Introduction to aberrations
OPTI 518
Lecture 14

\[ \bar{y}_o \cdot \vec{H} \]
\[ y_E \cdot (\vec{\rho} + \Delta \vec{\rho}) \]
\[ y'_E \cdot \vec{\rho} \]
\[ \bar{y}_I \cdot (\vec{H} + \Delta \vec{H}) \]

Object plane
Entrance pupil
Exit pupil
Image plane
Topics

• Structural aberration coefficients
• Examples
Structural coefficients

Application of the aberration coefficients to specific optical components shows that the coefficients can be written as a function of the Lagrange invariant $\mathcal{K}$ the optical power $\phi$, the marginal ray height $y_p$ at the principal planes, and the structural coefficients: $\sigma_I, \sigma_{II}, \sigma_{III}, \sigma_{IV}, \sigma_V, \sigma_L, \sigma_T$. The Seidel aberration coefficients can be expressed with the structural coefficients. The use of structural coefficients simplifies considerably the calculation of aberration coefficients and facilitates making trade-off studies.

Requires a focal system
Afocal systems can be treated with Seidel sums
<table>
<thead>
<tr>
<th>Seidel sums in terms of structural aberration coefficients</th>
</tr>
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<tbody>
<tr>
<td>Pupils located at principal planes</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$S_I = \frac{1}{4} y_P^4 \Phi^3 \sigma_I$</td>
</tr>
<tr>
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</tr>
<tr>
<td>$S_{II} = \frac{1}{2} \kappa y_P^2 \Phi^2 \sigma_{II}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$S_{III} = \kappa \Phi \sigma_{III}$</td>
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<tr>
<td></td>
</tr>
<tr>
<td>$S_{IV} = \kappa \Phi \sigma_{IV}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$S_V = \frac{2 \kappa \Phi^2 \sigma_V}{y_P^2}$</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$C_L = y_P^2 \Phi \sigma_L$</td>
</tr>
<tr>
<td></td>
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<tr>
<td>$C_T = 2 \kappa \sigma_T$</td>
</tr>
</tbody>
</table>
Stop shifting from principal planes

\[ \sigma_I^* = \sigma_I \]
\[ \sigma_{II}^* = \sigma_{II} + \bar{S}_\sigma \sigma_I \]
\[ \sigma_{III}^* = \sigma_{III} + 2 \bar{S}_\sigma \sigma_{II} + \bar{S}_\sigma^2 \sigma_I \]
\[ \sigma_{IV}^* = \sigma_{IV} \]
\[ \sigma_V^* = \sigma_V + \bar{S}_\sigma \left( \sigma_{IV} + 3 \sigma_{III} \right) + 3 \bar{S}_\sigma^2 \sigma_{II} + \bar{S}_\sigma^3 \sigma_I \]
\[ \sigma_L^* = \sigma_L \]
\[ \sigma_T^* = \sigma_T + \bar{S}_\sigma \sigma_L \]

\[ \bar{S}_\sigma = \frac{y_p y_p \Phi}{2 \kappa} \]
\[ \Delta \bar{S}_\sigma = \frac{y_p \Delta y_p \Phi}{2 \kappa} = \frac{y_\sigma^2 \Phi}{2 \kappa} \bar{S} \]
Structural stop shifting parameter

\[ \omega = nu = - (Y - 1) \cdot \varphi \cdot y / 2 \]

\[ S_\sigma = \frac{y_P \varphi}{2 \mathcal{K}} \]

Using \( \omega \) on we can express:

\[ S_\sigma = \frac{y_P \varphi}{2 \mathcal{K}} = \frac{\varphi \cdot s}{(Y - 1) \cdot \varphi \cdot s - 2n} = \frac{\varphi \cdot s'}{(Y + 1) \cdot \varphi \cdot s' - 2n'} \]

s is the distance from the front principal plane to entrance pupil

s’ is the distance from the rear principal plane to exit pupil

\[ \delta S_\sigma = \frac{y_P \delta y_P \varphi}{2 \mathcal{K}} = \frac{y_P^2 \varphi}{2 \mathcal{K}} \overline{S} \]
Structural coefficients of a system of $k$ components

\[
\sigma_I = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right)^3 \left( \frac{y_{p,k}}{y_p} \right)^4 \sigma_{I,k}
\]

\[
\sigma_{II} = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right)^2 \left( \frac{y_{p,k}}{y_p} \right)^2 \left( \sigma_{II,k} + \bar{S}_k \sigma_{I,k} \right)
\]

\[
\sigma_{III} = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right) \left( \sigma_{III,k} + 2 \bar{S}_k \sigma_{II,k} + \bar{S}_k^2 \sigma_{I,k} \right)
\]

\[
\sigma_{IV} = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right) \sigma_{IV,k}
\]

\[
\sigma_V = \sum_{i=0}^{k} \left( \frac{y_p}{y_{p,k}} \right)^2 \left( \sigma_{V,k} + \bar{S}_k \left( \sigma_{IV,k} + 3 \sigma_{III,k} \right) + 3 \bar{S}_k^2 \sigma_{II,k} + \bar{S}_k^3 \sigma_{I,k} \right)
\]

\[
\sigma_L = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right) \left( \frac{y_{p,k}}{y_p} \right)^2 \sigma_{L,k}
\]

\[
\sigma_T = \sum_{i=0}^{k} \left( \sigma_{T,k} + \bar{S}_k \sigma_{L,k} \right)
\]

\[
\bar{S}_k = \frac{\Phi_k \cdot y_{p,k} \cdot \bar{y}_{p,k}}{2 \mathcal{K}}
\]
Review of concepts

• Thin lens as the thickness tends to zero

\[ \phi = \phi_1 + \phi_2 - \phi_1 \phi_2 \frac{t}{n} \]

• Shape of a lens and shape factor

• Conjugate factor to quantify how the lens is used. Related to transverse magnification

• Must know well first-order optics
Shape and Conjugate factors

\[ Y = \frac{\omega' + \omega}{\omega' - \omega} = \frac{1 + m}{1 - m} \]

\( \omega = nu \)

\[ X = \frac{c_1 + c_2}{c_1 - c_2} = -\frac{R_1 + R_2}{R_1 - R_2} \]

Lens bending concept
Shape X

X=0

X=1

X=1.7

X=3.5

X=0

X=-1

X=-1.7

X=-3.5
Shape

X=-3  X=-2  X=-1  X=0

X=3  X=2  X=1  X=0
Shape or bending factor $X$

- Quantifies lens shape
- Optical power of thin lens is maintained
- Not defined for zero power, $R_1 = R_2$

$$X = \frac{c_1 + c_2}{c_1 - c_2} = -\frac{R_1 + R_2}{R_1 - R_2}$$
### Structural aberration coefficients of a surface (Stop at surface $y_P = 0$)

#### First-order identities

<table>
<thead>
<tr>
<th>Equation</th>
<th>Y</th>
<th>( \omega' + \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi = (n' - n) \cdot c )</td>
<td>( Y = \frac{\omega' + \omega}{\omega' - \omega} )</td>
<td></td>
</tr>
<tr>
<td>( \omega' = \omega - \Phi \cdot y_P )</td>
<td>( \omega = \omega' + \Phi \cdot y_P )</td>
<td></td>
</tr>
<tr>
<td>( \omega' = -\frac{Y + 1}{2} \Phi \cdot y_P )</td>
<td>( \omega = -\frac{Y - 1}{2} \Phi \cdot y_P )</td>
<td></td>
</tr>
</tbody>
</table>

\[ \delta n/n = (n_F - n_C)/n_d \]

#### Seidel sum arguments

<table>
<thead>
<tr>
<th>Equation</th>
<th>( \bar{A} = \bar{\omega} + n\bar{y}_P c = \bar{\omega} = \frac{\mathcal{K}}{y_P} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A = \omega + n y_P c = \left[ \frac{n' + n}{n' - n} - Y \right] \frac{\Phi \cdot y_P}{2} )</td>
<td>( \bar{A} = \bar{\omega} + n\bar{y}_P c = \bar{\omega} = \frac{\mathcal{K}}{y_P} )</td>
</tr>
<tr>
<td>( y_P \cdot \Delta \left( \frac{u}{n} \right) = y_P \cdot \Delta \left( \frac{\omega}{n^2} \right) )</td>
<td>( \bar{A} = \bar{\omega} + n\bar{y}_P c = \bar{\omega} = \frac{\mathcal{K}}{y_P} )</td>
</tr>
<tr>
<td>( = \left[ \frac{n'^2 - n^2}{n'^2 - n^2} - Y - \frac{n'^2 + n^2}{n'^2 - n^2} \right] \frac{\Phi \cdot y_P^2}{2} )</td>
<td>( P = -\frac{\Phi}{n n' y_P} )</td>
</tr>
</tbody>
</table>
Structural aberration coefficients of a surface

\[
\sigma_I = -\frac{1}{2} \left[ \frac{n' + n}{n' - n} - Y \right]^2 \left[ \frac{n'^2 - n^2}{n^2 n'^2} \cdot Y - \frac{n'^2 + n^2}{n^2 n'^2} \right]
\]

\[
\sigma_{\Pi} = -\frac{1}{2} \left[ \frac{n' + n}{n' - n} - Y \right] \left[ \frac{n'^2 - n^2}{n^2 n'^2} \cdot Y - \frac{n'^2 + n^2}{n^2 n'^2} \right]
\]

\[
\sigma_{\text{III}} = -\frac{1}{2} \left[ \frac{n'^2 - n^2}{n^2 n'^2} \cdot Y - \frac{n'^2 + n^2}{n^2 n'^2} \right]
\]

\[
\sigma_{\text{IV}} = \frac{1}{nn'}
\]

\[
\sigma_V = \frac{n'^2 - n^2}{n^2 n'^2}
\]

\[
\sigma_L = \frac{1}{2} \left[ Y - \frac{n' + n}{n' - n} \right] \cdot \Delta \left( \frac{\delta n}{n} \right)
\]

\[
\sigma_T = \Delta \left( \frac{\delta n}{n} \right)
\]
Example:
Refracting surface free from spherical aberration

Object at infinity $Y=1$

$$
\sigma_I = -\frac{1}{2} \left[ \frac{n' + n}{n' - n} - 1 \right]^2 \left[ \frac{n'^2 - n^2}{n'^2 n^2} - \frac{n'^2 + n^2}{n'^2 n^2} \right]
= -\frac{1}{2} \left[ \frac{2n}{n' - n} \right]^2 \left[ \frac{-2}{n'^2} \right]
= \left[ \frac{2n}{n' - n} \right]^2 \left[ \frac{1}{n'^2} \right]
$$

$$
S_I = \frac{1}{4} y_p^4 \phi^3 \sigma_I + \Delta(n) \frac{y_p^4}{r^3} K = \frac{1}{4} y_p^4 \phi^3 \sigma_I + \frac{1}{(n' - n)^2} \frac{y_p^4}{r^3} K
$$

$$
S_I = y_p^4 \phi^3 \left[ \frac{1}{4} \left[ \frac{2n}{n' - n} \right]^2 \left[ \frac{1}{n'^2} \right] + \frac{1}{(n' - n)^2} K \right]
= \frac{y_p^4}{r^3} \phi^3 \left[ \frac{n^2}{n'^2} + K \right]
$$

$$
S_I = 0 \Rightarrow \left[ \frac{n^2}{n'^2} = -K = \varepsilon^2 \right]
$$

Parabola for reflection
Ellipse for air to glass
Hyperbola for glass to air

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The contribution to the structural coefficient from the aspheric cap is

\[ S_I = \frac{1}{4} y_p^4 \phi^3 \sigma_I + \frac{1}{(n' - n)^2} y_p^4 \phi^3 K = \frac{1}{4} y_p^4 \phi^3 \left[ \sigma_I + \left( \frac{2}{n' - n} \right)^2 K \right] \]

For a reflecting surface is just the conic constant \( K \)

\[ \sigma_{\text{Icap}} = \alpha = \left( \frac{2}{n' - n} \right)^2 K \]
<table>
<thead>
<tr>
<th>Stop at surface</th>
<th>With stop shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_I = Y^2 + \alpha$</td>
<td>$\sigma_I = Y^2 + \alpha$</td>
</tr>
<tr>
<td>$\sigma_{II} = -Y$</td>
<td>$\sigma_{II} = -Y(1 - \overline{S}<em>\sigma Y) + \overline{S}</em>\sigma \cdot \alpha$</td>
</tr>
<tr>
<td>$\sigma_{III} = 1$</td>
<td>$\sigma_{III} = (1 - \overline{S}<em>\sigma Y)^2 + \overline{S}</em>\sigma^2 \cdot \alpha$</td>
</tr>
<tr>
<td>$\sigma_{IV} = -1$</td>
<td>$\sigma_{IV} = -1$</td>
</tr>
<tr>
<td>$\sigma_V = 0$</td>
<td>$\sigma_V = \overline{S}<em>\sigma \cdot (1 - \overline{S}</em>\sigma Y) \cdot (2 - \overline{S}<em>\sigma Y) + \overline{S}</em>\sigma^3 \cdot \alpha$</td>
</tr>
</tbody>
</table>

$$\sigma_{Icap} = \alpha = K$$
### Structural aberration coefficients of a thin lens in air (Stop at lens)

#### First-order identities

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi )</td>
<td>( (n-1) \cdot (c_1 - c_2) = (n-1) \cdot \left( \frac{1}{r_1} - \frac{1}{r_2} \right) )</td>
</tr>
<tr>
<td>( X )</td>
<td>( \frac{c_1 + c_2}{c_1 - c_2} = \frac{r_1 + r_2}{r_1 - r_2} )</td>
</tr>
<tr>
<td>( c_1 )</td>
<td>( \frac{1}{2} \frac{\Phi}{n-1} (X + 1) )</td>
</tr>
<tr>
<td>( \omega )</td>
<td>( \frac{1}{2} (Y - 1)(\Phi \cdot y_P) )</td>
</tr>
<tr>
<td>( Y )</td>
<td>( \frac{\omega' + \omega}{\omega' - \omega} = \frac{1+m}{1-m} )</td>
</tr>
<tr>
<td>( c_2 )</td>
<td>( \frac{1}{2} \frac{\Phi}{n-1} (X - 1) )</td>
</tr>
<tr>
<td>( \omega' )</td>
<td>( \frac{1}{2} (Y + 1)(\Phi \cdot y_P) )</td>
</tr>
</tbody>
</table>
### Structural aberration coefficients of a thin lens in air (Stop at lens)

<table>
<thead>
<tr>
<th>Term</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_I$</td>
<td>$AX^2 - BXY + CY^2 + D$</td>
</tr>
<tr>
<td>$\sigma_{II}$</td>
<td>$EX - FY$</td>
</tr>
<tr>
<td>$\sigma_{III}$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\sigma_{IV}$</td>
<td>$\frac{1}{n}$</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>$\frac{1}{\nu}$</td>
</tr>
<tr>
<td>$\sigma_T$</td>
<td>$0$</td>
</tr>
<tr>
<td>$A$</td>
<td>$\frac{n+2}{n(n-1)^2}$</td>
</tr>
<tr>
<td>$B$</td>
<td>$\frac{4(n+1)}{n(n-1)}$</td>
</tr>
<tr>
<td>$C$</td>
<td>$\frac{3n+2}{n}$</td>
</tr>
<tr>
<td>$D$</td>
<td>$\frac{n^2}{(n-1)^2}$</td>
</tr>
<tr>
<td>$E$</td>
<td>$\frac{n+1}{n(n-1)}$</td>
</tr>
<tr>
<td>$F$</td>
<td>$\frac{2n+1}{n}$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>$\frac{n_F - n_C}{n_d - 1}$</td>
</tr>
</tbody>
</table>
Contributions to the structural coefficients from a parallel plate of thickness $t$ and index $n$ into a system of optical power $\Phi$

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \sigma_I$</td>
<td>$-4 \left( \frac{Y \pm 1}{2} \right)^4 \left( \frac{n^2 - 1}{n^2} \right) \frac{\Phi t}{n}$</td>
</tr>
<tr>
<td>$\Delta \sigma_{II}$</td>
<td>$2 \left( \frac{Y \pm 1}{2} \right)^3 \left( \frac{n^2 - 1}{n^2} \right) \frac{\Phi t}{n}$</td>
</tr>
<tr>
<td>$\Delta \sigma_{III}$</td>
<td>$- \left( \frac{Y \pm 1}{2} \right)^2 \left( \frac{n^2 - 1}{n^2} \right) \frac{\Phi t}{n}$</td>
</tr>
<tr>
<td>$\Delta \sigma_{IV}$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\Delta \sigma'_I$</td>
<td>$\frac{1}{2} \left( \frac{Y \pm 1}{2} \right) \left( \frac{n^2 - 1}{n^2} \right) \frac{\Phi t}{n}$</td>
</tr>
<tr>
<td>$\Delta \sigma_L$</td>
<td>$- \left( \frac{Y \pm 1}{2} \right)^2 \left( \frac{n-1}{nv} \right) \frac{\Phi t}{n}$</td>
</tr>
<tr>
<td>$\Delta \sigma_T$</td>
<td>$\frac{1}{2} \left( \frac{Y \pm 1}{2} \right) \left( \frac{n-1}{nv} \right) \frac{\Phi t}{n}$</td>
</tr>
</tbody>
</table>

Positive sign + for image space
Negative sign - for object space
Spherical Mirror

A spherical mirror can be treated as a convex/concave plano lens with \( n = -1 \). The plano surface acts as an unfolding flat surface contributing no aberration.

\[
\begin{align*}
X &= \pm 1 \\
\sigma_I &= Y^2 \\
\sigma_{II} &= -Y \\
\sigma_{III} &= 1 \\
\sigma_{IV} &= -1 \\
\sigma_V &= 0 \\
\sigma_L &= 0 \\
\sigma_T &= 0 \\
A &= -\frac{1}{4} \\
B &= 0 \\
C &= 1 \\
D &= \frac{1}{4} \\
E &= 0 \\
F &= 1
\end{align*}
\]
Field curves

Field curve curvature in terms of structural coefficients

<table>
<thead>
<tr>
<th>Field</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Petzval</td>
<td>( C_{\text{Petzval}} = -n' \phi \cdot \sigma_IV )</td>
</tr>
<tr>
<td>Sagittal</td>
<td>( C_{\text{Sagittal}} = -n' \phi \cdot (\sigma_IV + \sigma_{III}) )</td>
</tr>
<tr>
<td>Medial</td>
<td>( C_{\text{Medial}} = -n' \phi \cdot (\sigma_IV + 2\sigma_{III}) )</td>
</tr>
<tr>
<td>Tangential</td>
<td>( C_{\text{Tangential}} = -n' \phi \cdot (\sigma_IV + 3\sigma_{III}) )</td>
</tr>
</tbody>
</table>
Field curves

\[ \text{Rt} \approx f/3.66 \]
\[ \text{Rm} \approx f/2.66 \]
\[ \text{Rs} \approx f/1.66 \]
\[ \text{Rp} \approx f/0.66 \]

<table>
<thead>
<tr>
<th>Field curve curvature in terms of structural coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{\text{Petzval}} = -n' \phi \cdot \sigma_{IV} )</td>
</tr>
<tr>
<td>( C_{\text{Sagittal}} = -n' \phi \cdot (\sigma_{IV} + \sigma_{III}) )</td>
</tr>
<tr>
<td>( C_{\text{Medial}} = -n' \phi \cdot (\sigma_{IV} + 2\sigma_{III}) )</td>
</tr>
<tr>
<td>( C_{\text{Tangential}} = -n' \phi \cdot (\sigma_{IV} + 3\sigma_{III}) )</td>
</tr>
</tbody>
</table>

Field curvatures do not change with object distance (stop at lens)
Thin lens
Spherical aberration and coma

\[ \sigma_I \quad \sigma_{II} \]

Fourth-order

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Spherical aberration of a F/4 lens

• Asymmetry
• For high index 4th order predicts well the aberration
Thin lens
spherical aberration
n=1.517

$\sigma_I$
Thin lens
coma aberration
n=1.517

$\sigma_{II}$
This lens
Aplanatic solutions

$X = \pm (2n+1)$
$Y = \pm \left(\frac{n+1}{n-1}\right)$

$Y = \frac{\omega^1 + \omega}{\omega^1 - \omega} = \frac{1 + m}{1 - m}$

$X = 4$
$Y = 5$
$n = 1.5$
Thin lens
Spherical and coma @ Y=0

Strong index dependence
Thin lens special cases

stop at lens

\[
\sigma_I = \text{MINIMUM}
\]

\[
X = \frac{B}{2A} \ Y = \frac{2(n+1)(n-1)}{n+2} \ Y
\]

\[
\sigma_I = \frac{n^2}{(n-1)^2} - \frac{n}{n+2} \ Y^2
\]

\[
= \frac{n^2}{(n-1)^2} - \frac{n(n+2)}{H(n+1)^2(n-1)^2} \ X^2
\]

\[
\sigma_{II} = -\frac{1}{n+2} \ Y
\]

\[
= -\frac{1}{2(n^2-1)} \ X
\]
Thin lens special cases

stop at lens

\[ \sigma_\Pi = 0 \]

\[ X = \frac{F}{E} \quad Y = \frac{(2n+1)(n-1)}{(n+1)} Y \]

\[ \sigma_x = \frac{n^2}{(n-i)^2} - \frac{h^2}{(n+1)^2} Y^2 \]

\[ = \frac{n^2}{(n-i)^2} - \frac{h^2}{(2n+1)^2(n-1)^2} X^2 \]

for \( \sigma_\Pi = 0 \) (Aplanatic Lens)

\[ X = \pm (2n+1) \]

\[ Y = \pm \left( \frac{n+1}{n-1} \right) \]
Thin lens special cases
stop at lens

\[ X = 0 \]
\[ \sigma_x = \frac{3n+2}{n} Y^2 + \frac{n^2}{(n-1)^2} \]
\[ \sigma_x = -\frac{2n+1}{n} Y \]
\[ Y = 1 \]
\[ \sigma_x = \frac{3n+2}{n} + \frac{n^2}{(n-1)^2} \]

For double convex lens (CX)
For double concave lens (CC)
Thin lens special cases
stop at lens

\[
X = 1
\]

\[
\sigma_x = \frac{3n+2}{n} \left( Y - \frac{2(n+1)}{(3n+2)(n-1)} \right)^2 + \frac{n(3n-1)(n+1)}{(3n+2)(n-1)^2}
\]

\[
\sigma_y = \frac{n+1}{n(n-1)} - \frac{2n+1}{n} Y
\]

\[
Y = 1
\]

\[
\sigma_x = 4 \left( 1 + \frac{2-n}{n(n-1)^2} \right)
\]

\[
\sigma_y = -2 + \frac{2}{n(n-1)}
\]
Thin lens special cases

\[ x = -1 \]

\[ \sigma_I = \frac{3n+2}{n} \left( Y + \frac{2(n+1)}{(3n+2)(n-1)} \right)^2 + \frac{h(3n-i)(n+1)}{(3n+2)(n-1)^2} \]

\[ \sigma_\Pi = -\frac{h+1}{n(n-1)} - \frac{2n+1}{n} Y \]

\[ Y = 1 \]

\[ \sigma_I = \left( \frac{2n}{n-1} \right)^2 \]

\[ \sigma_\Pi = -\frac{2n}{n-1} \]

Plano convex (concave),

Plane side forward,

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Achromatic doublet

Two thin lenses in contact
The stop is at the doublet

\[ \phi = \phi_1 + \phi_2 \]
\[ y_1 = y_2 \]
\[ \rho_1 = \frac{\phi_1}{\phi} \]
\[ \rho_2 = \frac{\phi_2}{\phi} \]
\[ \rho_1 + \rho_2 = 1 \]
\[ Y_1 = \frac{Y - \rho_2}{\rho_1} \]
\[ Y_2 = \frac{Y - \rho_1}{\rho_2} \]
Achromatic doublet

Correction for chromatic change of focus

\[ \sigma_L = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right) \left( \frac{y_{P,k}}{y_P} \right)^2 \sigma_{L,k} \]

\[ \sigma_L = \frac{\rho_1}{\nu_1} + \frac{\rho_2}{\nu_2} \]

\[ \rho_1 = \frac{\nu_1}{\nu_1 - \nu_2} \left( 1 - \nu_2 \sigma_L \right) \]

\[ \rho_2 = -\frac{\nu_2}{\nu_1 - \nu_2} \left( 1 - \nu_1 \sigma_L \right) \]

For an achromatic doublet:

\[ \sigma_L = 0 \]
Achromatic doublet

Correction for spherical aberration

\[ \sigma_I = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right)^3 \left( \frac{y_{p,k}}{y_p} \right)^4 \sigma_{I,k} \]

\[ \sigma_I = \rho_1^3 \left( A_1 X_1^2 - B_1 X_1 Y_1 + C_1 Y_1^2 + D_1 \right) + \rho_2^3 \left( A_2 X_2^2 - B_2 X_2 Y_2 + C_2 Y_2^2 + D_2 \right) \]

For a given conjugate factor \( Y \), spherical aberration is a function of the shape factors \( X_1 \) and \( X_2 \). For a constant value of spherical aberration we obtain a hyperbola as a function of \( X1 \) and \( X2 \).
Achromatic doublet

Correction for coma aberration

\[ \sigma_{II} = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right)^2 \left( \frac{y_{p,k}}{y_p} \right)^2 \left( \sigma_{II,k} + \bar{S}_k \sigma_{I,k} \right) \]

\[ \sigma_{II} = \rho_1^2 \left( E_1 X_1 - F_1 Y_1 \right) + \rho_2^2 \left( E_2 X_2 - F_2 Y_2 \right) \]

For a given conjugate factor Y and a constant amount of coma the graph of X1 and X2 is a straight line.
Achromatic doublet

Astigmatism aberration

\[ \sigma_{III} = \cdot \sum_{i=0}^{k} \left( \frac{\Phi_i}{\Phi} \right) \left( \sigma_{III,k} + 2\bar{S}_k \sigma_{II,k} + \bar{S}_k^2 \sigma_{I,k} \right) \]

\[ \sigma_{III} = \rho_1 (1) + \rho_2 (1) \]

Astigmatism is independent of the relative lens powers, shape factors, or conjugate factors.
Achromatic doublet

Field curvature aberration

$$\sigma_{IV} = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right) \sigma_{IV,k}$$

$$\sigma_{IV} = \frac{\rho_1}{n_1} + \frac{\rho_2}{n_2}$$

For an achromatic doublet there is no field curvature \( \sigma_{IV} = 0 \) if

$$\frac{n_2}{\nu_2} = \frac{n_1}{\nu_1}$$
Achromatic doublet

Distortion and chromatic change of magnification

\[
\sigma_V = \sum_{i=0}^{k} \left( \frac{y_p}{y_{p,k}} \right)^2 \left( \sigma_{V,k} + \bar{S}_k \left( \sigma_{IV,k} + 3\sigma_{III,k} \right) + 3\bar{S}_k^2 \sigma_{II,k} + \bar{S}_k^3 \sigma_{I,k} \right)
\]

\[
\sigma_T = \sum_{i=0}^{k} \left( \sigma_{T,k} + \bar{S}_k \sigma_{L,k} \right)
\]

\[
\sigma_V = 0
\]

\[
\sigma_T = 0
\]
Cemented achromatic doublet

\[ c_{12} = \frac{(X_1 - 1)\phi_1}{2(n_1 - 1)} = c_{21} = \frac{(X_2 + 1)\phi_2}{2(n_2 - 1)} \]

\[ \frac{(X_1 - 1)\rho_1}{(n_1 - 1)} = \frac{(X_2 + 1)\rho_2}{(n_2 - 1)} \]

\[ X_2 = \alpha (X_1 - 1) - 1 \]

\[ \alpha = \frac{(n_2 - 1)\rho_1}{(n_1 - 1)\rho_2} \]

\[ \alpha = -\frac{(n_2 - 1)v_1}{(n_1 - 1)v_2} \]

For a cemented achromatic lens the graph of \( X_1 \) and \( X_2 \) is a straight line.
Crown in front: BK7 and F8
Flint in front: BK7 and F8
Cemented achromatic doublet
Cemented doublet solutions

Crown in front

Flint in front
Doublets with no spherical aberration but with varying coma

Crown in front, no spherical aberration, F/5, f=100 mm, no chromatic aberration
Lister objective

XIII. On some properties in achromatic object-glasses applicable to the improvement of the microscope. By Joseph Jackson Lister, Esq. Communicated by Dr. Roget, Secretary.

Read January 21, 1830.
Lister objective

- Two achromatic doublets that are spaced
- Telecentric in image space
- Normalized system

\[ \mathcal{K} = 1 \]
\[ y_A = 1 \]
\[ \bar{u}_A = 1 \]
\[ \sigma_{IA} = 0 \]
\[ \sigma_{IB} = 0 \]
Lister objective

The aperture stop is at the first lens.
The system is telecentric

\[ \phi_B = 1 \]
\[ \phi_A = 1 - y_B \]
\[ \bar{y}_B = 1 \]
Lister objective

### Seidel sums in terms of structural aberration coefficients

<table>
<thead>
<tr>
<th>Pupils located at principal planes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_I = \frac{1}{4} y_P^4 \Phi \sigma_I$</td>
</tr>
<tr>
<td>$S_{II} = \frac{1}{2} y_P^2 \Phi^2 \sigma_{II}$</td>
</tr>
<tr>
<td>$S_{III} = \Phi^2 \Phi \sigma_{III}$</td>
</tr>
<tr>
<td>$S_{IV} = \Phi^2 \Phi \sigma_{IV}$</td>
</tr>
<tr>
<td>$S_V = \frac{2 \Phi^3 \sigma_V}{y_P}$</td>
</tr>
<tr>
<td>$C_L = y_P^2 \Phi \sigma_L$</td>
</tr>
<tr>
<td>$C_T = 2 \Phi \sigma_T$</td>
</tr>
</tbody>
</table>

### Stop shifting from principal planes

<table>
<thead>
<tr>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_I^* = \sigma_I$</td>
</tr>
<tr>
<td>$\sigma_{II}^* = \sigma_{II} + \overline{S}_\sigma \sigma_I$</td>
</tr>
<tr>
<td>$\sigma_{III}^* = \sigma_{III} + 2 \overline{S}<em>\sigma \sigma</em>{II} + \overline{S}_\sigma^2 \sigma_I$</td>
</tr>
<tr>
<td>$\sigma_{IV}^* = \sigma_{IV}$</td>
</tr>
<tr>
<td>$\sigma_{V}^* = \sigma_V + \overline{S}<em>\sigma \left( \sigma</em>{IV} + 3 \sigma_{III} \right) + 3 \overline{S}<em>\sigma^2 \sigma</em>{II} + \overline{S}_\sigma^3 \sigma_I$</td>
</tr>
<tr>
<td>$\sigma_L^* = \sigma_L$</td>
</tr>
<tr>
<td>$\sigma_T^* = \sigma_T + \overline{S}_\sigma \sigma_L$</td>
</tr>
<tr>
<td>$\overline{S}_\sigma = \frac{y_P \overline{y}_P \Phi}{2 \Phi}$</td>
</tr>
<tr>
<td>$\Delta \overline{S}_\sigma = \frac{y_P \Delta \overline{y}<em>P \Phi}{2 \Phi} = \frac{\gamma</em>\sigma \Phi}{2 \Phi} \overline{S}$</td>
</tr>
</tbody>
</table>
Condition for zero coma

\[
\sigma_{II} = \sum_{i=0}^{k} \left( \frac{\Phi_{k}}{\Phi} \right)^{2} \left( \frac{v_{P,k}}{v_{P}} \right)^{2} \left( \sigma_{II,k} + \bar{S}_{k} \sigma_{I,k} \right)
\]

\[
\sigma_{II} = \phi_{A}^{2} y_{A}^{2} \sigma_{IA} + \phi_{B}^{2} y_{B}^{2} \sigma_{IB}
\]

\[
\sigma_{IB} = 0
\]

\[
\sigma_{II} = (1 - y_{B})^{2} \sigma_{IA} + y_{B}^{2} \sigma_{IB} = 0
\]

\[
\sigma_{IA} = -\frac{y_{B}^{2}}{(1 - y_{B})^{2}} \sigma_{IB}
\]
Condition for zero astigmatism

\[
\sigma_{III} = \sum_{i=0}^{k} \left( \frac{\Phi_k}{\Phi} \right) \left( \sigma_{III,k} + 2\bar{S}_k \sigma_{II,k} + \bar{S}_k^2 \sigma_{I,k} \right)
\]

\[
\sigma_{III} = \phi_A + \phi_B \left( 1 + y_B \sigma_{IIB} \right) = 0
\]

\[
\sigma_{III} = \left( 1 - y_B \right) + \left( 1 + y_B \sigma_{IIB} \right) = 0
\]

\[
2 - y_B + y_B \sigma_{IIB} = 0
\]

\[
y_B = \frac{2}{1 - \sigma_{IIB}}
\]

\[
\sigma_{IIB} = -\frac{2 - y_B}{y_B}
\]

\[
\sigma_{IIA} = \frac{y_B \left( 2 - y_B \right)}{(1 - y_B)^2}
\]

\[
\bar{S}_k = \frac{\Phi_k \cdot y_{P,k} \cdot \bar{y}_{P,k}}{2 \mathcal{K}}
\]
Lister Objective

Choose:

\[ \sigma_{IIB} = -\sigma_{IIA} \]

\[ -\frac{2 - y_B}{y_B} = -\frac{y_B(2 - y_B)}{(1 - y_B)^2} \]

\[ (1 - y_B)^2 = y_B^2 \]

\[ 1 - 2y_B + y_B^2 = y_B^2 \]

\[ y_B = \frac{1}{2} \]

\[ \varphi_A = \frac{1}{2} \]

\[ \sigma_{IIA} = 3 \]

\[ \sigma_{IIB} = -3 \]

Seidel sums in terms of structural aberration coefficients

<table>
<thead>
<tr>
<th>Pupils located at principal planes</th>
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<tbody>
<tr>
<td>( S_I = \frac{1}{4} y_p^4 \Phi^3 \sigma_I )</td>
</tr>
<tr>
<td>( S_{II} = \frac{1}{2} \chi y_p^2 \Phi^2 \sigma_{II} )</td>
</tr>
<tr>
<td>( S_{III} = \chi^2 \Phi \sigma_{III} )</td>
</tr>
<tr>
<td>( S_{IV} = \chi^2 \Phi \sigma_{IV} )</td>
</tr>
<tr>
<td>( S_p = \frac{2 \chi^3 \sigma_p}{y_p^2} )</td>
</tr>
<tr>
<td>( C_L = y_p^2 \Phi \sigma_L )</td>
</tr>
<tr>
<td>( C_T = 2 \chi \sigma_T )</td>
</tr>
</tbody>
</table>
Plano convex lens

N-BK7: Petzval radius -151.7 mm

\[ C_{\text{Petzval}} = \frac{1}{\rho_{\text{Petzval}}} = -\phi \cdot \sigma_{IV} = -\left( \frac{\phi}{n} \right) \]
Wollaston meniscus lens

$W_{222} / W_{220P} = -0.8$

- Artificially flattening the field
- Periscopic lenses
Periskop lens

- Principle of symmetry
- No distortion
Field curvature

- Old achromat
- New achromat

N-BK7 : Petzval radius -151.7 mm

N-BK7 and N-F2: Petzval radius -139.99 mm (+139.99 for negative doublet)

N-BAK1 and N-LLF6: Petzval radius -185 mm

\[
C_{Petzval} = \frac{1}{\rho_{Petzval}} = -\phi \cdot \sigma_W = -\left(\frac{\phi_1 + \phi_2}{n_1 + n_2}\right) = -\frac{\phi}{v_1 - v_2} \left(\frac{v_1}{n_1} - \frac{v_2}{n_2}\right)
\]
Chevalier landscape lens

- F/5 telescope doublet used in reverse and with an aperture stop in front

\[ \frac{W_{222}}{W_{220P}} = -0.8 \]
Rapid rectilinear

- F/8
- Glass selection is key to minimize spherical aberration while artificially flattening the field

![RMS spot size graph]
Lister microscope objective

\[ \sigma_{IA} = \sigma_{IB} = 0 \]

\[ \sigma_{II} = \phi_A^2 y_A^2 \sigma_{IA} + \phi_B^2 y_B^2 \sigma_{IIB} \]

\[ \sigma_{IIA} = -\frac{y_B^2}{(1-y_B)^2} \sigma_{IIB} \]

\[ \sigma_{IIIB} = \frac{1}{2} \]

\[ \sigma_{IIA} = -\sigma_{IIB} \]

\[ \sigma_{III} = (1-y_B) + \left(1 + y_B \sigma_{IIB}\right) = 0 \]

\[ S_{III}^* = S_{III} + 2 \cdot \bar{S} S_{II} + \bar{S}^2 S_I \]
Lister microscope objective

Practical solution

- Two identical doublets
- Spherical aberration and coma are corrected
- Astigmatism is small
- Telecentric
- Less vignetting

RMS wavefront error in waves
Aplanatic concentric meniscus lens

- Optical speed is increased by an N factor
Petzval portrait objective

- Chromatic aberration and spherical aberration corrected at each doublet
- Positive coma in the first doublet corrected with negative coma of aberration of the second doublet
- Negative astigmatism introduced by the negative coma of the second doublet to artificially flatten the field of view.

\[ S_{III}^* = S_{III} + 2 \cdot \bar{S}S_{II} + \bar{S}^2 S_I \]

\( f' = 144 \text{ mm}; \ F/3.7; \ \text{FOV=}+/- \ 16.5^\circ. \)

\( W_{222} / W_{220p} = -0.8 \)
Concentric lens

- Use of new glasses
- Reduced Petzval sum
- Nearly flat field
- Surfaces nearly concentric
- Limited by spherical aberration due to strong curvatures.

N-BAK1 and N-LLF6: Petzval radius -185 mm

\[ C_{Petzval} = \frac{1}{\rho_{Petzval}} = -\phi \cdot \sigma_w = -\left(\frac{\phi_1 + \phi_2}{n_1 n_2}\right) = -\frac{\phi}{\nu_1 - \nu_2} \left(\frac{v_1}{v_2}\right) \]
Anastigmatic lens

- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is negligible
- Combination of an old achromat and a new achromat
Anastigmatic lens

- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is negligible
- Combination of an old achromat and a new achromat

RMS spot size
Telephoto lens

Telephoto lens with BK7 and SF5 glasses. 
f’=100 mm, F/4, FOV=+/- 6.2⁰, TTL/F=0.8.

\[ S_{iii}' = S_{iii} + 2 \cdot SS_{ii} + S^2 S_i \]
\[ S_{iii}' = S_{iii} = \kappa^2 \phi_B \sigma_{iii} = \kappa^2 \phi_B \]
\[ \phi_A = -\phi_B \]

\[ \bar{W}_{131} = W_{311} + \frac{1}{2} \kappa \cdot \Delta \left\{ \vec{u}^2 \right\} \]

- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is not corrected
- Telephoto ratio=\( \text{TTL}/f \)
Telephoto lens

Telephoto lens with BK7 and F6 glasses. 
f' = 100 mm, F/4, FOV = ± 6.2°, TTL/F = 0.8

- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is also corrected
Reverse telephoto lens

Reverse telephoto lens with BK7 and SF5 glasses. $f' = 100$ mm, BFL = 200 mm, TTL = 324 mm, FOV = ±12⁰, F/4

- Corrected for spherical aberration, coma, astigmatism, and field curvature
- Distortion is small ~1.5%
- Large back focal length/distance
Ray diffractive law (1D)

\[ n' \sin(I') \cdot \Delta y = n \sin(I) \cdot \Delta y \]

\[ n' \sin(I') \cdot \Delta y - n \sin(I) \cdot \Delta y = \Delta \phi(y) \]

\[ n' \sin(I') - n \sin(I) = \frac{\Delta \phi(y)}{\Delta y} \rightarrow \frac{\partial \phi(y)}{\partial y} \]

\[ \frac{\partial \phi(y)}{\partial y} = n' \sin(I') - n \sin(I) \]
Grating linear phase change

\[ n \sin(I') - n \sin(I) = \frac{m \lambda}{d} \]

\[ \phi(y) = n \sin(I') y - yn \sin(I) = \frac{m \lambda}{d} y \]
Diffractive optics
high-index model

Start with the diffraction grating equation

\[ n'\sin(I') - n\sin(I) = \left[ n'\cos(I') - n\cos(I) \right] \cdot \frac{m\lambda}{n'\cos(I') - n\cos(I)} \cdot \frac{1}{d} \]

\[ n'\sin(I') - n\sin(I) = \left[ n'\cos(I') - n\cos(I) \right] \cdot \tan(\alpha) \]

\[ n'\{\sin(I') - \cos(I')\tan(\alpha)\} = n\{\sin(I) - \cos(I)\tan(\alpha)\} \]

\[ n'\{\cos(\alpha)\sin(I') - \cos(I')\sin(\alpha)\} = n\{\cos(\alpha)\sin(I) - \cos(I)\sin(\alpha)\} \]

\[ n'\{\sin(I' - \alpha)\} = n\{\sin(I - \alpha)\} \]
Diffractive optics
high-index model

\[ n'\{\sin(I' - \alpha)\} = n\{\sin(I - \alpha)\} \]

\[ \tan(\alpha) = \frac{m\lambda}{n'\cos(I') - n\cos(I)} \]

For large \( n \)'s then \( \alpha \) is negligible and we have:

\[ n'\sin(I) = n\sin(I) \]

Thus for high index diffraction becomes like refraction!
Diffractive lens
(n very large @ x=0)

Structural aberration coefficients of a thin lens (Stop at lens)

<table>
<thead>
<tr>
<th>Paraxial identities</th>
<th>Structural aberration coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = (n'-n) \cdot (c_1 - c_2) = (n'-n) \cdot \left( \frac{1}{R_1} - \frac{1}{R_2} \right) )</td>
<td>( \sigma_I = 3Y^2 + 1 )</td>
</tr>
<tr>
<td>( X = \frac{c_1 + c_2}{c_1 - c_2} = \frac{R_1 + R_2}{R_1 - R_2} )</td>
<td>( \sigma_{II} = -2Y )</td>
</tr>
<tr>
<td>( c_1 = \frac{1}{2} \left( \frac{n}{n-1} \right)(X+1) )</td>
<td>( \sigma_{III} = 1 )</td>
</tr>
<tr>
<td>( c_2 = \frac{1}{2} \left( \frac{n}{n-1} \right)(X-1) )</td>
<td>( \sigma_{IV} = 0 )</td>
</tr>
<tr>
<td>( w = u = -\frac{1}{2}(Y-1)(\phi \cdot y) )</td>
<td>( \sigma_{V} = 0 )</td>
</tr>
<tr>
<td>( w' = u' = -\frac{1}{2}(Y+1)(\phi \cdot y) )</td>
<td>( \sigma_L = \frac{1}{n_d} )</td>
</tr>
<tr>
<td>( \sigma_T = 0 )</td>
<td>( \sigma_T = 0 )</td>
</tr>
</tbody>
</table>
Mirror Systems

Structural aberration coefficients of a mirror.\n\( K = -\varepsilon^2 \) is the conic constant and \( \varepsilon \) is the eccentricity.

<table>
<thead>
<tr>
<th>Stop at surface</th>
<th>With stop shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_I = Y^2 + K )</td>
<td>( \sigma_I = Y^2 + K )</td>
</tr>
<tr>
<td>( \sigma_{II} = -Y )</td>
<td>( \sigma_{II} = -Y \left( 1 - S_e Y \right) + S_e \cdot K )</td>
</tr>
<tr>
<td>( \sigma_{III} = 1 )</td>
<td>( \sigma_{III} = \left( 1 - S_e Y \right)^2 + S_e^2 \cdot K )</td>
</tr>
<tr>
<td>( \sigma_{IV} = -1 )</td>
<td>( \sigma_{IV} = -1 )</td>
</tr>
<tr>
<td>( \sigma_V = 0 )</td>
<td>( \sigma_V = S_e \cdot \left( 1 - S_e Y \right) \left( 2 - S_e Y \right) + S_e^3 \cdot K )</td>
</tr>
</tbody>
</table>

\[
\overline{S}_\sigma = \frac{y_P \overline{y}_P \varphi}{2K} = \frac{\varphi \cdot s}{(Y - 1) \cdot \varphi \cdot s - 2n} = \frac{\varphi \cdot s'}{(Y + 1) \cdot \varphi \cdot s' - 2n'}
\]
Two mirror afocal system

Application to a two mirror Mersenne system

In this section we determine the aberration coefficients of a two mirror afocal system as shown in the figure. We normalize the system parameters and set $\kappa = 1$, $\Phi_1 = 1$, $y_1 = 1$, $\overline{y}_1 = 0$ and set the magnification to be $m$ and therefore $y_2 = m$. We have that $\overline{y}_2 = 1 - m$, $\Phi_2 = -1/m$ and therefore we can write for the conjugate factors and stop shifting parameters,

\[
\begin{align*}
Y_1 &= 1 \\
Y_2 &= -1 \\
\overline{S}_1 &= 0 \\
\overline{S}_2 &= \frac{y_2 \overline{y}_2 \Phi_2}{2\kappa} = \frac{m-1}{2}.
\end{align*}
\]
Using the formulas in the table the structural coefficients of each mirror are calculated as:

<table>
<thead>
<tr>
<th>Structural aberration coefficients</th>
<th>Mirror 1</th>
<th>Mirror 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_I$</td>
<td>$1 + \alpha_1$</td>
<td>$1 + \alpha_2$</td>
</tr>
<tr>
<td>$\sigma_{II}$</td>
<td>$-1$</td>
<td>$\frac{m+1}{2} + \frac{m-1}{2} \alpha_2$</td>
</tr>
<tr>
<td>$\sigma_{III}$</td>
<td>$1$</td>
<td>$\left(\frac{m+1}{2}\right)^2 + \left(\frac{m-1}{2}\right)^2 \alpha_2$</td>
</tr>
<tr>
<td>$\sigma_{IV}$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>$0$</td>
<td>$\frac{m-1}{2} \frac{m+1}{2} \frac{m+3}{2} + \left(\frac{m-1}{2}\right)^3 \alpha_2$</td>
</tr>
</tbody>
</table>
Finally the Seidel sums for the two mirror afocal system are given by:

<table>
<thead>
<tr>
<th>Seidel sums for two mirror afocal system</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_I = \frac{1}{4} \sigma_{I1} + \frac{1}{4} m^4 \left(-\frac{1}{m}\right)^3 \sigma_{I2} = \frac{1}{4} \left((1+\alpha_1) - m(1+\alpha_2)\right) )</td>
</tr>
<tr>
<td>( S_{II} = \frac{1}{2} \sigma_{II1} + \frac{1}{2} m^2 \left(-\frac{1}{m}\right)^2 \sigma_{II2} = \frac{1}{4} (m-1)(1+\alpha_2) )</td>
</tr>
<tr>
<td>( S_{III} = \sigma_{III1} + \left(-\frac{1}{m}\right) \sigma_{III2} = \frac{-1}{4} \frac{(m-1)^2}{m} (1+\alpha_2) )</td>
</tr>
<tr>
<td>( S_{IV} = \sigma_{IV1} + \left(-\frac{1}{m}\right) \sigma_{IV2} = \frac{m-1}{m} )</td>
</tr>
<tr>
<td>( S_V = 2\sigma_{V1} + 2\left(\frac{1}{m}\right)^2 \sigma_{V2} = \frac{1}{4} \frac{m-1}{m^2} \left(8 + 6(m-1) + (m-1)^2 (1+\alpha_2)\right) )</td>
</tr>
</tbody>
</table>
For the particular case of having the mirror as parabolic in optical shape we have that the Seidel sums simplify as:

<table>
<thead>
<tr>
<th>Seidel sums for afocal system using parabolas</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_I = 0$</td>
</tr>
<tr>
<td>$S_{II} = 0$</td>
</tr>
<tr>
<td>$S_{III} = 0$</td>
</tr>
<tr>
<td>$S_{IV} = -\frac{m-1}{m}$</td>
</tr>
<tr>
<td>$S_V = \frac{1}{2} \frac{m-1}{m^2}(3m+1)$</td>
</tr>
</tbody>
</table>

When a system is free from spherical aberration, coma, and astigmatism it is called an anastigmatic system.
Two mirror systems

\[ \varphi = 1 \]
\[ y_p = 1 \]
\[ K = 1 \]

\[ L = \frac{y_2}{y_2} \]
\[ M = \frac{1 - y_2}{y_2} \]

\[ \varphi_1 / \varphi = 1 \]
\[ y_1 / y_p = 1 \]
\[ S_{\sigma_1} = 0 \]
\[ Y_1 = 0 \]

\[ \varphi_2 / \varphi = (1 - M)(1 + ML) \]
\[ y_2 / y_p = \frac{1}{1 + ML} \]
\[ S_{\sigma_2} = \frac{1}{2} \frac{(1 - ML)}{1 + ML} \]
\[ Y_2 = \frac{1 + M}{1 - M} \]
Two mirror systems

Structural coefficients of a two mirror system.

Stop at primary mirror.

Object at infinity; \( m \) is the transverse magnification of the secondary mirror, and \( L \) is the ratio of the mirror separation to the back focal distance.

\[
\sigma_I = m^3 (1 + K_1) + \frac{(1 - m)^3}{1 + mL} \left( \frac{1 + m}{1 - m} \right)^2 + K_2
\]

\[
\sigma_p = -m^2 + (1 - m)^3 \left( \frac{1 + m}{1 - m} \right)^2 \frac{1 - 1(1 - m)L}{2} \frac{1 + m}{1 - m} + \frac{1(1 - m)L}{2} K_2
\]

\[
\sigma_m = -m + (1 - m)(1 + mL) \left( \frac{1 - 1(1 - m)L}{2} \frac{1 + m}{1 - m} \right)^2 + \frac{(1 - m)L}{2} K_2
\]

\[
\sigma_r = -m - (1 - m)(1 + mL)
\]

\[
\sigma_r = \frac{1}{(1 + mL)^2} \left( \frac{1 - 1(1 - m)L}{2} \frac{1 + m}{1 - m} \right)^2 \frac{1 - 1(1 - m)L}{2} \frac{1 + m}{1 - m} + \frac{(1 - m)L}{2} K_2
\]

Conic constants of Cassegrain type configurations corrected for spherical aberration

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Primary mirror</th>
<th>Secondary mirror</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cassegrain</td>
<td>( K_1 = -1 )</td>
<td>( K_2 = -\left( \frac{1 + m}{1 - m} \right)^2 )</td>
</tr>
<tr>
<td>Dall-Kirkham</td>
<td>( K_1 = -1 - \frac{(1 - m)(1 + m)^2}{m^2(1 + mL)} )</td>
<td>( K_2 = 0 )</td>
</tr>
<tr>
<td>Pressman-Carmichael</td>
<td>( K_1 = 0 )</td>
<td>( K_2 = \left( \frac{1 + m}{1 - m} \right)^2 - \frac{m^2(1 + mL)}{(1 - m)^2} )</td>
</tr>
<tr>
<td>Ritchey-Chretien</td>
<td>( K_1 = -1 - \frac{2}{Lm^2} )</td>
<td>( K_2 = -\left( \frac{1 + m}{1 - m} \right)^2 - \frac{2(1 + mL)}{L(1 - m)^2} )</td>
</tr>
</tbody>
</table>

Prof. Jose Sasian

OPTI 518
Cassegrain type

- True Cassegrain
- Ritchey-Chretien: aplanatic
- Dall-Kirkham: spherical secondary
- Pressman-Camichel; spherical primary
- Olivier Guyon (no diffraction rings)
Principal surface

In an aplanat working at $m=0$
the equivalent refracting surface
is a hemisphere.
Cassegrain’s principal surface

Since the equivalent refracting surface in a Cassegrain telescope is a paraboloid then the coma of that Cassegrain is the same of a paraboloid mirror with the same focal length.
Schmidt camera
Merssene afocal system
Anastigmatic
Confocal paraboloids
Paul-Baker system
Anastigmatic-Flat field

Anastigmatic
Parabolic primary
Spherical secondary and tertiary
Curved field
Tertiary CC at secondary

Anastigmatic, Flat field
Parabolic primary
Elliptical secondary
Spherical tertiary
Tertiary CC at secondary
Meinel’s two stage optics concept (1985)

Large Deployable Reflector
Second stage corrects for errors of first stage; fourth mirror is at the exit pupil.
Aplanatic, Anastigmatic, Flat-field, Orthoscopic (free from distortion, rectilinear, JS 1987)

Spherical primary telescope.
The quaternary mirror is near the exit pupil. Spherical aberration and Coma are then corrected with a single aspheric surface. The Petzval sum is zero. If more aspheric surfaces are allowed then more aberrations can be corrected.
Summary

• Structural coefficients
• Basic treatment
• Analysis of simple systems