OPTI 518
Introduction to Aberrations
Lecture #4

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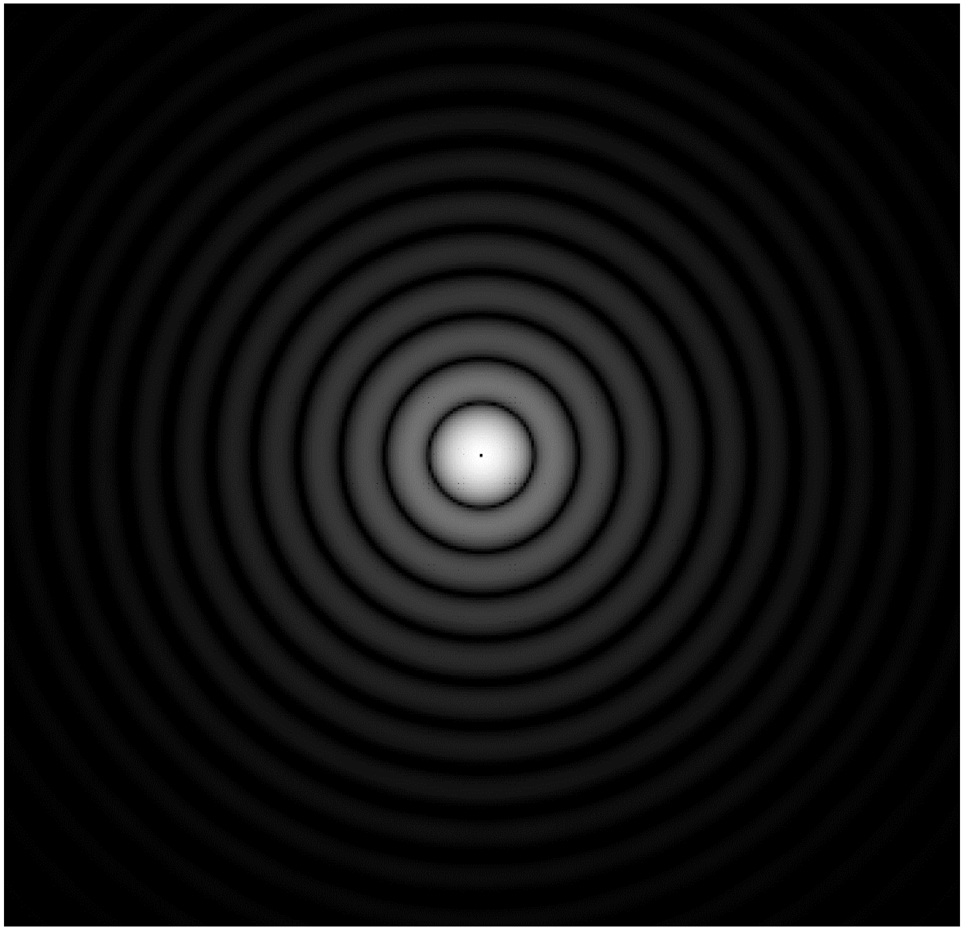
Airy's pattern



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Aberration

From the **Latin**, aberrare, to wander from; **Latin**, ab, away,

errare, to wander.

Symmetry properties

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Early optics

- Telescope
- Microscope and polarizing microscope
- Camera obscura
- Human eye

- The understanding of imaging defects in these optics lead to the discovery of aberrations

Historical aspects

- How the concept of aberration was developed?
- From looking at the individual trees first, and then to the forest
- Rather than looking at the forest first, and then at the individual trees

LECTIONES
OPTICÆ & *GEOMETRICÆ*:

In quibus

PHÆNOMENΩN OPTICORUM

Genuinæ *Rationes* investigantur, ac exponuntur:

ET

Generalia Curvarum Linearum *Symptomata declarantur.*

Auctore ISAACO BARROW,

Collegii *SS. Trinitatis* in Academia *Cantab.* Præfecto, Et *SOCIETATIS REGIÆ* Sodale.

Οἱ φύσει λογιστικοὶ εἰς πάντα τὰ μαθήματα, ὡς ἔπος εἰπεῖν, ὀξεῖς φαίνονται· οἵ τε βραδεῖς, ἂν ἐν τούτῳ παιδευθῶσι καὶ γυμνάσωνται, κἂν μηδὲν ἄλλο ὠφεληθῶσιν, ὅμως εἴς γε τὸ ὀξύτεροι αὐτοὶ αὑτῶν γίγνεσθαι πάντες ἐπιδιδόασιν. Plato de Repub.

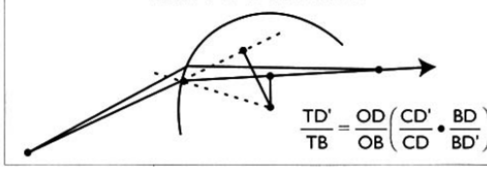
Ἀρκεῖ, εἰ τὰ μὲν οὐ χεῖρον. Arist.

LONDINI,

Typis *Guilielmi Godbid*, & prostant venales apud *Robertum Scott*, in vico Little-Britain. 1674.

FIGURE 1

Barrow's relation



$$\frac{TD'}{TB} = \frac{OD}{OB}\left(\frac{CD'}{CD} \cdot \frac{BD}{BD'}\right)$$

Brevitatis gratiâ notæ quædam adhibentur, quarum hîc subjungitur interpretatio.

A + B. *hoc est*	A & B *simul accepta.*
A — B.	A, *demptâ* B.
A —: B.	*differentia ipsarum* A, & B.
A × B.	A *multiplicata, vel ducta in* B.
A/B —	A *divisa per* B, *vel applicata ad* B.
A = B.	A *æquatur ipsi* B.
A ⊏ B.	A *major est quàm* B.
A ⊐ B	A *minor est quàm* B.
A . B :: C . D	A *ad* B *eandem rationem habet, quam* C *ad* D.
A, B, C, D ∺.	A, B, C, D *sunt continuè proportionales.*
A . B ⊏ C . D.	A *ad* B *majorem rationem habet, quàm* C *ad* D.
A . B ⊐ C . D.	A *ad* B *minorem rationem habet, quàm* C *ad* D.
A . B + C . D { = ⊏ ⊐ }	M.N. *Rationes* A ad B, & C ad D *composita* { *adæquant / excedunt / deficiunt à* } { *ratione* M ad N. }
A q.	*Quadratum ex* A.
√ A.	*Latus, vel radix quadrata ipsius* A.
A c.	*Cubus ex* A.
√ A q + B q.	*Latus compositi ex* A q & B q.

Reliquas, si quæ occurrunt, abbreviaturas Lector facili conjecturâ capiet, præsertim in analysi tantillùm versatus.

VI. Etiam hoc *Theorema* subdemus: Si fiat 2 CA . CN :: CN . E. & 2 CK . CN :: CN . F; & sumatur CQ = E + F; erit ducta NQ ad CA perpendicularis. vel reciprocè; posito quod sit NQ ad CA perpendicularis; erit CQ = E + F. || Nam (ut hoc posterius ostendamus) quoniam est 2 CA . CN :: CN . E. & CN . 2 CK :: F . CN. erit ex æquo perturbatè 2 CA . 2 CK :: F . E. vel CA . CK :: F . E. componendóque CA + CK . CK :: F + E . E. Porrò quoniam est ANq = ACq + CNq — 2 AC × CQ; erit 2 AC × CQ — CNq = ACq — ANq. itaque (juxta præcedentem) erit 2 AC × CQ — CNq . CNQ :: AC . CK. hoc est (ob CNq = 2 AC × E) 2 AC × CQ — 2 AC × E . 2 AC × E :: AC . CK. hoc est CQ — E . E :: AC . CK. vel componendo CQ . E :: AC + CK . CK. erat autem AC + CK . CK :: F + E . E. ergò CQ = F + E: Quod E. D.

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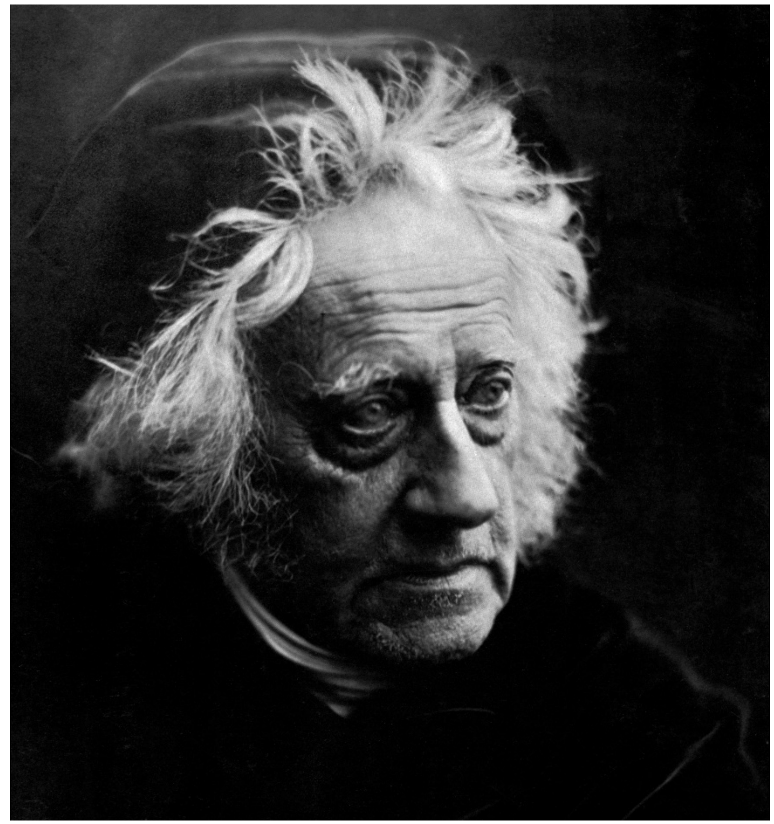
Dollond's achromat
18^{th} Century

- John Dollond's achromatic doublet became the novelty of the time.
- Alexis Claude Clairaut and Jean le Rond d'Alembert developed polynomial expansions for longitudinal aberrations

Early contributions from England

- Thomas Young
- George Airy
- John Herschel
- Henry Coddington
- William Wollaston

Late 1700's to early 1800's



John Herschel by
Julia Margaret Cameron

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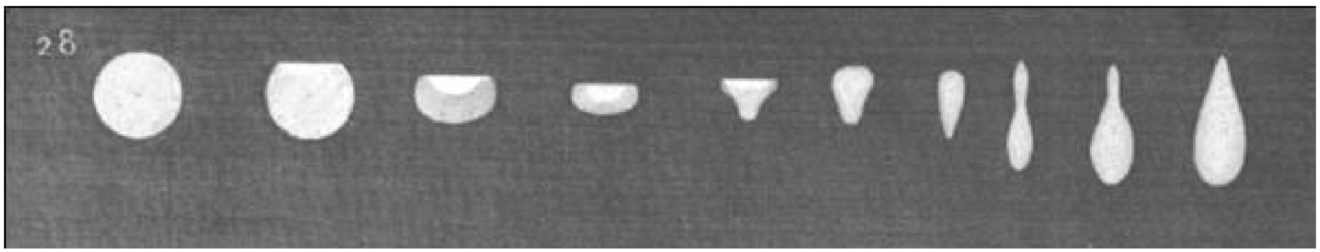
Thomas Young's sketch of the images produced by oblique rays passing through a lens, and at different distances from the lens (through focus).



~1801

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The discovery of aberrations

- Axial chromatic aberration
- Spherical aberration

- Astigmatism
- Coma

- Field curvature and distortion

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Aberration effects

- Spherical, coma, and astigmatism affect image sharpness
- Field curvature and distortion change the axial and lateral position of the point image form the ideal position.

Contributions from Germany

- Gauss first-order theory, 1839
- Seidel third-order theory, 1856
- Schwarzschild fifth-order theory, 1905

Their papers are translated into English from the German and are found in the class web site

Introduction to Aberrations
(Departures from ideal behavior)

- Central projection (collinear transformation) imaging as ideal imaging (Gaussian and Newtonian equations)
- Aberrations as departures from ideal behavior
- Aberration metrics: wave deformation, angular aberration, transverse ray aberration, longitudinal aberration
- Rays versus wave approach: combining aberrations with rays can be quite difficult. With waves is a simple superposition.

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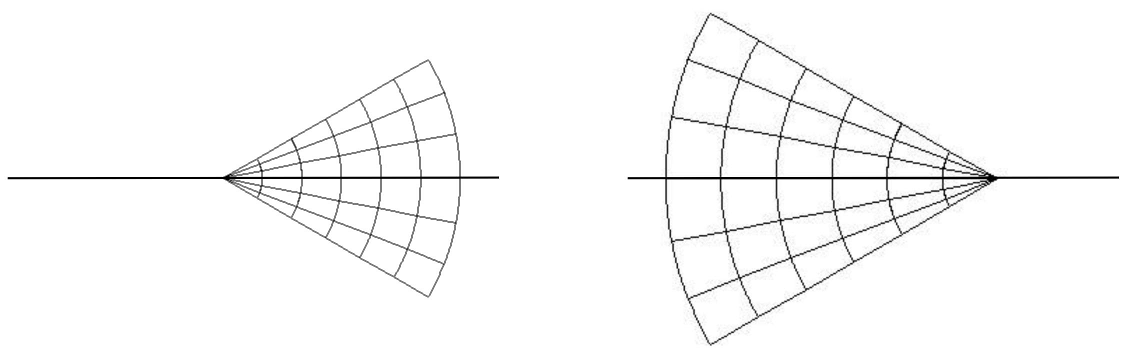
Symmetry

- Symmetry considerations are key to understand aberrations
- Smoothness of physical properties:
  - Surface
  - Index of refraction

Ideal wavefront shape

- Ideally wavefronts and rays converge to Gaussian image points. This implies that ideally wavefronts must be spherical and rays must be homocentric.



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Wavefront deformation determination

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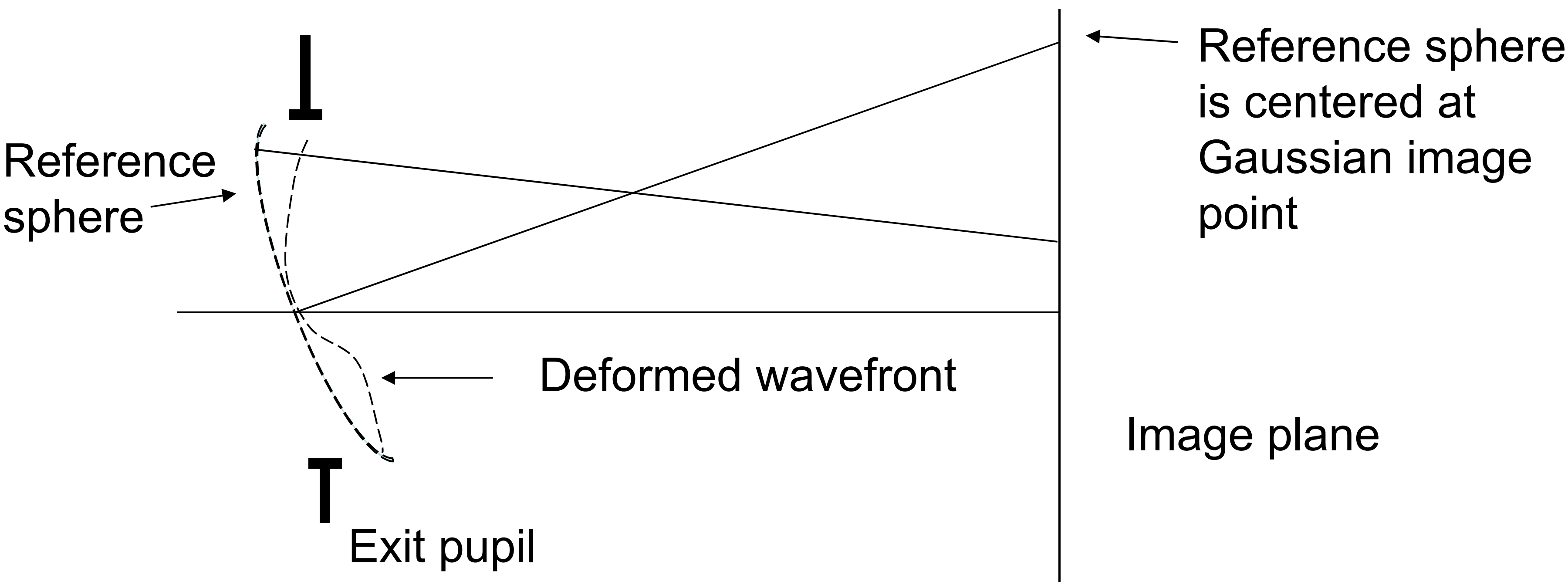
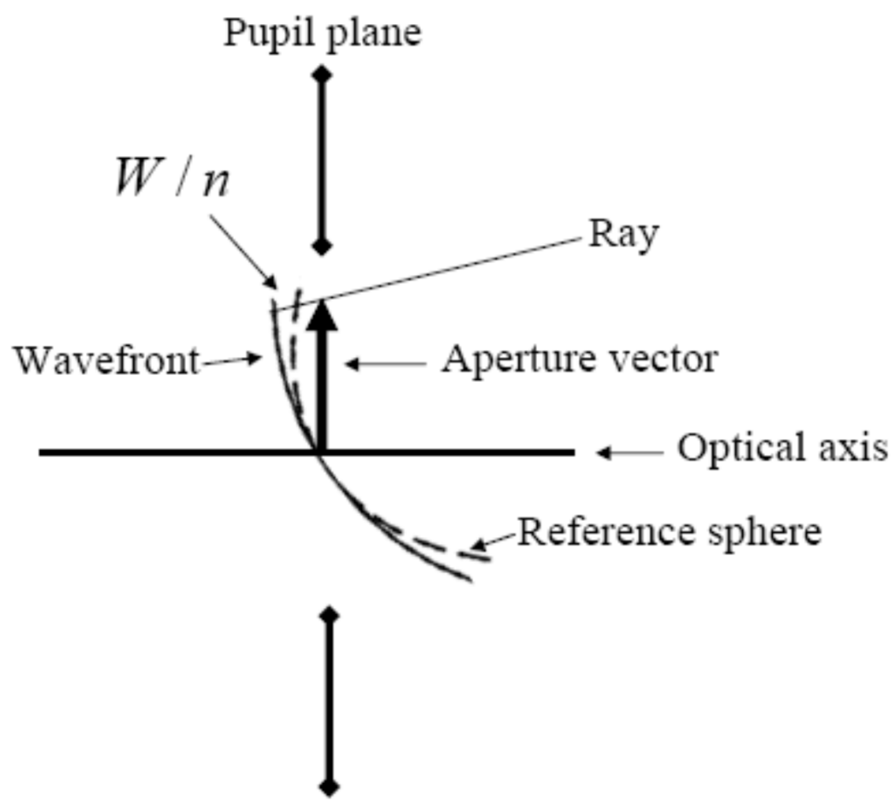
- The wavefront deformation is determined by the use of a reference sphere with center at the Gaussian image point and passing by the exit pupil on-axis point.



Aperture vector and ray detail



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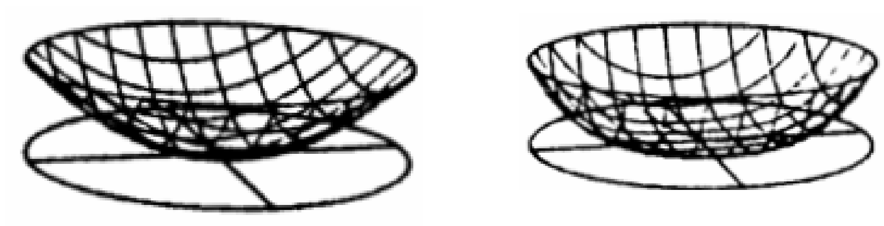
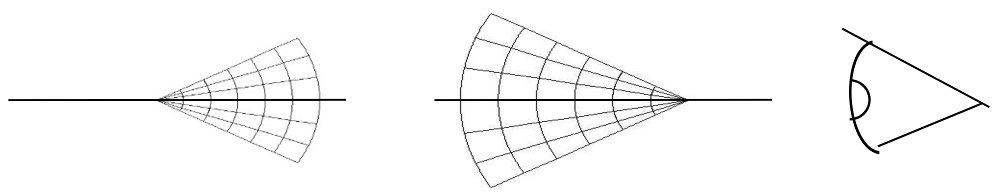
Wavefront deformation

- Actual image degradation by an optical system implies that Gaussian optics can not model accurately imaging. In the wave picture for light propagation we notice that geometrical wavefronts must be deformed from the ideal spherical shape.

Basic Reasoning: on-axis

- An axially symmetric system can only have an axially symmetric wavefront deformation for an object point on-axis. In its simplest form this deformation can be quadratic or quartic with respect to the aperture. If the reference sphere is centered in the Gaussian image point then the quadratic deformation can not be present for the design wavelength.





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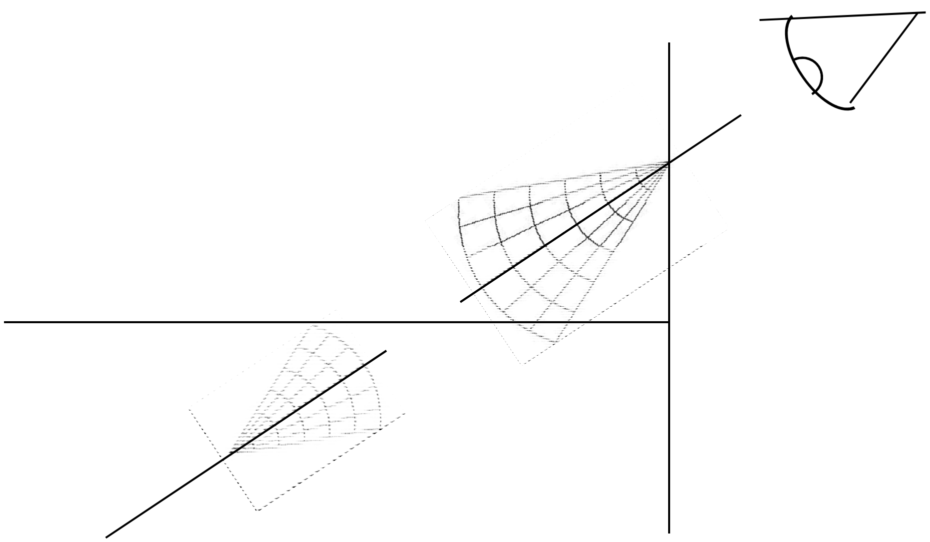
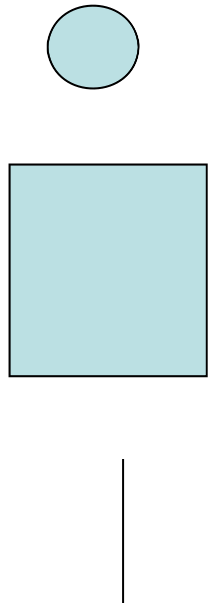
Basic Reasoning: off-axis

- For an object point that is off-axis the axial symmetry of the beam is lost and is reduced to plane symmetry. Therefore for that off-axis beam the wavefront deformation can have axial, plane, or double plane symmetry.





Wavefront deformation classification according to symmetry

- The simplest plane symmetric wavefront deformation shapes represent the primary aberrations. These are:

• Spherical aberration	Axially symmetric
• Coma	Plane symmetric
• Astigmatism	Double plane symmetric
• Field curvature	Axially symmetric
• Distortion	Plane symmetric

• Chromatic change of focus	Axially symmetric
• Chromatic change of magnification	Plane symmetric

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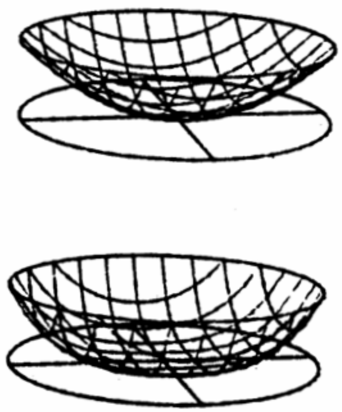
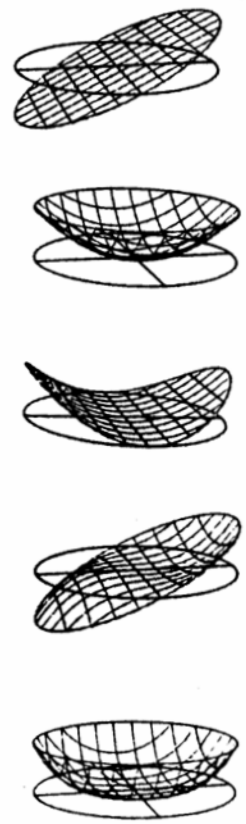
Aberration forms: graphics and symmetry considerations

Focus



Spherical aberration

On-axis



Distortion

Field curvature
Focus

Astigmatism

Coma

Spherical aberration

Off-axis

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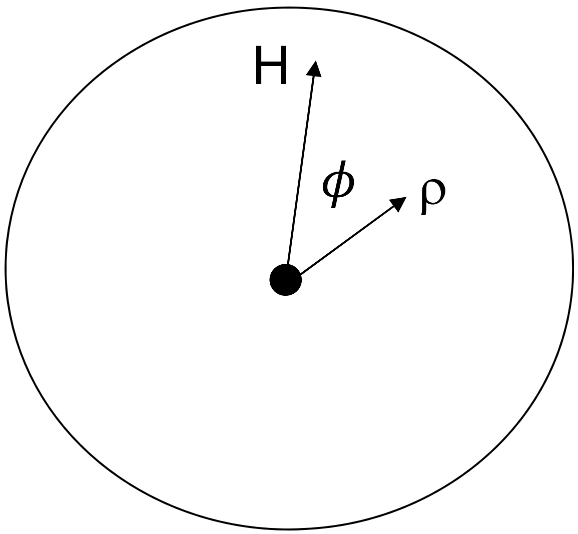
The wave aberration function

- The wave aberration function is a function of the field H and aperture ρ vectors. H and ρ define uniquely a ray. Because this function represents a scalar, which is the wavefront deformation at the exit pupil, it depends on the dot product of the field and aperture vectors. The assumed axial symmetry leads to a select set of terms.

$$W\{H,\rho\} = \sum_{j,m,n} W_{k,l,m} H^k \rho^l \cos^m \phi$$

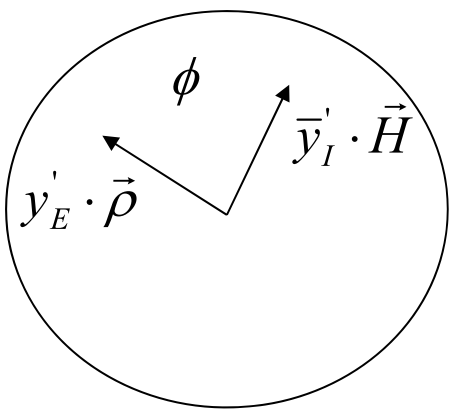


$$W(H,\rho,\phi) = W_{200}H^2 + W_{020}\rho^2 + W_{111}H\rho\cos\phi +$$
$$+W_{040}\rho^4 + W_{131}H\rho^3\cos\phi + W_{222}H^2\rho^2\cos^2\phi +$$
$$+W_{220}H^2\rho^2 + W_{311}H^3\rho\cos\phi + W_{400}H^4 +$$
$$+\ldots$$

Rotational invariants and additional plane symmetry

$$\vec{H}\cdot\vec{H}=H_x^2+H_y^2 \qquad \vec{\rho}\cdot\vec{\rho}=\rho_x^2+\rho_y^2$$

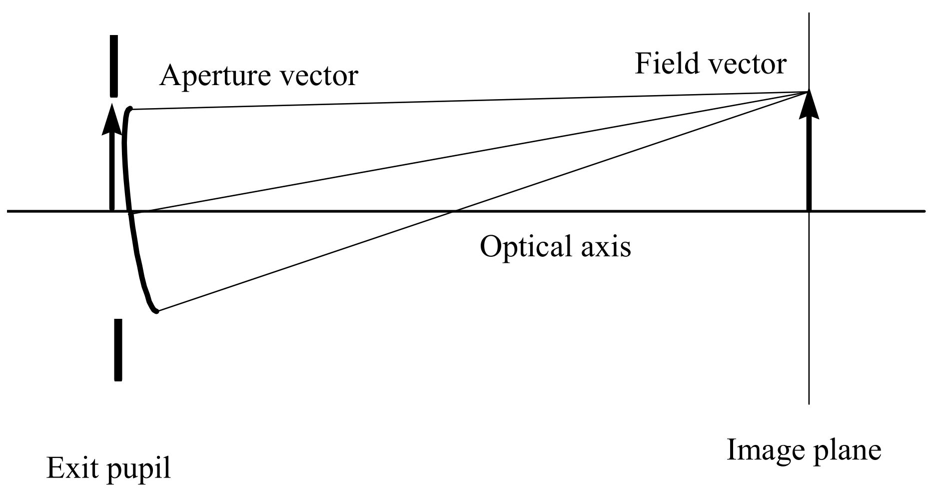
$$\vec{H}\cdot\vec{\rho}=H_x\rho_x+H_y\rho_y$$

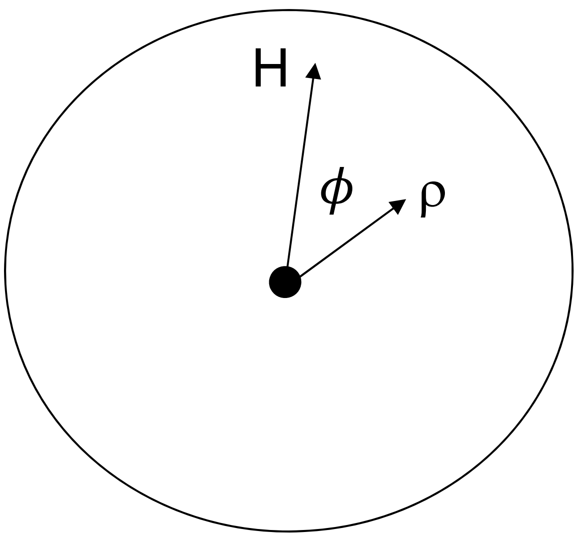


The field and aperture vectors

- The field vector has its foot at the center of the object plane and the aperture vector has its foot at the center of the exit pupil. Both are normalized. For convenience we draw the Gaussian image of the field vector in the image plane.

Aperture and field vectors





Second-order terms and piston terms

- Note that defocus W_{020} and the change of scale W_{111} terms may not be needed because Gaussian optics accurately predict the location and size of the image. The piston terms W_{200} and W_{400} represent a uniform phase change that does not degrade the image.

$$W(H,\rho,\phi) = W_{200}H^2 + W_{020}\rho^2 + W_{111}H\rho\cos\phi +$$
$$+W_{040}\rho^4 + W_{131}H\rho^3\cos\phi + W_{222}H^2\rho^2\cos^2\phi +$$
$$+W_{220}H^2\rho^2 + W_{311}H^3\rho\cos\phi + W_{400}H^4 +$$
$$+\ldots$$

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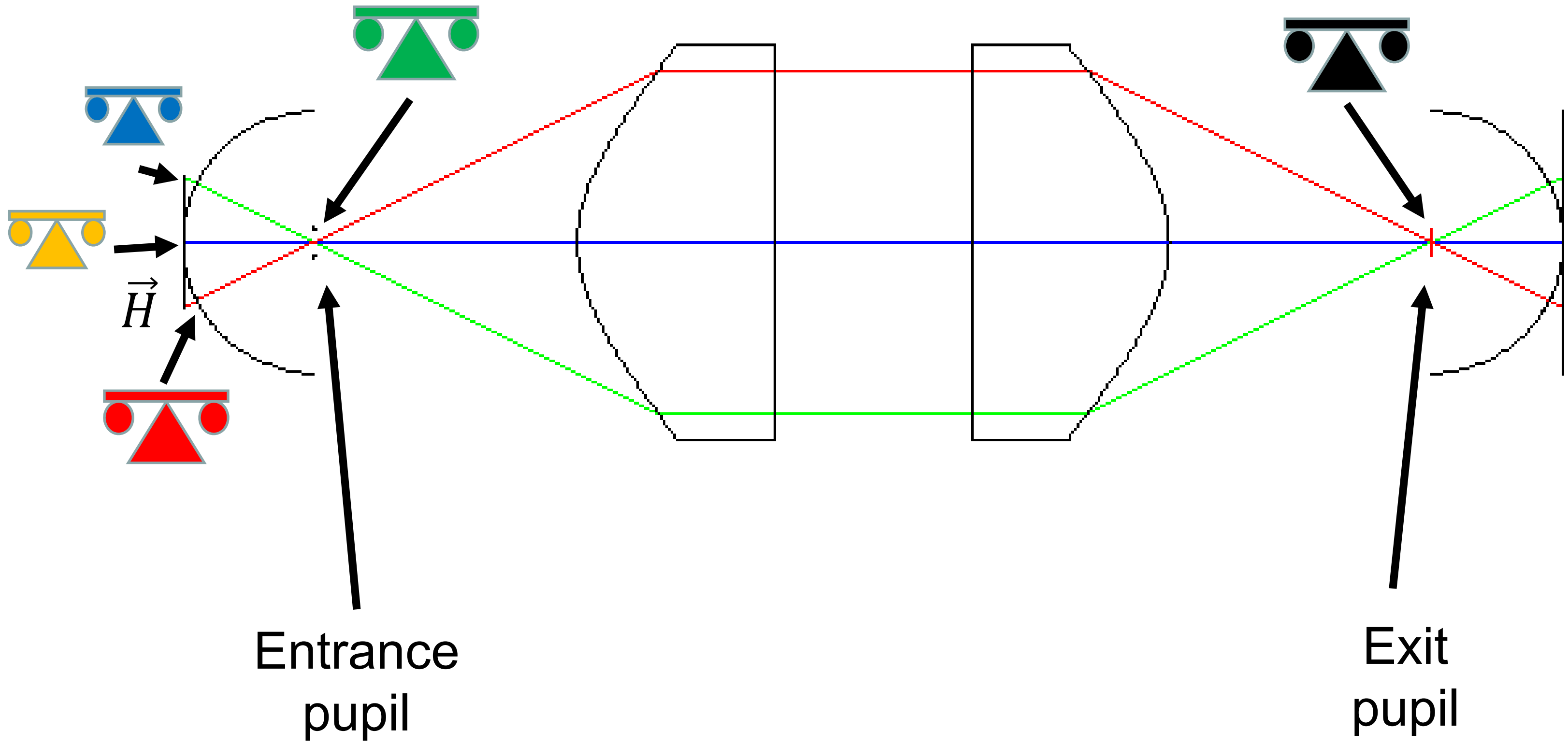
Piston terms



$$W\left(\vec{H},\vec{\rho}=0\right)=W_{000}+W_{200}\left(\vec{H}\cdot\vec{H}\right)+W_{400}\left(\vec{H}\cdot\vec{H}\right)^{2}+W_{600}\left(\vec{H}\cdot\vec{H}\right)^{3}+\ldots$$

Entrance pupil point is the reference to measure piston

$$W_{000}=\sum nt$$

$$W_{200}=0$$

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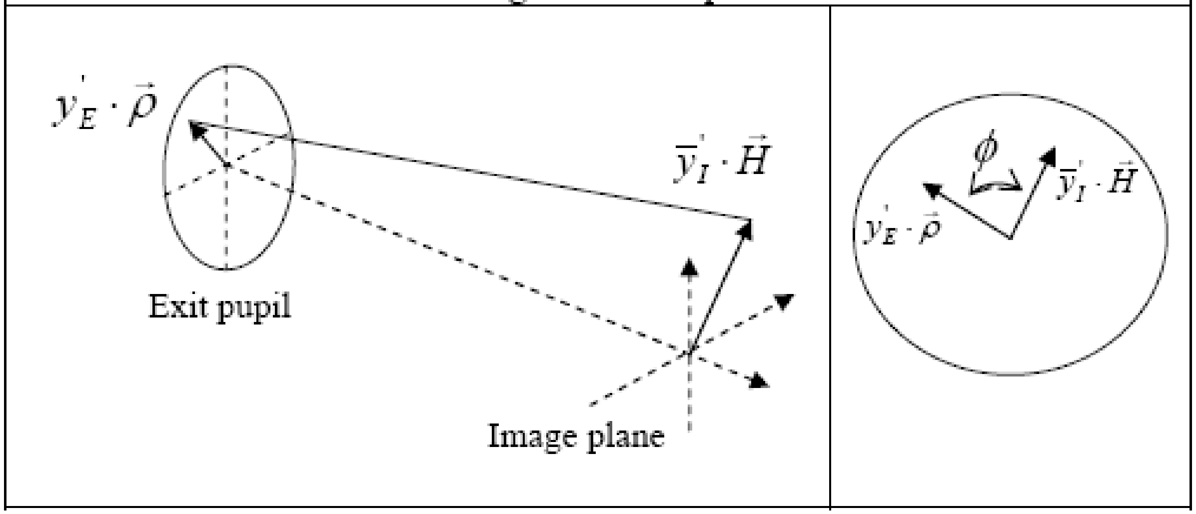
Comments on aberrations

- Note that the algebraic order of the terms as a function of the field and aperture vectors is zero, second, fourth-order, etc.
- To achieve sharp images with no mapping distortion an optical system has its fourth-order aberrations zero or nearly zero.
- Some simple systems are designed by formulas that relate fourth-order aberrations.
- The tip of the field vector indicates where a given ray originates from.
- The tip of the aperture vector indicates where the ray intersects the exit pupil plane.
- Note the organized way to present aberrations.
- Note that we are looking at a single wavefront located at the exit pupil. Actually, there is a train of wavefronts and it is a dynamic process of light propagation.

Review

The field and aperture vectors (scaled by the marginal ray height at the exit pupil and the chief ray height at the image plane) and the angle between them looking down the optical axis.



The field vector is located at the object plane.
The aperture vector is located at the exit pupil plane.
For convenience we draw the Gaussian image of the field vector in the image plane.

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The construction of the aberration function

Assuming that the optical surfaces and that the materials used to build a system are smooth in their optical properties, that is, that are continuous and that have a continuous first and second derivative as a function of the field and aperture vectors, then one would expect that the wavefront deformation itself would be a smooth function. There is no reason for the wavefront to behave in a non-smooth manner when the optical properties of the system are smooth. Therefore and under the circumstances of smoothness one can use a Taylor expansion to describe the wavefront deformation.

In the case of axially symmetric systems the wavefront deformation must be a scalar function of dot products of the field and aperture vectors, specifically $\vec{H}\cdot\vec{H}$, $\vec{H}\cdot\vec{\rho}$ and $\vec{\rho}\cdot\vec{\rho}$. These products only depend on the magnitude of the vectors and the angle between them and are used to describe axial symmetry. Therefore, the wavefront deformation, or as it is properly called, the wave aberration function $W(\vec{H},\vec{\rho})$ must have the form,

$$W(\vec{H},\vec{\rho})=\sum_{j,m,n} W_{k,l,m}(\vec{H}\cdot\vec{H})^j\cdot(\vec{H}\cdot\vec{\rho})^m\cdot(\vec{\rho}\cdot\vec{\rho})^n$$

where the sub-indices j,m,n represent integer numbers, $k=2j+m$, $l=2n+m$, and $W_{k,l,m}$ represent aberration coefficients.

The aberration function can be rewritten using the magnitude of the vectors as,

$$W(H,\rho,\cos\phi)=\sum_{j,m,n} W_{k,l,m} H^k \cdot \rho^l \cdot \cos^m(\phi)$$

$$= W_{000} + W_{200}H^2 + W_{111}H\rho\cos(\phi) + W_{020}\rho^2 +$$

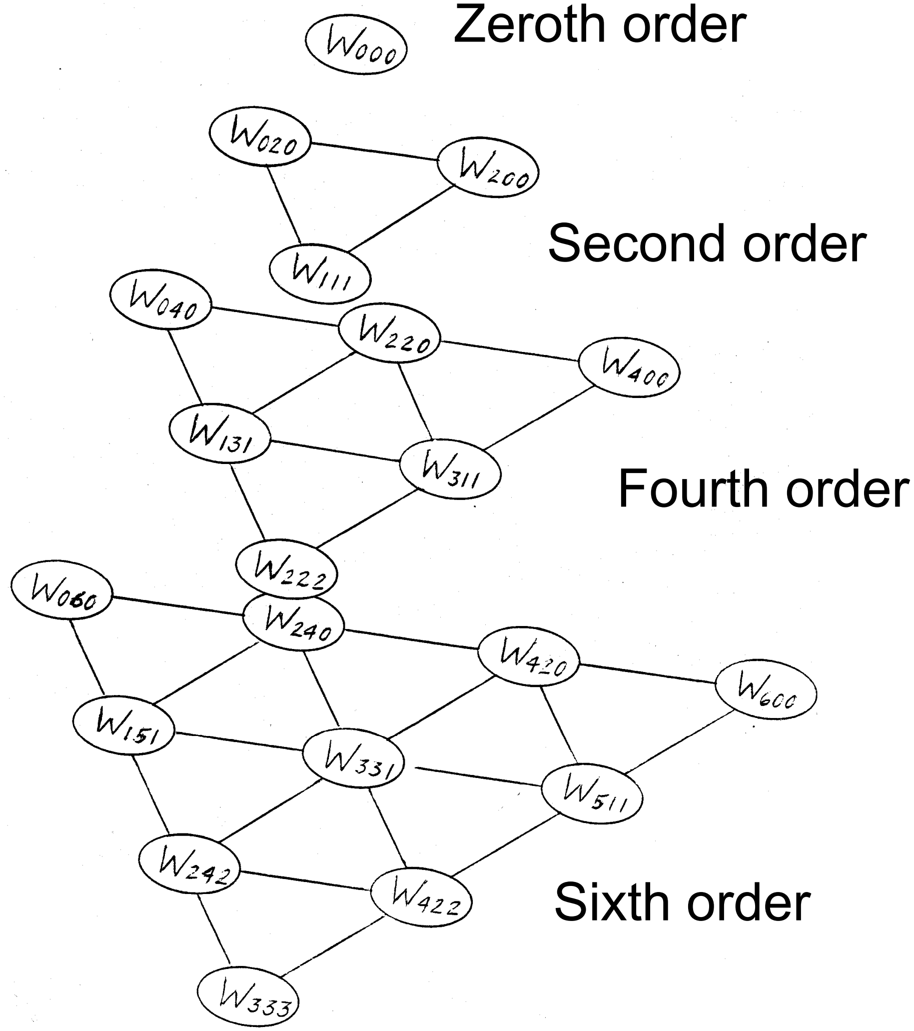
$$+ W_{040}\rho^4 + W_{131}H\rho^3\cos(\phi) + W_{222}H^2\rho^2\cos^2(\phi) + W_{220}H^2\rho^2 + W_{311}H^3\rho\cos(\phi) + W_{400}H^4 \ldots$$

where ϕ is the angle between the field and aperture vectors.

The terms in the Taylor expansion represent aberrations, that is, basic forms in which the wavefront can be deformed. The sum of all aberration terms produces the actual total wavefront deformation. The order of an aberration term is given by $2\cdot(j+m+n)$ which is always an even order. In the aberration function the field and aperture vectors are normalized so that when they are unity, the coefficients represent the maximum amplitude of each aberration which is expressed in wavelengths. The sub indices k,l,m in each coefficient indicate respectively the algebraic power of the field vector, the aperture vector, and the cosine of the angle ϕ between these vectors.

Wavefront aberrations					
Aberration name/order	Vector form	Algebraic form	j	m	n
Zero-order					
Uniform piston	W_{000}	W_{000}	0	0	0
Second-order					
Quadratic piston	$W_{200}(\vec{H}\cdot\vec{H})$	$W_{200}H^2$	1	0	0
Magnification	$W_{111}(\vec{H}\cdot\vec{\rho})$	$W_{111}H\rho\cos(\phi)$	0	1	0
Focus	$W_{020}(\vec{\rho}\cdot\vec{\rho})$	$W_{020}\rho^2$	0	0	1
Fourth-order,					
Spherical aberration	$W_{040}(\vec{\rho}\cdot\vec{\rho})^2$	$W_{040}\rho^4$	0	0	2
Coma	$W_{131}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$	$W_{131}H\rho^3\cos(\phi)$	0	1	1
Astigmatism	$W_{222}(\vec{H}\cdot\vec{\rho})^2$	$W_{222}H^2\rho^2\cos^2(\phi)$	0	2	0
Field curvature	$W_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$	$W_{220}H^2\rho^2$	1	0	1
Distortion	$W_{311}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})$	$W_{311}H^3\rho\cos(\phi)$	1	1	0
Quartic piston	$W_{400}(\vec{H}\cdot\vec{H})^2$	$W_{400}H^4$	2	0	0
Sixth-order					
Oblique spherical aberration	$W_{240}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})^2$	$W_{240}H^2\rho^4$	1	0	2
Coma	$W_{331}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$	$W_{331}H^3\rho^3\cos(\phi)$	1	1	1
Astigmatism	$W_{422}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})^2$	$W_{422}H^4\rho^2\cos^2(\phi)$	1	2	0
Field curvature	$W_{420}(\vec{H}\cdot\vec{H})^2(\vec{\rho}\cdot\vec{\rho})$	$W_{420}H^4\rho^2$	2	0	1
Distortion	$W_{511}(\vec{H}\cdot\vec{H})^2(\vec{H}\cdot\vec{\rho})$	$W_{511}H^5\rho\cos(\phi)$	2	1	0
Piston	$W_{600}(\vec{H}\cdot\vec{H})^3$	$W_{600}H^6$	3	0	0
Spherical aberration	$W_{060}(\vec{\rho}\cdot\vec{\rho})^3$	$W_{060}\rho^6$	0	0	3
Un-named	$W_{151}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})^2$	$W_{151}H\rho^5\cos(\phi)$	0	1	2
Un-named	$W_{242}(\vec{H}\cdot\vec{\rho})^2(\vec{\rho}\cdot\vec{\rho})$	$W_{242}H^2\rho^4\cos^2(\phi)$	0	2	1
Un-named	$W_{333}(\vec{H}\cdot\vec{\rho})^3$	$W_{333}H^3\rho^3\cos^3(\phi)$	0	3	0

Aberration orders



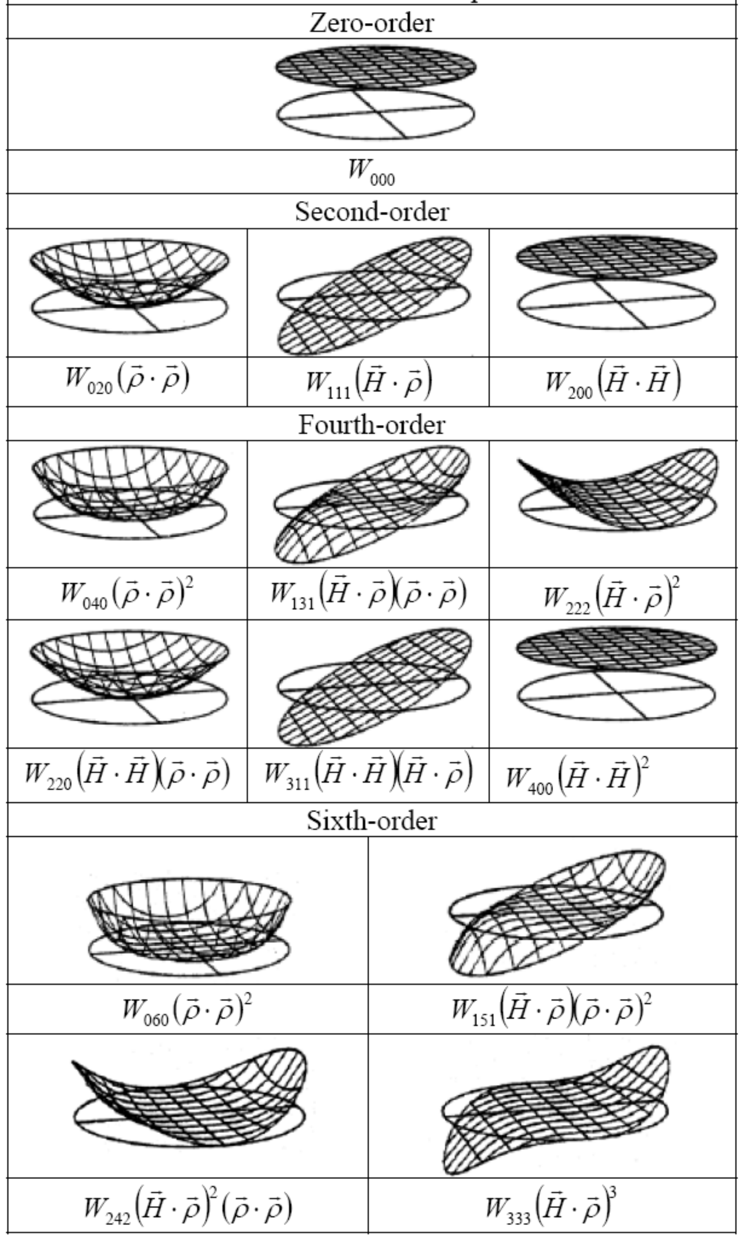


FIGURE 1
Wave aberration shapes

Aberration function using vector notation

$$W(\vec{H},\vec{\rho}) = \sum_{j,m,n} W_{k,l,m} (\vec{H}\cdot\vec{H})^j \cdot (\vec{H}\cdot\vec{\rho})^m \cdot (\vec{\rho}\cdot\vec{\rho})^n$$

$$= W_{000} + W_{200}(\vec{H}\cdot\vec{H}) + W_{111}(\vec{H}\cdot\vec{\rho}) + W_{020}(\vec{\rho}\cdot\vec{\rho})$$

$$+ W_{040}(\vec{\rho}\cdot\vec{\rho})^2 + W_{131}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{222}(\vec{H}\cdot\vec{\rho})^2$$

$$+ W_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho}) + W_{311}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho}) + W_{400}(\vec{H}\cdot\vec{H})^2$$

$$+ W_{240}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})^2 + W_{331}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho}) + W_{422}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})^2$$

$$+ W_{420}(\vec{H}\cdot\vec{H})^2(\vec{\rho}\cdot\vec{\rho}) + W_{511}(\vec{H}\cdot\vec{H})^2(\vec{H}\cdot\vec{\rho}) + W_{600}(\vec{H}\cdot\vec{H})^3$$

$$+ W_{060}(\vec{\rho}\cdot\vec{\rho})^3 + W_{151}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})^2 + W_{242}(\vec{H}\cdot\vec{\rho})^2(\vec{\rho}\cdot\vec{\rho}) + W_{333}(\vec{H}\cdot\vec{\rho})^3$$

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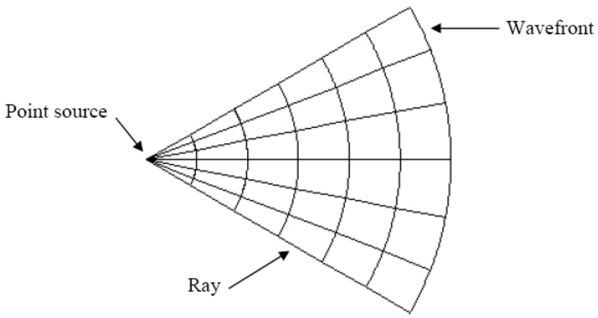
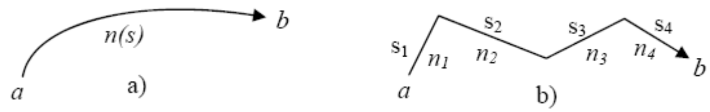
Transit time and wavefront

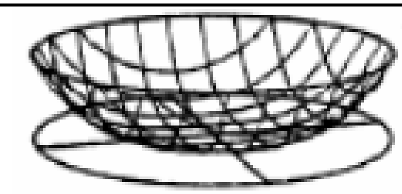
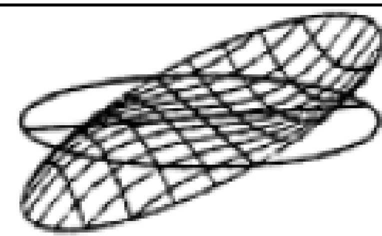
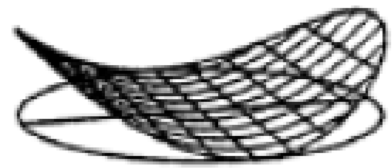
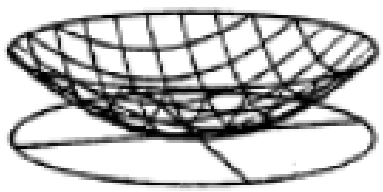
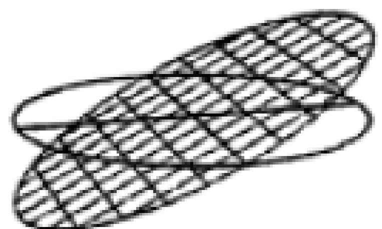


$$OPL = \int_a^b n \cdot ds \qquad OPL = \sum n \cdot s$$



- The geometrical wavefront is defined as the locus of equal optical path length (*OPL*). We have that *n* is the index of refraction and *ds* is the element of arc length. In a homogenous media *n* is constant, rays propagate in a straight line, and the optical path simplifies to a sum; *s* represents the length of the rays as they go from one point to the another. Insight about the *OPL* can be gained by dividing the *OPL* by the speed of light. Since the index of refraction is the ratio of the speed of light in vacuum *c* to the speed of light in the medium, then the factor *n/c* is the inverse speed of light in the medium. The term *n/c* is multiplied by the length s and the net result is a transit time *t*. Thus the optical path length divided by the speed of light is the transit time of a light particle traveling from point *a* to point *b*. The wavefront is therefore the locus of all light particles with the same transit time.

Primary aberrations

Fourth-order wavefront aberration shapes		
		
$W_{040}(\vec{\rho}\cdot\vec{\rho})^2$	$W_{131}(\vec{H}\cdot\vec{\rho})(\vec{\rho}\cdot\vec{\rho})$	$W_{222}(\vec{H}\cdot\vec{\rho})^2$
		
$W_{220}(\vec{H}\cdot\vec{H})(\vec{\rho}\cdot\vec{\rho})$	$W_{311}(\vec{H}\cdot\vec{H})(\vec{H}\cdot\vec{\rho})$	$W_{400}(\vec{H}\cdot\vec{H})^2$

Hamilton's characteristic function 1828

$$V\left(x_1, y_1, z_1, x_0, y_0, z_0\right)$$

$$\int_{P_0}^{P_1} n\,ds = V\left(x_1, y_1, z_1, x_0, y_0, z_0\right)$$

$$\frac{\partial V}{\partial x_1} = n_1\alpha_1 \qquad \frac{\partial V}{\partial y_1} = n_1\beta_1 \qquad \frac{\partial V}{\partial z_1} = n_1\gamma_1$$

$$\left(\frac{\partial V}{\partial x_1}\right)^2 + \left(\frac{\partial V}{\partial y_1}\right)^2 + \left(\frac{\partial V}{\partial z_1}\right)^2 = n_1^2$$

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Summary

- Historical aspects
- Aberration definition
- Aberration metrics
- The aberration function
- From the general to the particular
- Hamilton's work