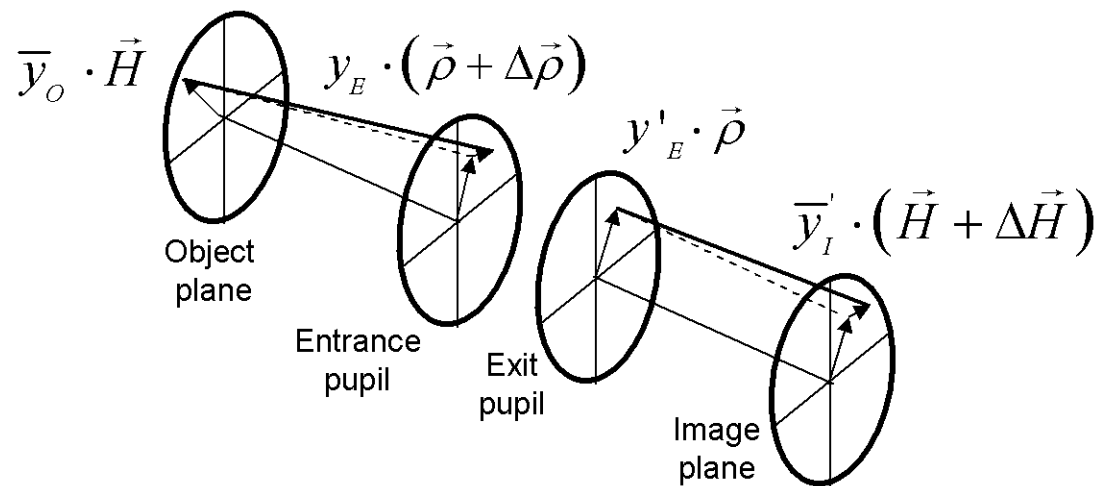


# Introduction to aberrations

OPTI 518

Lecture 14



# Topics

- Structural aberration coefficients
- Examples

# Structural coefficients

## Structural coefficients

Application of the aberration coefficients to specific optical components shows that the coefficients can be written as a function of the Lagrange invariant  $\mathcal{K}$ , the optical power  $\phi$ , the marginal ray height  $y_P$  at the principal planes, and the structural coefficients:  $\sigma_I, \sigma_{II}, \sigma_{III}, \sigma_{IV}, \sigma_V, \sigma_L, \sigma_T$ . The Seidel aberration coefficients can be expressed with the structural coefficients. The use of structural coefficients simplifies considerably the calculation of aberration coefficients and facilitates making trade-off studies.

Requires a focal system  
Afocal systems can be treated with Seidel sums

Seidel sums in terms of structural aberration coefficients
Pupils located at principal planes
$S_I = \frac{1}{4} y_P^4 \Phi^3 \sigma_I$
$S_{II} = \frac{1}{2} \mathcal{K} y_P^2 \Phi^2 \sigma_{II}$
$S_{III} = \mathcal{K}^2 \Phi \sigma_{III}$
$S_{IV} = \mathcal{K}^2 \Phi \sigma_{IV}$
$S_V = \frac{2 \mathcal{K}^3 \sigma_V}{y_P^2}$
$C_L = y_P^2 \Phi \sigma_L$
$C_T = 2 \mathcal{K} \sigma_T$

## Stop shifting from principal planes

$$\sigma_I^* = \sigma_I$$

$$\sigma_{II}^* = \sigma_{II} + \bar{S}_\sigma \sigma_I$$

$$\sigma_{III}^* = \sigma_{III} + 2\bar{S}_\sigma \sigma_{II} + \bar{S}_\sigma^2 \sigma_I$$

$$\sigma_{IV}^* = \sigma_{IV}$$

$$\sigma_V^* = \sigma_V + \bar{S}_\sigma (\sigma_{IV} + 3\sigma_{III}) + 3\bar{S}_\sigma^2 \sigma_{II} + \bar{S}_\sigma^3 \sigma_I$$

$$\sigma_L^* = \sigma_L$$

$$\sigma_T^* = \sigma_T + \bar{S}_\sigma \sigma_L$$

$$\bar{S}_\sigma = \frac{y_P \bar{y}_P \Phi}{2\mathcal{K}}$$

$$\Delta \bar{S}_\sigma = \frac{y_P \Delta \bar{y}_P \Phi}{2\mathcal{K}} = \frac{y_\sigma^2 \Phi}{2\mathcal{K}} \bar{S}$$

# Structural stop shifting parameter

$$\omega = nu = -(Y - 1) \cdot \varphi \cdot y / 2 \quad \bar{S}_\sigma = \frac{y_P \bar{y}_P \varphi}{2\mathcal{K}}$$

Using  $\omega$  on we can express:

$$\bar{S}_\sigma = \frac{y_P \bar{y}_P \varphi}{2\mathcal{K}} = \frac{\varphi \cdot s}{(Y - 1) \cdot \varphi \cdot s - 2n} = \frac{\varphi \cdot s'}{(Y + 1) \cdot \varphi \cdot s' - 2n'}$$

$s$  is the distance from the front principal plane to entrance pupil

$s'$  is the distance from the rear principal plane to exit pupil

$$\delta \bar{S}_\sigma = \frac{y_P \delta \bar{y}_P \varphi}{2\mathcal{K}} = \frac{y_P^2 \varphi}{2\mathcal{K}} \bar{S}$$

Structural coefficients of a system of $k$ components	
$\sigma_I = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right)^3 \left( \frac{y_{P,k}}{y_P} \right)^4 \sigma_{I,k}$	
$\sigma_{II} = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right)^2 \left( \frac{y_{P,k}}{y_P} \right)^2 (\sigma_{II,k} + \bar{S}_k \sigma_{I,k})$	
$\sigma_{III} = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right) (\sigma_{III,k} + 2\bar{S}_k \sigma_{II,k} + \bar{S}_k^2 \sigma_{I,k})$	
$\sigma_{IV} = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right) \sigma_{IV,k}$	
$\sigma_V = \sum_{i=0}^k \left( \frac{y_P}{y_{P,k}} \right)^2 (\sigma_{V,k} + \bar{S}_k (\sigma_{IV,k} + 3\sigma_{III,k}) + 3\bar{S}_k^2 \sigma_{II,k} + \bar{S}_k^3 \sigma_{I,k})$	
$\sigma_L = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right) \left( \frac{y_{P,k}}{y_P} \right)^2 \sigma_{L,k}$	
$\sigma_T = \sum_{i=0}^k (\sigma_{T,k} + \bar{S}_k \sigma_{L,k})$	
$\bar{S}_k = \frac{\Phi_k \cdot y_{P,k} \cdot \bar{y}_{P,k}}{2\mathcal{K}}$	

# Review of concepts

- Thin lens as the thickness tends to zero

$$\phi = \phi_1 + \phi_2 - \phi_1\phi_2 \frac{t}{n}$$

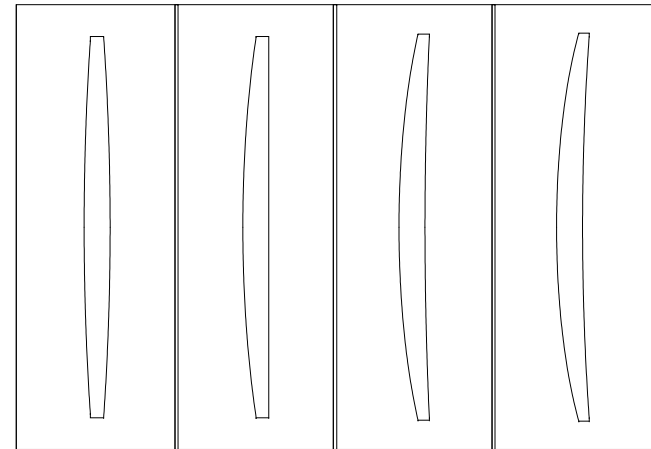
- Shape of a lens and shape factor
- Conjugate factor to quantify how the lens is used. Related to transverse magnification
- Must know well first-order optics



# Shape and Conjugate factors

$$Y = \frac{\omega' + \omega}{\omega' - \omega} = \frac{1 + m}{1 - m} \quad \omega = nu$$

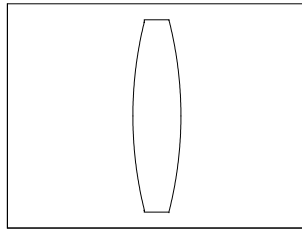
$$X = \frac{c_1 + c_2}{c_1 - c_2} = -\frac{R_1 + R_2}{R_1 - R_2}$$



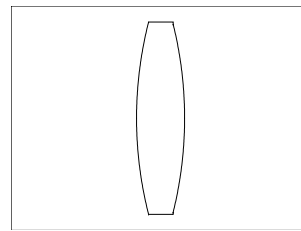
Lens bending concept

# Shape X

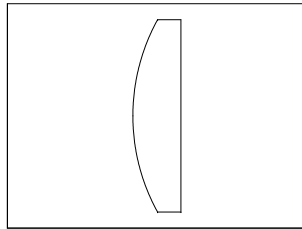
X=0



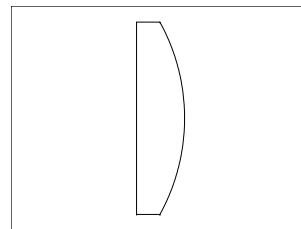
X=0



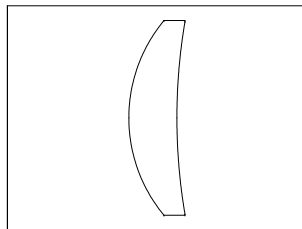
X=1



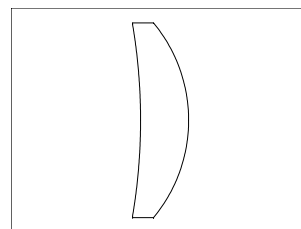
X=-1



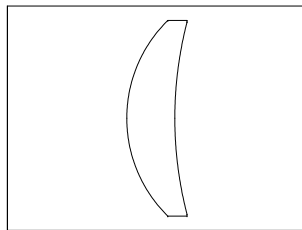
X=1.7



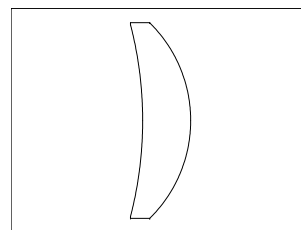
X=-1.7



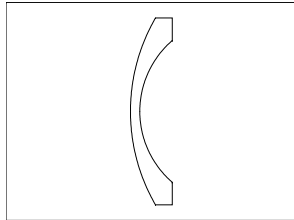
X=3.5



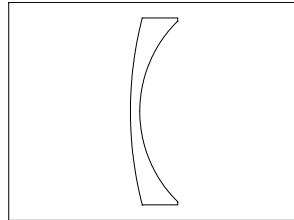
X=-3.5



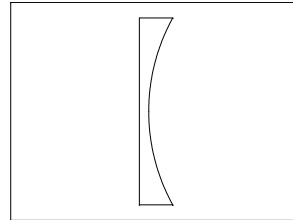
# Shape



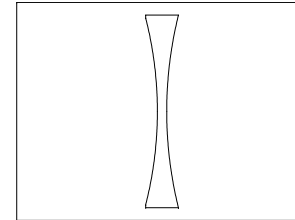
$X=-3$



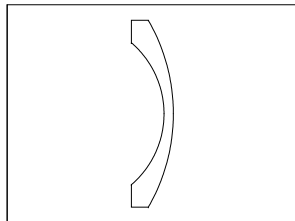
$X=-2$



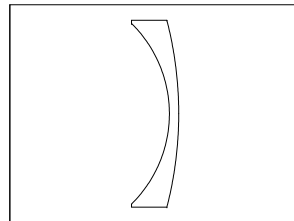
$X=-1$



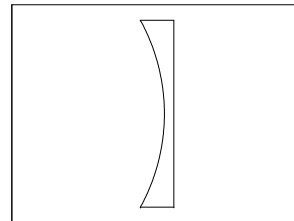
$X=0$



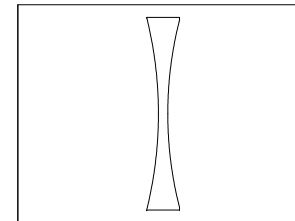
$X=3$



$X=2$



$X=1$



$X=0$

# Shape or bending factor $X$

- Quantifies lens shape
- Optical power of thin lens is maintained
- Not defined for zero power,  $R_1=R_2$

$$X = \frac{c_1 + c_2}{c_1 - c_2} = -\frac{R_1 + R_2}{R_1 - R_2}$$

Structural aberration coefficients of a surface (Stop at surface $\bar{y}_P = 0$ )			
First-order identities			
$\Phi = (n' - n) \cdot c$		$Y = \frac{\omega' + \omega}{\omega' - \omega}$	
$\omega' = \omega - \Phi \cdot y_P$		$\omega = \omega' + \Phi \cdot y_P$	
$\omega' = -\frac{Y+1}{2} \Phi \cdot y_P$		$\omega = -\frac{Y-1}{2} \Phi \cdot y_P$	
$\delta n / n = (n_F - n_C) / n_d$			
Seidel sum arguments			
$A = \omega + n y_P c = \left[ \frac{n' + n}{n' - n} - Y \right] \frac{\Phi \cdot y_P}{2}$		$\bar{A} = \bar{\omega} + n \bar{y}_P c = \bar{\omega} = \frac{\mathcal{K}}{y_P}$	
$y_P \cdot \Delta \left( \frac{u}{n} \right) = y_P \cdot \Delta \left( \frac{\omega}{n^2} \right)$  $= \left[ \frac{n'^2 - n^2}{n^2 n'^2} \cdot Y - \frac{n'^2 + n^2}{n^2 n'^2} \right] \frac{\Phi \cdot y_P^2}{2}$		$P = -\frac{\Phi}{nn'}$	

Structural aberration coefficients of a surface
$\sigma_I = -\frac{1}{2} \left[ \frac{n'+n}{n'-n} - Y \right]^2 \left[ \frac{n'^2-n^2}{n^2 n'^2} \cdot Y - \frac{n'^2+n^2}{n^2 n'^2} \right]$
$\sigma_{II} = -\frac{1}{2} \left[ \frac{n'+n}{n'-n} - Y \right] \left[ \frac{n'^2-n^2}{n^2 n'^2} \cdot Y - \frac{n'^2+n^2}{n^2 n'^2} \right]$
$\sigma_{III} = -\frac{1}{2} \left[ \frac{n'^2-n^2}{n^2 n'^2} \cdot Y - \frac{n'^2+n^2}{n^2 n'^2} \right]$
$\sigma_{IV} = \frac{1}{nn'}$
$\sigma_V = \frac{n'^2-n^2}{n^2 n'^2}$
$\sigma_L = \frac{1}{2} \left[ Y - \frac{n'+n}{n'-n} \right] \cdot \Delta \left( \frac{\delta n}{n} \right)$
$\sigma_T = \Delta \left( \frac{\delta n}{n} \right)$

# Example:

## Refracting surface free from spherical aberration

Object at infinity  $Y=1$

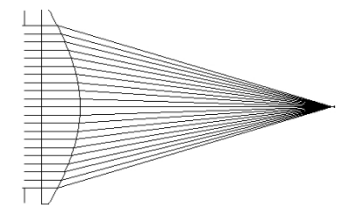
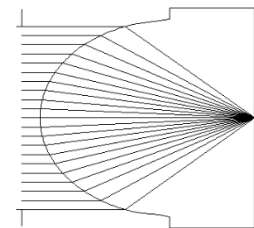
$$\sigma_I = -\frac{1}{2} \left[ \frac{n'+n}{n'-n} - 1 \right]^2 \left[ \frac{n'^2 - n^2}{n'^2 n^2} - \frac{n'^2 + n^2}{n'^2 n^2} \right] = -\frac{1}{2} \left[ \frac{2n}{n'-n} \right]^2 \left[ \frac{-2}{n'^2} \right] = \left[ \frac{2n}{n'-n} \right]^2 \left[ \frac{1}{n'^2} \right]$$

$$S_I = \frac{1}{4} y_P^4 \phi^3 \sigma_I + \Delta(n) \frac{y_P^4}{r^3} K = \frac{1}{4} y_P^4 \phi^3 \sigma_I + \frac{1}{(n'-n)^2} y_P^4 \phi^3 K$$

$$S_I = y_P^4 \phi^3 \left[ \frac{1}{4} \left[ \frac{2n}{n'-n} \right]^2 \left[ \frac{1}{n'^2} \right] + \frac{1}{(n'-n)^2} K \right] = \frac{y_P^4 \phi^3}{(n'-n)^2} \left[ \frac{n^2}{n'^2} + K \right]$$

$$S_I = 0 \Rightarrow \left[ \frac{n^2}{n'^2} = -K = \varepsilon^2 \right]$$

Parabola for reflection  
 Ellipse for air to glass  
 Hyperbola for glass to air



# Note

$$S_I = \frac{1}{4} y_P^4 \phi^3 \sigma_I + \frac{1}{(n' - n)^2} y_P^4 \phi^3 K = \frac{1}{4} y_P^4 \phi^3 \left[ \sigma_I + \left( \frac{2}{n' - n} \right)^2 K \right]$$

The contribution to the structural coefficient from the aspheric cap is

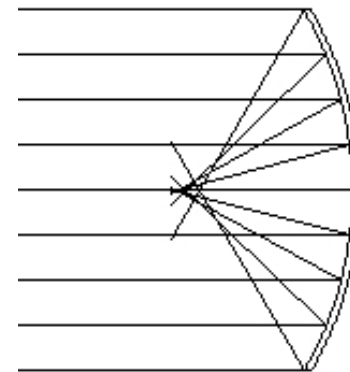
$$\sigma_{Icap} = \alpha = \left( \frac{2}{n' - n} \right)^2 K$$

For a reflecting surface is just the conic constant K



Structural aberration coefficients of a reflecting surface in air			
Stop at surface		With stop shift	
$\sigma_I = Y^2 + \alpha$		$\sigma_I = Y^2 + \alpha$	
$\sigma_{II} = -Y$		$\sigma_{II} = -Y(1 - \bar{S}_\sigma Y) + \bar{S}_\sigma \cdot \alpha$	
$\sigma_{III} = 1$		$\sigma_{III} = (1 - \bar{S}_\sigma Y)^2 + \bar{S}_\sigma^2 \cdot \alpha$	
$\sigma_{IV} = -1$		$\sigma_{IV} = -1$	
$\sigma_V = 0$		$\sigma_V = \bar{S}_\sigma \cdot (1 - \bar{S}_\sigma Y) \cdot (2 - \bar{S}_\sigma Y) + \bar{S}_\sigma^3 \cdot \alpha$	

$$\sigma_{Icap} = \alpha = K$$



Structural aberration coefficients of a thin lens in air (Stop at lens)	
First-order identities	
$\Phi = (n-1) \cdot (c_1 - c_2) = (n-1) \cdot \left( \frac{1}{r_1} - \frac{1}{r_2} \right)$	
$X = \frac{c_1 + c_2}{c_1 - c_2} = -\frac{r_1 + r_2}{r_1 - r_2}$	$Y = \frac{\omega' + \omega}{\omega' - \omega} = \frac{1 + m}{1 - m}$
$c_1 = \frac{1}{2} \frac{\Phi}{n-1} (X + 1)$	$c_2 = \frac{1}{2} \frac{\Phi}{n-1} (X - 1)$
$\omega = nu = -\frac{1}{2} (Y - 1) (\Phi \cdot y_P)$	$\omega' = n'u' = -\frac{1}{2} (Y + 1) (\Phi \cdot y_P)$

Structural aberration coefficients of a thin lens in air (Stop at lens)		
$\sigma_I = AX^2 - BXY + CY^2 + D$		$A = \frac{n+2}{n(n-1)^2}$
$\sigma_{II} = EX - FY$		$B = \frac{4(n+1)}{n(n-1)}$
$\sigma_{III} = 1$		$C = \frac{3n+2}{n}$
$\sigma_{IV} = \frac{1}{n}$		$D = \frac{n^2}{(n-1)^2}$
$\sigma_V = 0$		$E = \frac{n+1}{n(n-1)}$
$\sigma_L = \frac{1}{\nu}$		$F = \frac{2n+1}{n}$
$\sigma_T = 0$		$\nu = \frac{n_F - n_C}{n_d - 1}$



Contributions to the structural coefficients from a parallel plate of thickness $t$ and index $n$ into a system of optical power $\Phi$	
$\Delta\sigma_I = -4\left(\frac{Y \pm 1}{2}\right)^4 \left(\frac{n^2 - 1}{n^2}\right) \frac{\Phi t}{n}$	(0.1)
$\Delta\sigma_{II} = 2\left(\frac{Y \pm 1}{2}\right)^3 \left(\frac{n^2 - 1}{n^2}\right) \frac{\Phi t}{n}$	(0.2)
$\Delta\sigma_{III} = -\left(\frac{Y \pm 1}{2}\right)^2 \left(\frac{n^2 - 1}{n^2}\right) \frac{\Phi t}{n}$	(0.3)
$\Delta\sigma_{IV} = 0$	(0.4)
$\Delta\sigma_V = \frac{1}{2}\left(\frac{Y \pm 1}{2}\right) \left(\frac{n^2 - 1}{n^2}\right) \frac{\Phi t}{n}$	(0.5)
$\Delta\sigma_L = -\left(\frac{Y \pm 1}{2}\right)^2 \left(\frac{n - 1}{n\nu}\right) \frac{\Phi t}{n}$	(0.6)
$\Delta\sigma_T = \frac{1}{2}\left(\frac{Y \pm 1}{2}\right) \left(\frac{n - 1}{n\nu}\right) \frac{\Phi t}{n}$	(0.7)
Positive sign + for image space Negative sign - for object space	

# Spherical Mirror

A spherical mirror can be treated as a convex/concave plano lens with  $n=-1$ . The plano surface acts as an unfolding flat surface contributing no aberration.

$$X = \pm 1$$

$$\sigma_I = Y^2$$

$$\sigma_{II} = -Y$$

$$\sigma_{III} = 1$$

$$\sigma_{IV} = -1$$

$$\sigma_V = 0$$

$$\sigma_L = 0$$

$$\sigma_T = 0$$

$$A = -\frac{1}{4}$$

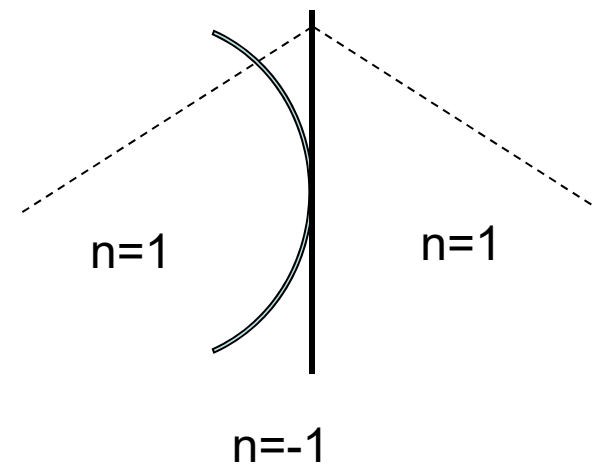
$$B = 0$$

$$C = 1$$

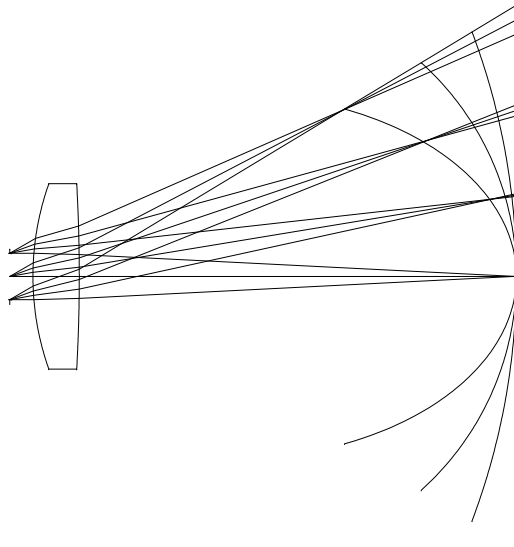
$$D = \frac{1}{4}$$

$$E = 0$$

$$F = 1$$



# Field curves



Field curve curvature in terms of structural coefficients

$$C_{Petzval} = -n' \phi \cdot \sigma_{IV}$$

$$C_{Sagittal} = -n' \phi \cdot (\sigma_{IV} + \sigma_{III})$$

$$C_{Medial} = -n' \phi \cdot (\sigma_{IV} + 2\sigma_{III})$$

$$C_{Tangential} = -n' \phi \cdot (\sigma_{IV} + 3\sigma_{III})$$

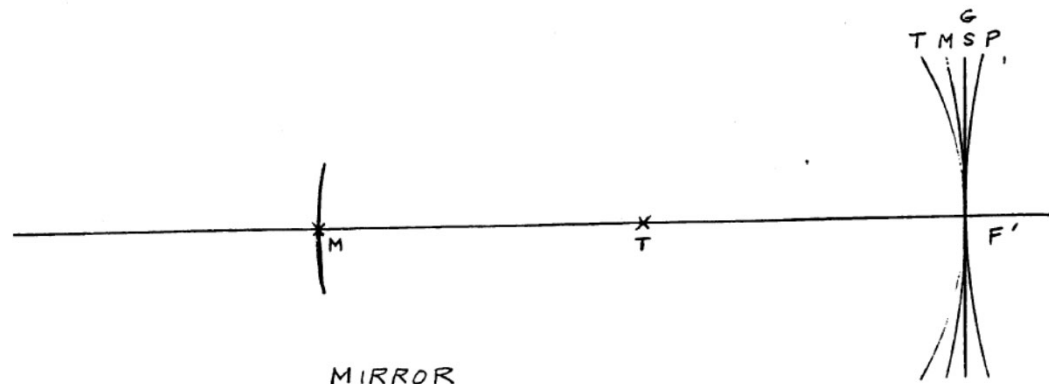
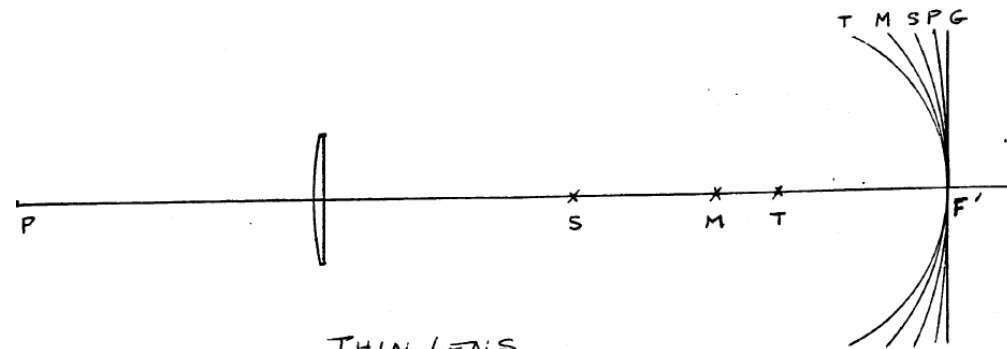
# Field curves

OBJECT AT INFINITY

STOP AT LENS

$R_t \sim f/3.66$   
 $R_m \sim f/2.66$   
 $R_s \sim f/1.66$   
 $R_p \sim f/0.66$

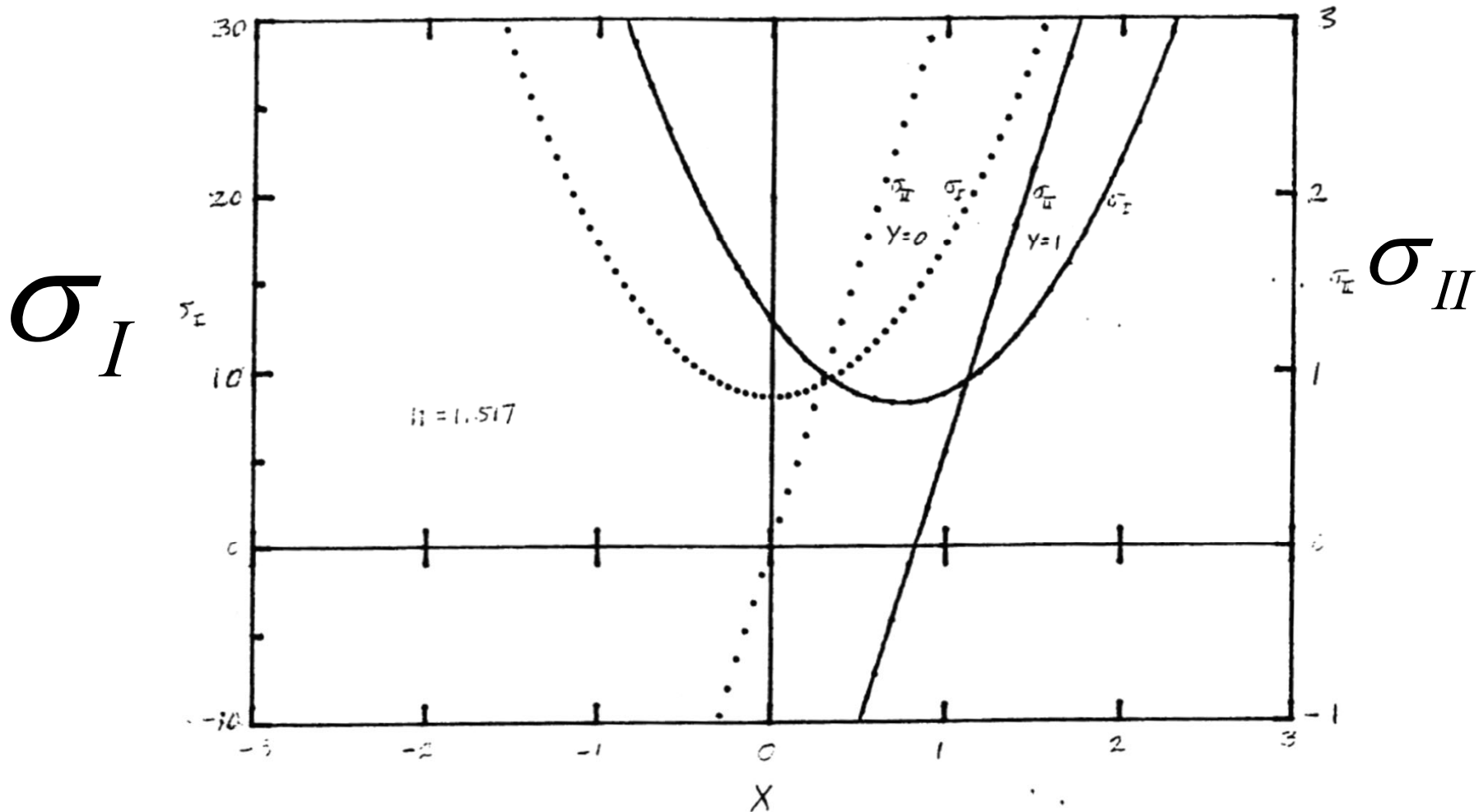
Field curve curvature in terms of structural coefficients
$C_{Petzval} = -n' \phi \cdot \sigma_{IV}$
$C_{Sagittal} = -n' \phi \cdot (\sigma_{IV} + \sigma_{III})$
$C_{Medial} = -n' \phi \cdot (\sigma_{IV} + 2\sigma_{III})$
$C_{Tangential} = -n' \phi \cdot (\sigma_{IV} + 3\sigma_{III})$



FIELD CURVATURES DO NOT CHANGE WITH  
OBJECT DISTANCE  
(STOP AT LENS)

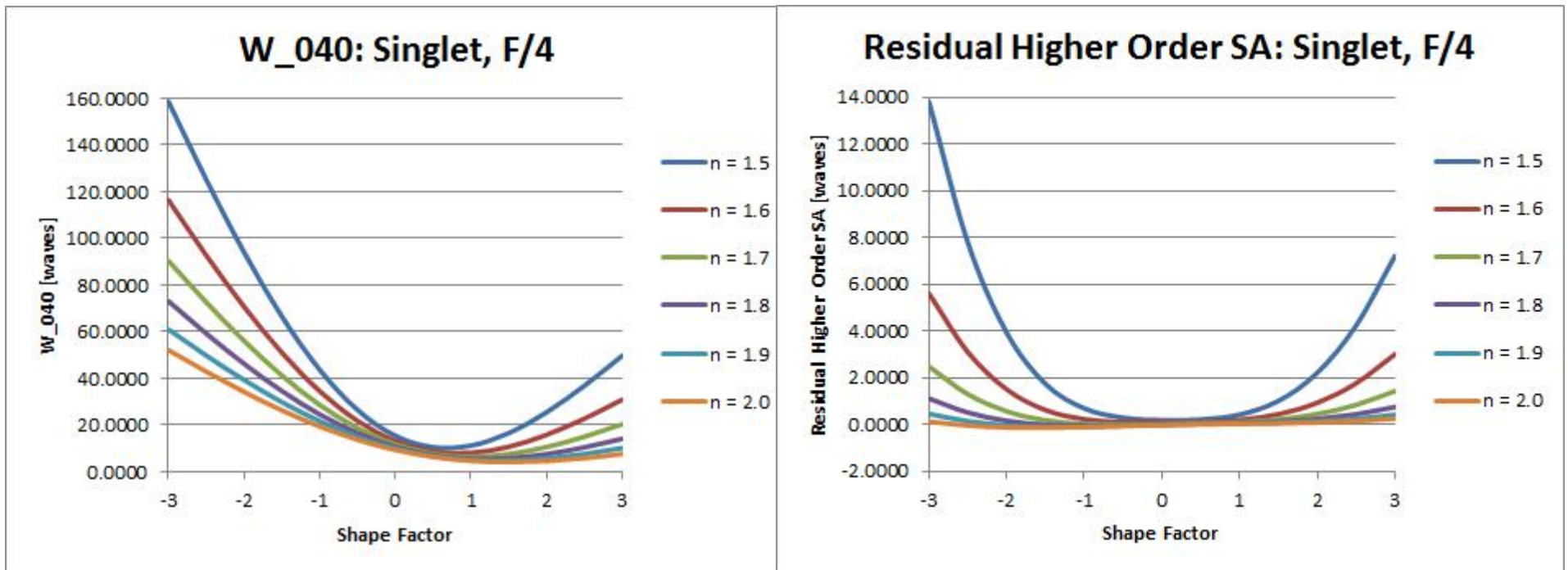
# Thin lens

## Spherical aberration and coma





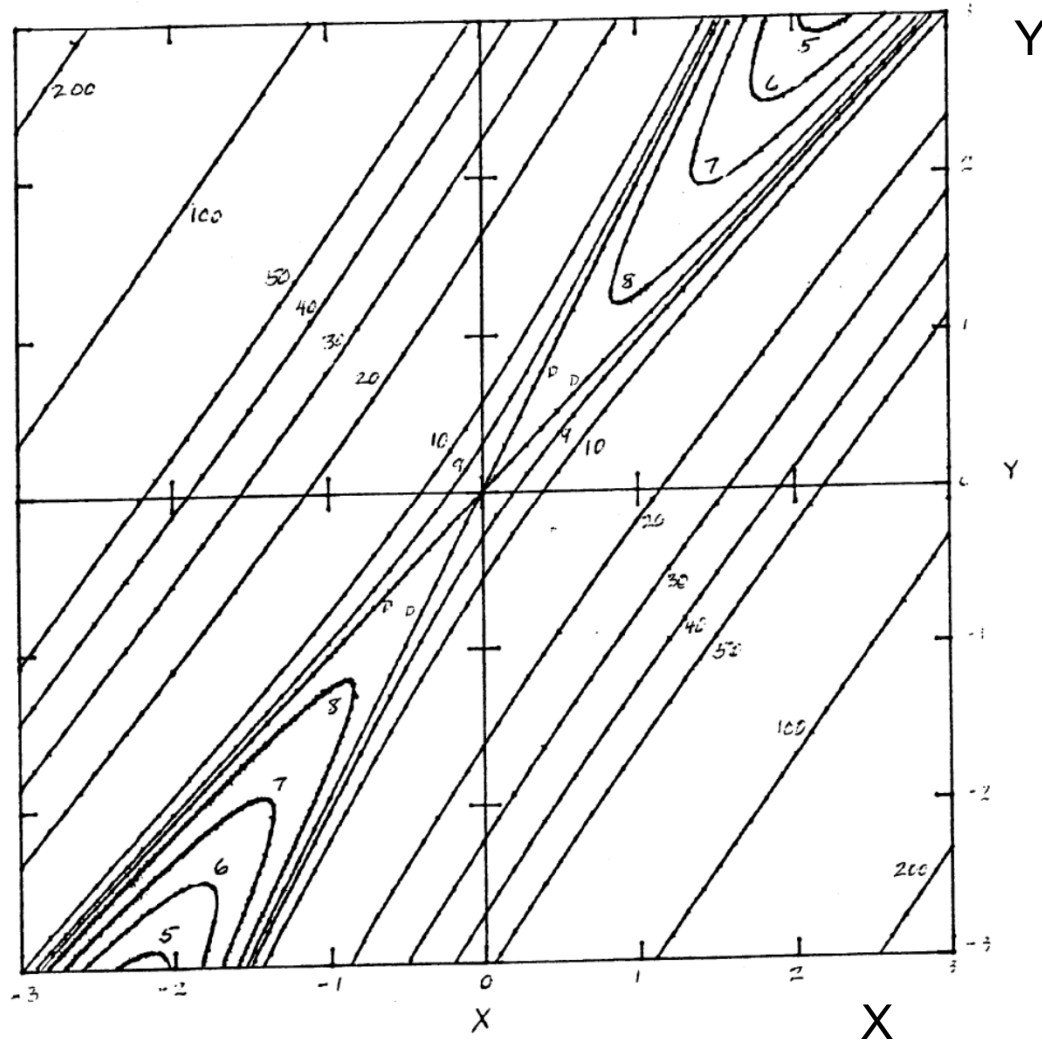
# Spherical aberration of a F/4 lens



- Asymmetry
- For high index 4<sup>th</sup> order predicts well the aberration

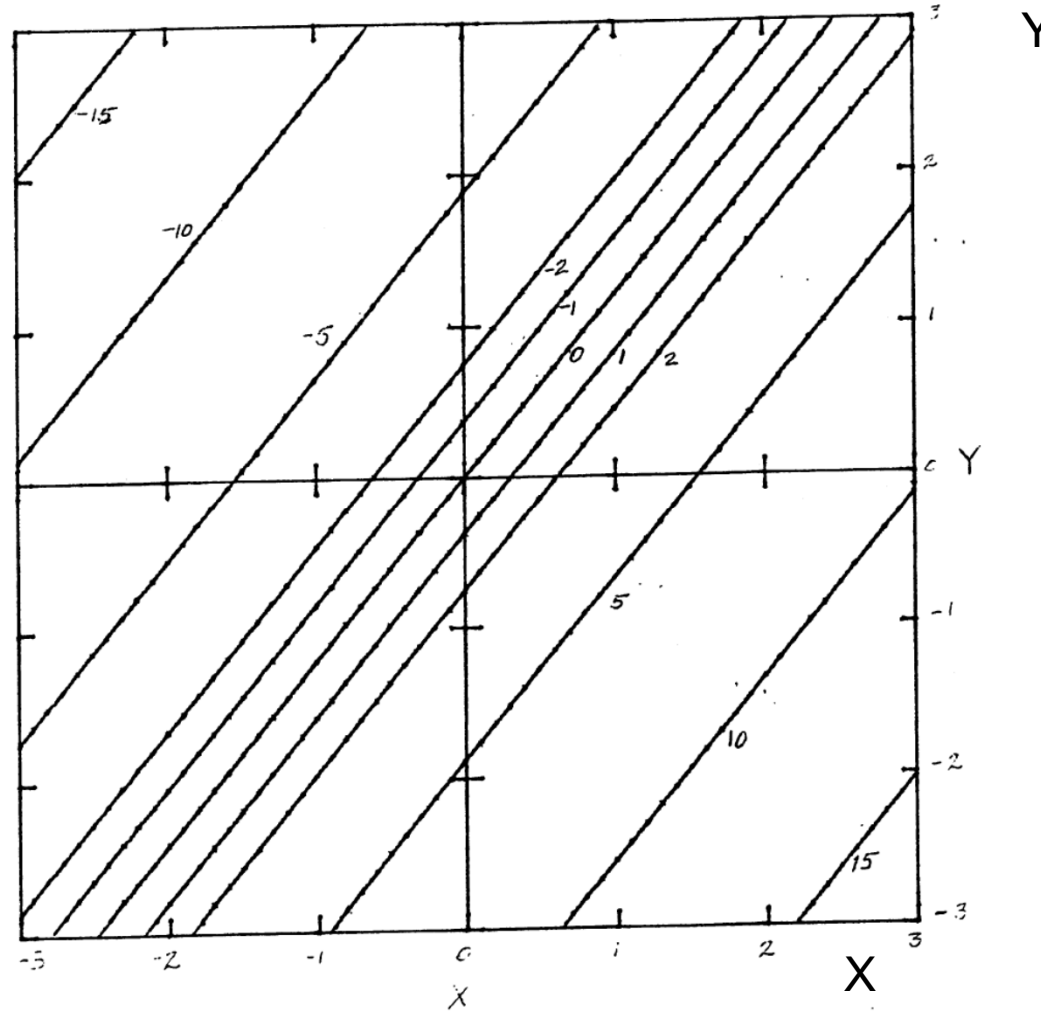
# Thin lens spherical aberration $n=1.517$

$\sigma_I$



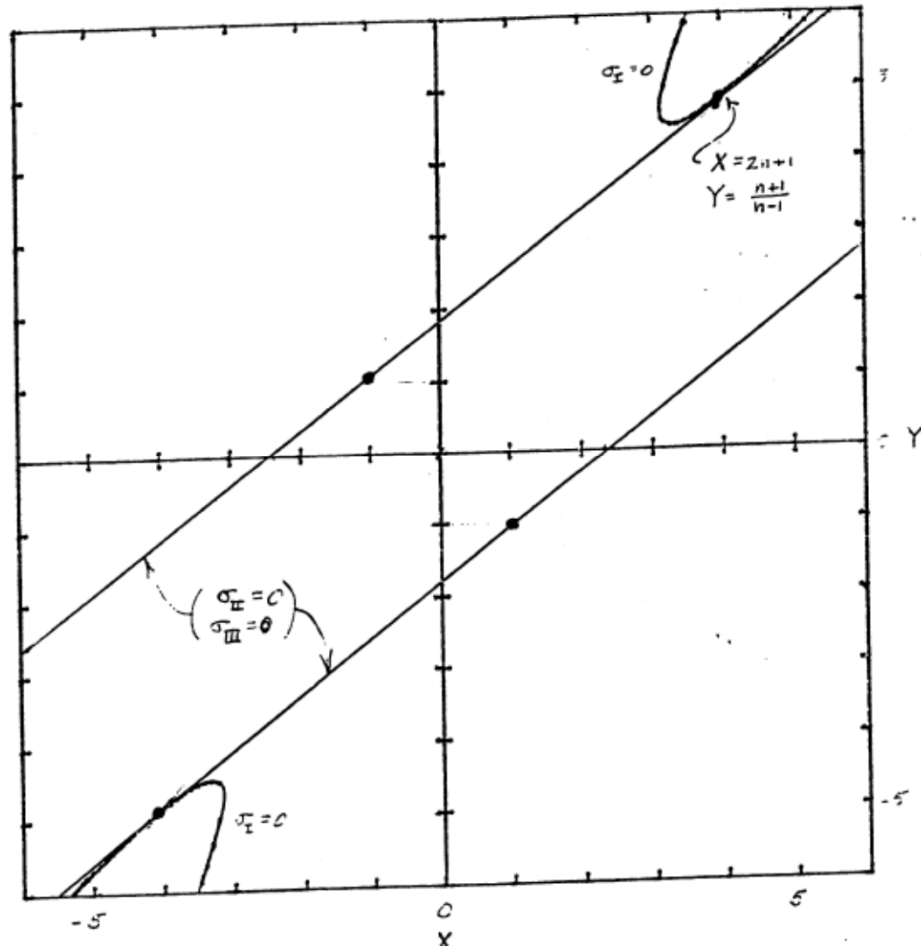
# Thin lens coma aberration $n=1.517$

$\sigma_{II}$



# This lens

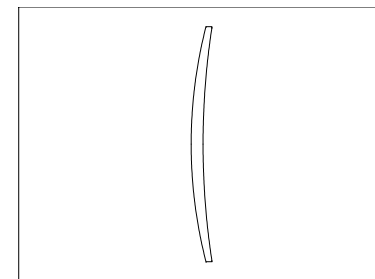
## Aplanatic solutions



$$X = \pm (2n+1)$$

$$Y = \pm \left( \frac{n+1}{n-1} \right)$$

$$Y = \frac{\omega' + \omega}{\omega' - \omega} = \frac{1+m}{1-m}$$



$$X=4$$

$$Y=5$$

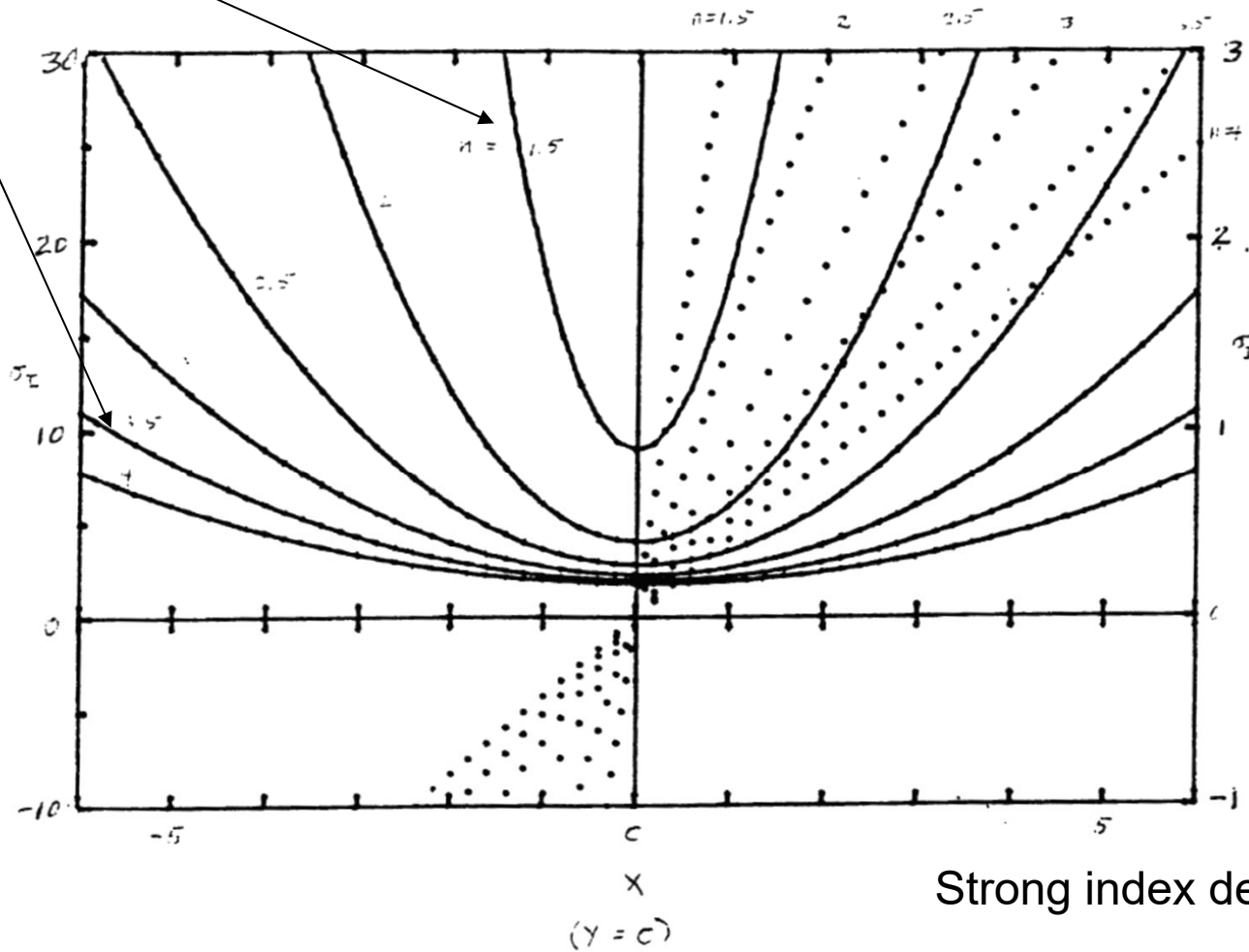
$$n=1.5$$

# Thin lens

Spherical and coma @  $Y=0$

- N=1.5
- N=2
- N=2.5
- N=3
- N=3.5
- N=4

$\sigma_I$



$\sigma_{II}$

Strong index dependence

# Thin lens special cases

## stop at lens

$$\underline{\sigma_{\text{I}} = \text{MINIMUM}}$$

$$X = \frac{B}{2A} Y = \frac{2(n+1)(n-1)}{n+2} Y$$

$$\begin{aligned}\sigma_{\text{I}} &= \frac{n^2}{(n-1)^2} - \frac{n}{n+2} Y^2 \\ &= \frac{n^2}{(n-1)^2} - \frac{n(n+2)}{4(n+1)^2(n-1)^2} X^2\end{aligned}$$

$$\begin{aligned}\sigma_{\text{II}} &= -\frac{1}{n+2} Y \\ &= -\frac{1}{2(n^2-1)} X\end{aligned}$$

# Thin lens special cases

## stop at lens

$$\underline{\sigma_{II} = 0}$$

$$X = \frac{F}{E} Y = \frac{(2n+1)(n-1)}{(n+1)} Y$$

$$\begin{aligned}\sigma_I &= \frac{n^2}{(n-1)^2} - \frac{n^2}{(n+1)^2} Y^2 \\ &= \frac{n^2}{(n-1)^2} - \frac{n^2}{(2n+1)^2(n-1)^2} X^2\end{aligned}$$

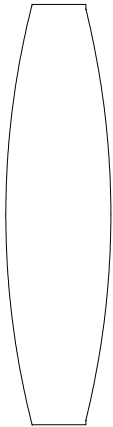
FOR  $\sigma_I = 0$  (APLANATIC LENS)

$$X = \pm (2n+1)$$

$$Y = \pm \left( \frac{n+1}{n-1} \right)$$

# Thin lens special cases

## stop at lens



$$\underline{X=0}$$

$$\sigma_{\text{I}} = \frac{3n+2}{n} Y^2 + \frac{n^2}{(n-1)^2}$$

$$\sigma_{\text{II}} = -\frac{2n+1}{n} Y$$

$$\underline{Y=1}$$

$$\sigma_{\text{I}} = \frac{3n+2}{n} + \frac{n^2}{(n-1)^2}$$

$$\sigma_{\text{II}} = -\frac{2n+1}{n}$$



For double convex lens (CX)  
For double concave lens (CC)



# Thin lens special cases

## stop at lens

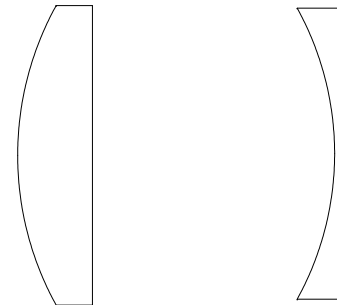
$$\frac{X=1}{\sigma_I = \frac{3n+2}{n} \left( Y - \frac{2(n+1)}{(3n+2)(n-1)} \right)^2 + \frac{n(3n-1)(n+1)}{(3n+2)(n-1)^2}}$$

$$\sigma_{II} = \frac{n+1}{n(n-1)} - \frac{2n+1}{n} Y$$

$$\frac{Y=1}{\sigma_I = 4 \left( 1 + \frac{2-n}{n(n-1)^2} \right)}$$

$$\sigma_{II} = -2 + \frac{2}{n(n-1)}$$

PLANO CONVEX (CONCAVE),  
CONVEX SIDE FORWARD  
(CONCAVE)



# Thin lens special cases

## stop at lens

$$\underline{X = -1}$$

$$\sigma_I = \frac{3n+2}{n} \left( Y + \frac{2(n+1)}{(3n+2)(n-1)} \right)^2 + \frac{n(3n-1)(n+1)}{(3n+2)(n-1)^2}$$

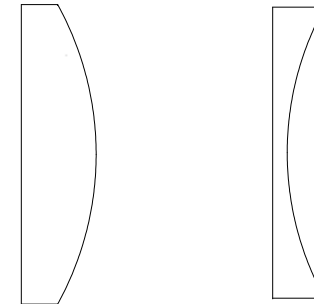
$$\sigma_{II} = -\frac{n+1}{n(n-1)} - \frac{2n+1}{n} Y$$

PLANO CONVEX (CONCAVE),  
PLANE SIDE FORWARD

$$\underline{Y = 1}$$

$$\sigma_I = \left( \frac{2n}{n-1} \right)^2$$

$$\sigma_{II} = -\frac{2n}{n-1}$$



# Achromatic doublet

Two thin lenses in contact  
The stop is at the doublet

$$\phi = \phi_1 + \phi_2$$

$$y_1 = y_2$$

$$\rho_1 = \frac{\phi_1}{\phi}$$

$$\rho_2 = \frac{\phi_2}{\phi}$$

$$\rho_1 + \rho_2 = 1$$

$$Y_1 = \frac{Y - \rho_2}{\rho_1}$$

$$Y_2 = \frac{Y - \rho_1}{\rho_2}$$

# Achromatic doublet

Correction for chromatic change of focus

$$\sigma_L = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right) \left( \frac{y_{P,k}}{y_P} \right)^2 \sigma_{L,k}$$

$$\sigma_L = \frac{\rho_1}{\nu_1} + \frac{\rho_2}{\nu_2}$$

$$\rho_1 = \frac{\nu_1}{\nu_1 - \nu_2} (1 - \nu_2 \sigma_L)$$

$$\rho_2 = -\frac{\nu_2}{\nu_1 - \nu_2} (1 - \nu_1 \sigma_L)$$

For an achromatic doublet:

$$\sigma_L = 0$$

# Achromatic doublet

Correction for spherical aberration

$$\sigma_I = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right)^3 \left( \frac{y_{P,k}}{y_P} \right)^4 \sigma_{I,k}$$

$$\sigma_I = \rho_1^3 \left( A_1 X_1^2 - B_1 X_1 Y_1 + C_1 Y_1^2 + D_1 \right) + \rho_2^3 \left( A_2 X_2^2 - B_2 X_2 Y_2 + C_2 Y_2^2 + D_2 \right)$$

For a given conjugate factor Y, spherical aberration is a function Of the shape factors  $X_1$  and  $X_2$  . For a constant value of spherical aberration we obtain a hyperbola as a function of X1 and X2.

# Achromatic doublet

Correction for coma aberration

$$\sigma_{II} = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right)^2 \left( \frac{y_{P,k}}{y_P} \right)^2 \left( \sigma_{II,k} + \bar{S}_k \sigma_{I,k} \right)$$

$$\sigma_{II} = \rho_1^2 \left( E_1 X_1 - F_1 Y_1 \right) + \rho_2^2 \left( E_2 X_2 - F_2 Y_2 \right)$$

For a given conjugate factor Y and a constant amount of coma the graph of X1 and X2 is a straight line.

# Achromatic doublet

Astigmatism aberration

$$\sigma_{III} = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right) (\sigma_{III,k} + 2\bar{S}_k \sigma_{II,k} + \bar{S}_k^2 \sigma_{I,k})$$

$$\sigma_{III} = \rho_1 (1) + \rho_2 (1)$$

Astigmatism is independent of the relative lens powers, shape factors, or conjugate factors.

# Achromatic doublet

Field curvature aberration

$$\sigma_{IV} = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right) \sigma_{IV,k}$$

$$\sigma_{IV} = \frac{\rho_1}{n_1} + \frac{\rho_2}{n_2}$$

For an achromatic doublet there is no field curvature  $\sigma_{IV} = 0$  if

$$\frac{n_2}{V_2} = \frac{n_1}{V_1}$$



# Achromatic doublet

Distortion and chromatic change of magnification

$$\sigma_V = \sum_{i=0}^k \left( \frac{y_P}{y_{P,k}} \right)^2 \left( \sigma_{V,k} + \bar{S}_k \left( \sigma_{IV,k} + 3\sigma_{III,k} \right) + 3\bar{S}_k^2 \sigma_{II,k} + \bar{S}_k^3 \sigma_{I,k} \right)$$

$$\sigma_T = \sum_{i=0}^k \left( \sigma_{T,k} + \bar{S}_k \sigma_{L,k} \right)$$

$$\sigma_V = 0$$

$$\sigma_T = 0$$

# Cemented achromatic doublet

$$c_{12} = \frac{(X_1 - 1)\phi_1}{2(n_1 - 1)} = c_{21} = \frac{(X_2 + 1)\phi_2}{2(n_2 - 1)}$$

$$\frac{(X_1 - 1)\rho_1}{(n_1 - 1)} = \frac{(X_2 + 1)\rho_2}{(n_2 - 1)}$$

$$X_2 = \alpha(X_1 - 1) - 1$$

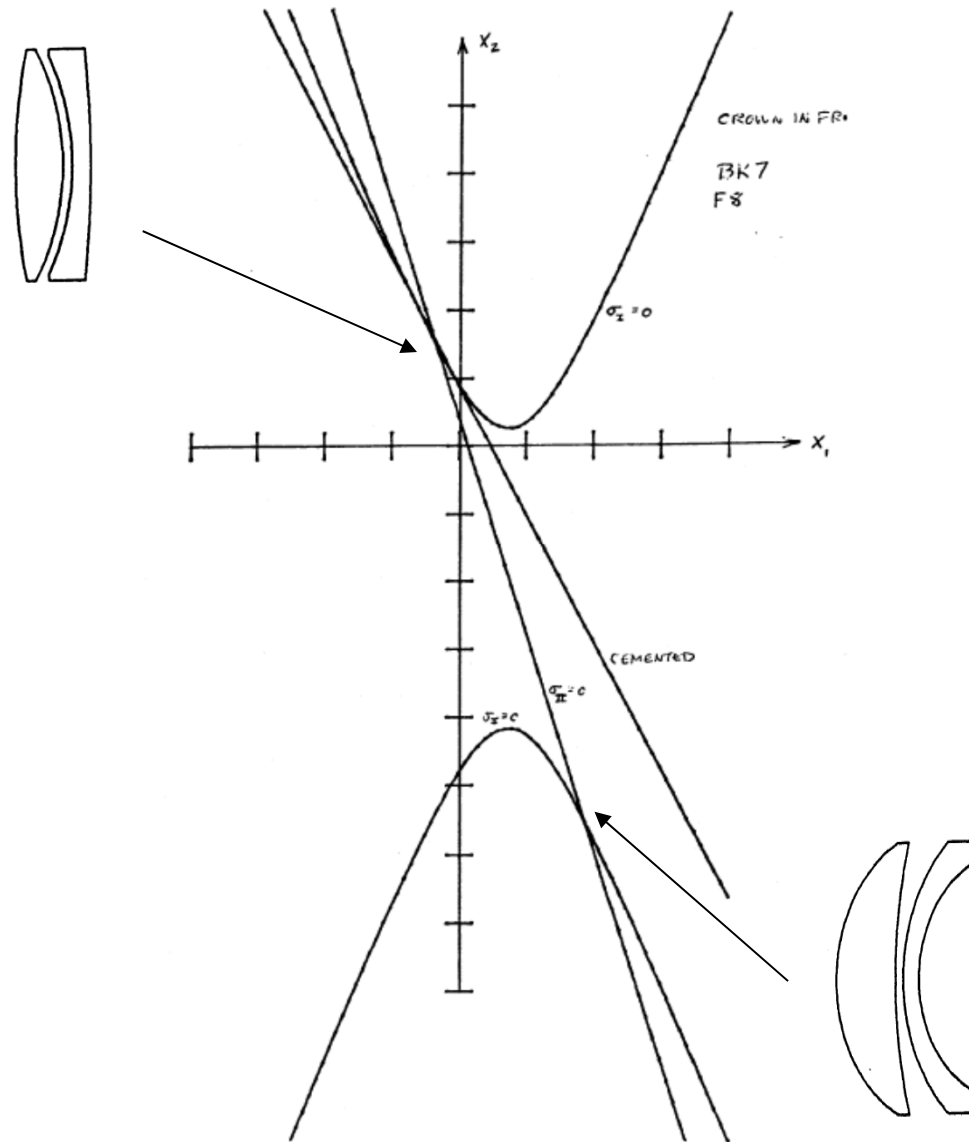
$$\alpha = \frac{(n_2 - 1)\rho_1}{(n_1 - 1)\rho_2}$$

$$\alpha = -\frac{(n_2 - 1)\nu_1}{(n_1 - 1)\nu_2}$$

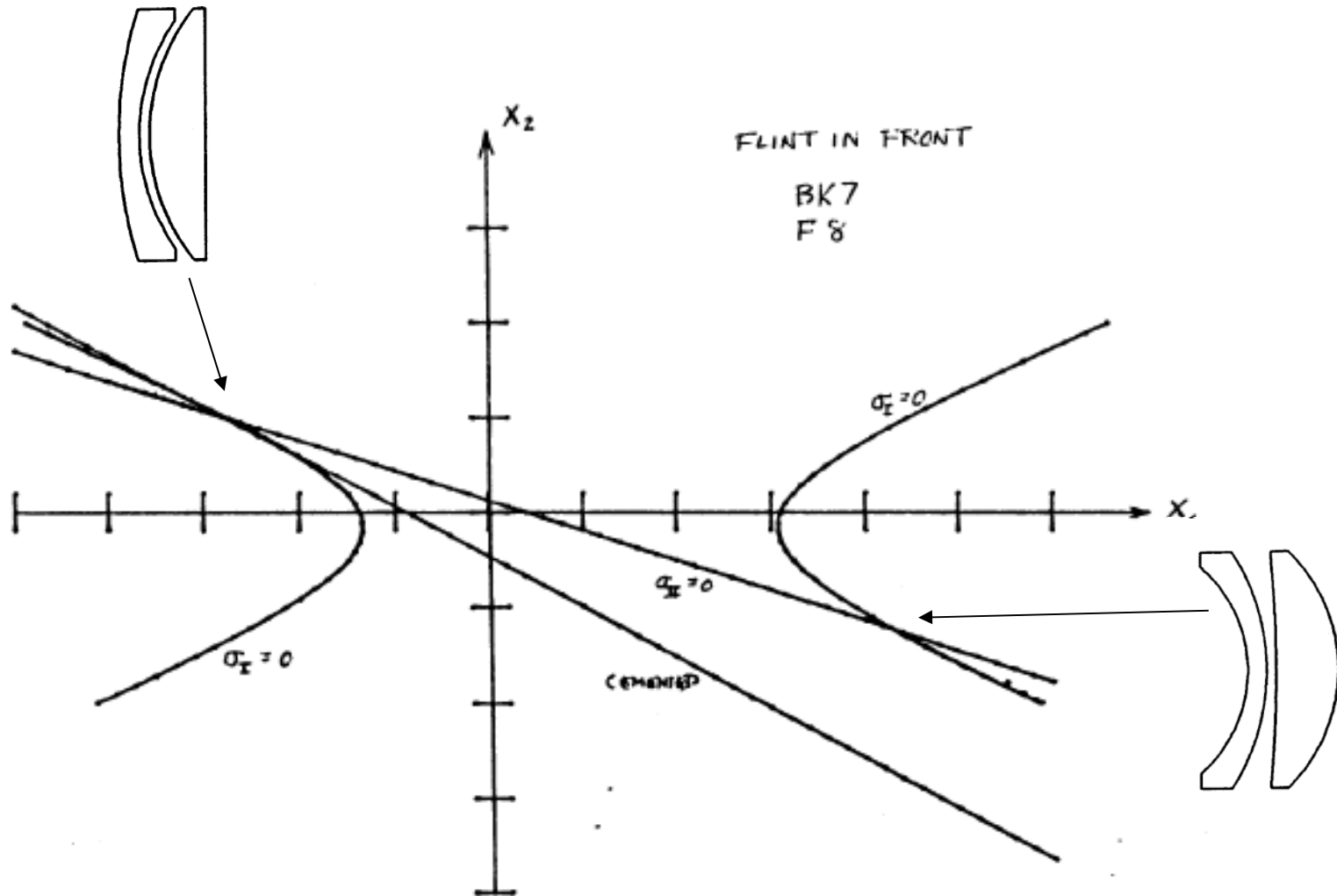
For achromat

For a cemented achromatic lens the graph of  $X_1$  and  $X_2$  is a straight line.

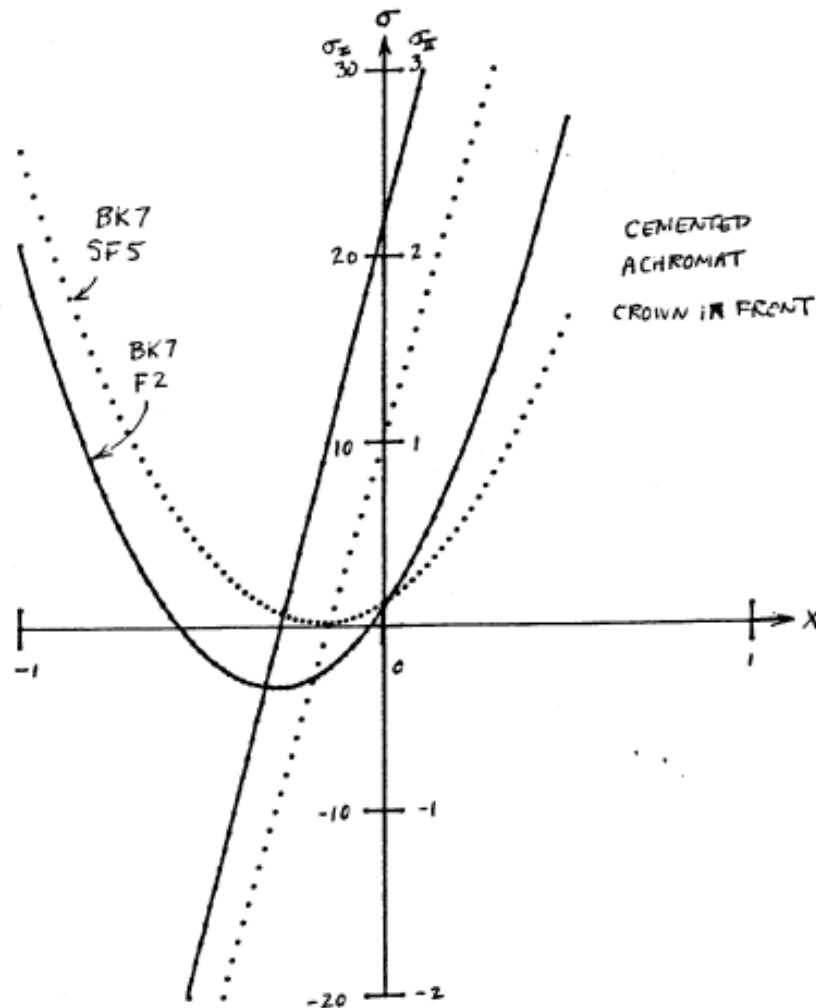
# Crown in front: BK7 and F8



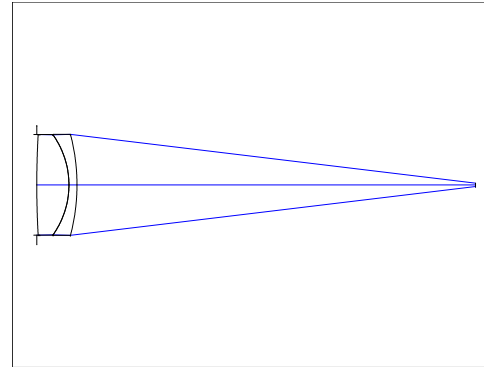
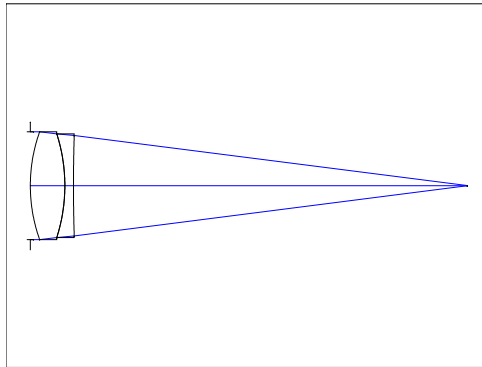
# Flint in front: BK7 and F8



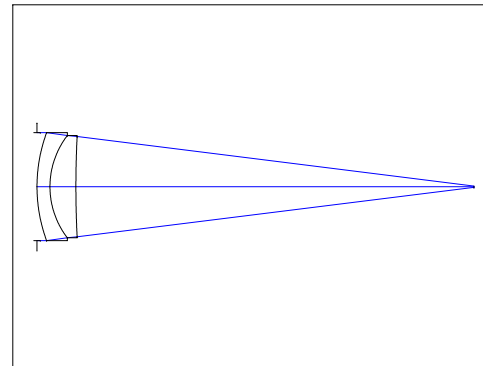
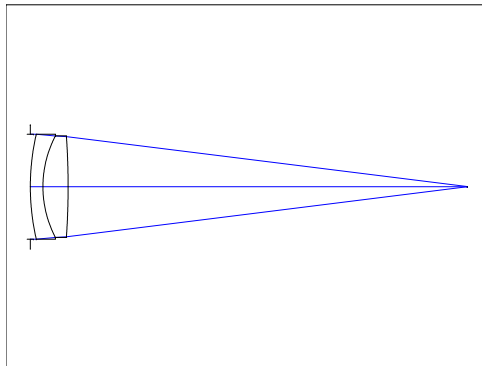
# Cemented achromatic doublet



# Cemented doublet solutions

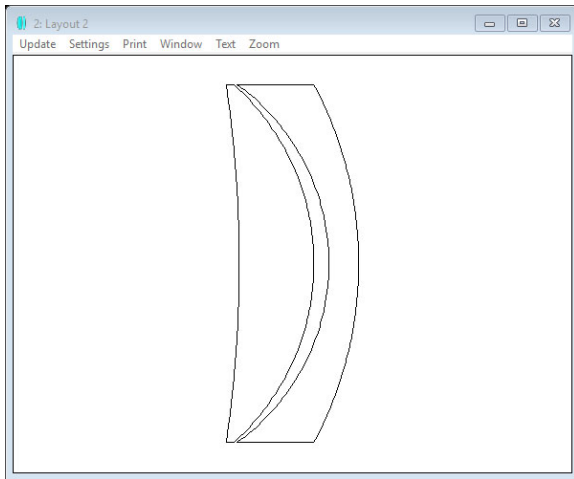


Crown in front

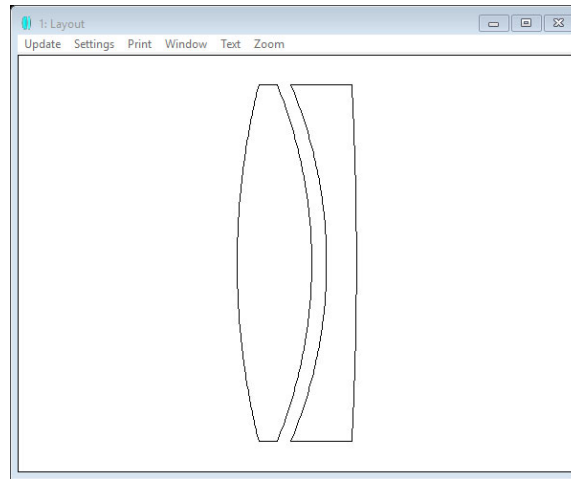


Flint in front

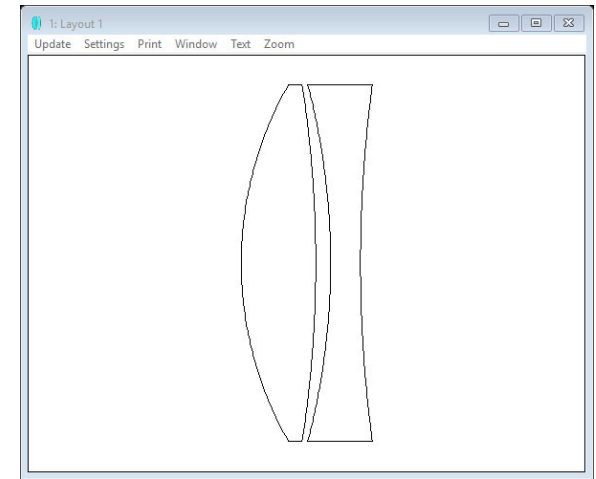
# Doublets with no spherical aberration but with varying coma



Coma=-10 waves



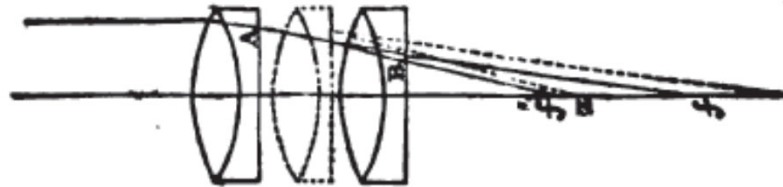
Coma=0



Coma=10 waves

Crown in front, no spherical aberration,  $F/5$ ,  $f=100$  mm, no chromatic aberration

# Lister objective



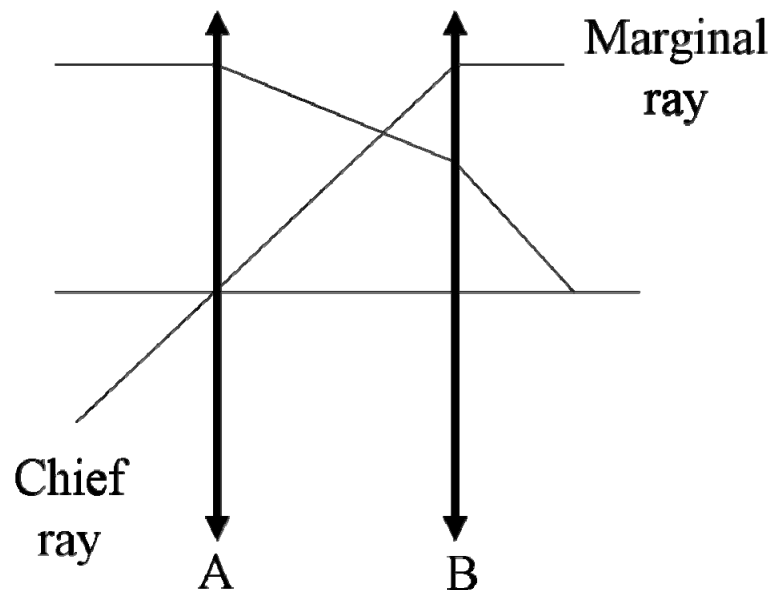
**XIII.** *On some properties in achromatic object-glasses applicable to the improvement of the microscope.* By **JOSEPH JACKSON LISTER, Esq.** Communicated by **Dr. ROGET, Secretary.**

**Read January 21, 1830.**



# Lister objective

- Two achromatic doublets that are spaced
- Telecentric in image space
- Normalized system



$$\mathcal{K} = 1$$

$$y_A = 1$$

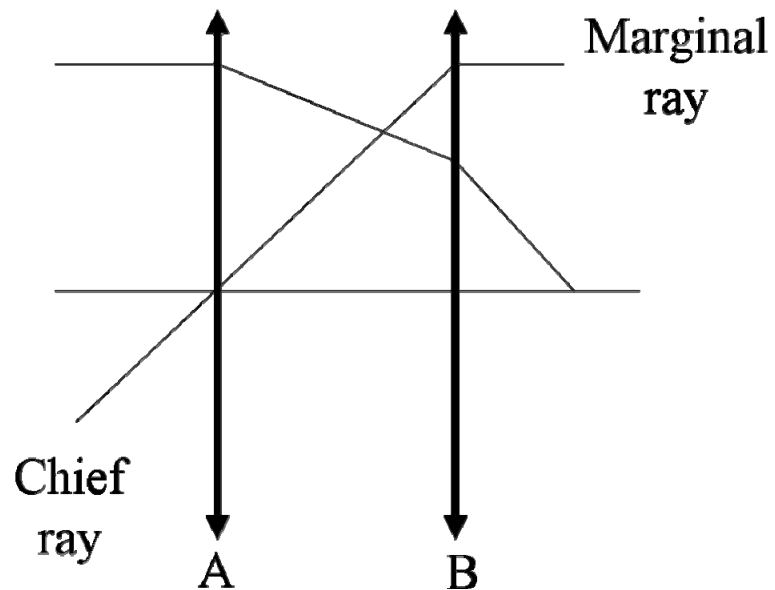
$$\bar{u}_A = 1$$

$$\sigma_{IA} = 0$$

$$\sigma_{IB} = 0$$

# Lister objective

The aperture stop is at the first lens.  
The system is telecentric



$$\phi_B = 1$$

$$\phi_A = 1 - y_B$$

$$\bar{y}_B = 1$$

# Lister objective

Seidel sums in terms of structural aberration coefficients
Pupils located at principal planes
$S_I = \frac{1}{4} y_P^4 \Phi^3 \sigma_I$
$S_{II} = \frac{1}{2} \mathcal{K} y_P^2 \Phi^2 \sigma_{II}$
$S_{III} = \mathcal{K}^2 \Phi \sigma_{III}$
$S_{IV} = \mathcal{K}^2 \Phi \sigma_{IV}$
$S_V = \frac{2 \mathcal{K}^3 \sigma_V}{y_P^2}$
$C_L = y_P^2 \Phi \sigma_L$
$C_T = 2 \mathcal{K} \sigma_T$

Stop shifting from principal planes
$\sigma_I^* = \sigma_I$
$\sigma_{II}^* = \sigma_{II} + \bar{S}_\sigma \sigma_I$
$\sigma_{III}^* = \sigma_{III} + 2 \bar{S}_\sigma \sigma_{II} + \bar{S}_\sigma^2 \sigma_I$
$\sigma_{IV}^* = \sigma_{IV}$
$\sigma_V^* = \sigma_V + \bar{S}_\sigma (\sigma_{IV} + 3 \sigma_{III}) + 3 \bar{S}_\sigma^2 \sigma_{II} + \bar{S}_\sigma^3 \sigma_I$
$\sigma_L^* = \sigma_L$
$\sigma_T^* = \sigma_T + \bar{S}_\sigma \sigma_L$
$\bar{S}_\sigma = \frac{y_P \bar{y}_P \Phi}{2 \mathcal{K}}$
$\Delta \bar{S}_\sigma = \frac{y_P \Delta \bar{y}_P \Phi}{2 \mathcal{K}} = \frac{y_\sigma^2 \Phi}{2 \mathcal{K}} \bar{S}$

# Condition for zero coma

$$\sigma_{II} = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right)^2 \left( \frac{y_{P,k}}{y_P} \right)^2 (\sigma_{II,k} + \bar{S}_k \sigma_{I,k})$$

$$\sigma_{II} = \phi_A^2 y_A^2 \sigma_{IIA} + \phi_B^2 y_B^2 \sigma_{IIB}$$

$$\sigma_{IB} = 0$$

$$\sigma_{II} = (1 - y_B)^2 \sigma_{IIA} + y_B^2 \sigma_{IIB} = 0$$

$$\sigma_{IIA} = -\frac{y_B^2}{(1 - y_B)^2} \sigma_{IIB}$$

# Condition for zero astigmatism

$$\sigma_{III} = \sum_{i=0}^k \left( \frac{\Phi_k}{\Phi} \right) (\sigma_{III,k} + 2\bar{S}_k \sigma_{II,k} + \bar{S}_k^2 \sigma_{I,k})$$

$$\sigma_{III} = \phi_A + \phi_B (1 + y_B \sigma_{IIB}) = 0$$

$$\bar{S}_k = \frac{\Phi_k \cdot y_{P,k} \cdot \bar{y}_{P,k}}{2\mathcal{K}}$$

$$\sigma_{III} = (1 - y_B) + (1 + y_B \sigma_{IIB}) = 0$$

$$2 - y_B + y_B \sigma_{IIB} = 0$$

$$y_B = \frac{2}{1 - \sigma_{IIB}}$$

$$\sigma_{IIB} = -\frac{2 - y_B}{y_B}$$

$$\sigma_{IIA} = \frac{y_B (2 - y_B)}{(1 - y_B)^2}$$

# Lister Objective

Choose:

$$\sigma_{IIB} = -\sigma_{IIA}$$

$$-\frac{2 - y_B}{y_B} = -\frac{y_B(2 - y_B)}{(1 - y_B)^2}$$

$$(1 - y_B)^2 = y_B^2$$

$$1 - 2y_B + y_B^2 = y_B^2$$

$$y_B = \frac{1}{2}$$

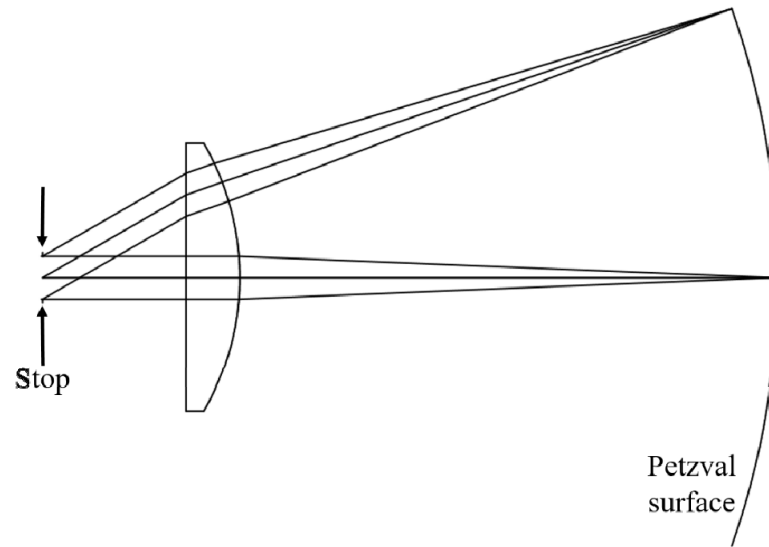
$$\varphi_A = \frac{1}{2}$$

$$\sigma_{IIA} = 3$$

$$\sigma_{IIB} = -3$$

Seidel sums in terms of structural aberration coefficients
Pupils located at principal planes
$S_I = \frac{1}{4} y_P^4 \Phi^3 \sigma_I$
$S_{II} = \frac{1}{2} \mathcal{K} y_P^2 \Phi^2 \sigma_{II}$
$S_{III} = \mathcal{K}^2 \Phi \sigma_{III}$
$S_{IV} = \mathcal{K}^2 \Phi \sigma_{IV}$
$S_V = \frac{2 \mathcal{K}^3 \sigma_V}{y_P^2}$
$C_L = y_P^2 \Phi \sigma_L$
$C_T = 2 \mathcal{K} \sigma_T$

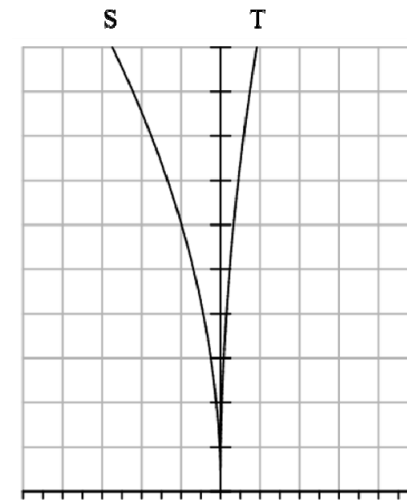
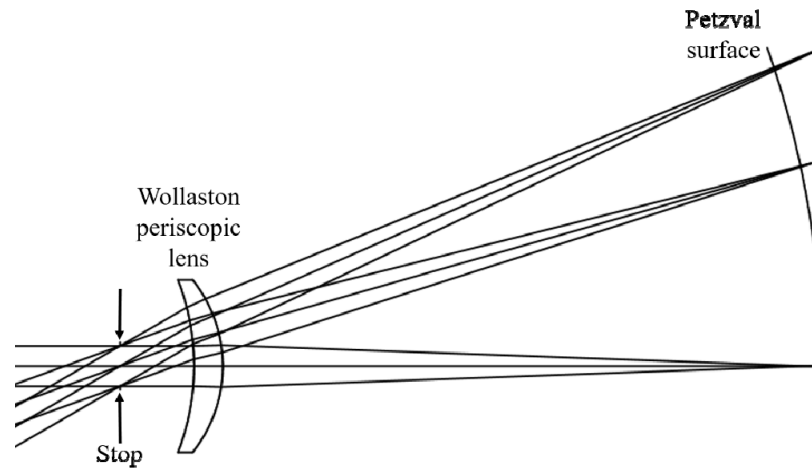
# Plano convex lens



N-BK7 : Petzval radius -151.7 mm

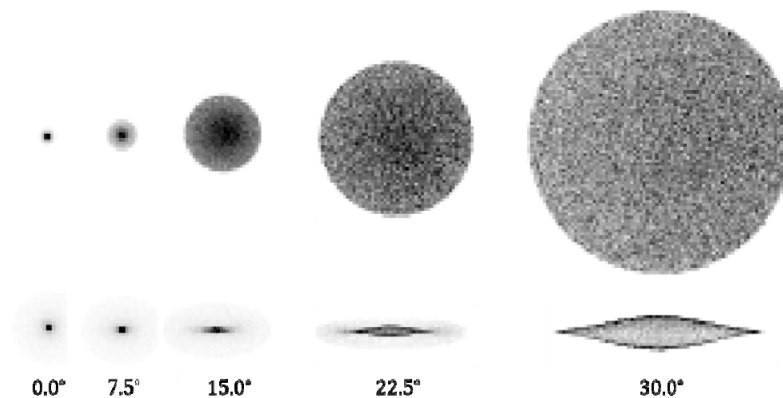
$$C_{Petzval} = \frac{1}{\rho_{Petzval}} = -\phi \cdot \sigma_{IV} = -\left(\frac{\phi}{n}\right)$$

# Wollaston meniscus lens



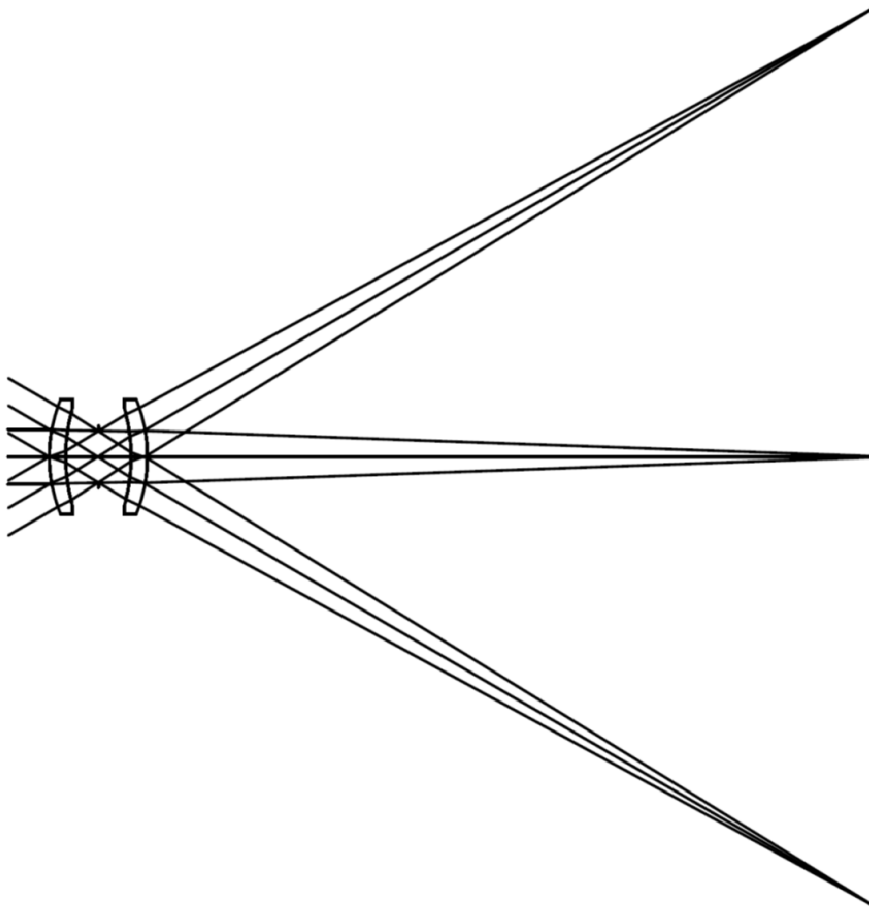
$$W_{222} / W_{220P} = -0.8$$

- Artificially flattening the field
- Periscopic lenses



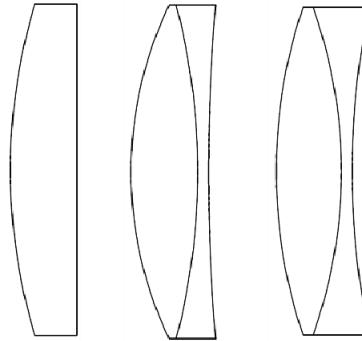


# Periskop lens



- Principle of symmetry
- No distortion

# Field curvature



- Old achromat
- New achromat

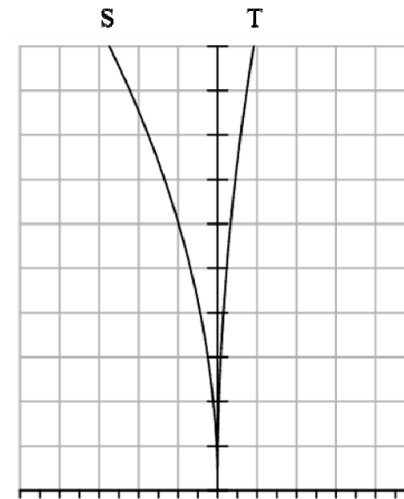
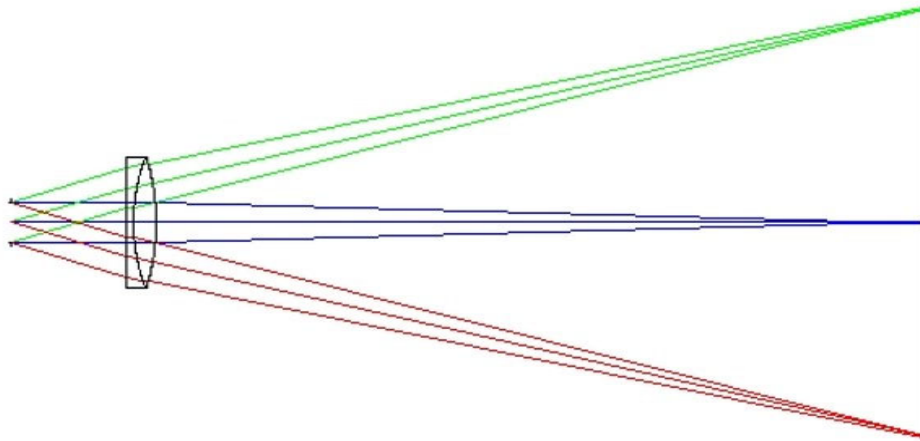
N-BK7 : Petzval radius -151.7 mm

N-BK7 and N-F2: Petzval radius -139.99 mm (+139.99 for negative doublet)

N-BAK1 and N-LLF6: Petzval radius -185 mm

$$C_{\text{Petzval}} = \frac{1}{\rho_{\text{Petzval}}} = -\phi \cdot \sigma_{IV} = -\left(\frac{\phi_1}{n_1} + \frac{\phi_2}{n_2}\right) = -\frac{\phi}{v_1 - v_2} \left(\frac{v_1}{n_1} - \frac{v_2}{n_2}\right)$$

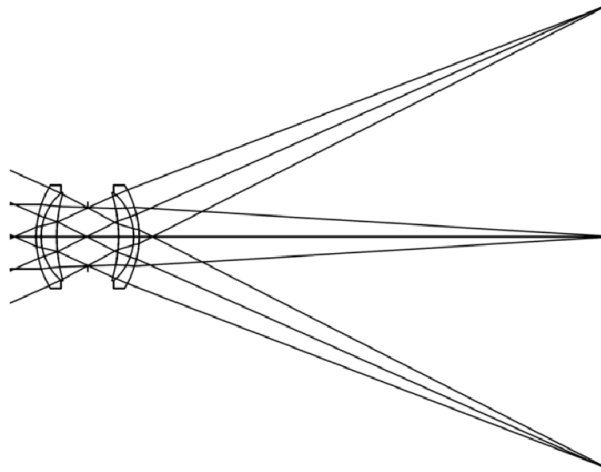
# Chevalier landscape lens



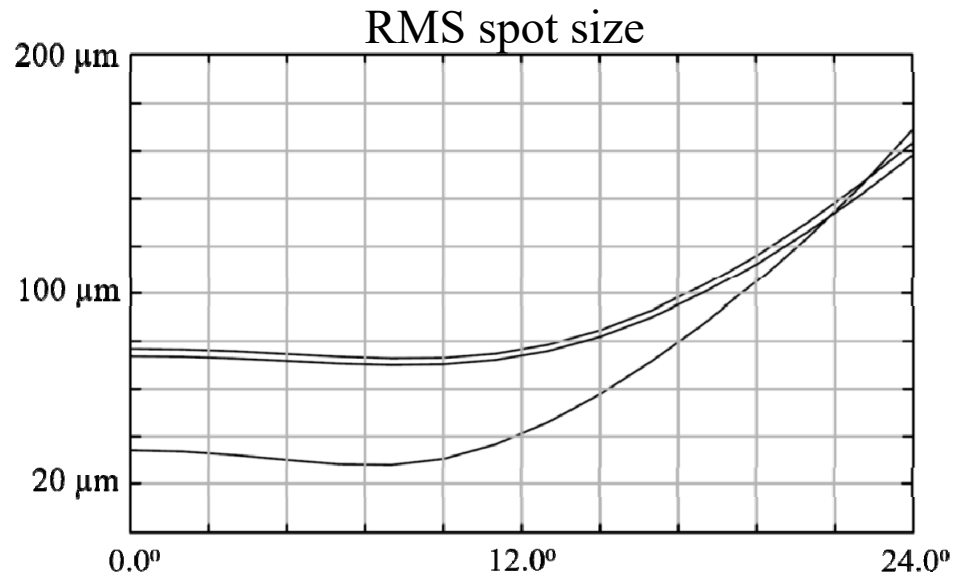
$$W_{222} / W_{220P} = -0.8$$

- F/5 telescope doublet used in reverse and with an aperture stop in front

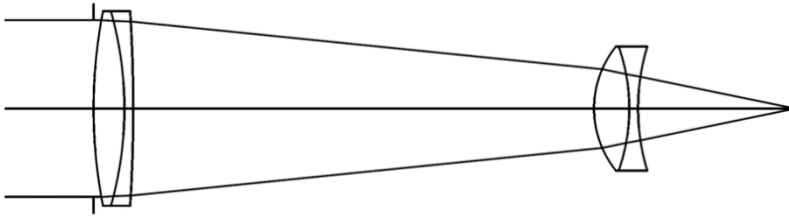
# Rapid rectilinear



- F/8
- Glass selection is key to minimize spherical aberration while artificially flattening the field



# Lister microscope objective



- Telecentric

$$\sigma_{IA} = \sigma_{IB} = 0$$

$$y_B = 1/2$$

$$\sigma_{II} = \phi_A^2 y_A^2 \sigma_{IIA} + \phi_B^2 y_B^2 \sigma_{IIB}$$

$$\sigma_{IIA} = -\sigma_{IIB}$$

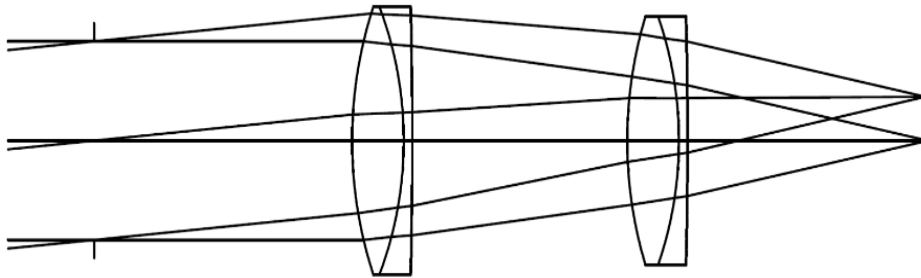
$$\sigma_{IIA} = -\frac{y_B^2}{(1-y_B)^2} \sigma_{IIB}$$

$$\sigma_{III} = (1-y_B) + (1+y_B \sigma_{IIB}) = 0$$

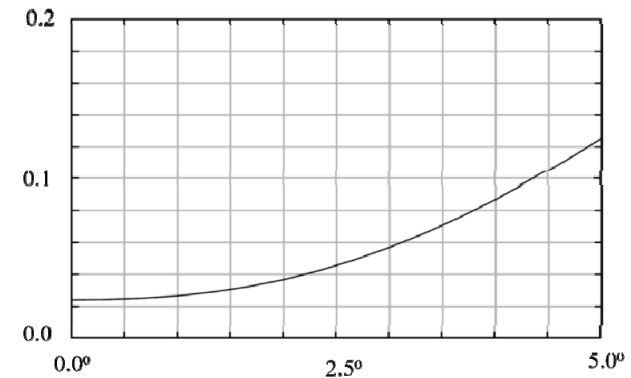
$$S_{III}^* = S_{III} + 2 \cdot \bar{S} S_{II} + \bar{S}^2 S_I$$

# Lister microscope objective

Practical solution

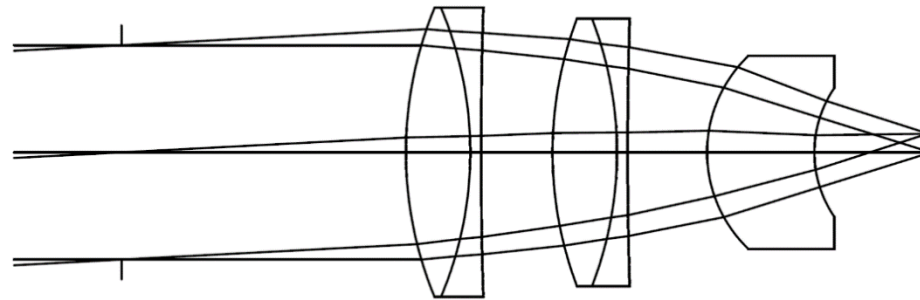


- RMS wavefront error in waves



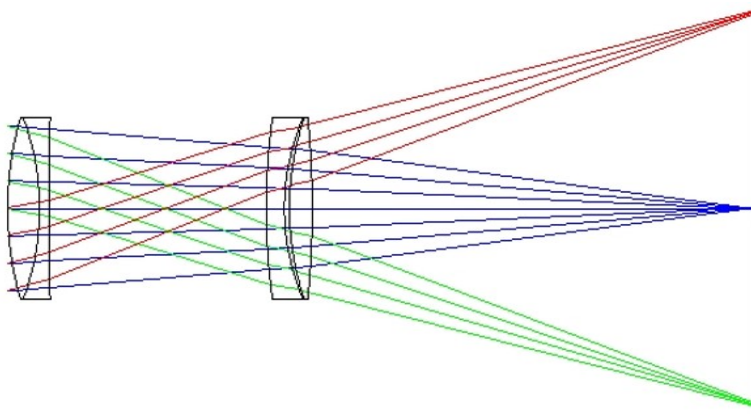
- Two identical doublets
- Spherical aberration and coma are corrected
- Astigmatism is small
- Telecentric
- Less vignetting

# Aplanatic concentric meniscus lens

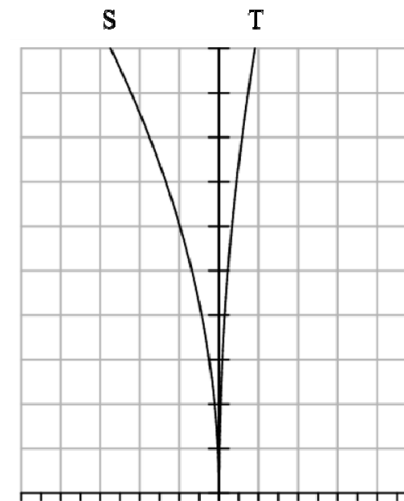


- Optical speed is increased by an N factor

# Petzval portrait objective



$f' = 144 \text{ mm}$ ;  $F/3.7$ ;  $\text{FOV} = \pm 16.5^\circ$ .



$$W_{222} / W_{220P} = -0.8$$

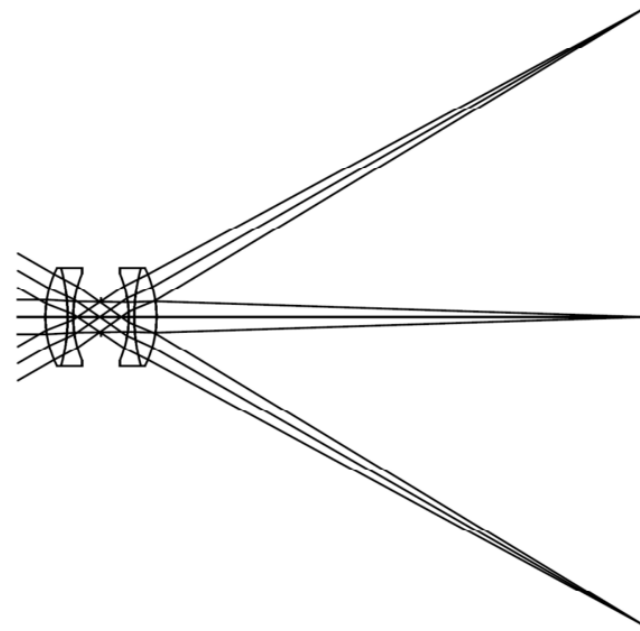
- Chromatic aberration and spherical aberration corrected at each doublet
- Positive coma in the first doublet corrected with negative coma of aberration of the second doublet
- Negative astigmatism introduced by the negative coma of the second doublet to artificially flatten the field of view.

$$S_{III}^* = S_{III} + 2 \cdot \bar{S} S_{II} + \bar{S}^2 S_I$$



# Concentric lens

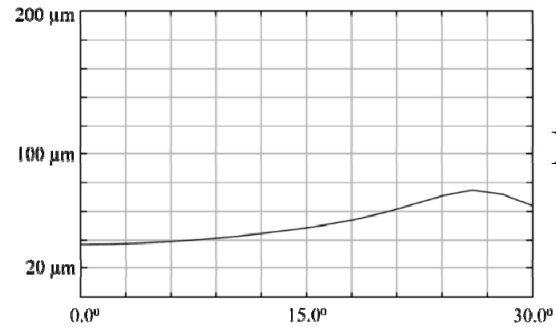
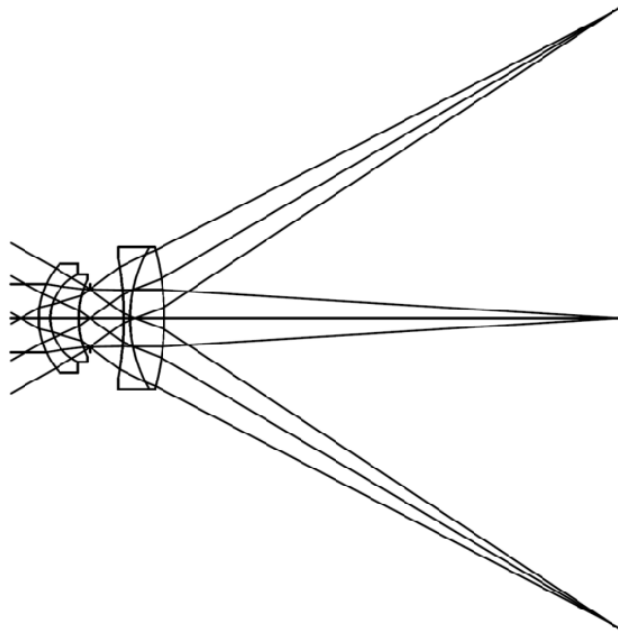
- Use of new glasses
- Reduced Petzval sum
- Nearly flat field
- Surfaces nearly concentric
- Limited by spherical aberration due to strong curvatures.



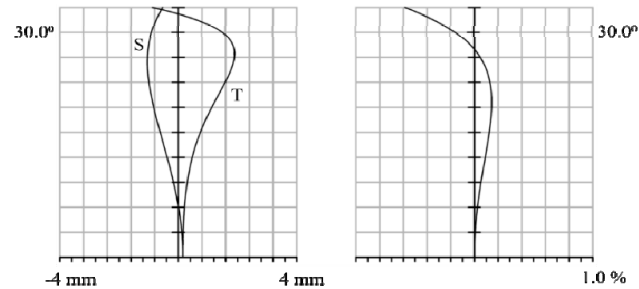
N-BAK1 and N-LLF6: Petzval radius -185 mm

$$C_{\text{Petzval}} = \frac{1}{\rho_{\text{Petzval}}} = -\phi \cdot \sigma_{IV} = -\left(\frac{\phi_1}{n_1} + \frac{\phi_2}{n_2}\right) = -\frac{\phi}{v_1 - v_2} \left(\frac{v_1}{n_1} - \frac{v_2}{n_2}\right)$$

# Anastigmatic lens

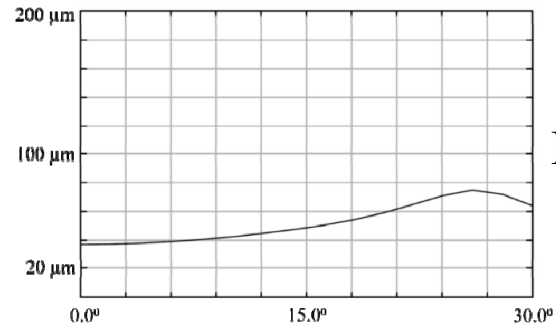
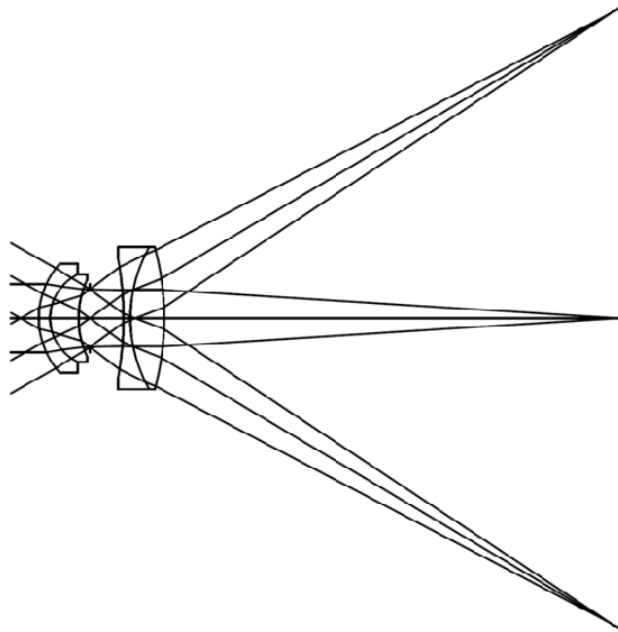


RMS spot size

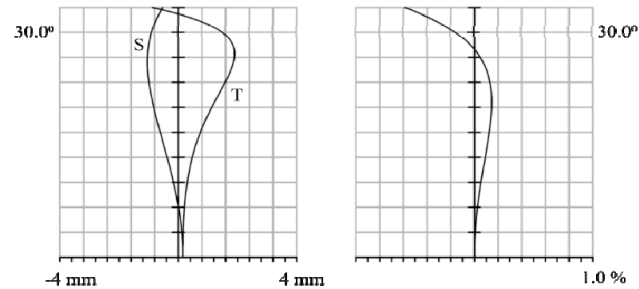


- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is negligible
- Combination of an old achromat and a new achromat

# Anastigmatic lens



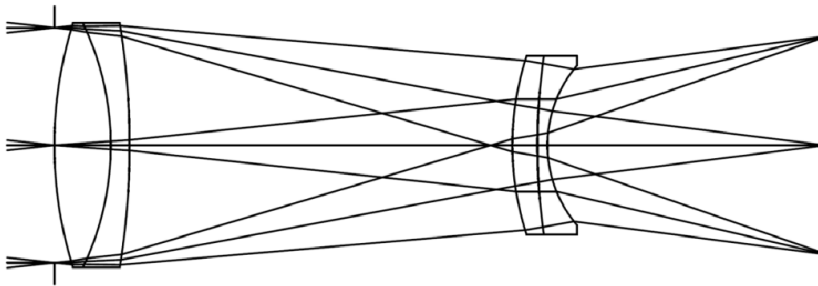
RMS spot size



- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is negligible
- Combination of an old achromat and a new achromat

# Telephoto lens

Telephoto lens with BK7 and SF5 glasses.  
 $f^*=100$  mm, F/4, FOV= $\pm 6.2^\circ$ , TTL/F=0.8.



$$S_{III}^* = S_{III} + 2 \cdot \bar{S} S_{II} + \bar{S}^2 S_I$$

$$S_{III}^* = S_{III} = \mathcal{K}^2 \phi_B \sigma_{III B} = \mathcal{K}^2 \phi_B$$

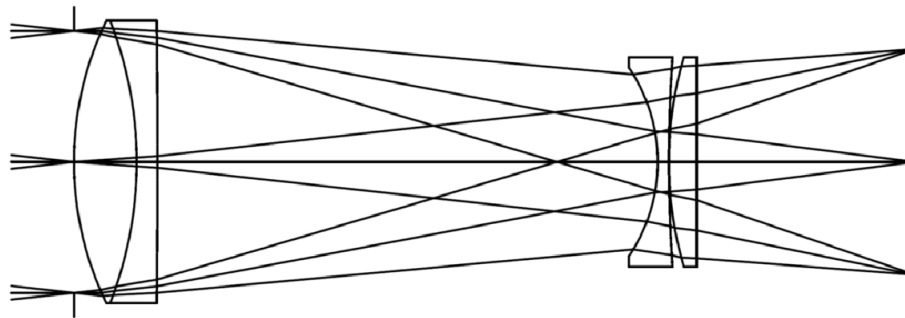
$$\phi_A = -\phi_B$$

$$\bar{W}_{131} = W_{311} + \frac{1}{2} \mathcal{K} \cdot \Delta \left\{ u^{-2} \right\}$$

- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is not corrected
- Telephoto ratio=TTL/f

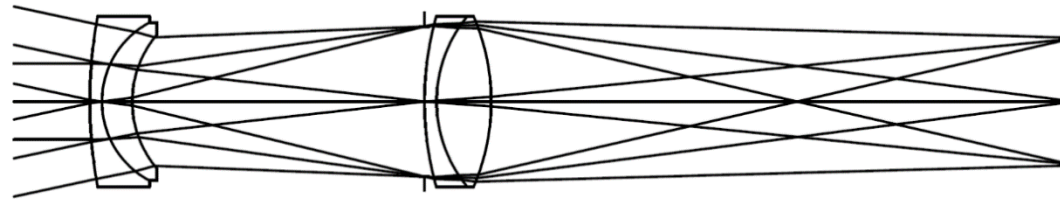
# Telephoto lens

Telephoto lens with BK7 and F6 glasses.  
 $f' = 100$  mm,  $F/4$ ,  $FOV = \pm 6.2^\circ$ ,  $TTL/F = 0.8$



- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is also corrected

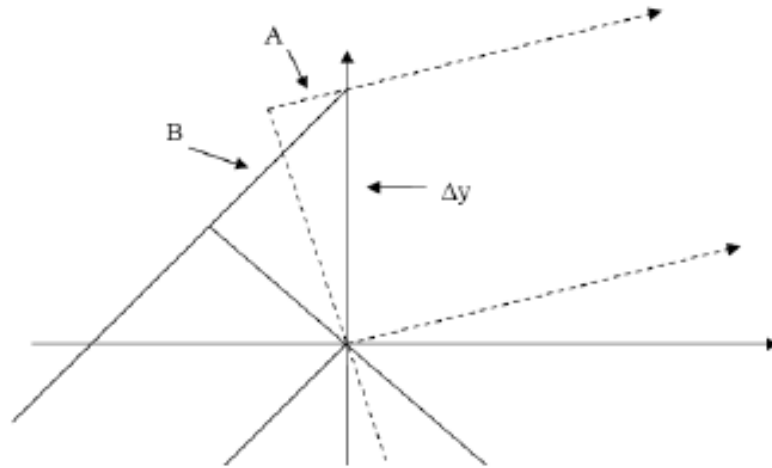
# Reverse telephoto lens



Reverse telephoto lens with BK7 and SF5 glasses.  $f' = 100$  mm, BFL = 200 mm, TTL = 324 mm, FOV =  $\pm 12^\circ$ , F/4

- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is small  $\sim -1.5\%$
- Large back focal length/distance

# Ray diffractive law (1D)



$$n' \sin(I') \cdot \Delta y = n \sin(I) \cdot \Delta y$$

$$n' \sin(I') \cdot \Delta y - n \sin(I) \cdot \Delta y = \Delta \phi(y)$$

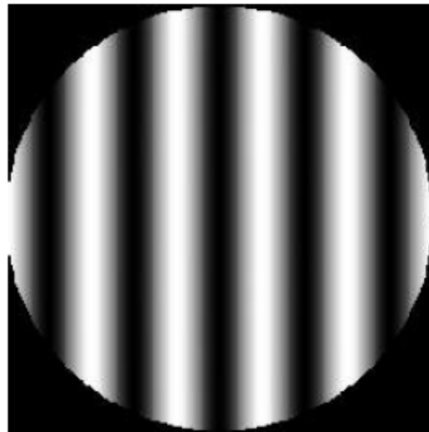
$$n' \sin(I') - n \sin(I) = \frac{\Delta \phi(y)}{\Delta y} \rightarrow \frac{\partial \phi(y)}{\partial y}$$

$$\frac{\partial \phi(y)}{\partial y} = n' \sin(I') - n \sin(I)$$

# Grating linear phase change

$$n \sin(I') - n \sin(I) = \frac{m\lambda}{d}$$

$$\phi(y) = n \sin(I') y - y n \sin(I) = \frac{m\lambda}{d} y$$





# Diffraction optics high-index model

Start with the diffraction grating equation

$$n' \sin(I') - n \sin(I) = [n' \cos(I') - n \cos(I)] \cdot \frac{m\lambda}{n' \cos(I') - n \cos(I)} \cdot 1/d$$

$$n' \sin(I') - n \sin(I) = [n' \cos(I') - n \cos(I)] \cdot \tan(\alpha)$$

$$n' \{ \sin(I') - \cos(I') \tan(\alpha) \} = n \{ \sin(I) - \cos(I) \tan(\alpha) \}$$

$$n' \{ \cos(\alpha) \sin(I') - \cos(I') \sin(\alpha) \} = n \{ \cos(\alpha) \sin(I) - \cos(I) \sin(\alpha) \}$$

$$n' \{ \sin(I' - \alpha) \} = n \{ \sin(I - \alpha) \}$$

# Diffraction optics high-index model

$$n' \{ \sin(I' - \alpha) \} = n \{ \sin(I - \alpha) \}$$

$$\tan(\alpha) = \frac{m\lambda}{n' \cos(I') - n \cos(I)} \quad 1/d$$

For large  $n$ 's then  $\alpha$  is negligible and we have:

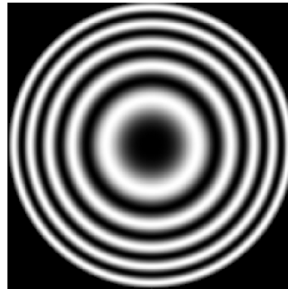
$$n' \sin(I) = n \sin(I)$$

Thus for high index diffraction becomes like refraction!

# Diffractive lens

(n very large @ X=0)

Structural aberration coefficients of a thin lens (Stop at lens)	
Paraxial identities	
$\phi = (n'-n) \cdot (c_1 - c_2) = (n'-n) \cdot \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$	
$X = \frac{c_1 + c_2}{c_1 - c_2} = -\frac{R_1 + R_2}{R_1 - R_2}$	$Y = \frac{w'+w}{w'-w} = \frac{1+m}{1-m}$
$c_1 = \frac{1}{2} \frac{\phi}{n-1} (X+1)$	$c_2 = \frac{1}{2} \frac{\phi}{n-1} (X-1)$
$w = u = -\frac{1}{2} (Y-1)(\phi \cdot y)$	$w' = u' = -\frac{1}{2} (Y+1)(\phi \cdot y)$
Structural aberration coefficients	
$\sigma_I = AX^2 - BXY + CY^2 + D$	$A = \frac{n+2}{n(n-1)^2}$
$\sigma_{II} = EX - FY$	$B = \frac{4(n+1)}{n(n-1)}$
$\sigma_{III} = 1$	$C = \frac{3n+2}{n}$
$\sigma_{IV} = \frac{1}{n}$	$D = \frac{n^2}{(n-1)^2}$
$\sigma_V = 0$	$E = \frac{n+1}{n(n-1)}$
$\sigma_L = \frac{1}{v}$	$F = \frac{2n+1}{n}$
$\sigma_T = 0$	



A=0  
B=0  
C=3  
D=1  
E=0  
F=2

$$\sigma_I = 3Y^2 + 1$$

$$\sigma_{II} = -2Y$$

$$\sigma_{III} = 1$$

$$\sigma_{IV} = 0$$

$$\sigma_V = 0$$

$$\sigma_L = \frac{1}{v_{\text{diffractive}}}$$

$$\sigma_T = 0$$



# Mirror Systems

Structural aberration coefficients of a mirror. $K = -\varepsilon^2$ is the conic constant and $\varepsilon$ is the eccentricity.	
Stop at surface	With stop shift
$\sigma_I = Y^2 + K$	$\sigma_I = Y^2 + K$
$\sigma_{II} = -Y$	$\sigma_{II} = -Y(1 - \bar{S}_\sigma Y) + \bar{S}_\sigma \cdot K$
$\sigma_{III} = 1$	$\sigma_{III} = (1 - \bar{S}_\sigma Y)^2 + \bar{S}_\sigma^2 \cdot K$
$\sigma_{IV} = -1$	$\sigma_{IV} = -1$
$\sigma_V = 0$	$\sigma_V = \bar{S}_\sigma \cdot (1 - \bar{S}_\sigma Y) \cdot (2 - \bar{S}_\sigma Y) + \bar{S}_\sigma^3 \cdot K$

$$\bar{S}_\sigma = \frac{y_P \bar{y}_P \varphi}{2\mathcal{K}} = \frac{\varphi \cdot s}{(Y-1) \cdot \varphi \cdot s - 2n} = \frac{\varphi \cdot s'}{(Y+1) \cdot \varphi \cdot s' - 2n'}$$

# Two mirror afocal system

## Application to a two mirror Mersenne system

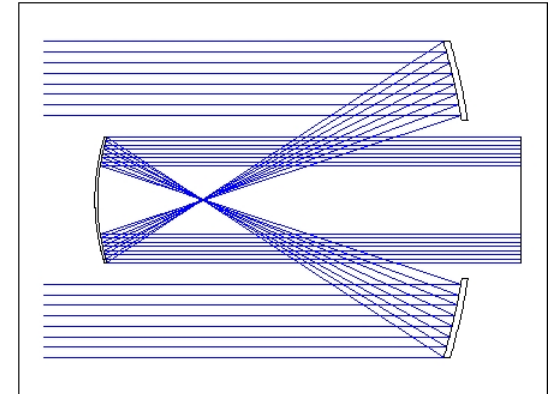
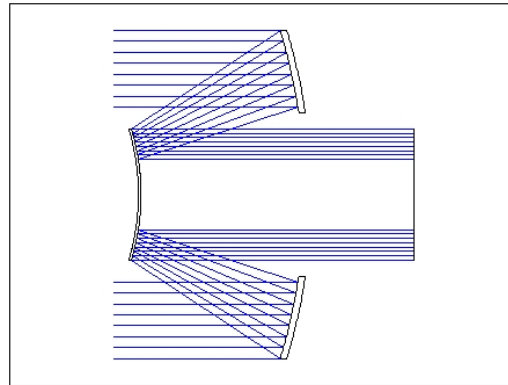
In this section we determine the aberration coefficients of a two mirror afocal system as shown in the figure. We normalize the system parameters and set  $\mathcal{K}=1$ ,  $\Phi_1=1$ ,  $y_1=1$ ,  $\bar{y}_1=0$  and set the magnification to be  $m$  and therefore  $y_2=m$ . We have that  $\bar{y}_2=1-m$ ,  $\Phi_2=-1/m$  and therefore we can write for the conjugate factors and stop shifting parameters,

$$Y_1 = 1$$

$$Y_2 = -1$$

$$\bar{S}_1 = 0$$

$$\bar{S}_2 = \frac{y_2 \bar{y}_2 \Phi_2}{2\mathcal{K}} = \frac{m-1}{2}.$$



Using the formulas in the table the structural coefficients of each mirror are calculated as:

Structural aberration coefficients		
	Mirror 1	Mirror 2
$\sigma_I$	$1 + \alpha_1$	$1 + \alpha_2$
$\sigma_{II}$	-1	$\frac{m+1}{2} + \frac{m-1}{2} \alpha_2$
$\sigma_{III}$	1	$\left(\frac{m+1}{2}\right)^2 + \left(\frac{m-1}{2}\right)^2 \alpha_2$
$\sigma_{IV}$	-1	-1
$\sigma_V$	0	$\frac{m-1}{2} \frac{m+1}{2} \frac{m+3}{2} + \left(\frac{m-1}{2}\right)^3 \alpha_2$

Finally the Seidel sums for the two mirror afocal system are given by:

Seidel sums for two mirror afocal system
$S_I = \frac{1}{4} \sigma_{I1} + \frac{1}{4} m^4 \left( -\frac{1}{m} \right)^3 \sigma_{I2} = \frac{1}{4} ((1 + \alpha_1) - m(1 + \alpha_2))$
$S_{II} = \frac{1}{2} \sigma_{II1} + \frac{1}{2} m^2 \left( -\frac{1}{m} \right)^2 \sigma_{II2} = \frac{1}{4} (m - 1)(1 + \alpha_2)$
$S_{III} = \sigma_{III1} + \left( -\frac{1}{m} \right) \sigma_{III2} = -\frac{1}{4} \frac{(m - 1)^2}{m} (1 + \alpha_2)$
$S_{IV} = \sigma_{IV1} + \left( -\frac{1}{m} \right) \sigma_{IV2} = -\frac{m - 1}{m}$
$S_V = 2\sigma_{V1} + 2 \left( \frac{1}{m} \right)^2 \sigma_{V2} = \frac{1}{4} \frac{m - 1}{m^2} (8 + 6(m - 1) + (m - 1)^2 (1 + \alpha_2))$

For the particular case of having the mirror as parabolic in optical shape we have that the Seidel sums simplify as:

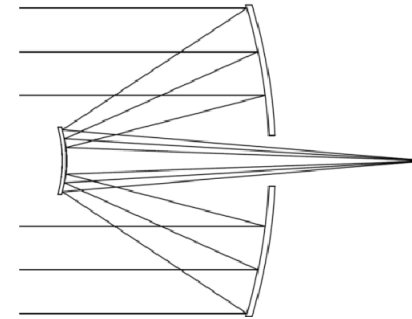
Seidel sums for afocal system using parabolas
$S_I = 0$
$S_{II} = 0$
$S_{III} = 0$
$S_{IV} = -\frac{m-1}{m}$
$S_V = \frac{1}{2} \frac{m-1}{m^2} (3m+1)$

When a system is free from spherical aberration, coma, and astigmatism it is called an anastigmatic system.

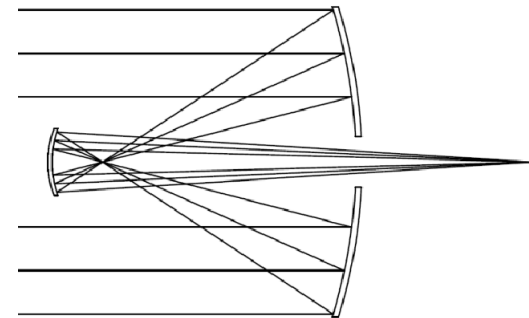


# Two mirror systems

$$\begin{aligned} \varphi &= 1 & L &= \frac{\bar{y}_2}{y_2} \\ y_P &= 1 & M &= \frac{1 - y_2}{\bar{y}_2} \\ \mathcal{K} &= 1 \end{aligned}$$



$$\begin{aligned} \varphi_1 / \varphi &= 1 & \varphi_2 / \varphi &= (1 - M)(1 + ML) \\ y_1 / y_P &= 1 & y_2 / y_P &= \frac{1}{1 + ML} \\ \bar{S}_{\sigma 1} &= 0 & \bar{S}_{\sigma 2} &= \frac{1}{2} \frac{(1 - ML)}{1 + ML} \\ Y_1 &= 0 & Y_2 &= \frac{1 + M}{1 - M} \end{aligned}$$



# Two mirror systems

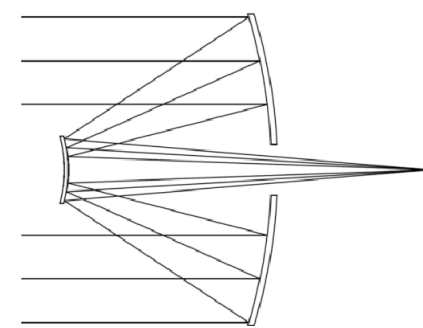
Structural coefficients of a two mirror system.  
Stop at primary mirror.  
Object at infinity;  $m$  is the transverse magnification of the secondary mirror, and  $L$  is the ratio of the mirror separation to the back focal distance.

$$\sigma_I = m^3(1 + K_1) + \frac{(1 - m)^3}{1 + mL} \left( \frac{1 + m}{1 - m} + K_2 \right)$$

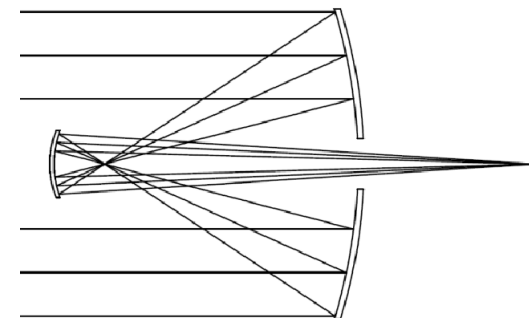
$$\sigma_{II} = -m^2 + (1 - m)^2 \left( -\frac{1 + m}{1 - m} \left( 1 - \frac{1}{2} \frac{(1 - m)L}{1 + mL} \frac{1 + m}{1 - m} \right) + \frac{1}{2} \frac{(1 - m)L}{1 + mL} K_2 \right)$$

$$\sigma_{III} = -1 + (1 - m)(1 + mL) \left( \left( 1 - \frac{1}{2} \frac{(1 - m)L}{1 + mL} \frac{1 + m}{1 - m} \right)^2 + \left( \frac{1}{2} \frac{(1 - m)L}{1 + mL} \right)^2 K_2 \right)$$

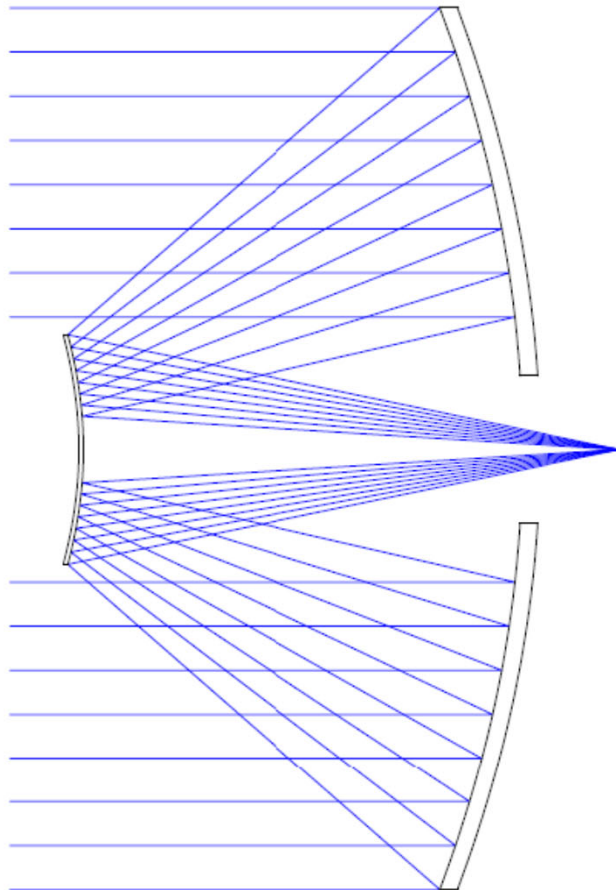
$$\sigma_{IV} = -m - (1 - m)(1 + mL)$$

$$\sigma_V = \frac{1}{(1 + mL)^2} \left( \frac{1}{2} \frac{(1 - m)L}{1 + mL} \right) \left( 1 - \frac{1}{2} \frac{(1 - m)L}{1 + mL} \frac{1 + m}{1 - m} \right) \left( 2 - \frac{1}{2} \frac{(1 - m)L}{1 + mL} \frac{1 + m}{1 - m} \right) + \left( \frac{1}{2} \frac{(1 - m)L}{1 + mL} \right)^3 K_2$$


Conic constants of Cassegrain type configurations corrected for spherical aberration		
Configuration	Primary mirror	Secondary mirror
Cassegrain	$K_1 = -1$	$K_2 = -\left(\frac{1 + m}{1 - m}\right)^2$
Dall-Kirkham	$K_1 = -1 - \frac{(1 - m)(1 + m)^2}{m^3(1 + mL)}$	$K_2 = 0$
Pressman-Carmichel	$K_1 = 0$	$K_2 = -\left(\frac{1 + m}{1 - m}\right)^2 - \frac{m^3(1 + mL)}{(1 - m)^3}$
Ritchey-Chretien (aplanatic)	$K_1 = -1 - \frac{2}{Lm^3}$	$K_2 = -\left(\frac{1 + m}{1 - m}\right)^2 - \frac{2(1 + mL)}{L(1 - m)^3}$

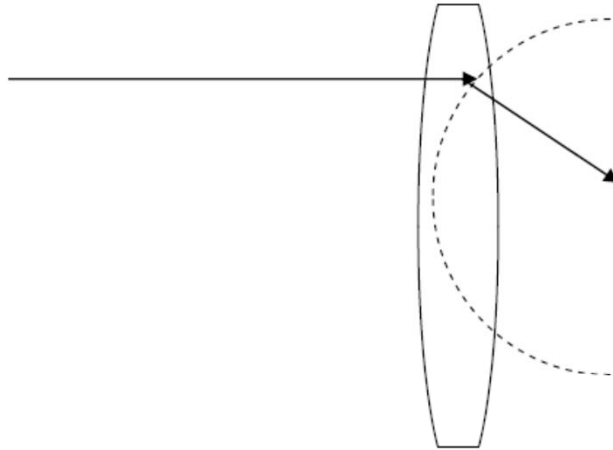


# Cassegrain type

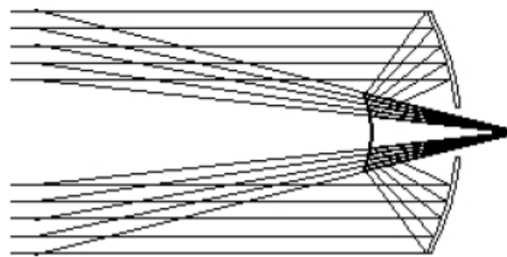
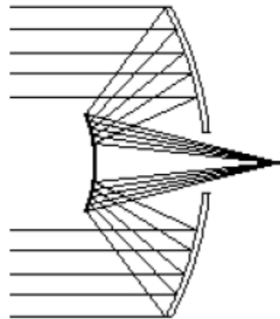


- True Cassegrain
- Ritchey-Chretien: aplanatic
- Dall-Kirkham: spherical secondary
- Pressman-Camichel; spherical primary
- Olivier Guyon (no diffraction rings)

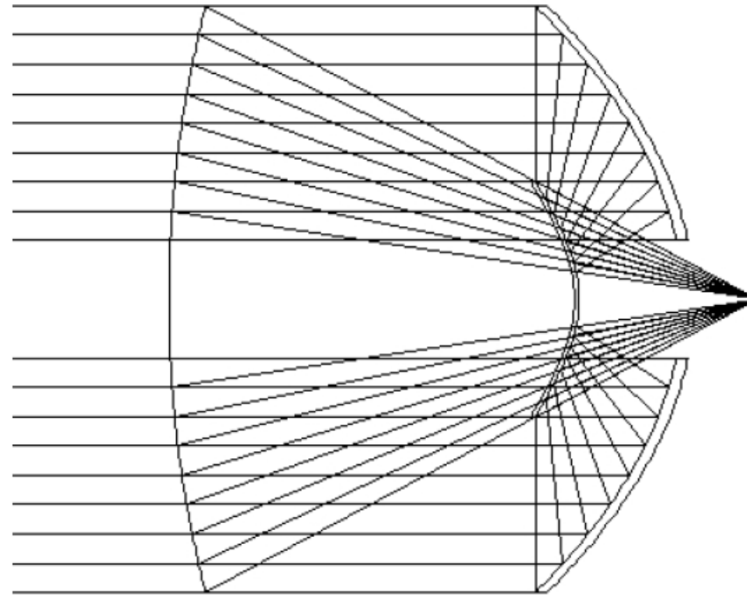
# Principal surface



In an aplanat working at  $m=0$   
the equivalent  
refracting surface  
is a hemisphere

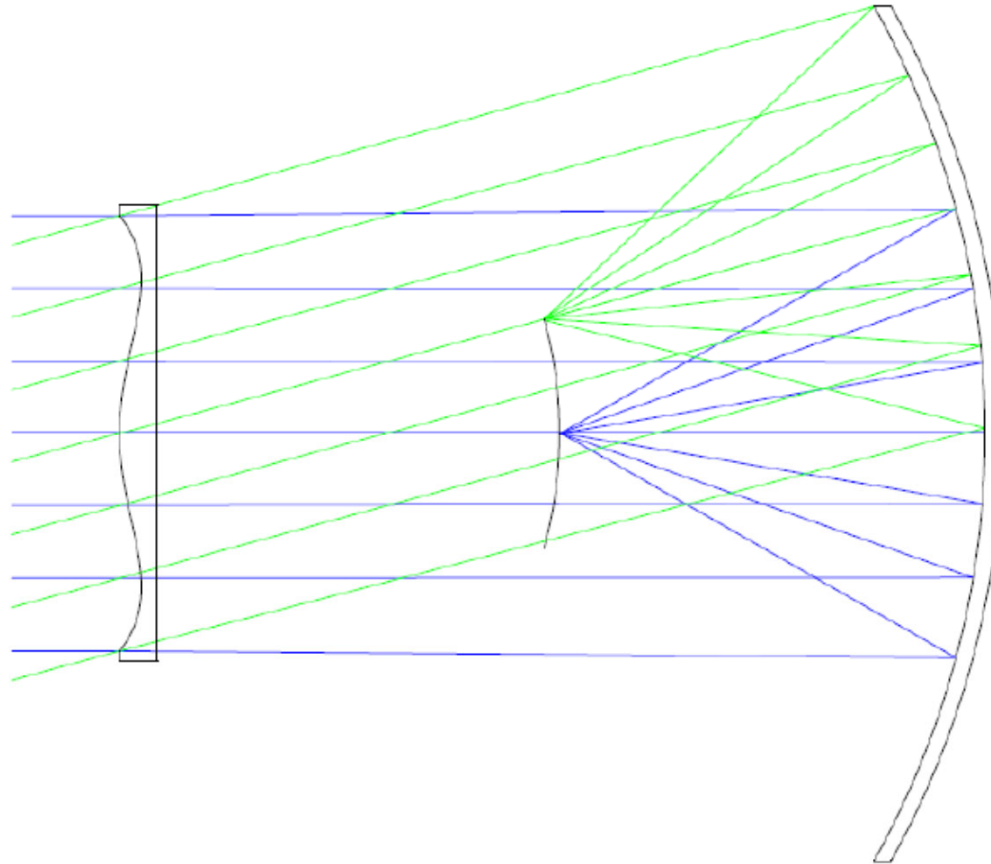


# Cassegrain's principal surface



Since the equivalent refracting surface in a Cassegrain telescope is a paraboloid then the coma of that Cassegrain is the same of a paraboloid mirror with the same focal length.

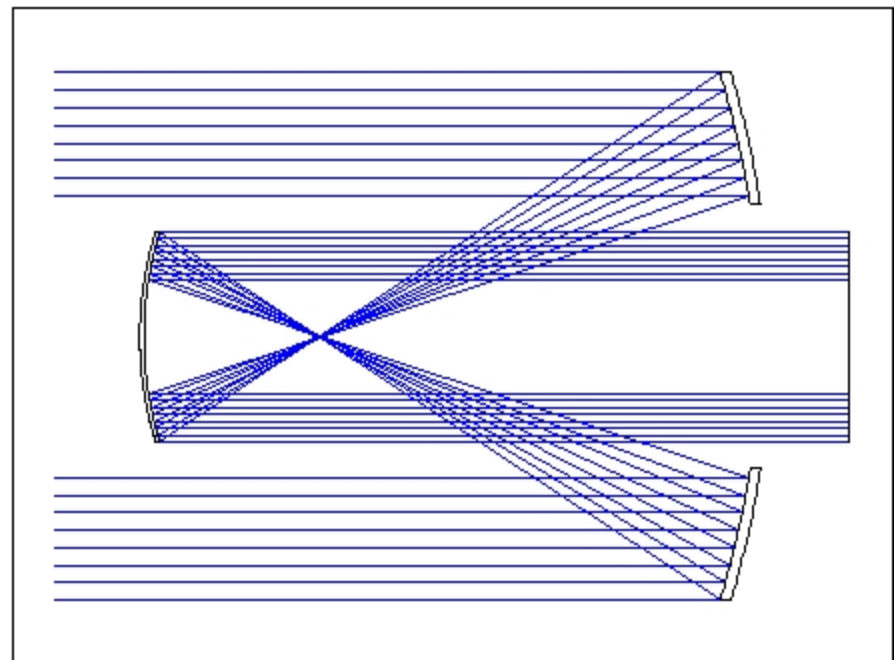
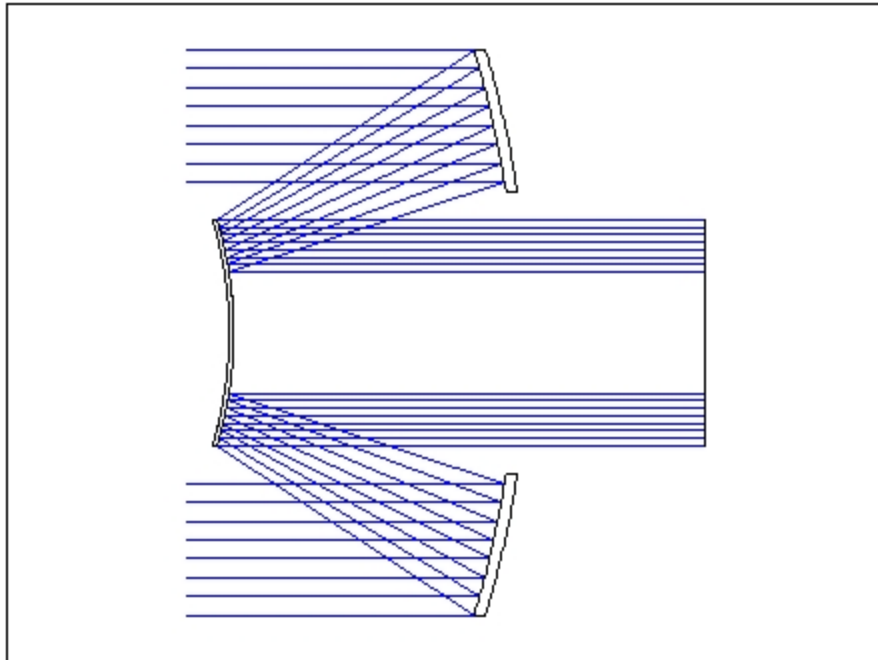
# Schmidt camera



# Merssene afocal system

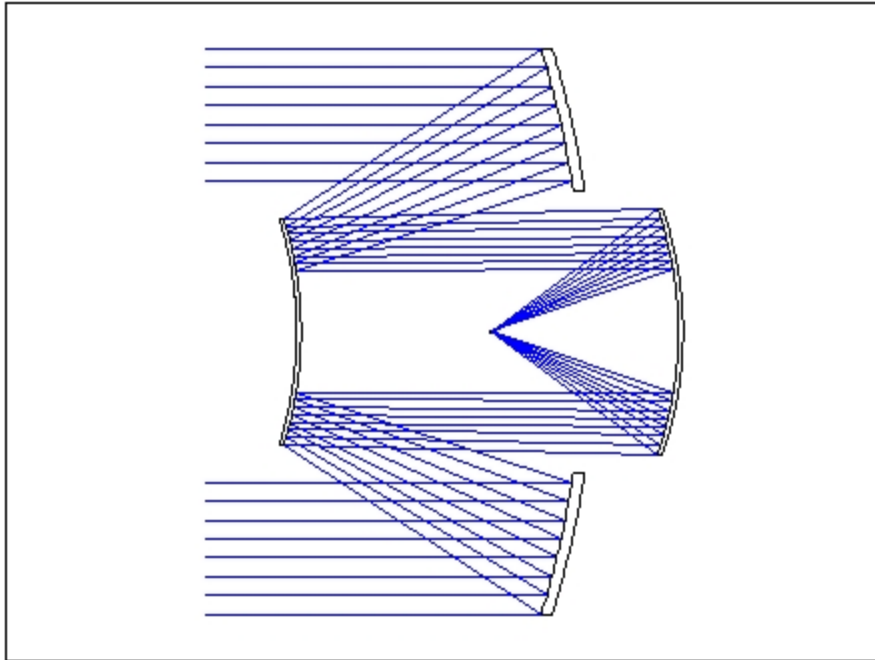
## Anastigmatic

### Confocal paraboloids



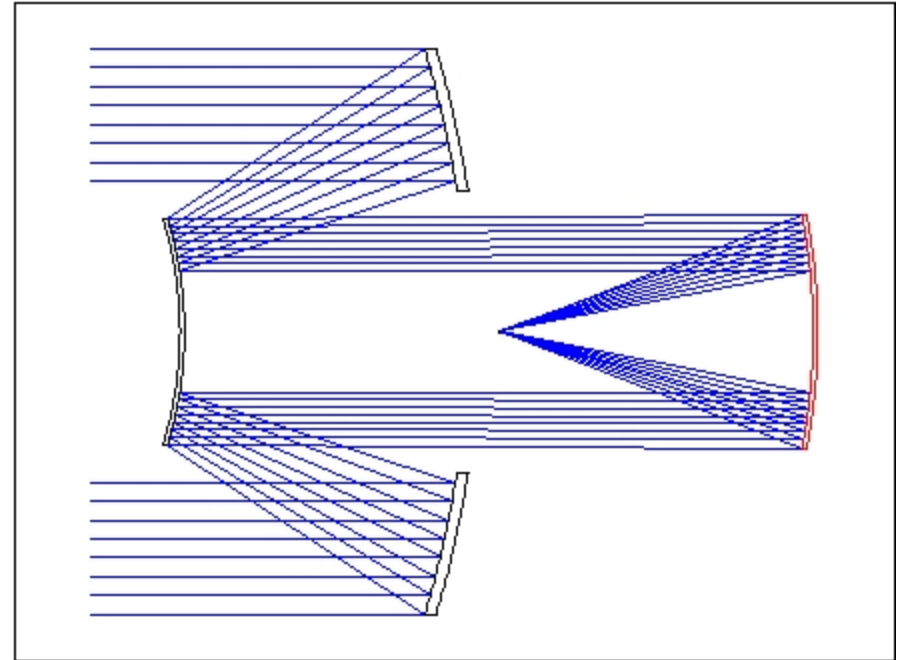
# Paul-Baker system

## Anastigmatic-Flat field



Anastigmatic  
Parabolic primary  
Spherical secondary and tertiary  
Curved field  
Tertiary CC at secondary

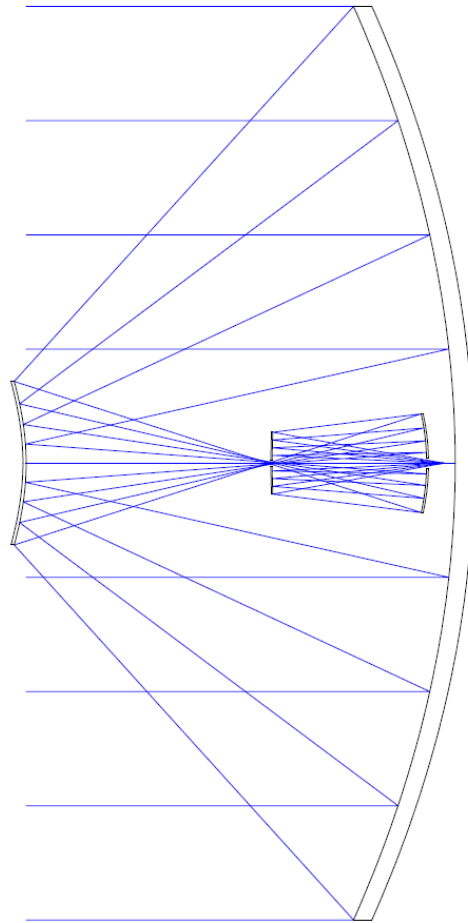
Prof. Jose Sasian  
OPTI 518



Anastigmatic, Flat field  
Parabolic primary  
Elliptical secondary  
Spherical tertiary  
Tertiary CC at secondary

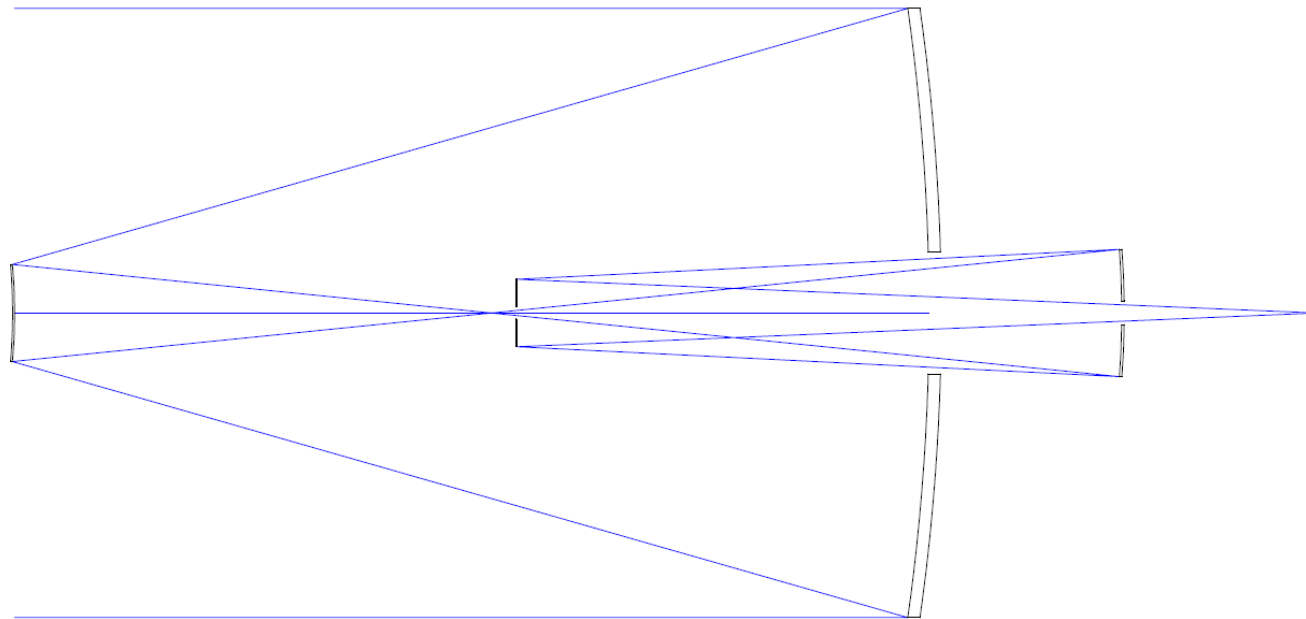


# Meinel's two stage optics concept (1985)



Large Deployable  
Reflector  
Second stage corrects  
for errors of first stage;  
fourth mirror is at the  
exit pupil.

Aplanatic, Anastigmatic, Flat-field, Orthoscopic (free from distortion, rectilinear, JS 1987)



Spherical primary telescope.

The quaternary mirror is near the exit pupil. Spherical aberration and Coma are then corrected with a single aspheric surface. The Petzval sum is zero. If more aspheric surfaces are allowed then more aberrations can be corrected.

# Summary

- Structural coefficients
- Basic treatment
- Analysis of simple systems