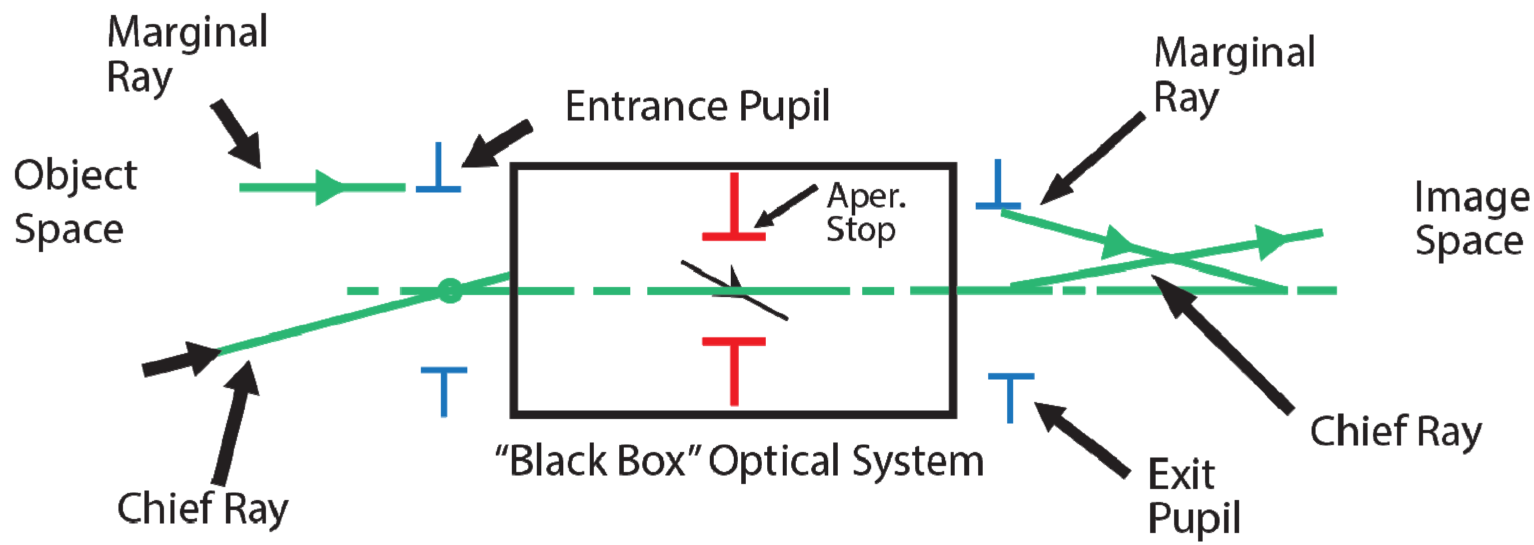
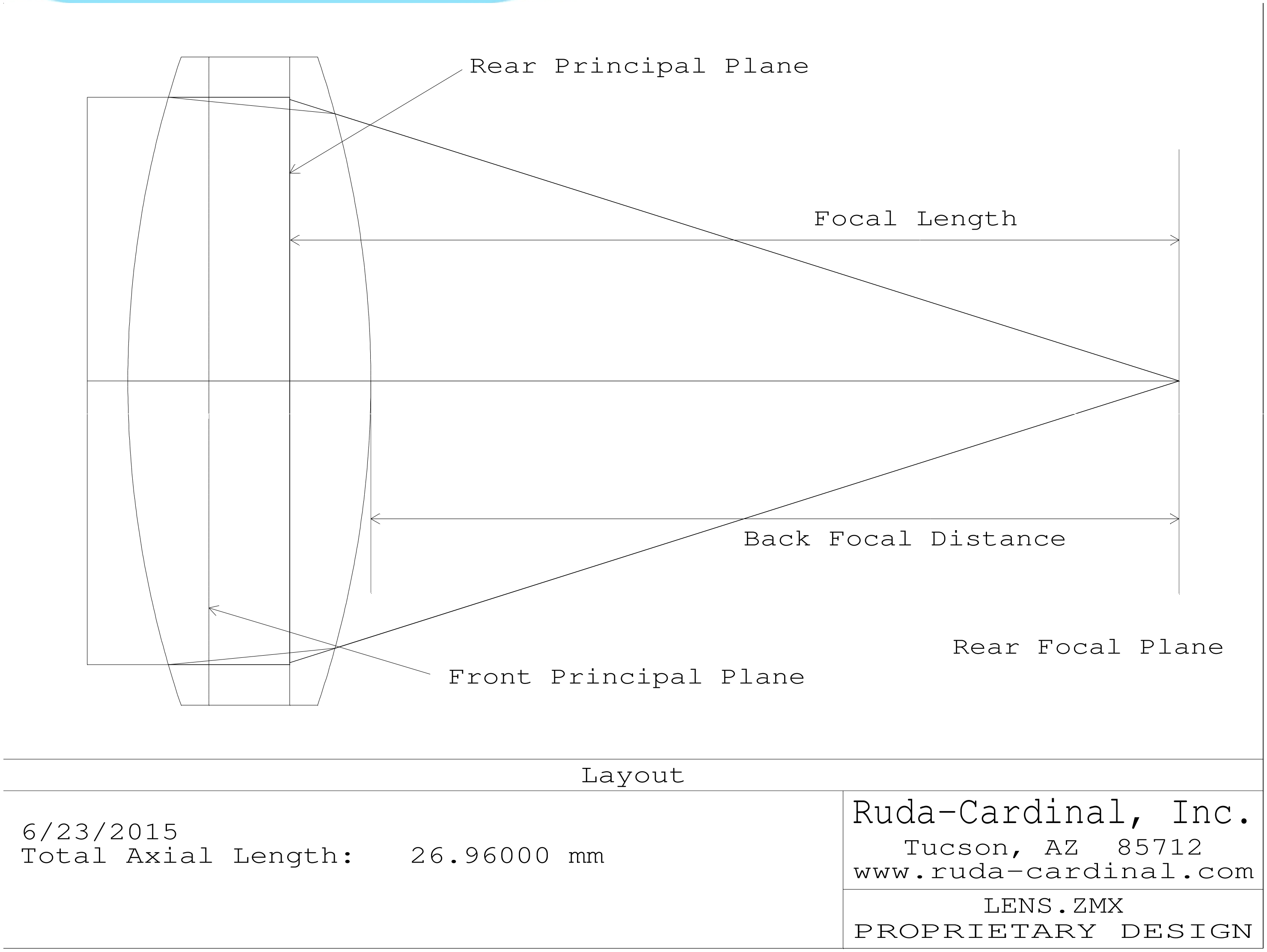
Cardinal Points of an Optical System--and Other Basic Facts

- The fundamental feature of any optical system is the aperture stop. Thus, the most fundamental optical system is the pinhole camera.
- The image of the aperture stop from the lenses on the object space side of the aperture stop is called the entrance pupil.

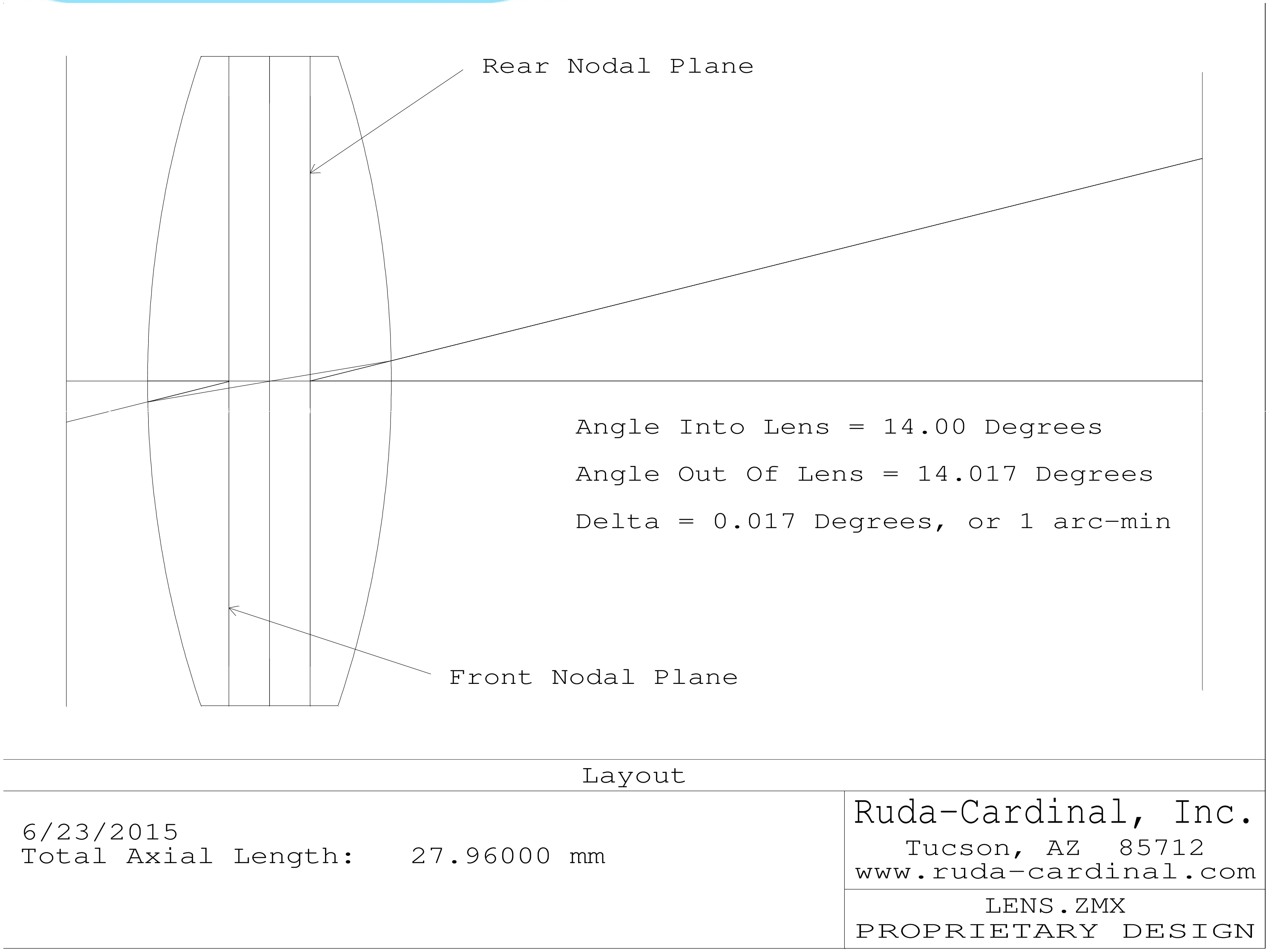


- The image of the aperture stop from the lenses on the image space side of the aperture stop is called the exit pupil.

Focal and Principal Planes



Nodal Points Illustrated



Numerical Aperture and F-Number

- Numerical Aperture (N.A.): A measure of the converging light cones half angle

  N.A. = 1/2F#

- F-Number (f#) = Note the F# sometimes refers to the cone at the actual working conjugate.
- For example, the F# at the center of curvature of a sphere is R/D, not f/D.
- Be Aware!!

F# = f/D

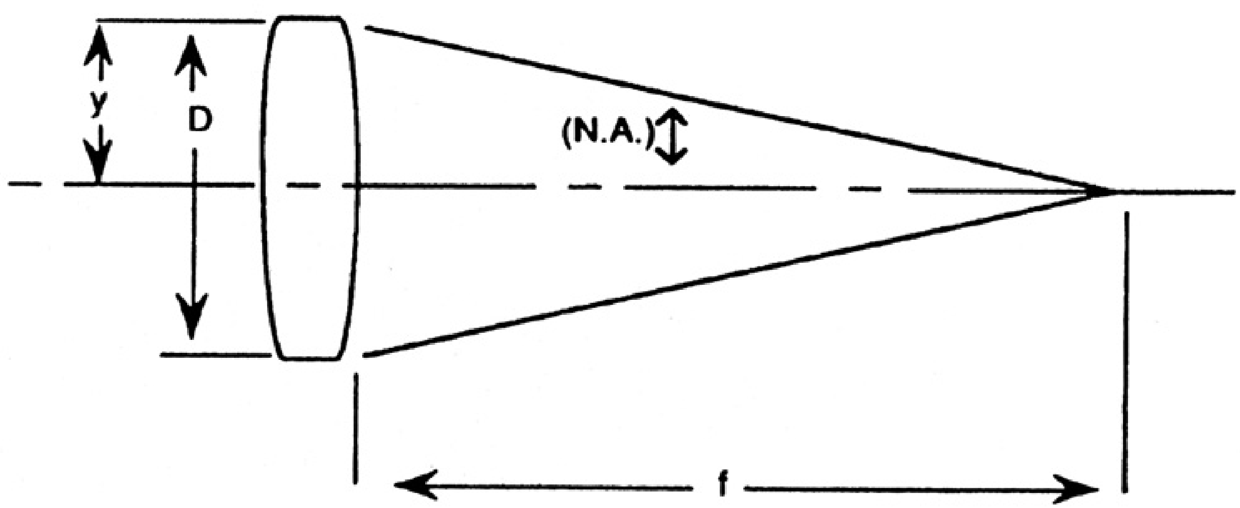
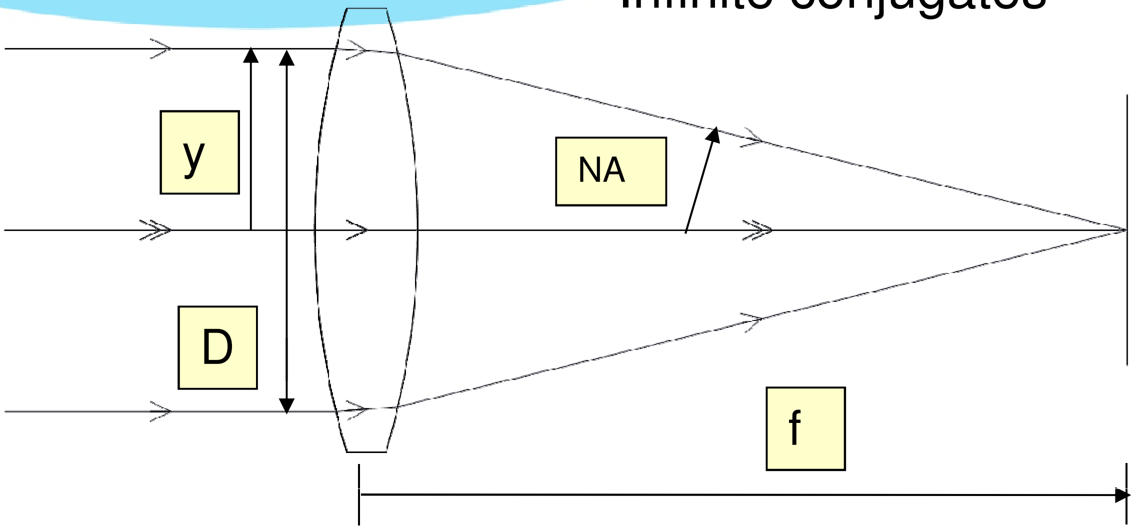


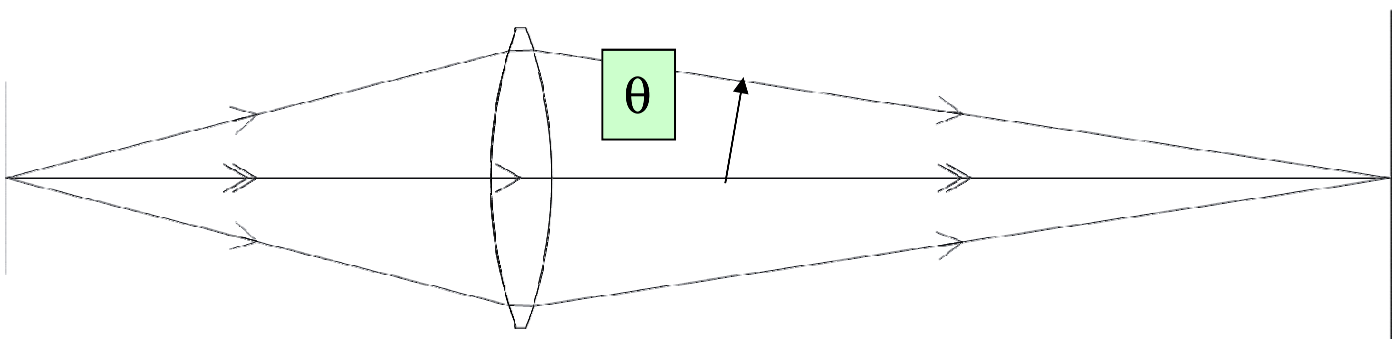
Illustration of F/#'s

Infinite conjugates



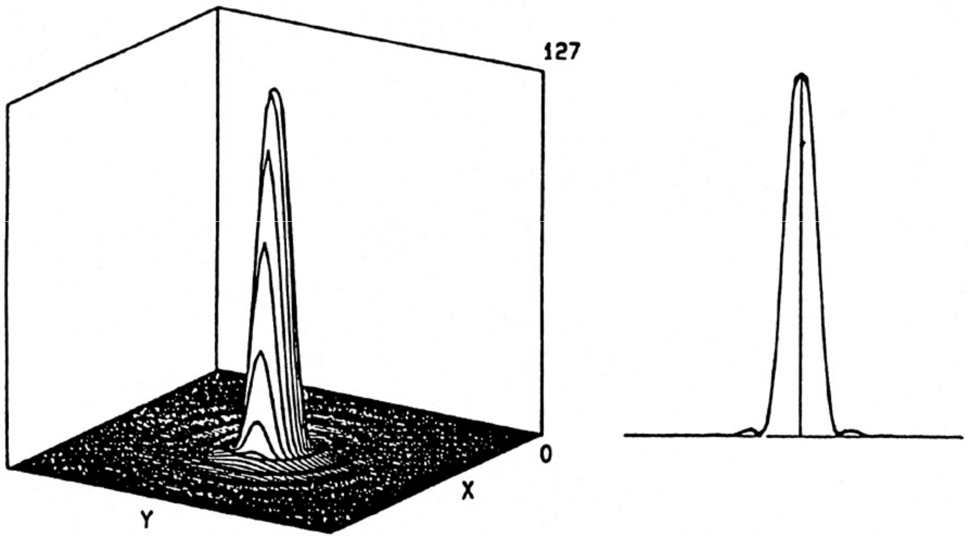
$F/\# = f/D$

As-used conjugates



The Airy Disk

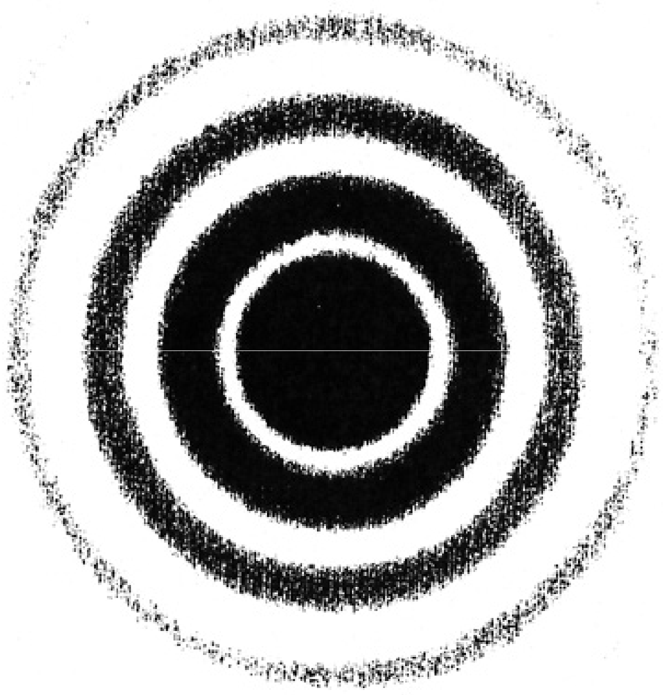
- The Airy disk
  - The image of a point source is called the Airy disk, and is shown in the figure below:



The first zero occurs at a diameter of: $2.44\lambda(F\#)$

  - 84% of the energy occurs in the central core; 91% is contained in the core plus the first ring.

The Airy Disk



- 84% of energy in central core, 91% to first ring

RUDA CARDINAL

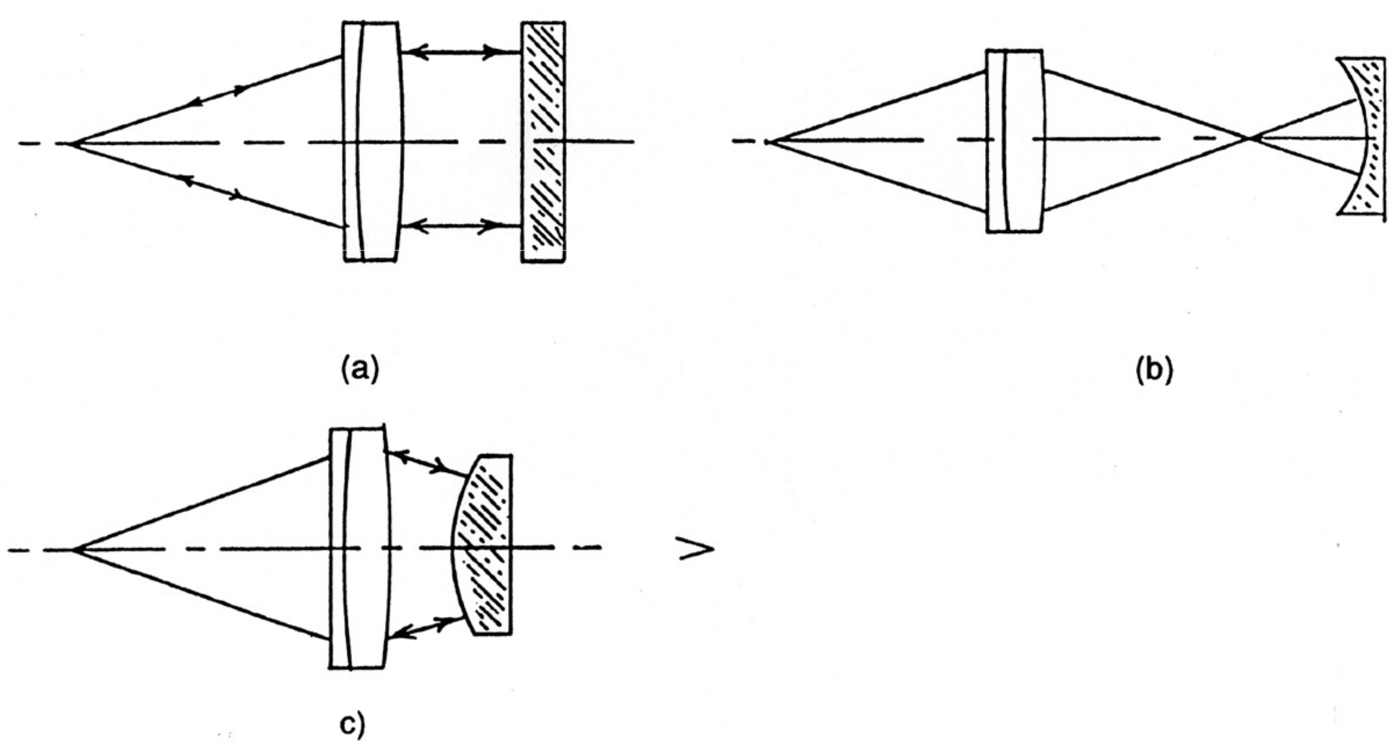
Auto-reflection

This is really important!

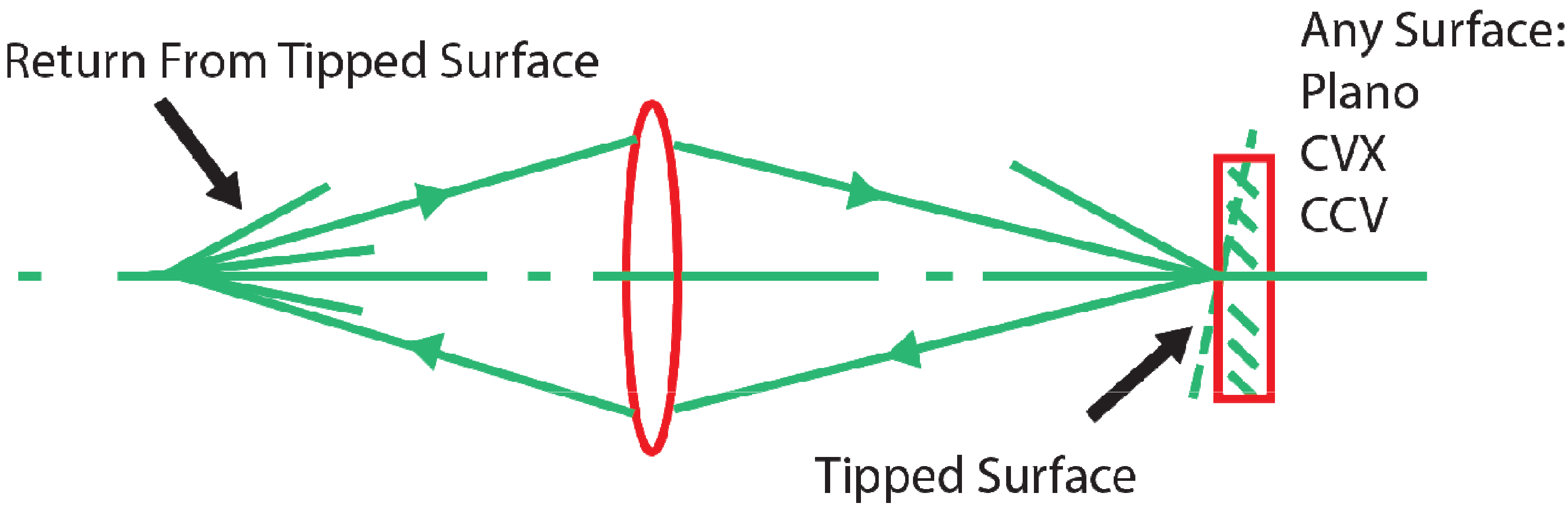
(a) In collimated space (autocollimator)

(b) With concave mirror

(c) With convex mirror

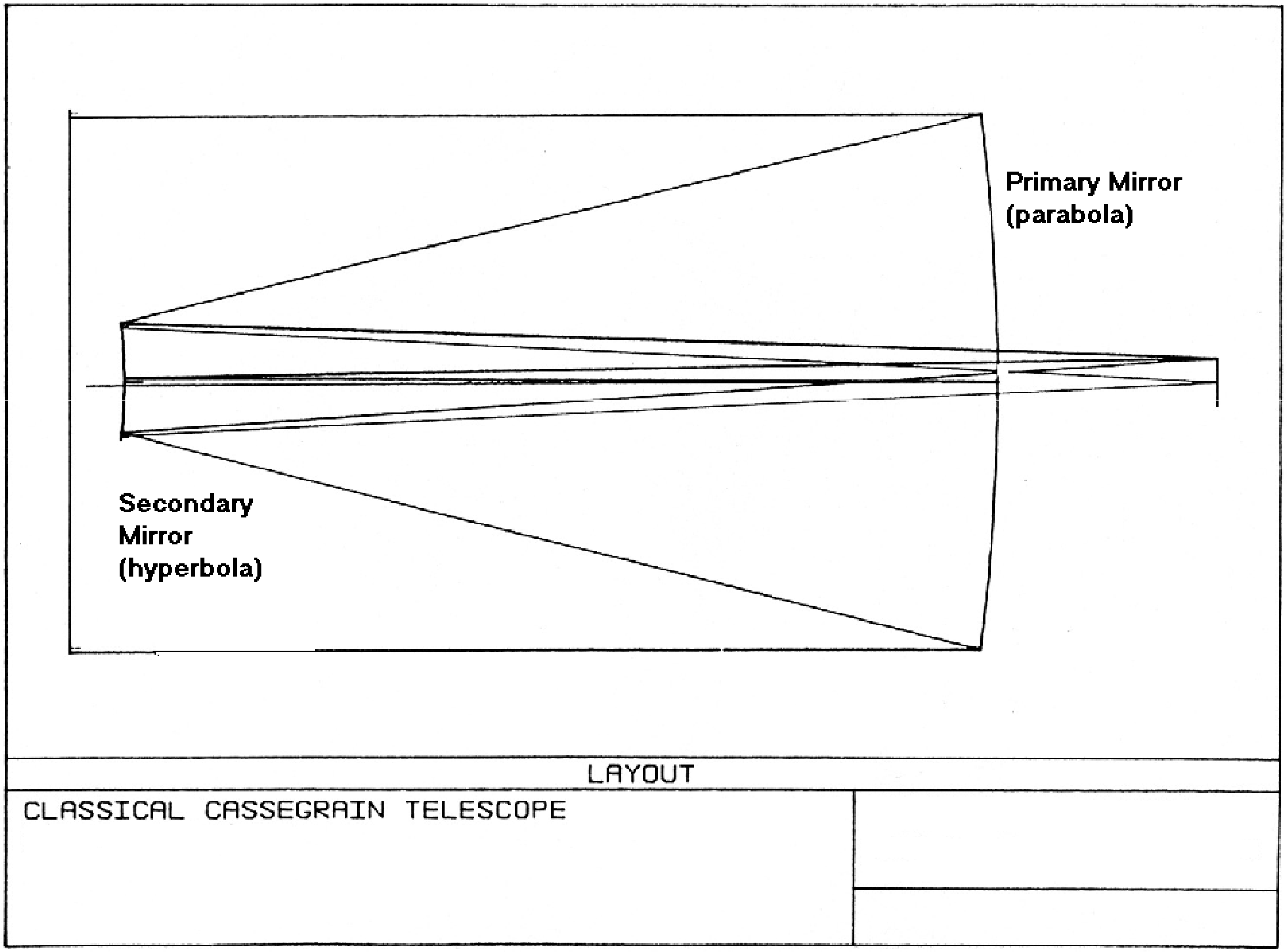


Retro-reflection -- Also Important

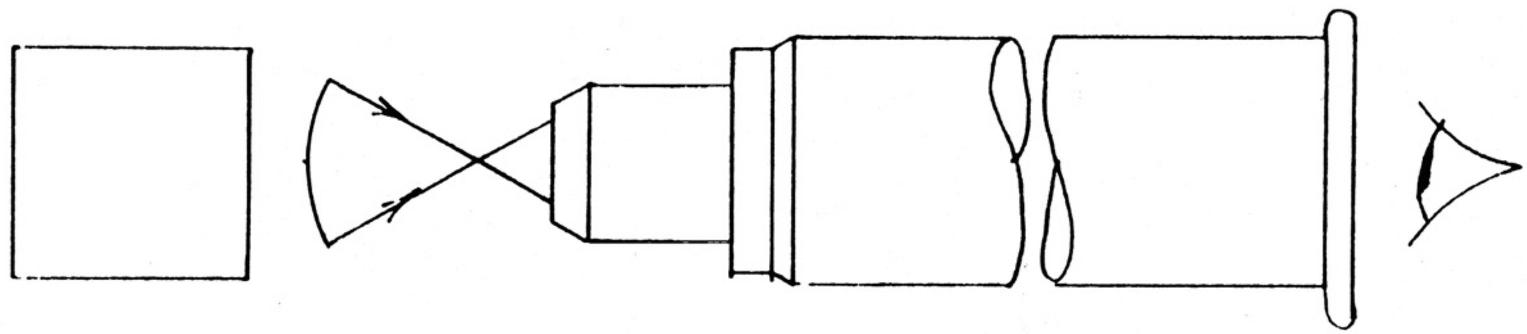


- Retro-reflection occurs when an image focuses on an optical surface.
- Interesting facts:
  - Image and surface are in focus
  - Image motion is tilt insensitive (displacement insensitive for flats
  - Illumination varies with tilt
  - Return image quality is very scratch/dig sensitive

The Cassegrain Telescope



The Star Test Definition



- The star test is simply a visual inspection of a point image formed by an optical system. Qualitative and quantitative information can be obtained by:
  - Observing the image
  - Measuring its size
  - Orientation of asymmetries
  - Examining the through-focus appearance
- Note: Microscope NA must be > system NA or microscope F# must be faster than system F#!

2.0 Recognizing the Elementary Aberrations and their Role in Optical Alignment

- Wavefront error basics
- Wavefront and image errors vs. field and pupil position
- Focus error
- Longitudinal magnification
- Spherical aberration
- Coma
- Astigmatism
- Boresight error
- Determining the cause of aberration -- Misalignment or otherwise

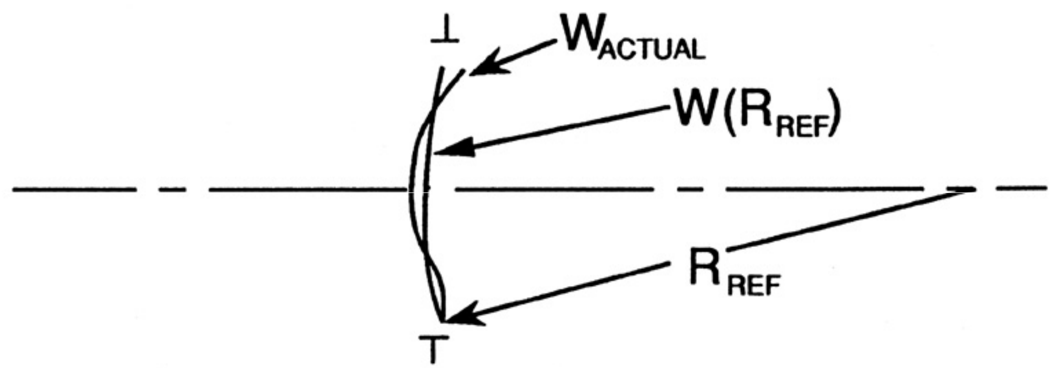
Recognizing the Elementary Aberrations

- Recognizing and understanding the elementary aberrations is a vital aid in optical alignment. Both the qualitative recognition and quantitative measurement of these aberrations provide powerful diagnostic tools for correcting misaligned systems
- There are three approaches we can take in understanding these aberrations:
  - Mathematical understanding: For our purposes only a summary will be necessary
  - Understanding how aberrations affect wavefront shape: This will clarify why interference fringes have specific patterns for each aberration
  - How aberrations affect the image of a point source: This so-called "star test" can be an extremely powerful alignment tool

Wavefront Error Basics

- Since we usually want light to converge to a perfect focus, a perfect wavefront is usually considered to be either a sphere or a plane.
- Optical path difference (OPD) is the difference between the actual and best fit reference wavefront. OPD is the yardstick for defining wavefront error.

$$OPD = (W^{(x,y)}_{ACTUAL} - W\ (R_{REF}\ )$$



- The types of wavefront errors that occur due to the normal aberrations in an optical design and those caused be misalignment are very similar, if not identical.
- Mathematically, design and misalignment errors are expressed by a polynomial expansion. Design errors are often carried out to higher order terms; however, misalignment errors, because they are generally small, are carried out to only the fourth order.

Wavefront Error Basics

- For this expression of wavefront error in polynomial form, let:

  H = Field Height of Object

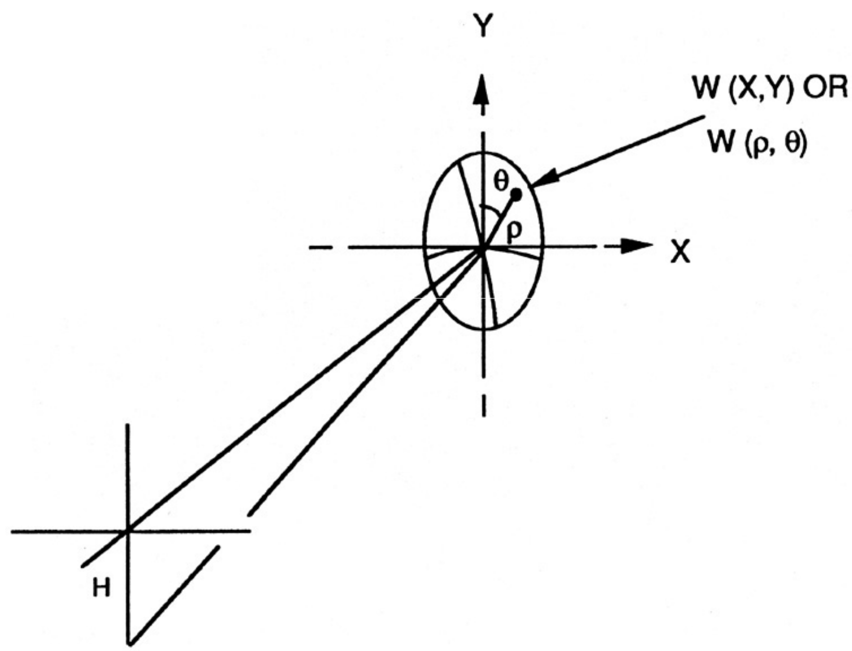
  X,Y = Cartesian Coordinates

  ρ, θ = Polar Coordinates

  Z = Optical Axis

  R_{REF} = Reference Sphere Radius

  *W(X,Y) or W(ρ, θ) = Functional Description of Wavefront



*Usually described as a polynomial expansion and includes the field dependence (H).
Then, $W(H, \rho, \cos\theta) = \Sigma a_{ijk} H^i \rho^j \cos^k(\theta)$

Wavefront Error Basics

To 3rd Error

$$W(H, \rho, \cos\theta) = \underbrace{a_{000}}_{\text{piston error}} + \underbrace{a_{011}}_{\text{tilt}}\rho\cos\theta + \underbrace{a_{020}}_{\text{defocus}}\rho^2 + \underbrace{a_{222}}_{\text{astigmatism}}H^2\rho^2\cos^2\theta$$

$$+ \underbrace{a_{131}}_{\text{coma}}H\rho^3\cos\theta + \underbrace{a_{040}}_{\text{spherical aberration}}\rho^4 + \underbrace{a_{311}}_{\text{distortion}}H^3\rho\cos\theta$$

The spot size or image blur varies as the derivative of the above expression, and can be expressed as:

$$\varepsilon = -\frac{\lambda}{n'u'}\frac{\partial W}{\partial \rho}$$

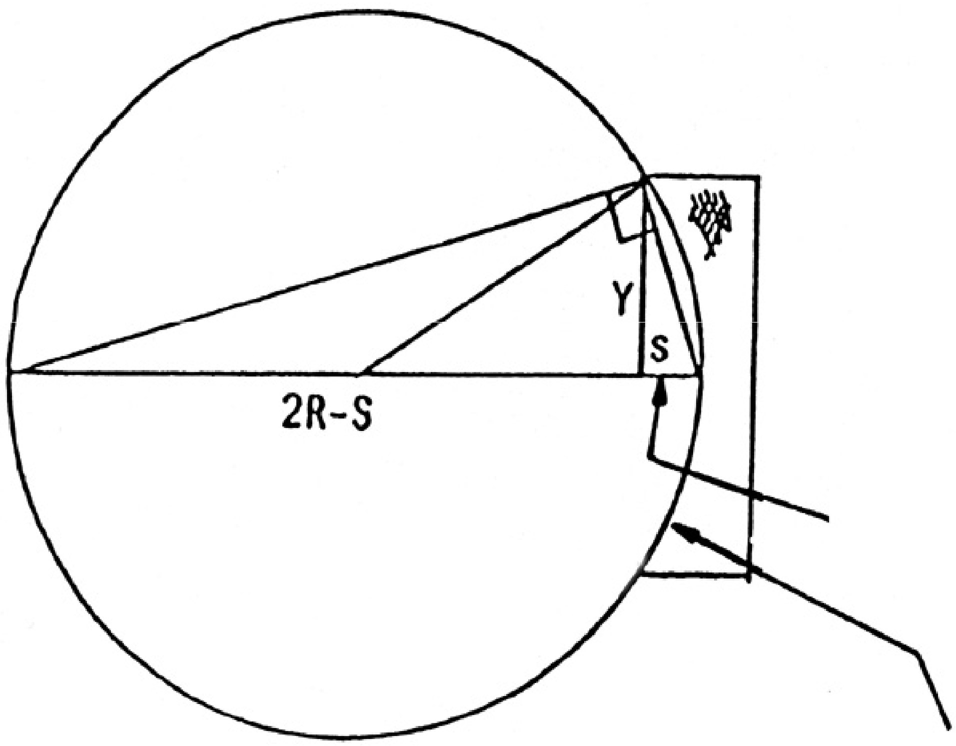
A Quick Summary of the Field and Pupil Dependencies of Wavefront and Image Errors

	Aberration	Power of H (H^m)	Power of ρ (ρ^n)
Wavefront Error Dependence	Defocus (a_{020})	----	Squared (n=2)
	Spherical Aberration (a_{040})	----	Fourth (n=4)
	Coma (a_{131})	Linear (m=1)	Cubed (n=3)
	Astigmatism (a_{222})	Squared (m=2)	Squared (n=2)
Spot size dependence Spot size $\propto \partial W/\partial \rho$	Defocus	----	Linear
	Spherical Aberration	----	Cubed
	Coma	Linear	Squared
	Astigmatism	Squared	Linear

Focus Error--Very Important

- Defocus is probably the most common misalignment error
- Mathematically:
  - $W = W_{020} = a_{020}\rho^2$
    Field *Independent*--No Surprise
- Interferometrically:
  - Concentric rings, the number increasing as ρ^2
- Star Test:
  - Uniform blur on either side of focus (geometric regime).
  - Black dot in the center of the airy disk for $\pm 1\lambda$ defocus
- We will now derive a very useful formula and relate Wavefront Defocus Error to defocus of image plane

Relationship Between Sag, Diameter, and Radius of Curvature



$$\frac{S}{Y} = \frac{Y}{2R - S}$$

$$S^2 - 2RS + Y^2 = 0$$

$$S = R - \sqrt{R^2 - Y^2} \quad \text{EXACT}$$

BUT, IF 2R >> S, THEN

$$S \approx Y^2/2R \quad \text{APPX.}$$

Surface of lens or mirror
wavefront, exit pupil or wavefront

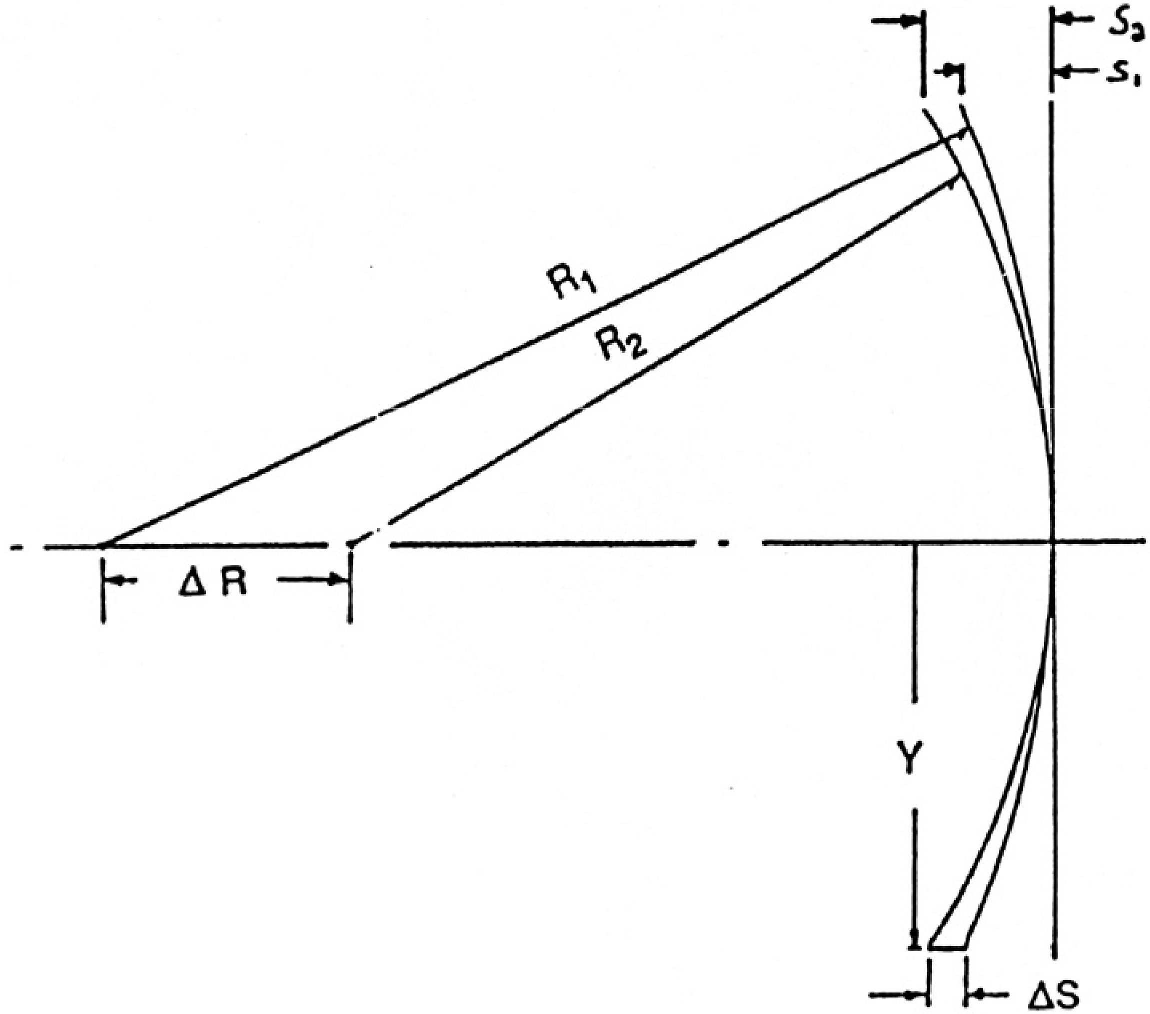
A Warm-up to an Important Equation

$$S = Y^2/2R$$

$$\Delta S = \frac{Y^2}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

ΔS IS LIKE a_{020}

ΔR IS LIKE Δf



Derivation of the Defocus Equation

¥ From the previous page, let's substitute f (focal length) for R and Δf for ΔR, then :

$$\Delta S = \frac{y^2}{2}\left(\frac{1}{f_1} - \frac{1}{f_2}\right) \qquad (1)$$

¥ ΔS = # of waves of sag difference (a_{020}) × length of a wavelength (λ), i.e.:

$$\Delta S = a_{020}\,\lambda \qquad (2)$$

¥ $\left(\frac{1}{f_1} - \frac{1}{f_2}\right) = \frac{f_2 - f_1}{f_1 f_2} = \frac{\Delta f}{f_1 f_2}$, but since $\Delta f << f_1$ or f_2, $f_1 \approx f_2 = f$

THEN $$\left(\frac{1}{f_1} - \frac{1}{f_2}\right) \approx \frac{\Delta f}{f^2} \qquad (3)$$

¥ Now, F# = f/D = f/2y

Thus, $f^2/y^2 = 4(F\#)^2$ (4)

Derivation of the Defocus Equation (continued)

- Substituting equations 2 and 3 into 1, we get

$$a_{020}\,\lambda = \frac{y^2 \Delta f}{2 f^2} \qquad (5)$$

- Substituting 4 into 5 and solving for Δf:

$$\mathbf{\Delta f = \pm\, 8\, \lambda\, a_{020}\, F\#^2}$$ **The Defocus Equation**

Thus we have related focus error in the *wavefront* to *shift in focus* at the image

Reminder a_{020} is dimensionless and is the number of waves of defocus (sag departure of the wavefront).

Three Handy Formulas

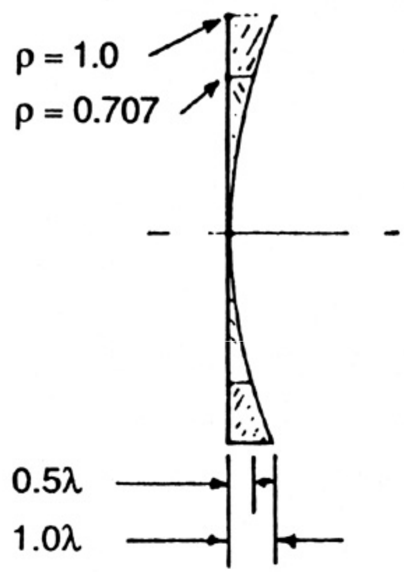
- General Expression

  $\Delta f = \pm\, 8\, \lambda\, a_{020}\, F\#^2$
- Rayleigh Criterion ($a_{020} = 1/4$)

  $\Delta f = \pm\, 2\, \lambda\, F\#^2$
- In the visible ($\lambda \sim 0.5$ Microns):

  $\Delta f = \pm\, F\#^2$ (In microns)

Why is There a Black Dot when Defocus = 1λ?



$W = W_{020} = a_{020}\rho^2$

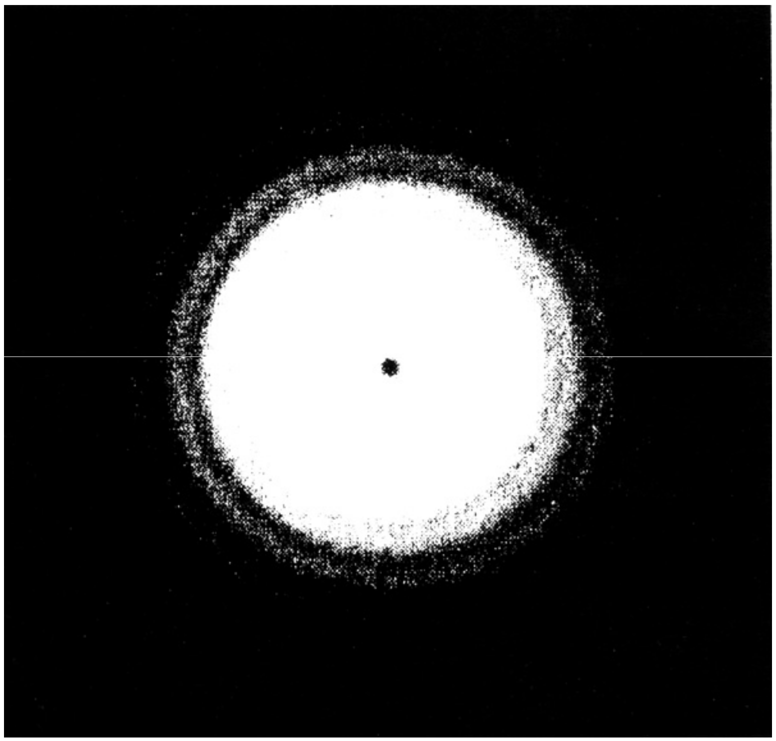
When $a_{020} = 1\lambda$

- Sag of the wavefront at full aperture ($\rho = 1$) = 1λ
- Sag of the wavefront at $\rho = 0.707 = 0.5\lambda$
- Area of the pupil from $\rho = 0$ to $\rho = 0.707$ equals area of annular pupil from $\rho = 0.707$ to $\rho = 1.0$

Therefore, for every point within r = 0.707, there is a point in annulus that is $\lambda/2$ out of phase

Consequently, on-axis everything cancels! No light! Black Dot!

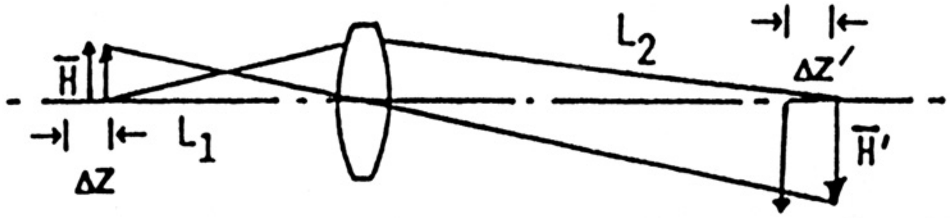
The Black Dot



- When a perfect circular wavefront is defocused exactly 1λ, a black dot appears at the center of the spot
- Find the $\pm 1\lambda$ planes, then split the difference for sharp focusing

Longitudinal Magnification

- First what is transverse magnification?



$M = L_2/L_1$

- Longitudinal Magnification is simple. If we move the object along the axis by an amount, DZ, the image will move DZ'
- Longitudinal Magnification is defined as:

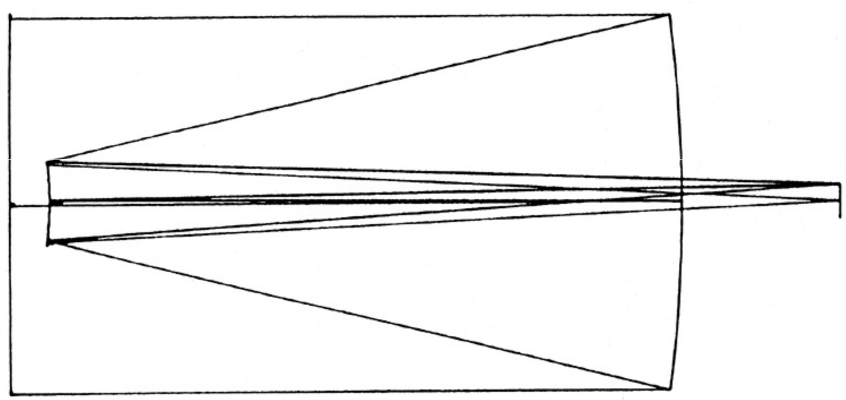
$$M_L = \frac{\Delta Z'}{\Delta Z}$$

- It can be shown that
  - $M_L = M^2$

- So what?
  It is useful for measuring despacing sensitivities of optical components.
  It is also why your nose looks so big on a doorknob!

The Usefulness of Depth of Focus and Longitudinal Magnification Formulas (example)

Problem: Find the spacing tolerance between the primary and secondary mirror of a diffraction limited, F/10 Cassegrain telescope. The primary mirror is F/2, the telescope is used in the visible (λ = 0.5μ).



1. The diffraction limited (λ/4) depth of focus at the large image plane of the Cassegrain is given by

   $\Delta f = \pm F/\#^2$ (in microns) (when λ = 0.5μ)
   $= \pm 100$ microns

2. The secondary mirror converts an F/2 beam into an F/10 beam. Thus, the magnification of the secondary mirror is

$$M = \frac{F/10}{F/2} = 5$$

3. The longitudinal magnification $M_L = M^2 = 5^2 = 25$

4. A 100μ change in focus will occur if ΔS changes by 100/25 = 4μ = spacing tolerance