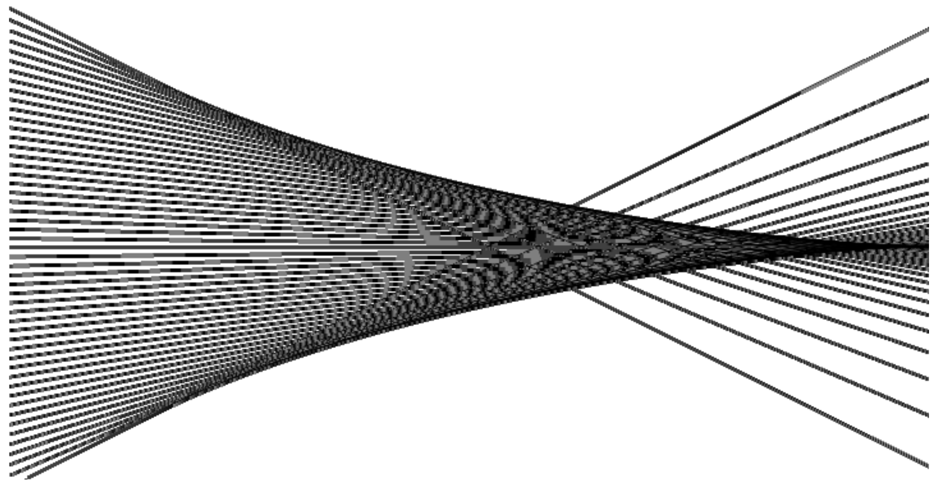
$$z(x, y) = \frac{x^2}{2R_s} + \frac{y^2}{2R_t}$$

Curvature of a curve

$$c = \frac{\dfrac{\partial^2 y}{\partial x^2}}{\left(1 + \left(\dfrac{\partial y}{\partial x}\right)^2\right)^{3/2}} \approx \frac{\partial^2 y}{\partial x^2}$$

Spherical aberration case





Need to determine the caustic shape

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Spherical aberration external caustic

- The external caustic is the locus of meridional rays centers of curvature
- On axis the wavefront deformation has no curvature (no second-order terms)
- The caustic is found by adding focus so that for a given aperture zone, the wavefront has no curvature.

$$c = \frac{\dfrac{\partial^2 y}{\partial x^2}}{\left(1 + \left(\dfrac{\partial y}{\partial x}\right)^2\right)^{3/2}} \approx \frac{\partial^2 y}{\partial x^2}$$

External caustic

$$0=\left(12W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)+2W_{020}\right)$$

$$W_{020}=-6W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)$$

- We add defocus to bring the caustic point to the ideal image plane for a given aperture zone ρ.

In longitudinal terms this is, $\Delta z'=-2\frac{W_{020}}{n'}\frac{1}{u'^2}$

$$\Delta z'=-\frac{n'}{\left(n'u'\right)^2}2W_{020}=\frac{n'}{\left(n'u'\right)^2}12W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)$$

External caustic

The transverse ray aberration is,

$$\bar{y}_I \Delta\vec{H} = \frac{1}{n'u'}\vec{\nabla}_\rho W\left(\vec{H},\vec{\rho}\right)$$

$$= \frac{1}{n'u'}\vec{\nabla}_\rho \left(W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2 + W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)\right)$$

$$= \frac{1}{n'u'}\left(4W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} + 2W_{020}\vec{\rho}\right)$$

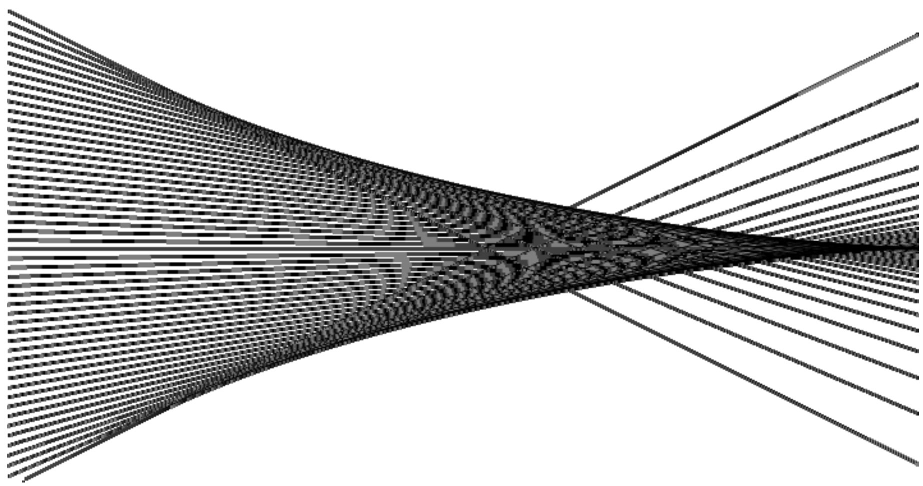
$$= \frac{1}{n'u'}\left(4W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} - 12W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}\right)$$

$$= -\frac{1}{n'u'}8W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}$$

Caustic parametric equations

$$\Delta z' = -\frac{n'}{(n'u')^2} 12 W_{040} (\vec{\rho} \cdot \vec{\rho})$$

$$\bar{y}_I \Delta \vec{H} = -\frac{1}{n'u'} 8 W_{040} (\vec{\rho} \cdot \vec{\rho}) \vec{\rho}$$



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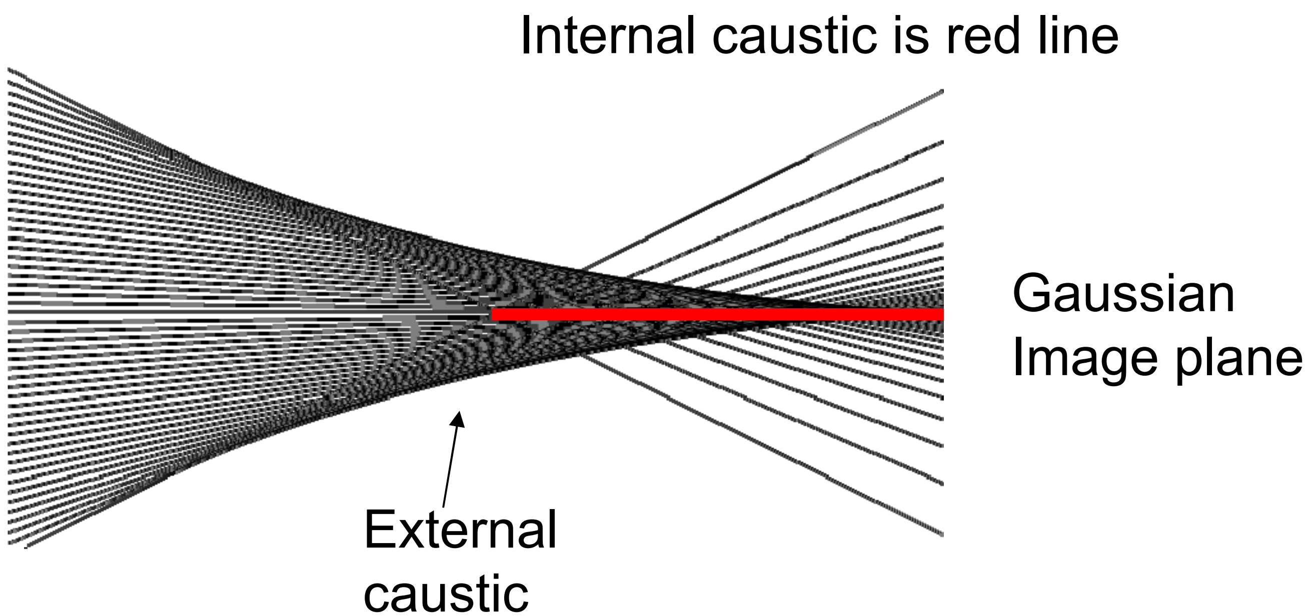
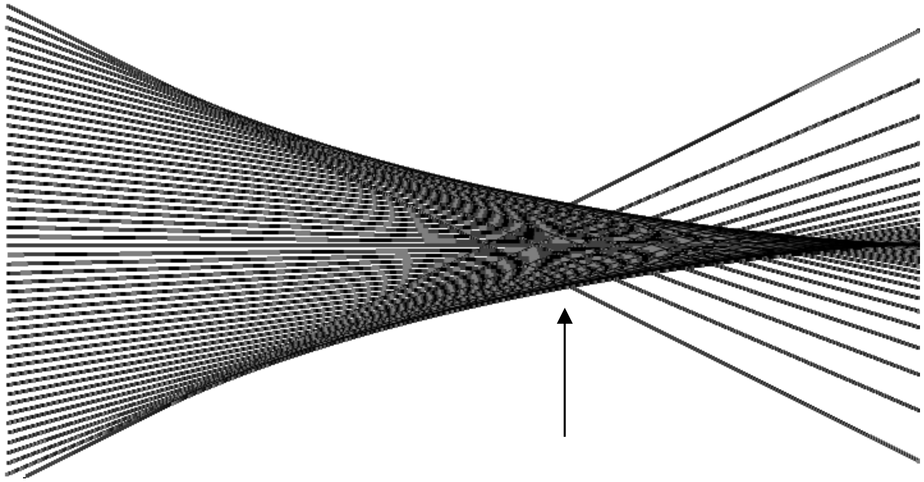
Internal caustic

The second sheet is the internal caustic and it is degenerated into a line that coincides with the optical axis



Minimum Circle

$$\vec{\rho} = \rho \vec{g}$$



The minimum circle is located
where the external caustic meets the marginal ray

$$\bar{y}_I \Delta \vec{H} = -\frac{1}{n'u'} 8W_{040} \rho_c^3 \vec{g} = -\left(4W_{040}\left(1^3\right) + 2W_{020}\left(1\right)\right)\vec{g}$$

Caustic

Marginal
ray

Minimum Circle

$$\bar{y}_I \Delta \vec{H} = -\frac{1}{n'u'} 8W_{040}\rho_c^3 \vec{g} = -\frac{1}{n'u'}\left(4W_{040}\left(1^3\right) + 2W_{020}\left(1\right)\right)\vec{g}$$

For caustic $$W_{020} = -6W_{040}\rho_c^2$$

$$8W_{040}\rho_c^3 = 4W_{040} - 12W_{040}\rho_c^2$$

$$\rho_c = -1$$

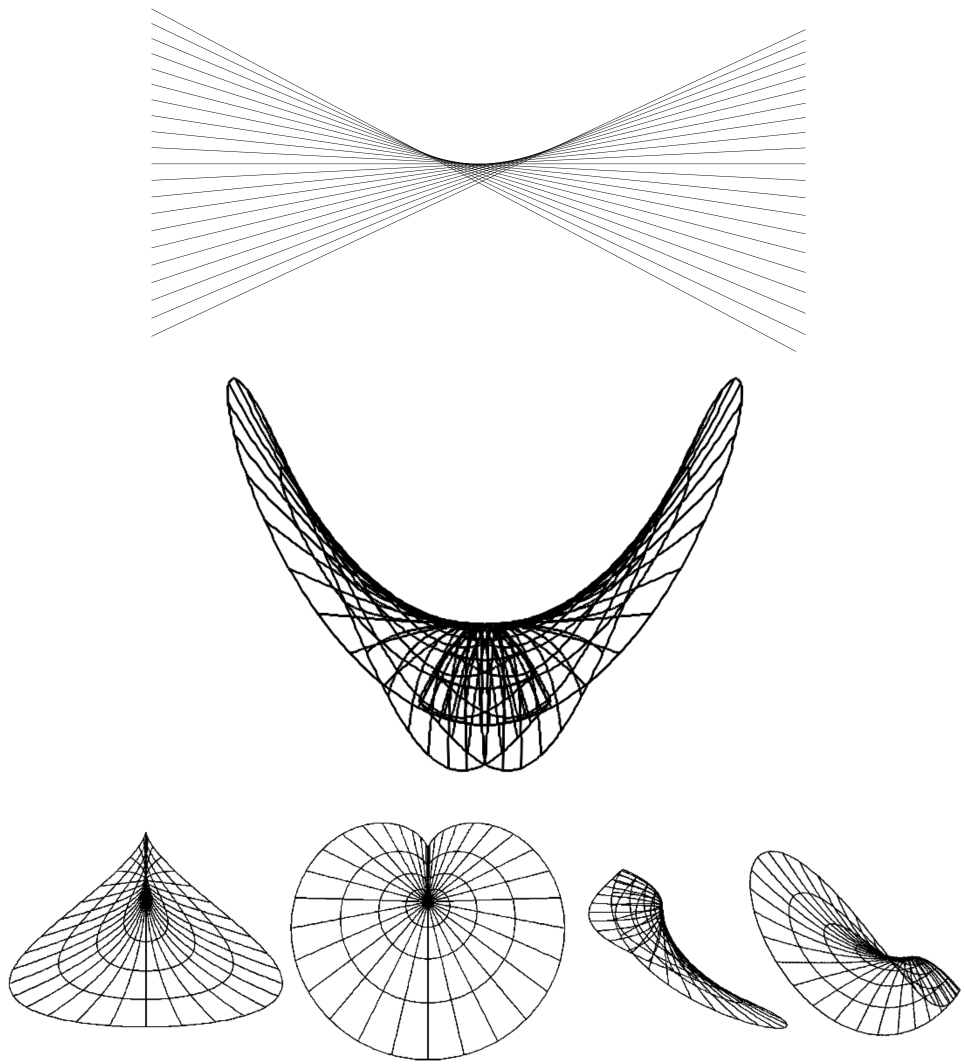
$$\rho_c = \frac{1}{2}$$

$$W_{020} = -6W_{040}\left(\frac{1}{4}\right) = -\frac{3}{2}W_{040}$$

Significant locations

	W_{020}	$\frac{(n'u')^2 \Delta z'}{n'}$	$(n'u')\bar{y}_I \lvert\Delta\vec{H}\rvert$
Ideal focus	0	0	$-4W_{040}$
Minimum circle	$-\frac{3}{2}W_{040}$	$-3W_{040}$	$-W_{040}$
Marginal focus	$-2W_{040}$	$-4W_{040}$	$-\frac{8}{3\sqrt{3}}W_{040}$

Coma caustic



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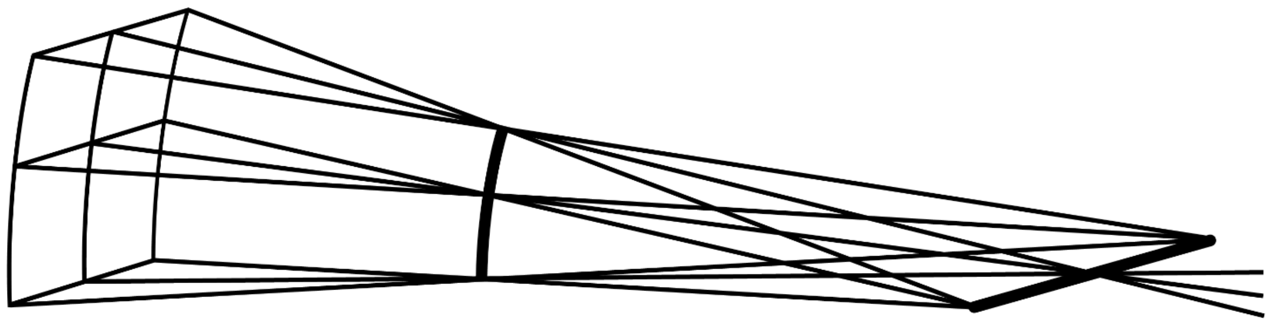
Astigmatism



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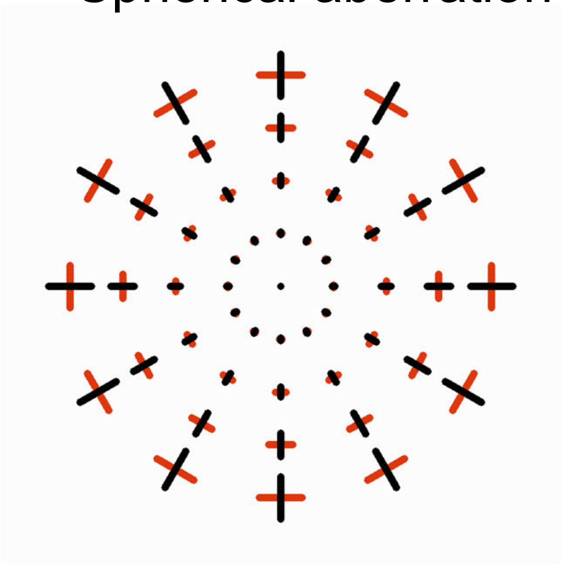
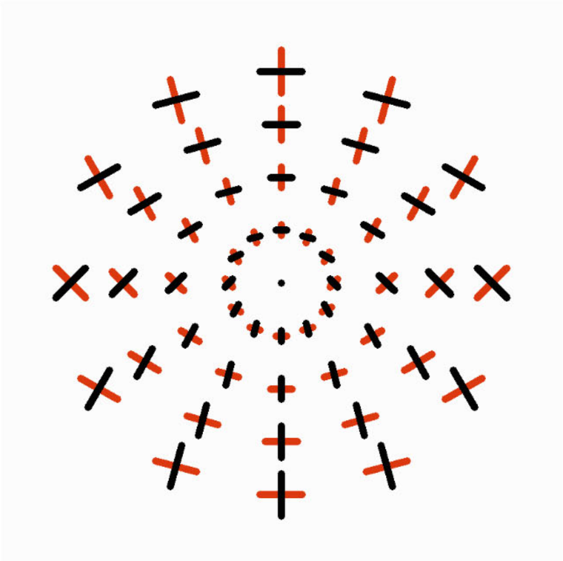
Wavefront principal curvatures orientation and magnitude

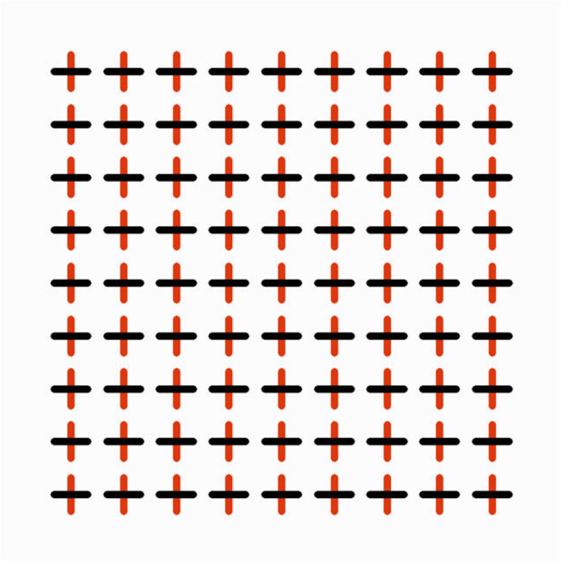
Spherical aberration



Coma



Astigmatism



As a function of the exit pupil position

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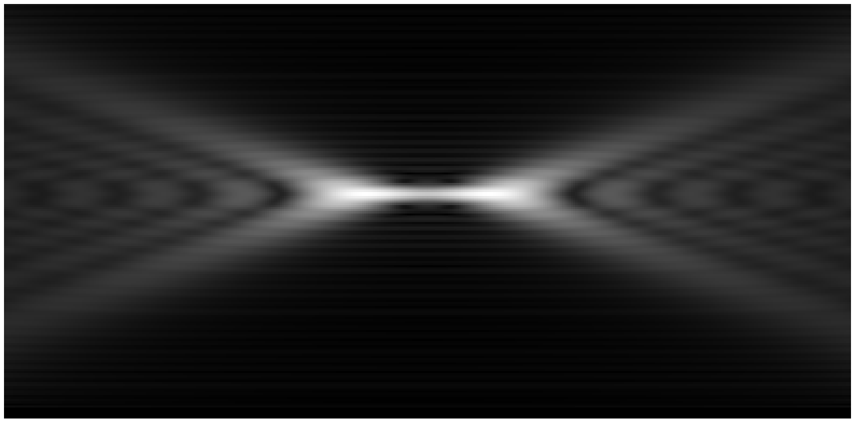
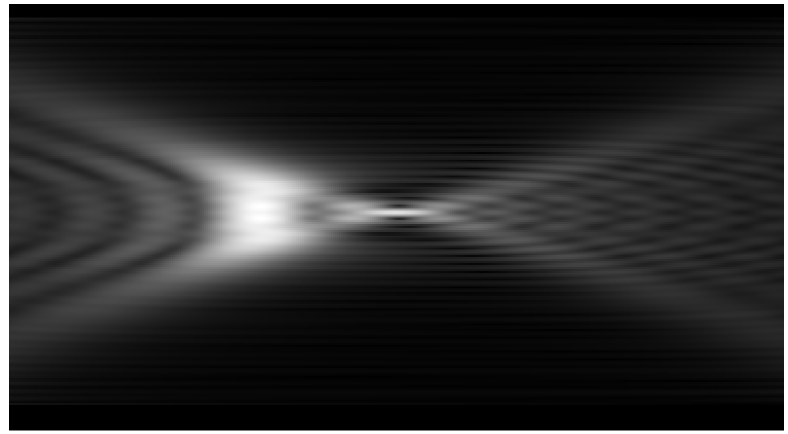
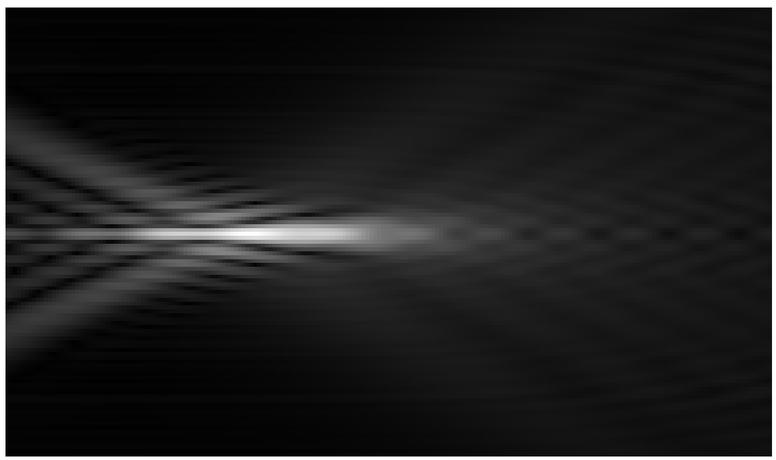
Diffraction images along the axis





Two waves:
Spherical
Coma
Astigmatism



Which one is which?

Sine condition from optical flux conservation and radiance theorem

Etendue considerations

On-axis irradiance

Relative illumination

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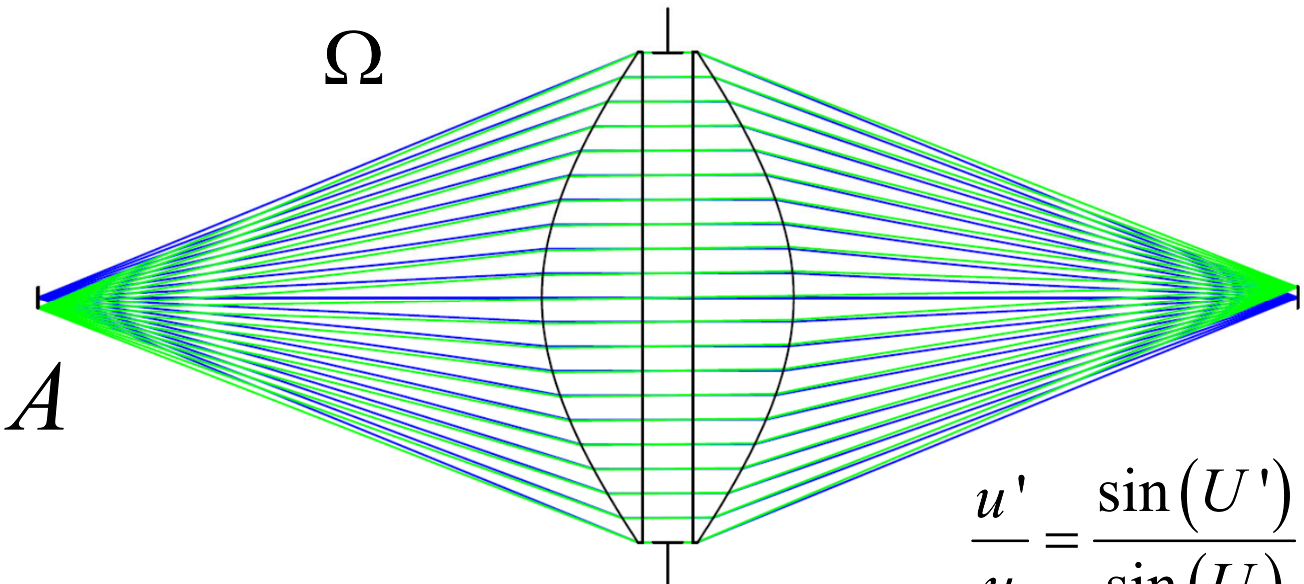
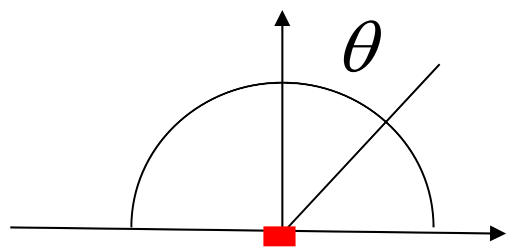
Sine condition



$$\frac{u'}{u} = \frac{\sin(U')}{\sin(U)}$$

Optical flux from a Lambertian source

$$\Phi(\theta) = 2\pi A L_0 \int_0^{\theta} \cos(\zeta)\sin(\zeta)\,d\zeta = \pi A L_0 \sin^2(\theta)$$



L_0 is assumed uniform

Compare with isotropic source

$$\Phi(\theta) = 2\pi A L_0 \int_0^{\theta} \sin(\zeta)\,d\zeta = 2\pi A L_0 \left(1 - \cos(\theta)\right) = A L_0 \Omega$$

Optical flux=radiance x throughput

$$\Phi(\theta) = \frac{L_0}{n^2} T \qquad \Phi(\theta') = \frac{L_0'}{n'^2} T$$

$$\frac{n^2}{L_0} \Phi(\theta) = \frac{n'^2}{L_0'} \Phi(\theta') = T$$

$$\frac{n^2}{L_0} = \frac{n'^2}{L_0'}$$

Radiance theorem

$$U = \theta$$

$$\frac{n^2}{L_0} \Phi(U) = \frac{n'^2}{L_0'} \Phi(U')$$

Sine condition from optical flux conservation

$$\frac{n^2}{L_0}\Phi(U)=\frac{n'^2}{L_0'}\Phi(U')$$

$$n^2 A\sin^2(U)=n'^2 A'\sin^2(U')$$

$$\pi n^2 h^2 \sin^2(U)=\pi h'^2 n'^2 \sin^2(U')$$

$$n^2 h^2 \sin^2(U)=h'^2 n'^2 \sin^2(U')$$

$$nh\sin(U)=n'h'\sin(U')$$

$$\frac{u'}{u}=\frac{\sin(U')}{\sin(U)}$$

Sine condition

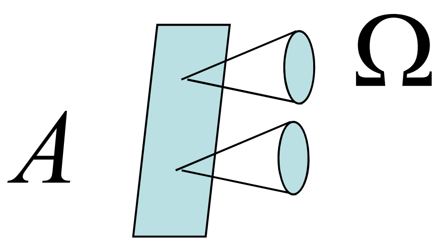
Throughput
Etendue

(area-omega product)

$$\varepsilon = n^2 A\Omega = n'^2 A'\Omega'$$

$$T = \pi Ж^2$$

“Capacity to transfer optical flux”



$$\Omega = 2\pi\left(1-\cos\left(\theta\right)\right)$$

Even better

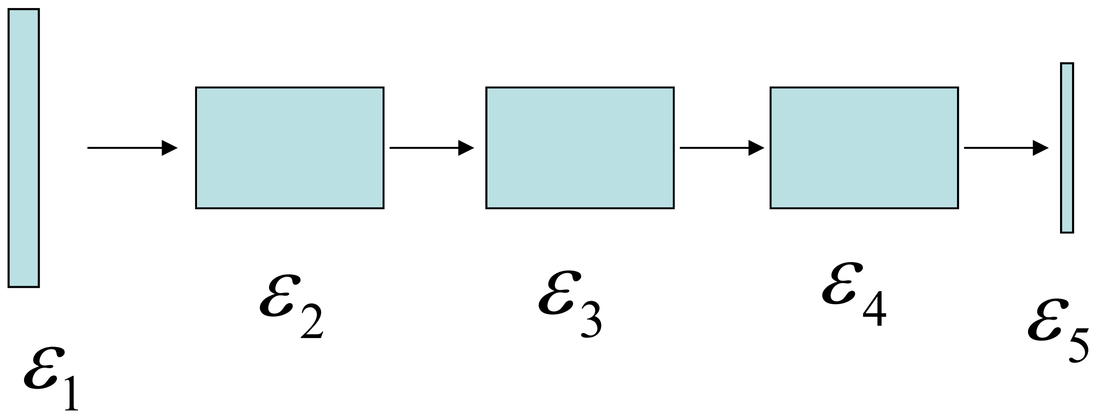
$$\varepsilon = n^2 A\Omega = n'^2 A'\Omega'$$

For isotropic source

$$\varepsilon = A(NA)^2 = n^2 A \sin^2(\theta)$$

For Lambertian source

Etendu considerations are key to design an optical system



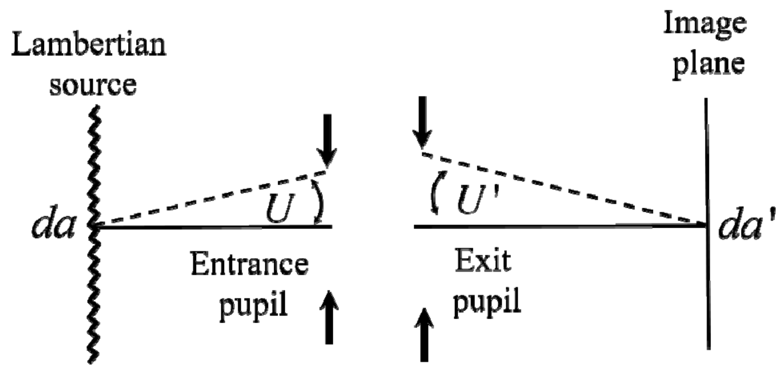
$$\varepsilon_5 \geq \varepsilon_4 \geq \varepsilon_3 \geq \varepsilon_2 \geq \varepsilon_1$$

Start with the sensor at the end $\varepsilon_5 = A_5 \left(NA\right)^2$

Every optics component has associated an etendue. The component with the smallest etendue limits the amount of optical flux transferred by the system.

Assumption: lossless and not active components.

Ratio of on-axis irradiance to exitance



$$d\phi = 2\pi L_0 da \int_0^U \sin(\xi)\cos(\xi) d\xi = \pi L_0 da \sin^2(U)$$

$$\bar{W}_{311} = W_{131} + \frac{1}{2}\text{Ж}\Delta(u^2)$$

Exitance $I_0 = \pi L_0 \sin^2(U)$

$$\frac{\Delta u'}{u'} = \frac{\bar{W}_{311}}{\text{Ж}}$$

On-axis irradiance
Is proportional
to the F/# square!

Irradiance $I_0' = \pi \frac{L_0'}{n'^2} NA^2 = \pi L_0' \sin^2(U')$

$$\sin^2(U') \simeq \frac{1}{4(F/\#)^2}$$

$$\frac{I_0'}{I_0} = \frac{L_0'}{L_0}\frac{\sin^2(U')}{\sin^2(U)}$$

$$\frac{I_0'}{I_0} = \frac{n'^2}{n^2}\frac{\sin^2(U')}{\sin^2(U)} \simeq \frac{1+u^2}{u^2}\frac{(u'+\Delta u')^2}{1+(u'+\Delta u')^2} \simeq \frac{1}{m^2}\left(1 + 2\frac{W_{131}}{\text{Ж}}\right)$$

Relative illumination

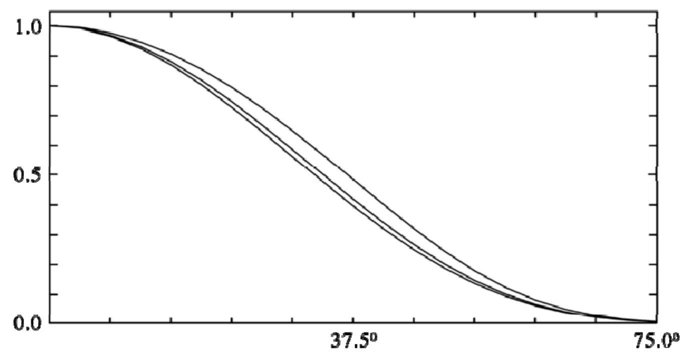
- Ratio of the off-axis irradiance to the on-axis irradiance

$$dI' = \frac{L_0' ds'}{s'^2}\cos^4(\theta')$$

$$RI = \cos^4(\theta')$$

Pinhole camera RI

$$RI(\vec{H}) = 1 - 2\bar{u}'^2(\vec{H}\cdot\vec{H})$$



Relative illumination of pinhole cameras at F/100 (bottom curve), F/2 (intermediate curve), and at F/1 top curve. The field of view is in degrees in the horizontal axis from 0º to 75º.

Lens relative illumination

Stop at the entrance pupil

$$RI\left(\vec{H}\right)=1-\left(2\bar{u}'^{2}-\frac{4}{Ж}\bar{W}_{131}\right)\left(\vec{H}\cdot\vec{H}\right)$$

Pupil coma effect on relative illumination

Summary

- The concept of caustic
- Principal curvatures
- Significant locations along the axis
- Spherical aberration caustic
- Coma caustic
- Astigmatism caustic
- Radiometric aspects