Optical Engineering

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Jose Sasian, Weichuan Gao, Yufeng Yan, "Method to design apochromat and superachromat objectives," *Opt. Eng.* **56**(10), 105106 (2017), doi: 10.1117/1.OE.56.10.105106.

Optical Engineering 56(10), 105106 (October 2017)

Method to design apochromat and superachromat objectives

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Abstract. We present a simple method to design apochromat and superachromat objectives. The chromatic focal shift is used to determine glass combinations that yield three and four crossings in the chromatic focal shift curve. The method can be extended to design superachromats with more than four crossings. [DOI: 10.1117/1.OE.56.10.105106]

Keywords: achromatic lens; apochromatic lens; superachromat lens; secondary spectrum; tertiary spectrum.

Paper 171415 received Sep. 8, 2017; accepted for publication Oct. 6, 2017; published online Oct. 31, 2017.

1 Introduction

There are several papers in the optics literature on the design of apochromatic and superachromat objectives. One of the earlier methods to design an apochromatic lens objective[1] selects glasses according to the triangle they form in the glass P–V diagram. Other diagrams for glass selection have been devised with advantages over the P–V diagram.[2] The concept of a superachromatic objective was introduced by Herzberger and McClure[3] in which an objective has more than three crossings in the chromatic focal shift curve. Some papers use glass dispersion to provide formulas for glass selection. A number of papers provided computer-aided methods to design apochromats and superachromats. Some of these use chromatic coordinates.[4–14]

Currently, the systematic design of a superachromatic objective is an elaborated process as most methods in papers involve glass dispersion equations and computer optimization. Finding superachromatic solutions can be more complicated in the infrared or UV where, for example, the glass dispersion values change.

In this paper, we provide a simple and powerful method to design apochromat and superachromat objectives. Our method separates an apochromat into two achromatic objectives, which have secondary spectra that cancel and separate four-crossing superachromats as two apochromats, which have tertiary spectra that cancel. Our method relies on the ability of lens design programs to plot the chromatic focal shift curve. The method, in addition, provides insight into the ways superachromat objectives can be formed.

2 Design of a Thin Lens Apochromat

We separate an apochromatic objective into two achromatic doublets. Because the combination of these two doublets forms an apochromat, it follows that their secondary spectra must cancel. According to the theory, the secondary spectrum for each achromatic doublet is given by[2]

$$\partial_\lambda W_{020} = \frac{1}{f}\frac{(P_a - P_b)}{(V_a - V_b)}\frac{y^2}{2}, \tag{1}$$

where the algebraic signs of the difference between $P_a - P_b$ and f provide the sign of $\partial_\lambda W_{020}$, and the secondary spectrum is expressed in wave aberration terms. As is well known, if the glass partial dispersions are equal then a two-glass apochromat solution is obtained. In Eq. (1), f is the doublet focal length, P_a and P_b are the glasses' partial dispersion ratios, V_a and V_b are the glasses V-numbers, and y is the first-order marginal ray height.

The sign of the secondary spectrum also depends on the sign of the doublet focal length. Thus, an apochromatic objective with positive focal length can be obtained as types I, II, or III according to the sign of the focal length of the achromatic doublets used and the sign of their respective secondary spectra. Table 1 summarizes these doublet type combinations.

The design of an apochromatic objective requires plotting the chromatic focal shift for doublets made of different types of glasses. The focal shift is plotted for the desired spectral range as shown for some glass combinations in Fig. 1.

Once the sign and the magnitude of the chromatic focal shift are known for a given set of glasses, then the focal length of one doublet is scaled as to satisfy the equation (see Appendix)

$$\frac{f_1}{f_2} = -\frac{SS_1}{SS_2}, \tag{2}$$

so when the doublets are combined their secondary spectra for one wavelength are canceled. In Eq. (2), f_1 and f_2 are the focal lengths of the constituent doublets and SS_1 and SS_2 are the peak values of the secondary spectrum when the doublets have the same focal length.

Figure 2 shows the three types of apochromatic objective solutions using the doublets in Fig. 1. The curvature of the lens elements will depend on the glass V-number difference in the doublets. In turn, the amount of spherochromatism aberration will depend on the surface curvatures. Thus, lens elements with weak curvatures in their surfaces are desirable, which requires large V-number differences.

The effect of scaling in Eq. (2) is shown in Fig. 3 with a type I apochromat consisting of two doublets made with

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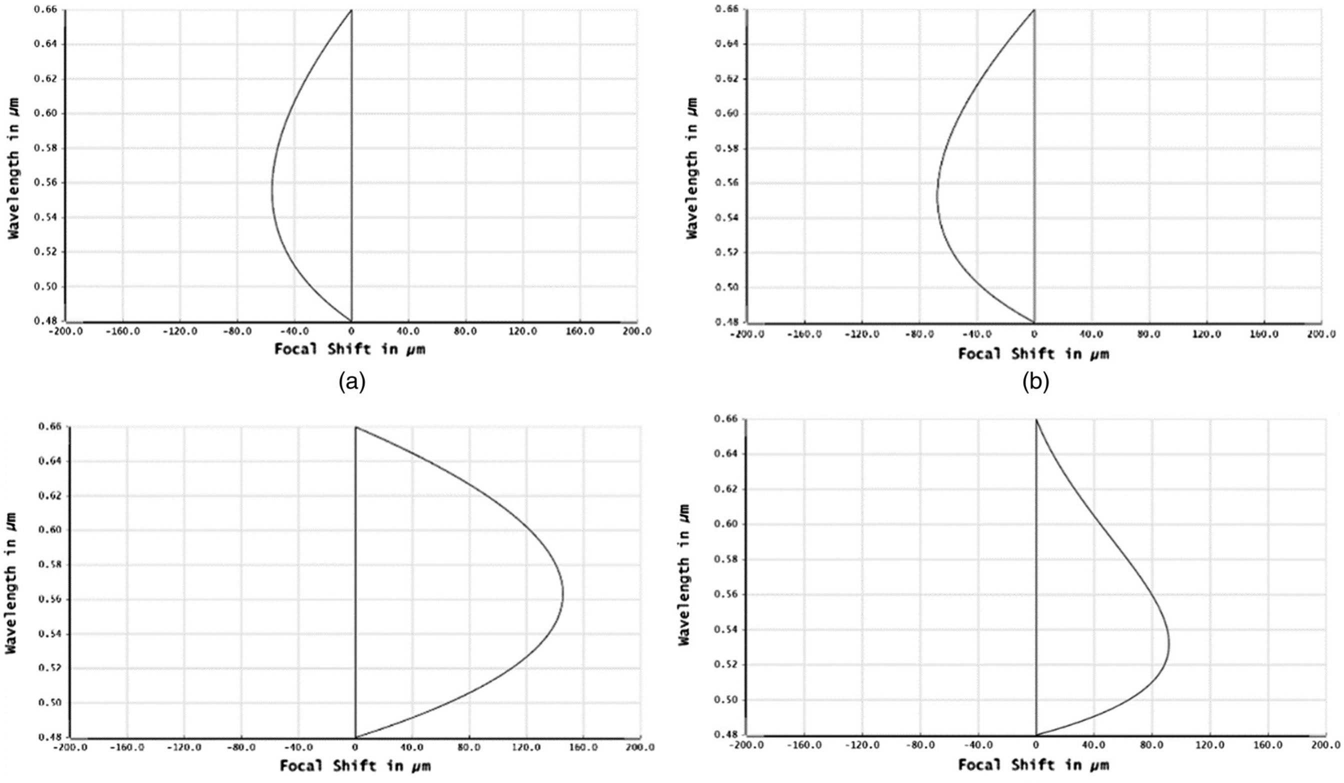
Table 1 Types of apochromatic objectives formed with two achromatic doublets.

	Sign of focal length of doublet A	Sign of focal length of doublet B	Sign of secondary spectrum A	Sign of secondary spectrum B
Type I	+	+	+	−
Type II	+	−	−	−
Type III	+	−	+	+

N-BK7/N-PSK57 and N-BK7/N-LAF33, respectively. As the scaling factor changes, the third crossing moves along the axis of wavelength and eventually the apochromat becomes an achromat.

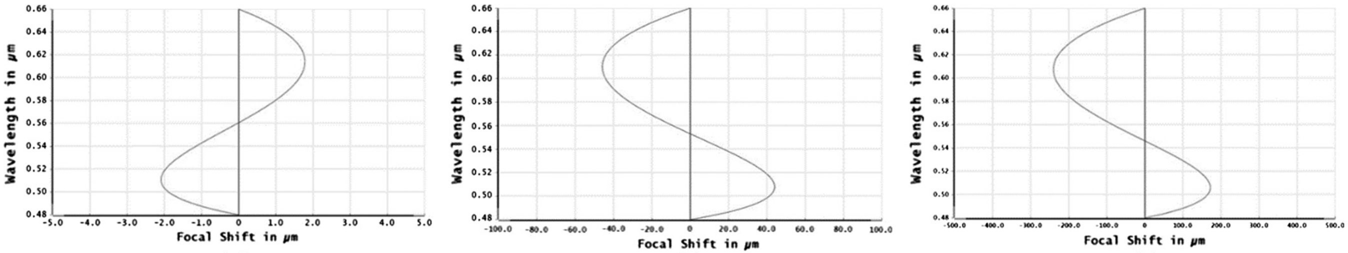
3 Design of a Thin Lens Superachromat

We separate a superachromatic objective with four crossings in the chromatic focal shift curve into two apochromatic triplets. Because the combination of these two triplets forms a superachromat, it follows that their tertiary spectra must cancel for two wavelengths. Each triplet is adjusted to



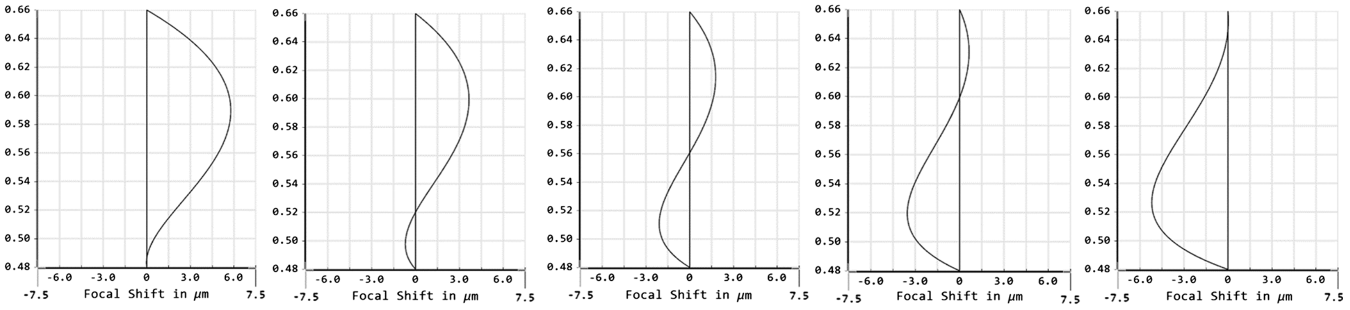
(a) (b) (c) (d)

Fig. 1 Chromatic focal shift for achromatic doublets with a focal length of 100 mm. The peak focal shift *SS* is given in micrometers according to glass. Achromat glass selections are (a) N-BK7/N-LAF33 ($SS = -55.7$), (b) N-BK7/N-BAF52 ($SS = -67.7$), (c) N-BK7/N-PSK57 ($SS = 145.7$), and (d) N-LLF1/N-LAF33 ($SS = 91.5$).



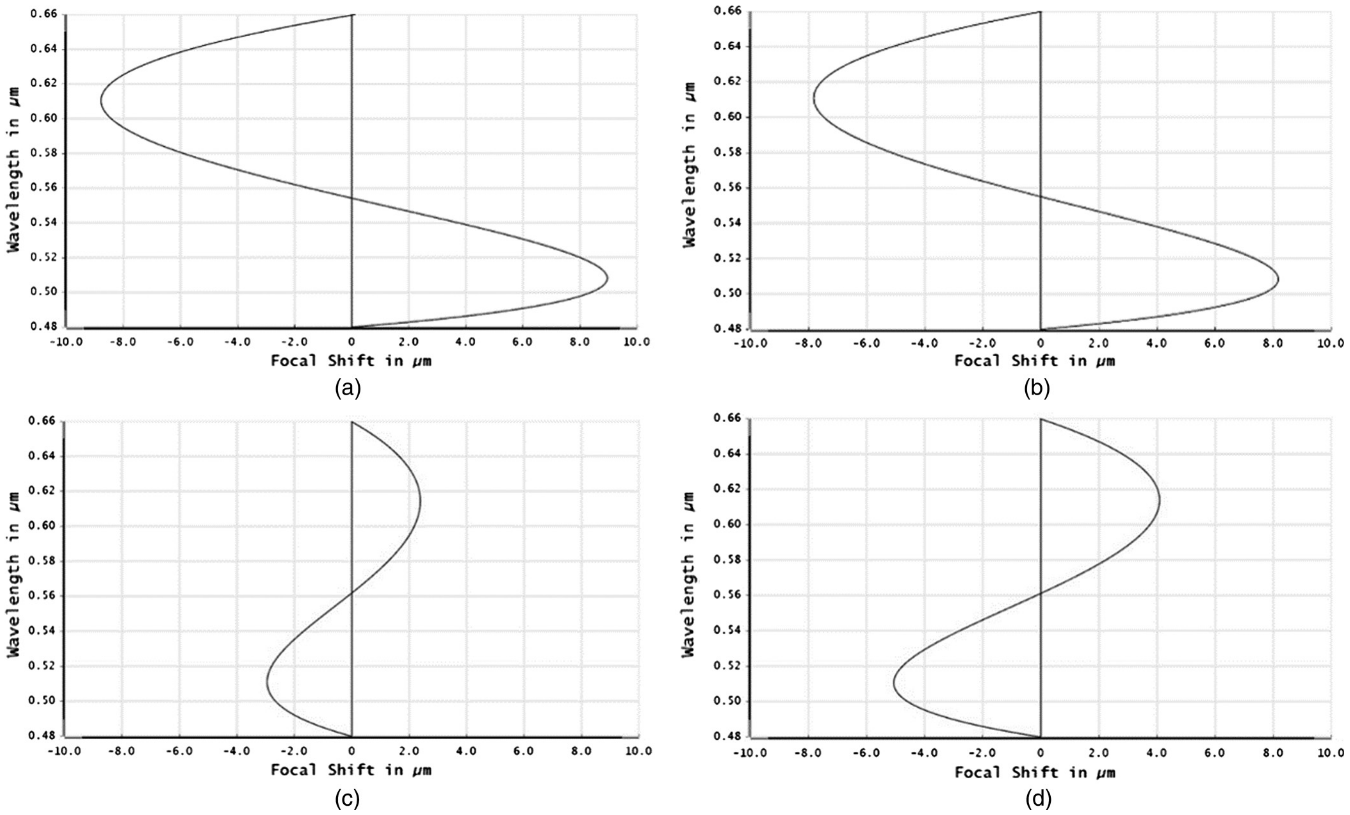
(a) (b) (c)

Fig. 2 Chromatic focal shift of apochromatic objectives built with doublets from Fig. 1. (a) Type I, N-BK7/N-PSK57 ($f = 261.6$ mm) and N-BK7/N-LAF33 ($f = 100$ mm); (b) type II, N-BK7/N-LAF33 ($f = 100$ mm) and N-BK7/N-BAF52 ($f = -121.5$ mm); and (c) type III, N-LLF1/N-LAF33 ($f = 100$ mm) and N-BK7/N-PSK57 ($f = -159.2$ mm).



(a) (b) (c) (d) (e)

Fig. 3 Chromatic focal shift of a type I apochromatic objectives made using N-BK7/N-PSK57 and N-BK7/N-LAF33 doublets, and where the focal length of the N-BK7/N-PSK57 doublet is scaled to (a) 220 mm, (b) 240 mm, (c) 261.6 mm, (d) 280 mm, and (e) 300 mm, respectively.



(a) (b) (c) (d)

Fig. 4 Chromatic focal shift for apochromatic triplets with a focal length of 100 mm. Glass selections are (a) N-LLF1/N-LAF33/N-BK7, (b) N-BAF52/N-BK7/N-LAF33, (c) N-LAF33/N-BK7/N-PSK57, and (d) N-F2/N-BK7/N-PSK57.

have at least two intermediate wavelengths with the same magnitude of tertiary spectrum but with opposite sign. Then when combining the two triplets, four crossings in the chromatic focal shift take place; two from the wavelength extremes and two from the intermediate wavelengths. The adjustment is carried out by slightly changing the ratio of the focal lengths of the two achromatic doublets forming each triplet.

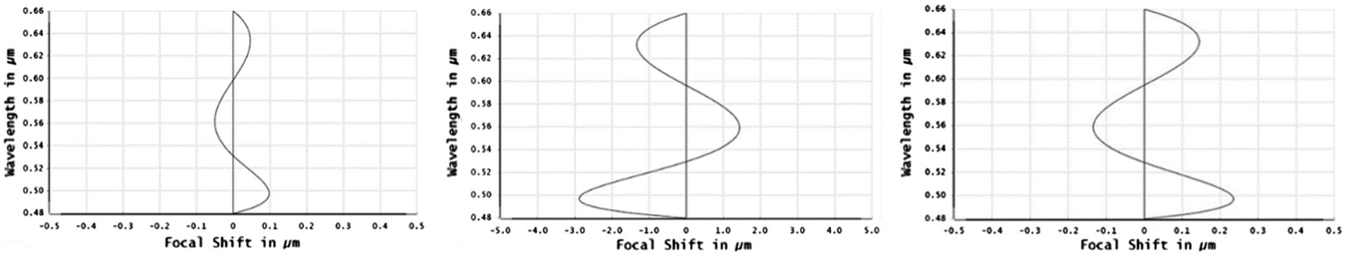
According to the sign of the focal length of the triplets and the sign of their tertiary spectra, three types of solutions are also possible. The chromatic shift curves for four apochromatic triplets are shown in Fig. 4. Figure 5 shows the three types of superachromat solutions from the apochromats used for Fig. 4.

As can be noted, two glasses in the triplets are common so that the superachromat is synthesized to have four lens elements with four different glasses. The solution does not depend on the lens elements shape and this allows the designer to merge lens elements with the same glass into a single lens element.

4 Example in the Infrared

Table 2 provides the chromatic focal shift for several materials in the range of 3- to 5-μm in the infrared.

Four achromatic doublets are chosen with the materials ZnSe/Ge, IRG25/Ge, AMTIR1/Ge, and AMTIR/ZnSe, respectively. Then two apochromatic triplets are formed with material combinations of ZnSe/Ge/IRG25 and AMTIR1/



(a) (b) (c)

Fig. 5 Chromatic focal shift of superachromatic objectives built with triplets in Fig. 3. (a) Type I, N-LLF1/N-LAF33/N-BK7 ($f = 339$ mm) and N-LAF33/N-BK7/N-PSK57 ($f = 100$ mm); (b) type II, N-BAF52/N-BK7/N-LAF33 ($f = 100$ mm) and N-LLF1/N-LAF33/N-BK7 ($f = -111.5$ mm); and (c) type III, N-LAF33/N-BK7/N-PSK57 ($f = 100$ mm) and N-F2/N-BK7/N-PSK57 ($f = -171.1$ mm).

Table 2 Secondary spectrum for several achromatic doublets according to material.

3- to 5-μm MWIR band ($f = 100$ mm)		
Material 1	Material 2	Peak value of secondary spectrum (μm)
Si	Ge	−3.876
Si	ZnSe	235.666
Si	AMTIR1	208.918
Si	IRG25	105.556
ZnSe	Ge	−201.470
IRG25	Ge	−109.826
AMTIR1	Ge	−92.610
AMTIR1	ZnSe	285.973
ZnSe	IRG25	−1342.340
AMTIR1	IRG25	−48.668

ZnSe/IRG25. Finally, these two triplets are combined to form a superachromat as shown by the chromatic focal shift curve in Fig. 6.

Table 3 provides the lens prescription for the IR superachromat objective. With lens thicknesses added, the IR superachromat objective is made aplanatic and offers near-diffraction-limited image quality across ±2- deg field of view.

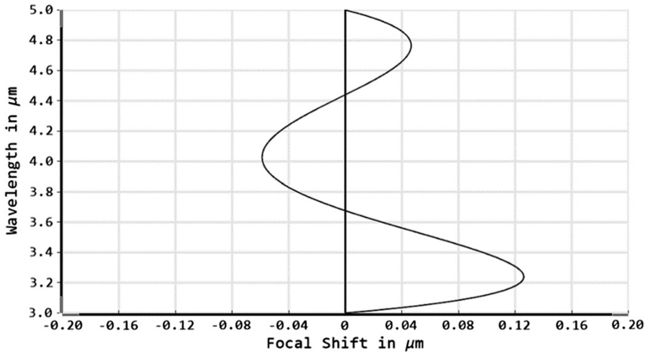


Fig. 6 Chromatic focal shift curve with material combination of ZnSe/Ge/IRG25/AMTIR1.

Table 3 Lens prescription for an IR aplanatic superachromat.

Objective in the 3- to 5-μm MWIR band ($f = 200$ mm, $F/5$)		
Radius (mm)	Thickness (mm)	Material
161.581	3	ZnSe
69.582	1	
72.787	5	AMTIR1
Infinity	2	
−205.893	4	IRG25
−188.255	1	
−2021.492	3	Germanium
797.646	188.860	

The design method can be summarized as follows:

Step 1: Select two achromatic doublets

Select two achromatic doublets from a look-up table like Table 2 with different secondary spectra SS_1 and SS_2. Such table can be generated for any design wavelength band using a preferred material collection. Combinations of weaker elements tend to generate less residual spherochromatism and monochromatic aberrations.

Step 2: Scale one doublet and combine

Using Eq. (1), selected achromats in the look-up table can be combined to form an apochromatic lens. Following the same formula, two apochromats can be further combined to form a superachromat. Lens elements with the same glass type are combined into a single lens element.

Step 3: Optimize for thickness and monochromatic aberrations

Lens thickness can be added and the chromatic correction is restored by changing the relative optical power of the constituent achromats or apochromats. By maintaining the individual power of each element, the chromatic correction is kept while the monochromatic aberrations can be corrected using the shape of the lens elements. The objective can now be scaled to obtain the final objective focal length. The sequence

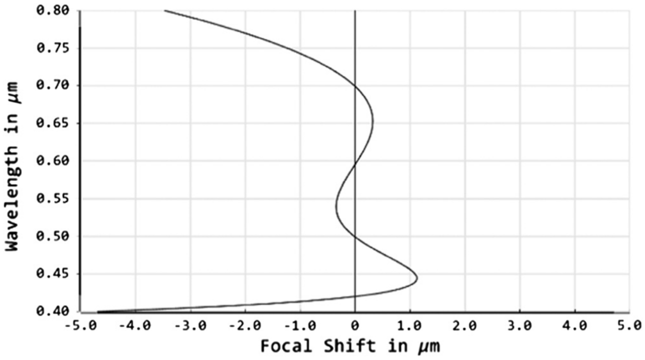


Fig. 7 Chromatic focal shift curve for three glass superachromat (N-FK51, N-KZFS4, and N-SF15).

of the materials can be optimized to minimize aberration residuals.

5 Three Glass Superachromat

It is possible to build a four-crossing superachromat with only three materials rather than four. The secondary spectrum SS_1 for an achromatic doublet about a central wavelength λ_0 can be written to fourth order as

$$SS_1 = A_1(\lambda - \lambda_0)^2 + B_1(\lambda - \lambda_0)^3 + C_1(\lambda - \lambda_0)^4, \tag{3}$$

and for a second doublet as

$$SS_2 = A_2(\lambda - \lambda_0)^2 + B_2(\lambda - \lambda_0)^3 + C_2(\lambda - \lambda_0)^4, \tag{4}$$

where A, B, and C are coefficients.

When combining the doublets to form an apochromatic objective, the tertiary spectrum can be written as

$$SS_3 = B_3(\lambda - \lambda_0)^3 + C_3(\lambda - \lambda_0)^4. \tag{5}$$

Around the central wavelength λ_0 and for a reduced bandwidth, the cubic term may dominate and for that bandwidth the objective is an apochromat. However, over a larger bandwidth the fourth-order term might dominate and then the objective becomes a superachromat.

Using the glasses N-FK51, N-KZFS4, and N-SF15, an apochromat objective can be designed for the bandwidth of 0.5 to 0.7 μm. However, when the bandwidth is increased from 0.4 to 0.8 μm, the triplet becomes a superachromat given that the fourth-order term in the spectrum dominates. This is shown in the chromatic focal shift in Fig. 7.

6 Designing an Athermal Achromatic Triplet

The method can be extend to design achromatic triplets that are athermal. For this a table of equivalent optothermal coefficients γ would be built with achromatic doublets sharing a common glass and with the same focal length. Then to obtain an achromatic triplet that is athermal, one would select two achromats with different optothermal coefficients, γ_1 and γ_2, and form the triplet with the two doublets having a ratio of their focal lengths equal to the ratio of γ_1/γ_2. This can be extended to form apochromatic objectives that are athermal.

7 Conclusions

This paper presents a simple and insightful method to design thin apochromatic and superachromatic objective lenses. An apochromat is separated into two achromatic doublets, and a superachromat is separated into two apochromatic triplets. Using the method outlined in this paper, an optical designer can plot the chromatic focal shift for achromatic doublets out of materials available, chose appropriate pairs of doublets, and readily construct apochromat and superachromat objectives.

The method shows that there are several ways to build an apochromatic or superachromatic objective according to the sign of the focal lengths of the constituent doubles or triplets, and to the sign of their residual spectrum; we call these as types I, II, and III.

The paper derives and provides a scaling formula to find out the focal length of the second doublet given the focal length of the first doublet, and the ratio of secondary spectra when the doublets have the same focal length.

The method does not rely on glass V-values, partial dispersions, elaborated formulas or chromatic coordinates, and can be applied to a given bandwidth. In addition, the method can be extended to find superachromats with more crossings in the chromatic focal shift.

Appendix: Derivation of Scaling Formula

The secondary spectrum for an achromatic doublet in terms of wave aberration is given by

$$\partial_\lambda W_{020} = \frac{1}{f}\frac{(P_a - P_b)}{(V_a - V_b)}\frac{y^2}{2}.$$

And for two achromatic doublets in contact by

$$\partial_\lambda W_{020} = \left[\frac{1}{f_1}\frac{(P_{1,a} - P_{1,b})}{(V_{1,a} - V_{1,b})} + \frac{1}{f_2}\frac{(P_{2,a} - P_{2,b})}{(V_{2,a} - V_{2,b})}\right]\frac{y^2}{2}.$$

Assume $f_2 = k \cdot f_1$ and $\partial_\lambda W_{020} = 0$, then we have

$$0 = \frac{1}{f_1}\frac{(P_{1,a} - P_{1,b})}{(V_{1,a} - V_{1,b})} + \frac{1}{f_2}\frac{(P_{2,a} - P_{2,b})}{(V_{2,a} - V_{2,b})},$$

or

$$0 = SS_1 + \frac{SS_2}{k},$$

which leads to

$$\frac{f_1}{f_2} = -\frac{SS_1}{SS_2},$$

where SS_1 and SS_2 are the peak values in the chromatic focal shift and are calculated when the focal lengths of the achromatic doublets are the same.

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