

Double-curvature surfaces in mirror system design

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Abstract. The use in mirror system design of double-curvature surfaces such as sections of toroids or ellipsoids is discussed. Four design examples are presented that illustrate how double-curvature surfaces could be used to advantage in mirror system design. © 1997 Society of Photo-Optical Instrumentation Engineers.

Subject terms: optical design; reflective optics; aspheric mirrors; toroids.

Paper 31056 received May 28, 1996; revised manuscript received Aug. 26, 1996; accepted for publication Aug. 26, 1996.

1 Introduction

In mirror system design, as in lens design, the surfaces that are desirable to use are spherical in shape. Spherical surfaces, or better, sections of surfaces that are nearly spherical, are easy to manufacture given the great degree of symmetry of such geometrical forms. Very often, however, the performance of reflective systems is limited when only spherical surfaces are used and therefore, to improve performance, one or more mirrors are made aspheric in shape. The fabrication of aspheric mirrors that are axially symmetric has been in development since Newton¹ explained how he made the mirrors for his reflecting telescope. This technology has advanced to the point that very large and fast mirrors can be specified with the certainty that they can be manufactured.

In the case of unobstructed reflective system design, surfaces that are off-axis sections of axially symmetric aspheric surfaces are used to obtain the desirable image quality. These off-axis sections have only one plane of symmetry. By this we mean that there is only one plane contained by the normal at the physical center about which the surface has mirror symmetry. This reduced symmetry makes the fabrication of off-axis sections elaborated, expensive, and some times impractical. Therefore, the design of systems involving off-axis aspherics or other nonaxially symmetric surfaces is done typically with the concerns of expense, plausibility of fabrication, or because no other design alternative exists.

The lack of an axis of rotational symmetry complicates the fabrication of optical surfaces. However, surfaces that have two planes of symmetry such as toroids and cylinders can still be fabricated without major difficulty. Ophthalmic lenses and a large variety of cylindrical lenses commercially available are examples. Surfaces with two planes of symmetry have two principal sections with different radii of curvature and therefore we can call them double-curvature surfaces.

A useful way to envision a nonaxially symmetric surface is as being constituted by the superposition of a spherical surface and several deformations representing basic surface asphericities. Thus, the superposition of a spherical surface and a cylindrical (quadratic) deformation produces a double-curvature surface. An off-axis paraboloid can be

thought of as the superposition of a spherical surface, a cylindrical deformation, and a comatic (cubic) deformation. When the surface description includes cubic terms representing comatic asphericities, then the fabrication becomes considerably more difficult given that the double-plane symmetry is lost and only plane symmetry is retained.

There are several methods for generating double-curvature surfaces.²⁻⁸ The fundamental principle for fabricating such surfaces is that the relative movements between the surface and its grinding or polishing tool must be only translational; except for a rotation of 180 deg, which preserves symmetry, no rotational movements are allowed. The symmetry of the polishing movements must be in accord with the symmetry of the surface to be generated.

Since some moderately aspheric, double-curvature surfaces such as toroids and cylinders are not very difficult to manufacture, it is of interest to explore their use in the design of unobstructed reflective optical systems. A double-curvature surface enables the designer to control astigmatism at the center of the field of view and anamorphic distortion, leaving the curvature and the surface tilt to correct other aberrations. This paper presents some examples of systems with simplified fabrication through the use of one or more double-curvature surfaces that are weak aspheres. The purpose of the paper is to illustrate the use and advantages of some double-curvature surfaces in mirror system design.

2 Description of Double-Curvature Surfaces

For the purpose of defining what is meant by a double-curvature surface, some surfaces are considered in this section. From a mathematical point of view, the simplest surface is an elliptical paraboloid with surface sag z expressed in Cartesian coordinates xyz as

$$z = \frac{1}{2} \left(\frac{x^2}{R_s} + \frac{y^2}{R_t} \right), \quad (1)$$

where R_s and R_t are the principal radii of curvature. Cross sections of this surface parallel to the xy plane are ellipses and cross sections perpendicular to that plane are parabolic. It is assumed that an optical surface with this shape will

have its physical center at the pole of the surface $x=y=0$ and its limiting edge will be symmetrical with respect to the normal to the pole. The plane that coincides with the principal section of vertex radius R_s is called the sagittal plane, and the plane that coincides with the section of vertex radius R_t is called the tangential plane; we assume that R_t is larger than R_s . These two planes, the sagittal and the tangential, are perpendicular to each other. Since Eq. (1) is even in the powers of the x and y coordinates, the surface has double-plane symmetry. This is with respect to the sagittal and tangential planes.

From a fabrication point of view, perhaps the simplest double-curvature surface is a toroid, described as

$$z = R_t - \{[R_t - R_s + (R_s^2 - x^2)^{1/2}]^2 - y^2\}^{1/2}. \quad (2)$$

In Eq. (2) R_t and $R_t - R_s$ are the radii of the main cross sections of the toroid. The sag of the toroid to fourth-order approximation is

$$z = \frac{1}{2} \left(\frac{x^2}{R_s} + \frac{y^2}{R_t} \right) + \frac{1}{8} \left(\frac{x^4}{R_s^3} + \frac{2x^2y^2}{R_t^2R_s} + \frac{y^4}{R_t^3} \right). \quad (3)$$

Since a toroid has axial symmetry, a polishing tool that is moved around the axis of symmetry will have uniform contact. This is a feature that simplifies the fabrication of a toroid section. When the radius R_t approaches infinity, the toroid becomes a cylinder. In this case, a polishing tool will keep uniform contact when it is moved along the two main cross sections.

From an optical testing point of view, the simplest double curvature is the ellipsoid, as described by

$$\frac{x^2}{R_s^2} + \frac{y^2}{R_tR_s} + \frac{z^2}{R_s^2} = 1. \quad (4)$$

The sag of this surface described about the oblate section and to fourth-order approximation is

$$z = \frac{1}{2} \left(\frac{x^2}{R_s} + \frac{y^2}{R_t} \right) + \frac{1}{8} \left(\frac{x^4}{R_s^3} + \frac{2x^2y^2}{R_s^2R_t} + \frac{y^4}{R_sR_t^2} \right). \quad (5)$$

The testing of this surface is simple because it possesses two foci where it can be null tested. Since the difference between the elliptical paraboloid, the toroid, and the ellipsoid is of fourth order, from a paraxial point of view, all these surfaces can be considered the same and tested at the foci of the corresponding ellipsoid with minimum aberration residual.

The off-axis angle with respect to the pole normal at which the surface will not introduce astigmatism is found by means of Coddington equations applied to reflection:

$$\frac{1}{s'} - \frac{1}{s} = \frac{2 \cos(\theta)}{R_s}, \quad \frac{1}{t'} - \frac{1}{t} = \frac{2}{R_t \cos(\theta)}, \quad (6)$$

where s and t and s' and t' are the object and image distances for rays in the principal sections. By subtracting one equation from the other and equating $s=t$ and $s'=t'$, the cosine of the angle θ is found to be

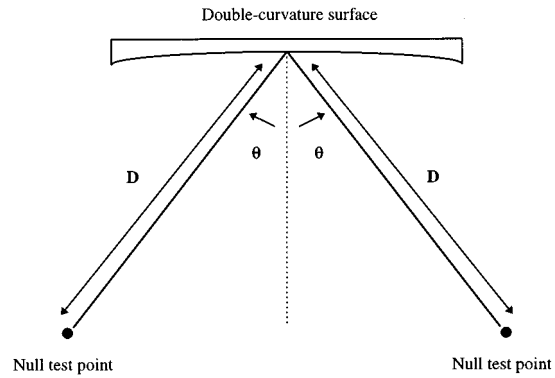


Fig. 1 Geometry illustrating the angle θ in relation to the mirror normal and the incident and reflected rays. The distance D is also shown.

$$\cos(\theta) = \left(\frac{R_s}{R_t} \right)^{1/2}. \quad (7)$$

This expression is independent of the object and image distances. Therefore, no astigmatism is generated irrespective of the object and image location along the ray in the tangential plane that is incident at the surface vertex and at an angle θ ; this geometry is illustrated in Fig. 1. The distance D from the surface vertex to the foci where the double-curvature surface can be tested is

$$D = \frac{R_s}{\cos(\theta)}. \quad (8)$$

3 Optical Systems with Double-Curvature Surfaces

In this section, four unobstructed reflective systems are discussed, emphasizing the use of double-curvature surfaces. Central to the discussion is the way the imaging aberrations are corrected. The aberrations that affect the systems of interest have been discussed elsewhere⁹ and the main ones are coma, astigmatism, and spherical aberration at the field center, linear astigmatism and coma, anamorphic distortion, and field tilt for field positions away from the field center. In all the examples presented here, the aperture stop coincides with the primary mirror.

3.1 Simulation of an Off-Axis Paraboloid

The off-axis paraboloid shown in Fig. 2 and specified in Table 1 is a useful optical component since it can collimate or focus a beam and enable at the same time an unobstructed path. When properly aligned, the off-axis paraboloid provides an imaging field that is aberration free at its center and affected mainly by linear astigmatism and coma across the field extent, as shown in Fig. 3. This figure shows a composite spot diagram for an off-axis paraboloid working at $f/8$ and an aperture of 39 mm. The spot diagrams correspond to the field center and to eight positions around a semifield of 1 deg. These spot diagrams were generated with respect to a plane perpendicular to the ray reflected at the physical center of the mirror or zero aperture, zero field ray. It is of interest to note that the astig-

Table 1 Specifications of the off-axis paraboloid.

Vertex radius	623 mm
Diameter	38 mm
Off-axis distance (from vertex to physical center)	51 mm

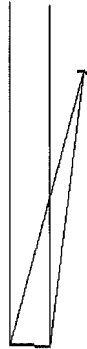


Fig. 2 Parallel beam of light is focused by an off-axis paraboloid mirror.

SPOT DIAGRAM

F/number 8.03

Aperture 38.97 mm

Scale 0.0576 mm

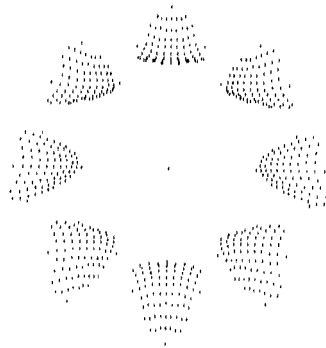


Fig. 3 Composite spot diagram illustrating the performance of the off-axis paraboloid. The dashed line represents the scale length to determine spot size.

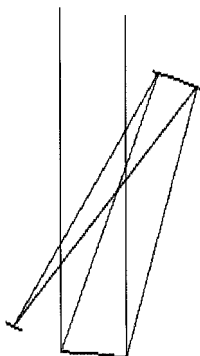


Fig. 4 Two double-curvature mirrors simulating an off-axis paraboloid.

Table 2 Specifications of the double-curvature mirror system (millimeters).

Mirror	Sagittal Radius	Tangential Radius	Spacing	Tilt Angle
Primary	-1000	-1017.332	-192.754	7.5 deg
Secondary	1000	1017.332		7.5 deg

matic focal lines that are produced when an object at infinity is imaged with the system stop at the mirror, are at angles I and $-I$ with respect to this zero field, zero aperture ray. Thus the so-called medial astigmatic image surface is perpendicular to such a ray.

Many unobstructed reflective systems benefit from using an off-axis paraboloid. However, the fabrication of an off-axis section of a paraboloid is elaborated due to the cubic or comatic asphericities involved. Thus, it is of interest to investigate the replacement of such parabolic mirror with two double-curvature surfaces; the replacement with a single double-curvature surface has been previously suggested.^{10,11} Naturally, a double-curvature surface provides control over astigmatism, but since no cubic asphericities are involved, there is no control over coma. Using two double-curvature mirrors it is possible to control coma and line coma aberrations by having a symmetrical arrangement in which the coma of one mirror cancels the contribution from the other.

The two-mirror system illustrated in Fig. 4 is formed with two double-curvature mirrors; its construction data are given in Table 2. These mirrors have the same radii of curvature and are tilted the same angle with respect to the zero aperture, zero field ray called the optical axis ray (OAR). Thus, this mirror configuration is symmetrical so that its performance is independent of the mirror in which light is reflected first. By the proper selection of radii of curvature this system is corrected for astigmatism at the field center and for anamorphic distortion. In addition, by proper selection of the mirror separation, coma at the field center is corrected, and interestingly enough, line coma is also corrected (line coma produces an image that is shaped like a straight line). The composite spot diagram in Fig. 5 shows the performance of this two-mirror system at the

SPOT DIAGRAM

F/number 8.00

Aperture 38.97 mm

Scale 0.0573 mm

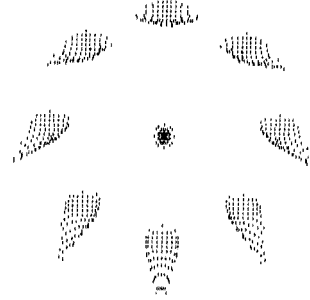


Fig. 5 Composite spot diagram illustrating the performance of the two double-curvature mirrors.



Fig. 6 Off-axis paraboloid and a double-curvature mirror.

field center and eight positions around a semifield of 1 deg. The spot diagram at the field center reveals the presence of spherical aberration, which has not been corrected. This aberration could be corrected by including a fourth-order asphericity in one or both of the mirrors.

With respect to the equivalent off-axis paraboloid mirror of Fig. 2, that the angle of tilt of the off-axis paraboloid is smaller in comparison to the angle of tilt of the mirrors in the compound double-curvature system in Fig. 4. For a proper system comparison, the distance from the focal point to the incoming beam must be the same. To fulfill this requirement, the angle of tilt of the off-axis paraboloid must be reduced. Another difference between these systems is that the compound system is shorter and that it suffers a lesser amount of linear astigmatism. In the compound system, the sagittal astigmatic field is almost perpendicular to the OAR and the distance from the second mirror to the focal point is equal to the mirror spacing.

The compound two-mirror system may be a good substitute for the off-axis paraboloid in systems where cost is more important than simplicity or packaging. The use of two double-curvature mirrors results in better field performance with respect to the performance of the off-axis paraboloid.

3.2 Field Improvement of an Off-Axis Paraboloid

As mentioned, the field performance of an off-axis paraboloid mirror is degraded mainly by linear astigmatism. In optical systems where this aberration is objectionable, it is possible to correct it by the use of a double-curvature mirror. This is illustrated in Fig. 6, where the image formed by an off-axis paraboloid is relayed at unit magnification by a double-curvature mirror. In this system, by construction there is no astigmatism at the field center or anamorphic magnification. Since the double-curvature mirror is used at

SPOT DIAGRAM

F/number 8.85

Aperture 38.97 mm

Scale 0.0577 mm

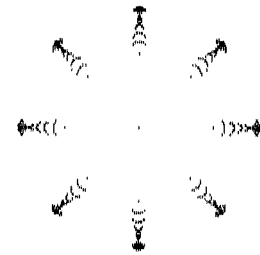


Fig. 7 Composite spot diagram of the improved off-axis paraboloid.

negative unit magnification, there is also no on-axis coma introduced. However, this mirror contributes the same amount of linear astigmatism that the paraboloid generates but with opposite sign; so the result is that the combined system is corrected for linear astigmatism. In addition, the image plane is perpendicular to the OAR, which is the ray passing through the physical center of the mirrors. The specifications for this system are given in Table 3. The field performance is illustrated in Fig. 7 using a composite spot diagram for the field center and eight positions around a semifield of 1 deg. In this system, the image quality is mainly degraded by linear coma and quadratic astigmatism. Note also that the diameter of the secondary mirror is equal to the diameter of the primary when no field of view is considered; for a finite field of view, the diameter of the mirrors will increase depending on where the system stop is placed.

3.3 Wide Field Three-Mirror System

The Schwarzschild two-mirror, flat-field, anastigmatic telescope is capable of covering wide fields of view with sharp imagery. However, this system suffers from both obscuration and stray light problems. In its unobstructed version, these two problems are solved at the expense of having at least one of the mirrors as an off-axis section. The fabrication of an off-axis mirror is elaborated, as mentioned, and it is desirable to substitute this off-axis mirror with two double-curvature mirrors, as illustrated in Fig. 8.

In this design, all the mirrors are double-curvature mirrors and the contribution to on-axis astigmatism from each

Table 3 Specifications of the off-axis paraboloid and double-curvature mirror system (millimeters).

Mirror	Sagittal Radius	Tangential Radius	Spacing	Tilt Angle
Primary	-623.923	-628.231	-624	4.75 deg
Secondary	311	311.575		-2.38 deg

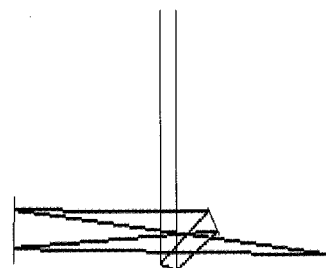


Fig. 8 Three-mirror design to simulate a two-mirror Schwarzschild telescope.

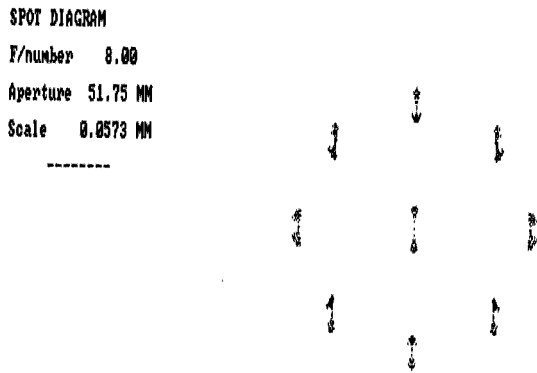


Fig. 9 Composite spot diagram illustrating the performance of the three-mirror system.

mirror is avoided by the proper selection of the radii of curvature according to Eq. (7). This correction also ensures that the system is free from anamorphic distortion. The primary and secondary mirrors replace the primary mirror of an unobstructed Schwarzschild system and contribute coma at the field center and linear astigmatism. These aberrations exactly cancel the corresponding ones generated by the concave tertiary mirror. The Seidel aberrations are controlled and nullified by the choice of fourth-order asphericities (conic constants), mirror spacing, and sagittal curvatures. The result is a system that, except for distortion, is corrected for all fourth-order aberrations and can produce sharp imagery over a wide and flat field of view. The performance is illustrated in Fig. 9 using spot diagrams at the center of the field and around a semifield of view of 2.5 deg. As can be appreciated, the image quality is uniform over the field of view and is limited by high-order aberrations. Table 4 gives the specifications and characteristics of such a three-mirror system. The conic constants associated with the primary and secondary mirrors specify an axially symmetric deformation in the usual way done for conic mirrors. Note that all the mirrors have two planes of symmetry, which is a feature that makes their fabrication simpler.

3.4 Unobstructed Telescope

Telescope objectives are an excellent application of unobstructed reflecting systems. The advantages they have over achromatic and apochromatic objectives are the lower price

Table 4 Specifications of the three-mirror system (millimeters).

Mirror	Sagittal Radius	Tangential Radius	Spacing	Tilt Angle
Primary	1500	1757.359	-200	22.5 deg
Secondary	-3691.708	-4279.945	625	21.58 deg
Tertiary	-1125	-1128.627		3.30 deg
Primary conic constant:	11.63			
Secondary conic constant:	0.16			

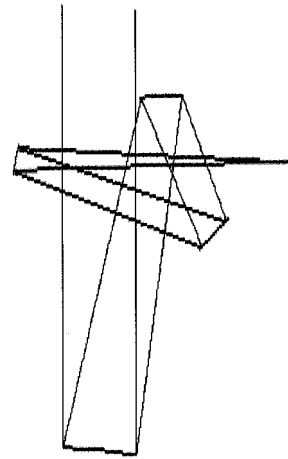


Fig. 10 Four-mirror telescope with one mirror being a folding flat and another a double-curvature mirror.

and the possibility of having large diameters. The first unobstructed telescope was probably the Schiefspiegler,¹² which consisted of two spherical mirrors, one concave and one convex, arranged tilted to avoid the boring of the primary and the beam obstruction. The performance of the Schiefspiegler is limited by coma at the field center, which leads to telescopes with a slow speed of $f/24$ and apertures in the order of 100 mm. An improvement over the Schiefspiegler is the tri-Schiefspiegler telescope, which uses a weak tertiary spherical mirror to improve the image quality at the field center. However, linear astigmatism and field tilt limit the performance of this design to speeds around $f/20$ and diameters of 300 mm. Except for the primary mirror, the secondary and tertiary mirrors of this design are spherical. The primary mirror can be aspheric (axially symmetric) to control spherical aberration. The main design trust in these designs is to specify spherical surfaces, or nearly spherical surfaces to ease their fabrication.

One way to improve these unobstructed telescopes and make them faster and more compact is to use a double-curvature mirror. In this way, an additional degree of freedom is gained to control optical aberrations. The design presented in Fig. 10 uses a quaternary mirror that has a double curvature, as specified in Table 5. The short radius of curvature of this mirror coincides with the plane of sym-

Table 5 Specifications of the four-mirror telescope (millimeters).

Mirror	Sagittal Radius	Tangential Radius	Spacing	Tilt Angle
Primary	-10353.26	-10353.26	-2438.859	5 deg
Secondary	-10353.26	-10353.26	1000	-15.5 deg
Tertiary	flat	flat	-1438.859	24 deg
Quaternary	8967.396	8238.862	1888.527	-10 deg
Primary conic constant:	-1.92			
Secondary conic constant:	-17			

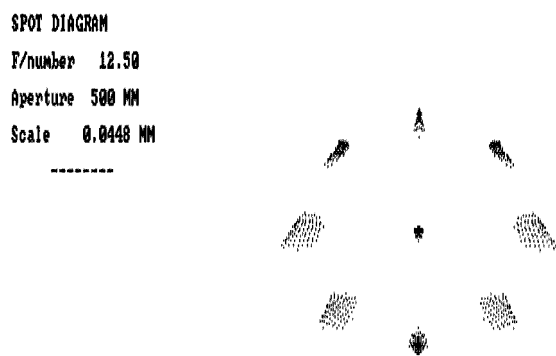


Fig. 11 Composite spot diagram illustrating the performance of the telescope.

metry of the telescope. The tertiary mirror is flat to make the configuration easier to package and mount. The primary and secondary mirrors are axially symmetric hyperboloids; the shapes of these mirrors correct spherical aberration and linear coma, making the telescope aplanatic. The performance of this telescope is shown in Fig. 11 using spot diagrams for the center of the field and eight positions around a field of $1/2$ deg. By using a double-curvature mirror the speed of the telescope has been decreased to $f/12.5$ and the aperture has been increased to 500 mm. In addition, the image plane has been made perpendicular to the OAR of the telescope.

4 Conclusion

The design examples presented in this paper illustrate how double-curvature surfaces could be used to avoid the use of optical surfaces with greater fabrication difficulty such as off-axis paraboloids. These design examples prove that in some applications good imaging performance can be obtained without the need of specifying complex surface shapes.

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