Introduction to aberrations
OPTI518
Lecture 6

Chromatic aberrations

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OPTI 518

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Chromatic aberrations

$$\partial_\lambda W\left(\vec{H},\rho\right)=\partial_\lambda W_{000}+\partial_\lambda W_{200}H^2+\partial_\lambda W_{111}H\rho\cos\left(\varphi\right)+\partial_\lambda W_{020}\rho^2$$

- Consider up to second order terms
- Change of image location with λ (axial or longitudinal chromatic aberration)
- Change of magnification with λ (transverse or lateral chromatic aberration)
- We start with a single surface

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Second-order: single surface

$$W(0,\vec{\rho}) = \frac{y^2}{2}\left(\left(\frac{n'-n}{r}\right) - \left(\frac{n'}{s'} - \frac{n}{s}\right)\right)(\vec{\rho}\cdot\vec{\rho}) = W_{020}(\vec{\rho}\cdot\vec{\rho})$$

$$A = n\left(\frac{1}{r} - \frac{1}{s}\right)y = A' = n'\left(\frac{1}{r} - \frac{1}{s'}\right)y$$

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Chromatic change of focus: single surface

$$\partial_\lambda W_{020} = \frac{y^2}{2}\left(\partial n'\left(\frac{1}{r}-\frac{1}{s'}\right)-\partial n\left(\frac{1}{r}-\frac{1}{s}\right)\right)$$

$$= \frac{y}{2}A\left(\frac{\partial n'}{n'}-\frac{\partial n}{n}\right) = \frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y$$

$$\lambda_F = 486.1\ nm$$

$$\lambda_C = 656.3\ nm$$

$$\Delta\left(\frac{\partial n}{n}\right) = \frac{\partial n'}{n'}-\frac{\partial n}{n}$$

$$\lambda_d = 587.6\ nm$$

$$n = n_d$$

$$\partial n = n_F - n_C$$

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Change of image location with λ (Chromatic change of focus)

$$W_{020} = \frac{y^2}{2}\left\{n'\left[\frac{1}{R}-\frac{1}{l'}\right]-n\left[\frac{1}{R}-\frac{1}{l}\right]\right\} = 0$$

$$\partial_\lambda W_{020} = \frac{y^2}{2}\left\{(n'+\partial n')\left[\frac{1}{R}-\frac{1}{l'}\right]-(n+\partial n)\left[\frac{1}{R}-\frac{1}{l}\right]\right\}$$

$$\partial_\lambda W_{020} = \frac{y^2}{2}\left\{\partial n'\left[\frac{1}{R}-\frac{1}{l'}\right]-\partial n\left[\frac{1}{R}-\frac{1}{l}\right]\right\}$$

$$= \frac{y}{2}\left\{\partial n'\left[\frac{y}{R}-\frac{y}{l'}\right]-\partial n\left[\frac{y}{R}-\frac{y}{l}\right]\right\}$$

$$= \frac{y}{2}\left\{\partial n'\left[\alpha+u'\right]-\partial n\left[\alpha+u\right]\right\}$$

$$= \frac{y}{2}\left\{\partial n'\, i'-\partial n\, i\right\}$$

$$\boxed{\partial_\lambda W_{020} = \frac{y}{2} A\, \Delta\left\{\frac{\partial n}{n}\right\}}$$

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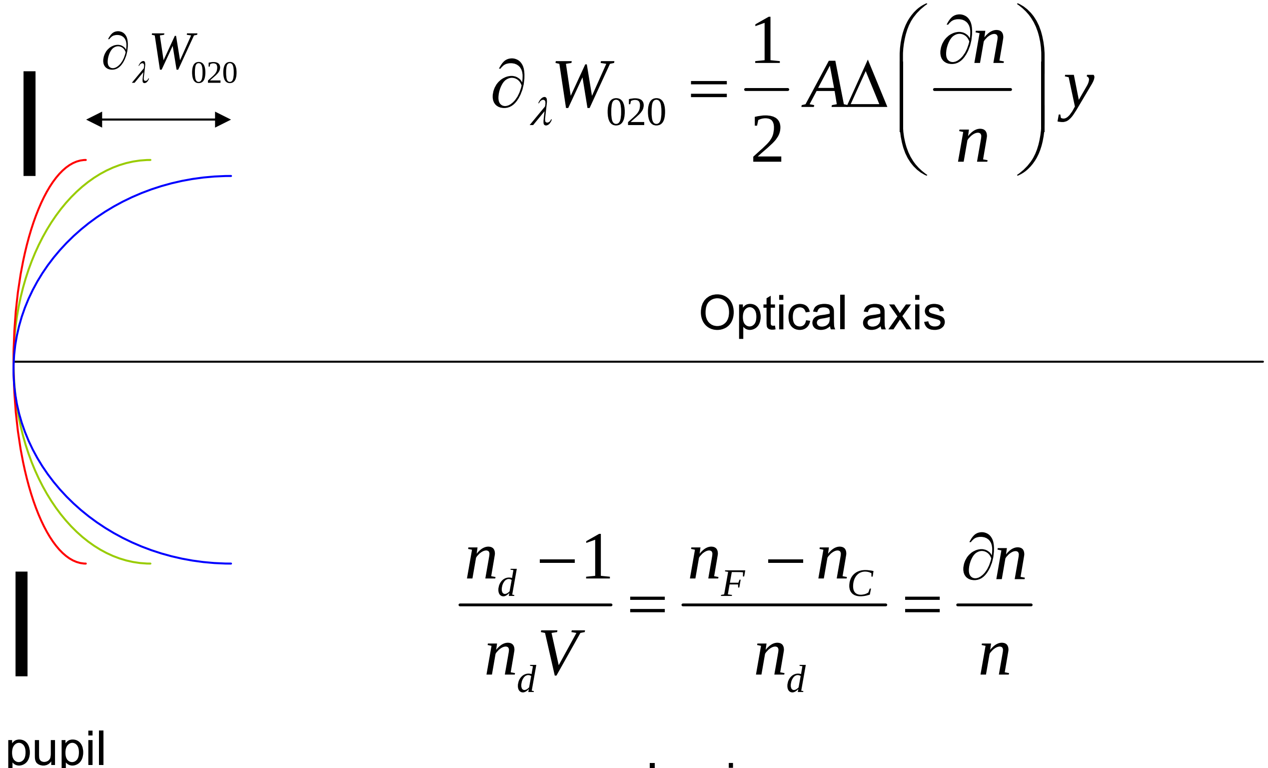
Chromatic coefficient meaning

$$\partial_\lambda W_{020} = \frac{1}{2} A \Delta \left(\frac{\partial n}{n} \right) y$$



$$\frac{n_d - 1}{n_d V} = \frac{n_F - n_C}{n_d} = \frac{\partial n}{n}$$

In air case

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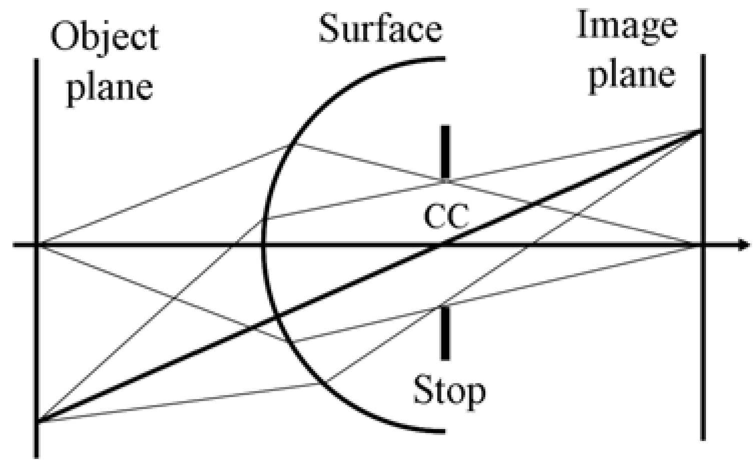
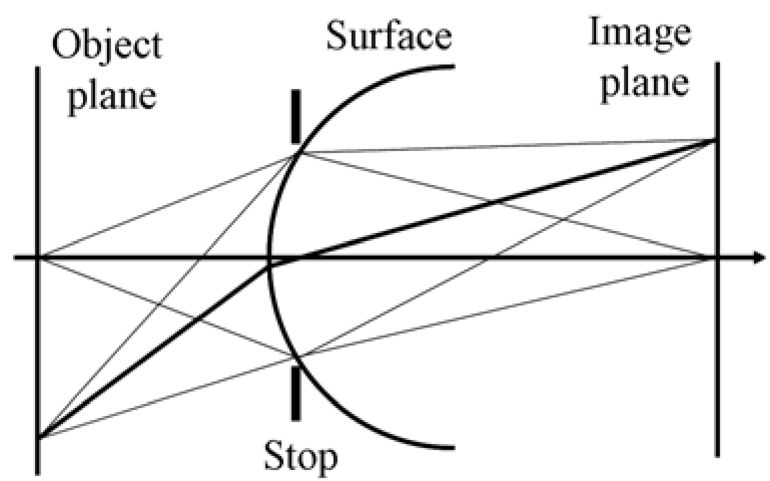
Chromatic change of magnification





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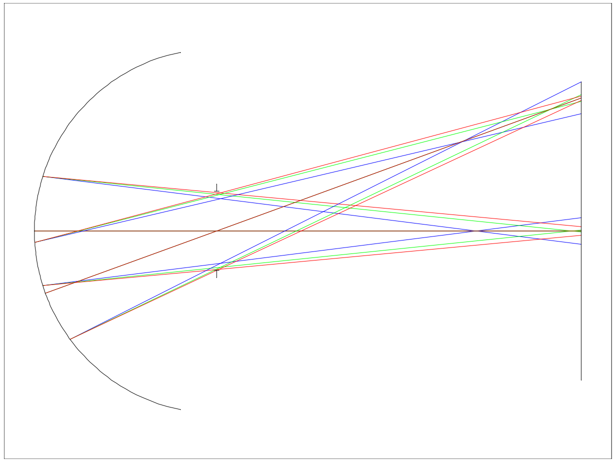
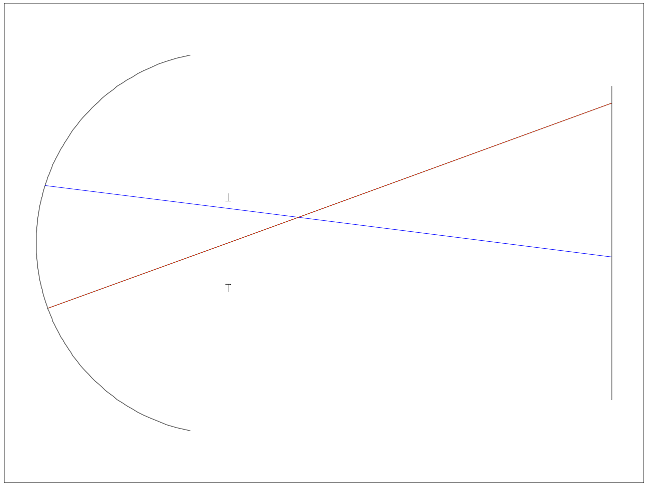
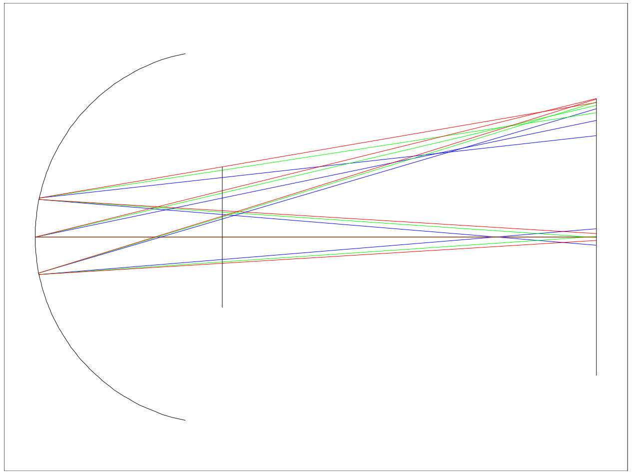
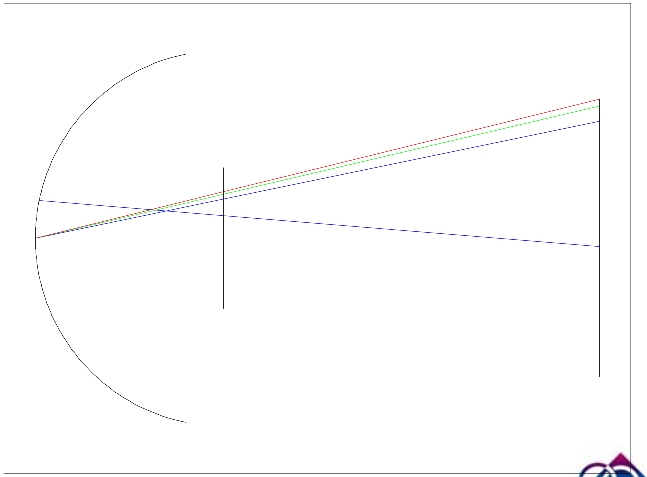
With stop at CC





With stop at surface





With stop at cc chief rays is not refracted and therefore there is no change of magnification with λ

$$\partial_\lambda W_{111} = 0$$

$$\partial_\lambda W\left(\vec{H}, \rho\right) = \partial_\lambda W_{000} + \partial_\lambda W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)$$

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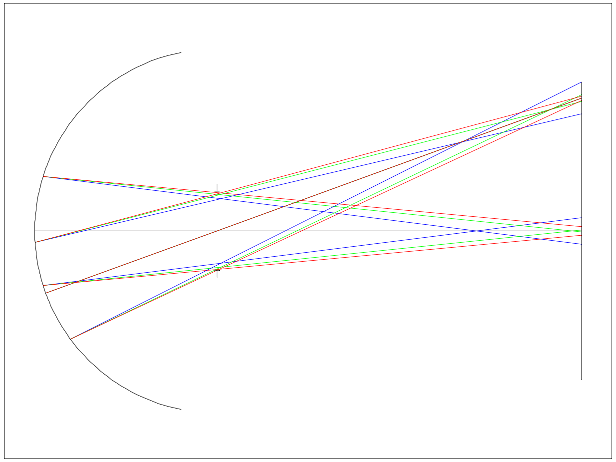
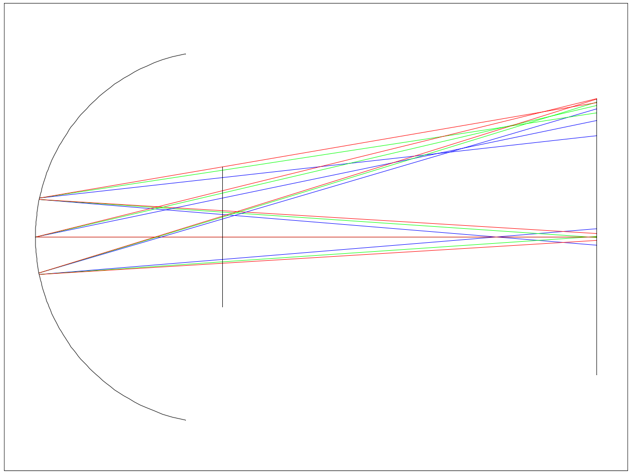
Description at other stop location





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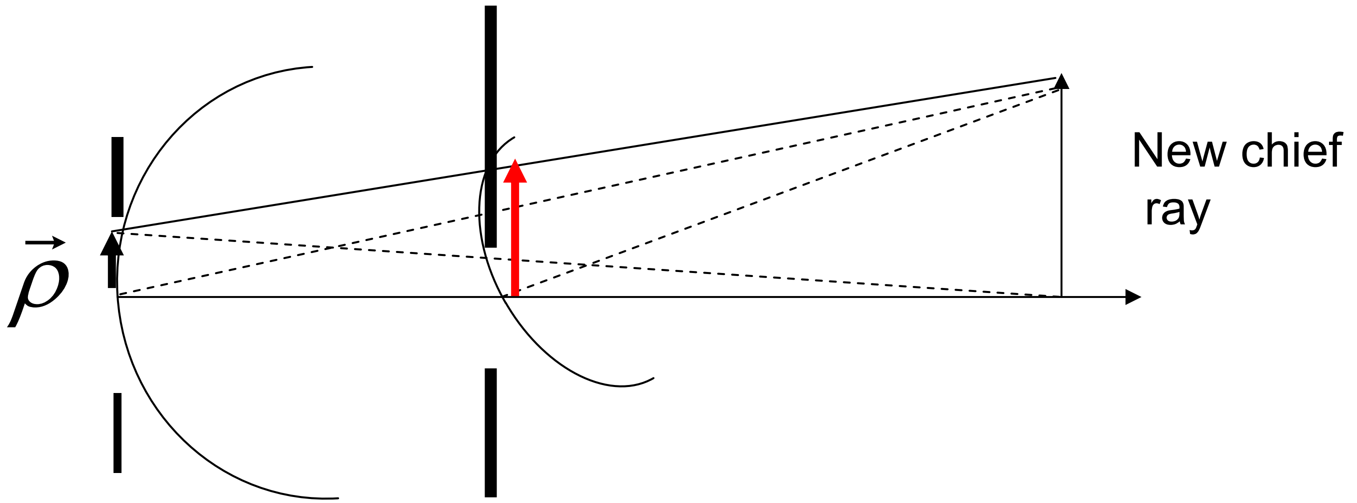
Stop shifting



New stop   Old stop plane at CC

New chief ray height at old pupil $\bar{y}_E$

Marginal ray height at old pupil y_E

$$y_E \vec{\rho}_{shift} = y_E \vec{\rho} + \bar{y}_E \vec{H}$$

$$\vec{\rho}_{shift} = \vec{\rho} + \frac{\bar{y}_E}{y_E} \vec{H} = \vec{\rho} + \frac{\bar{A}}{A} \vec{H}$$

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Perform stop shifting

$$\partial_\lambda W\left(\vec{H},\rho\right)=\partial_\lambda W_{000}+\partial_\lambda W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)$$

$$\vec{\rho}\rightarrow\vec{\rho}+\bar{S}\vec{H}$$

$$\partial_\lambda W\left(\vec{H},\rho\right)=\partial_\lambda W_{000}+\partial_\lambda W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)=\partial_\lambda W_{000}+\frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{\rho}\cdot\vec{\rho}\right)$$

$$\partial_\lambda W\left(\vec{H},\rho\right)\rightarrow\partial_\lambda W\left(\vec{H},\rho\right)=\partial_\lambda W_{000}+\frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{\rho}\cdot\vec{\rho}\right)$$

$$=\partial_\lambda W_{000}+\frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{\rho}\cdot\vec{\rho}\right)+A\bar{S}\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{H}\cdot\vec{\rho}\right)+\frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y\bar{S}^2\left(\vec{H}\cdot\vec{H}\right)$$

$$=\partial_\lambda W_{000}+\frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{\rho}\cdot\vec{\rho}\right)+\bar{A}\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{H}\cdot\vec{\rho}\right)+\frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y\left(\frac{\bar{A}}{A}\right)^2\left(\vec{H}\cdot\vec{H}\right)$$

Chromatic changes

$$\partial_{\lambda} W\left(\vec{H},\rho\right)=\partial_{\lambda} W_{000}$$

$$+\frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{\rho}\cdot\vec{\rho}\right)+\bar{A}\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{H}\cdot\vec{\rho}\right)+\frac{1}{2}A\Delta\left(\frac{\partial n}{n}\right)y\left(\frac{\bar{A}}{A}\right)^{2}\left(\vec{H}\cdot\vec{H}\right)$$

Chromatic change of focus
Chromatic change of magnification
Chromatic change of piston

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For a system of j surfaces

$$\partial_\lambda W_{000} = \sum_{i=0}^{j} t_i \left(n'_F - n'_C \right)_i$$

$$\partial_\lambda W_{200} = \frac{1}{2}\sum_{i=1}^{j} \frac{\left(\bar{A}_j\right)^2}{A_j} \Delta_j \left(\partial n / n\right) y_j$$

$$\partial_\lambda W_{111} = \sum_{i=1}^{j} \bar{A}_j \Delta_j \left(\partial n / n\right) y_j$$

$$\partial_\lambda W_{020} = \frac{1}{2}\sum_{i=1}^{j} A_j \Delta_j \left(\partial n / n\right) y_j$$

- Optical paths add
- Aberration coefficients add
- Though there might be other very small terms, called extrinsic, that are missing.

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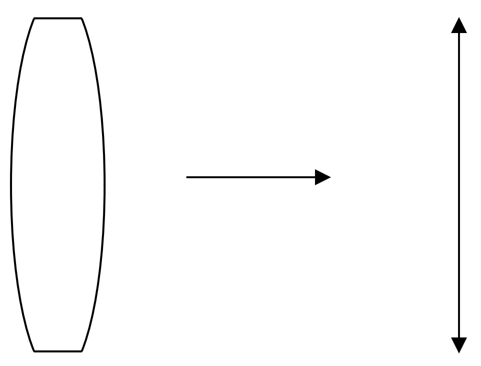
Thin lens concept

- Has no thickness
- Properties are found by setting t=0 in pertinent equations
- For example y is the same for both surfaces



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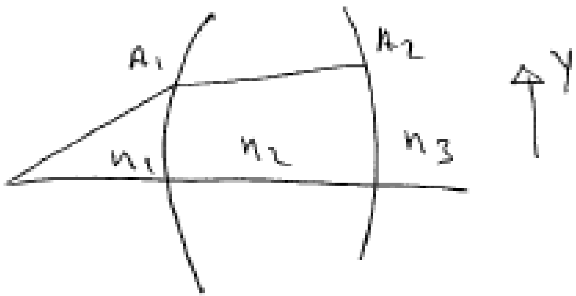
For a system of thin lenses



$$A = n i = n u + n \alpha \quad ; \quad \frac{\delta n}{n} = \frac{n_F - n_C}{n_d}$$

$$\phi = \frac{n' - n}{R} \quad ; \quad = \frac{n_d - 1}{n_d V}$$

$$n' u' = n u - \phi y \quad ; \quad \phi = (n-1)(c_1 - c_2)$$

$$A_2 = n_2 i_2 = n_2 u_2 + n_2 \alpha_2 = (n_1 u_1 - \phi_1 y) + \frac{n_2}{R_2} y$$

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$A_1 - A_2$

$$A_1 - A_2 = A_1 - \overbrace{\left[n_1 u_1 + \frac{n_1 y}{R_1}\right]}^{A_1} - \frac{n_1 y}{R_1} - \phi_1 y + \frac{n_2 y}{R_2}$$

$$= -\frac{n_1 y}{R_1} - \frac{n_2 - n_1}{R_1} y + \frac{n_2 y}{R_2}$$

$$= \frac{-n_2 y}{R_1} + \frac{n_2 y}{R_2} = +n_2 y \left[c_1 - c_2\right]$$

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$\partial_\lambda W_{020}$

$$\frac{1}{2}\sum_{i=1}^{i=2} A_j \Delta\left\{\frac{\delta n}{n}\right\}_j y_j = \frac{1}{2}\,\frac{n_d-1}{n_d V}\, y_j \left[A_1 - A_2\right]$$

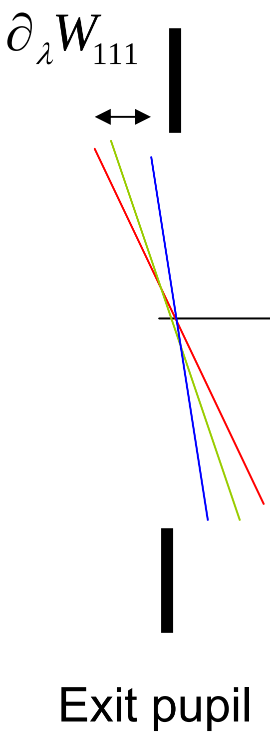
$$= \frac{1}{2}\,\frac{n_d-1}{n_d V}\, y \left[\cancel{n_2 u n}\; c_1 - c_2\right] n_2 y = \frac{1}{2}\,\phi\,\frac{y^2}{V}$$

$$\delta_\lambda W_{020} = \frac{1}{2}\sum_{i=1}^{n}\left\{\frac{\phi_i\, y_i^2}{V_i}\right\}$$

Does not depend on conjugate!

Chromatic change of magnification coefficient meaning

$$\partial_{\lambda} W_{111} = \bar{A} \Delta \left(\frac{\partial n}{n} \right) y$$



Optical axis

$$\frac{n_d - 1}{n_d V} = \frac{n_F - n_C}{n_d} = \frac{\partial n}{n}$$

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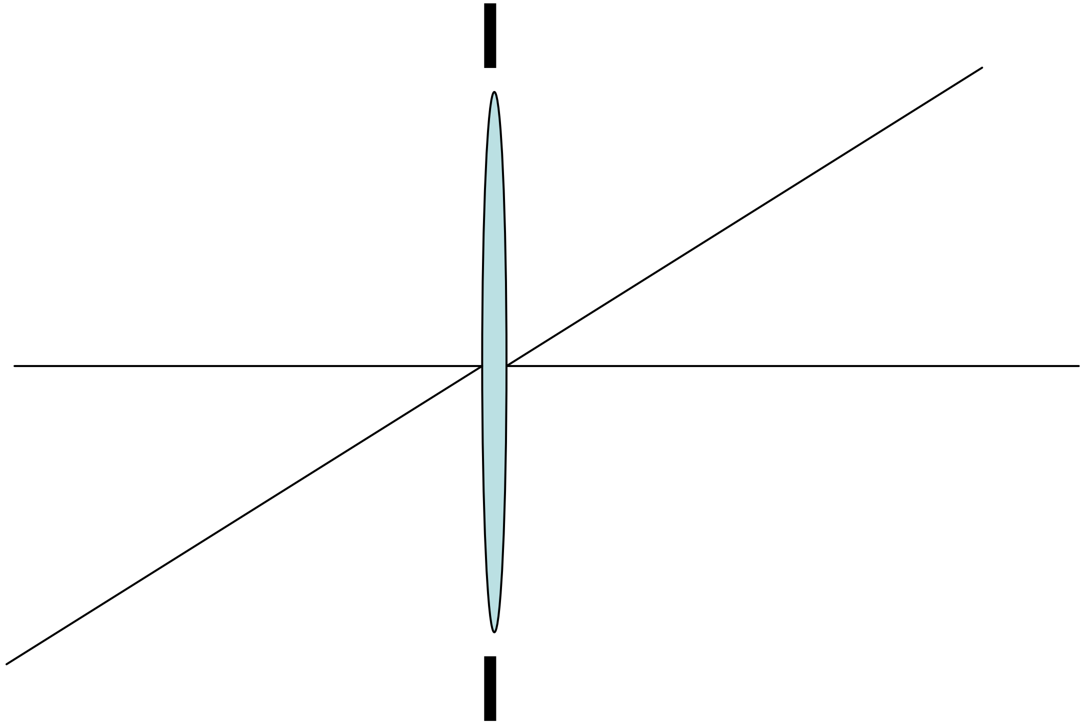
Thin lens: stop at lens



Any ray at any wavelength through the center of the thin lens is un-deviated as it sees a parallel plate of thickness zero. Thus,

$$\partial_{\lambda} W_{111} = 0$$

Aberration function for thin lens with stop at the lens and with stop shifting

$$\partial_\lambda W\left(\vec{H},\rho\right)=\partial_\lambda W_{000}+\partial_\lambda W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)=\partial_\lambda W_{000}+\frac{1}{2}\frac{\phi}{\nu}y^2\left(\vec{\rho}\cdot\vec{\rho}\right)$$

$$\partial_\lambda W\left(\vec{H},\rho\right)\rightarrow$$

$$=\partial_\lambda W_{000}+\frac{1}{2}\frac{\phi}{\nu}y^2\left(\vec{\rho}\cdot\vec{\rho}\right)+\frac{\phi}{\nu}y^2\bar{S}\Delta\left(\frac{\partial n}{n}\right)y\left(\vec{H}\cdot\vec{\rho}\right)+\frac{1}{2}\frac{\phi}{\nu}y^2\bar{S}^2\left(\vec{H}\cdot\vec{H}\right)$$

$$\bar{S}=\bar{y}/y$$

$$=\partial_\lambda W_{000}+\frac{1}{2}\frac{\phi}{\nu}y^2\left(\vec{\rho}\cdot\vec{\rho}\right)+\frac{\phi}{\nu}y\bar{y}\left(\vec{H}\cdot\vec{\rho}\right)+\frac{1}{2}\frac{\phi}{\nu}y^2\left(\frac{\bar{y}}{y}\right)^2\left(\vec{H}\cdot\vec{H}\right)$$

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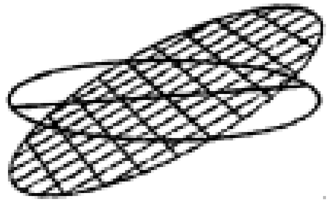
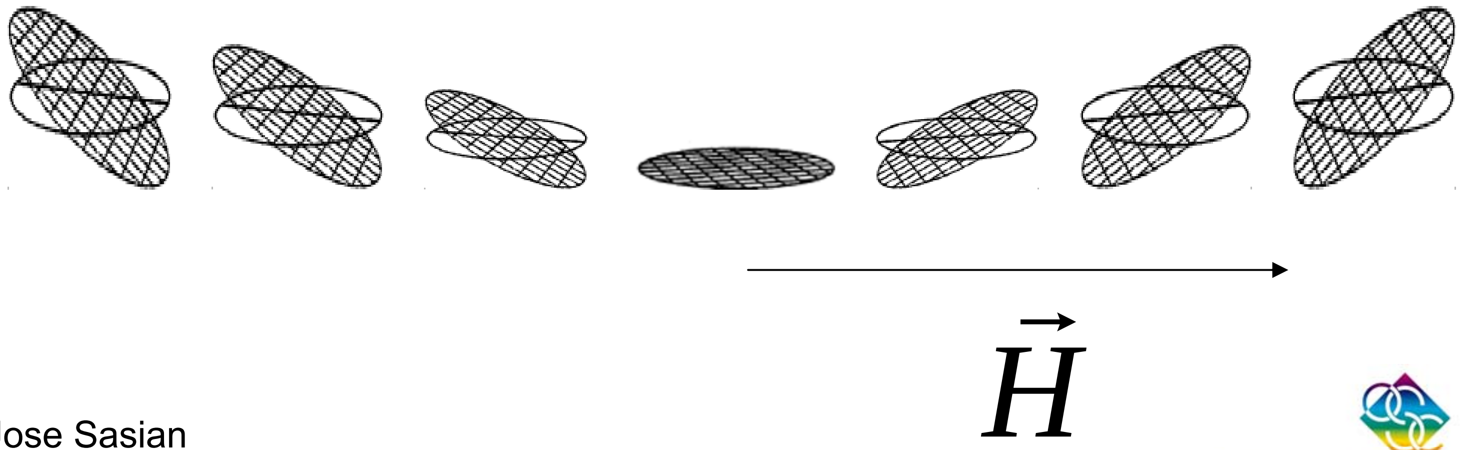
Change of Magnification

$$\Delta W_{111}\left(\vec{H}\cdot\vec{\rho}\right)$$



- Change of magnification depends linearly with the field of view.



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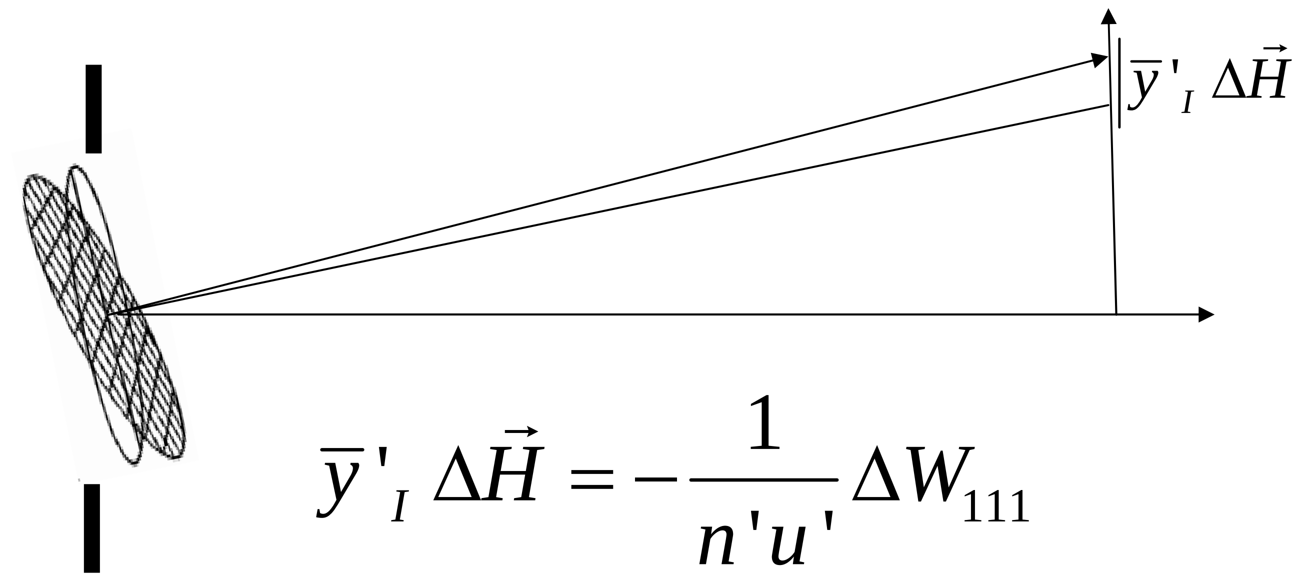
Change of magnification



$$\bar{y}'_I \Delta\vec{H} = -\frac{1}{n'u'}\Delta W_{111}$$

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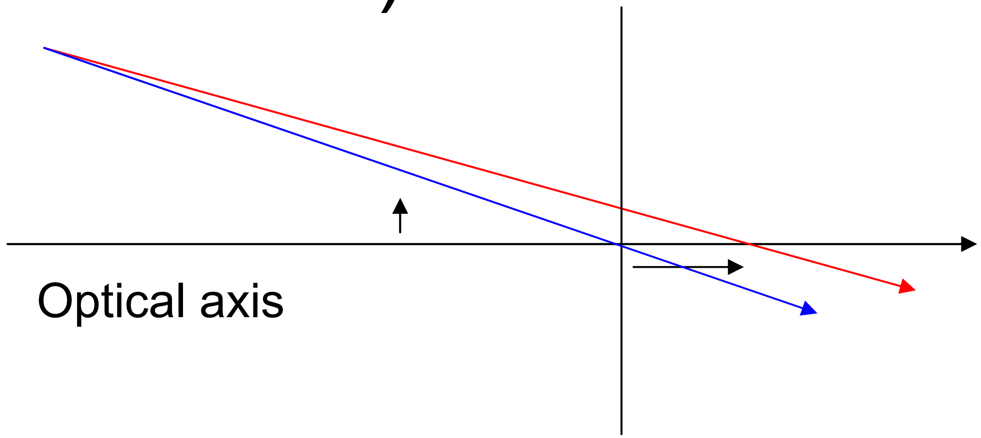
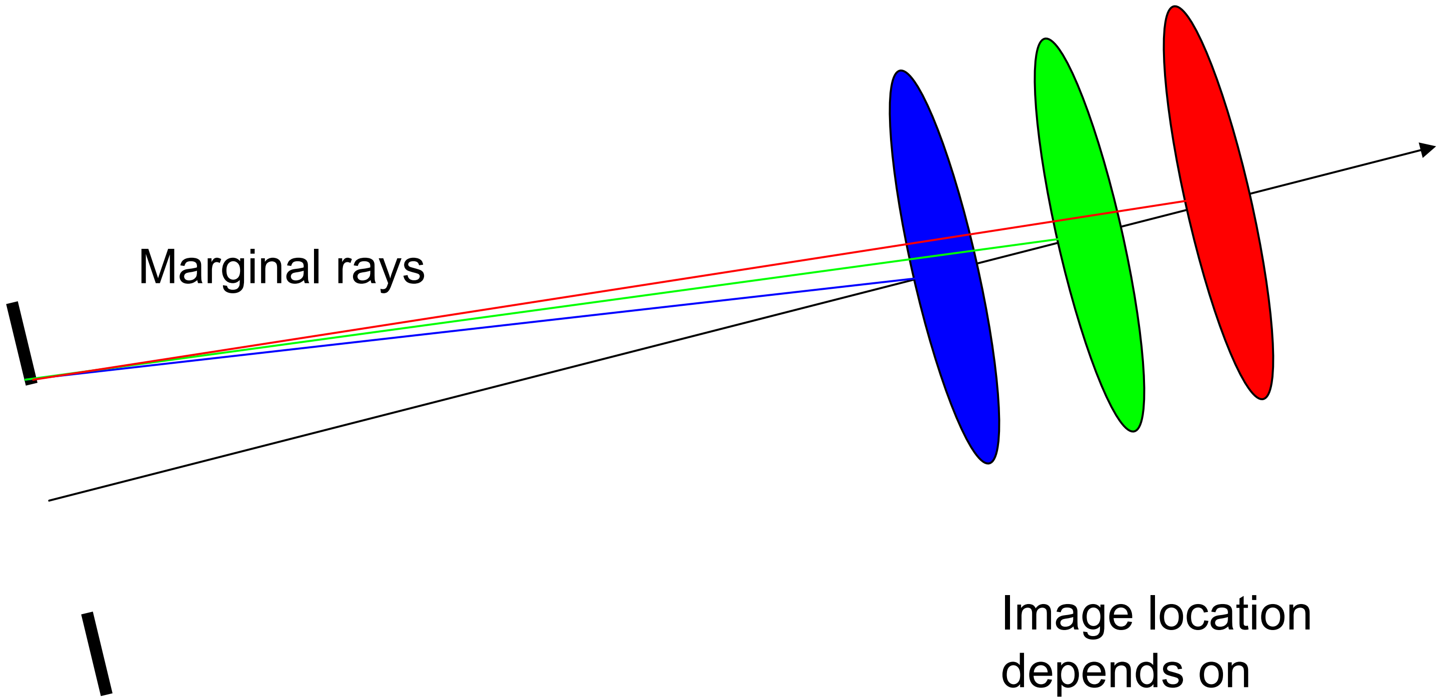
Clear terminology

- Chromatic change of magnification (older: lateral, transverse color)
- Chromatic change of focus (older: axial, longitudinal color)



Chromatic change of focus



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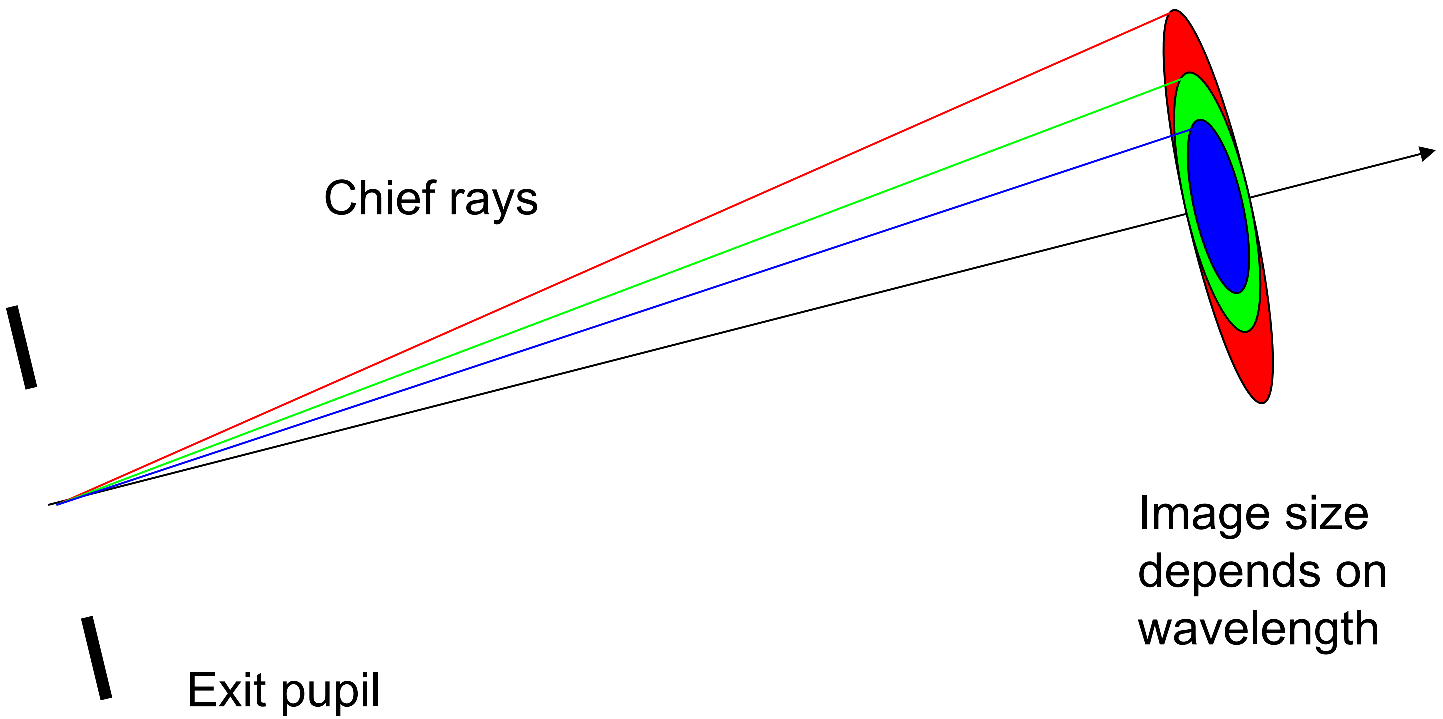
Chromatic change of magnification



Equation summary

For a system of j surfaces

$$\partial_\lambda W_{000} = \sum_{i=0}^{j} t_i \left(n'_F - n'_C \right)_i$$

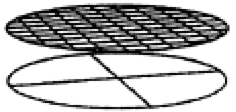
$$\partial_\lambda W_{200} = \frac{1}{2} \sum_{i=1}^{j} \frac{\left(\bar{A}_j\right)^2}{A_j} \Delta_j \left(\partial n / n \right) y_j$$

$$\partial_\lambda W_{111} = \sum_{i=1}^{j} \bar{A}_j \Delta_j \left(\partial n / n \right) y_j$$

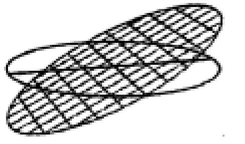
$$\partial_\lambda W_{020} = \frac{1}{2} \sum_{i=1}^{j} A_j \Delta_j \left(\partial n / n \right) y_j$$

For a system of thin lenses

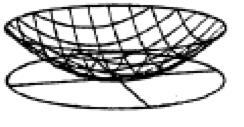
$$\partial_\lambda W_{200} = \frac{1}{2} \sum_{i=1}^{j} \left[\frac{\phi}{\nu} \bar{y}^2 \right]_i$$



$$\partial_\lambda W_{111} = \sum_{i=1}^{j} \left[\frac{\phi}{\nu} \bar{y} y \right]_i$$



$$\partial_\lambda W_{020} = \frac{1}{2} \sum_{i=1}^{j} \left[\frac{\phi}{\nu} y^2 \right]_i$$



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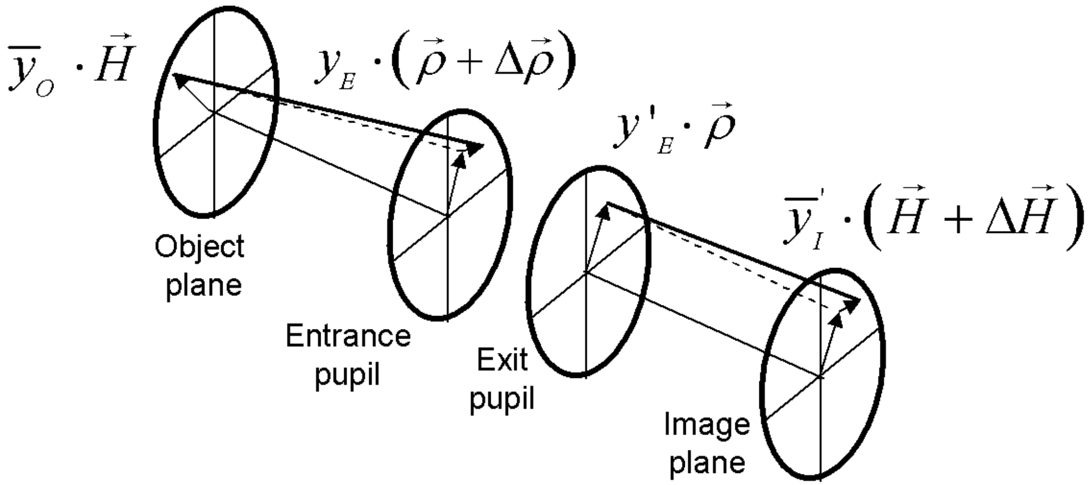
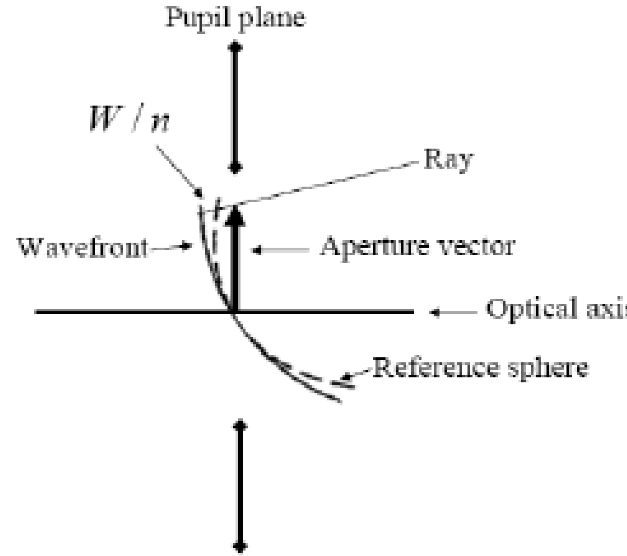
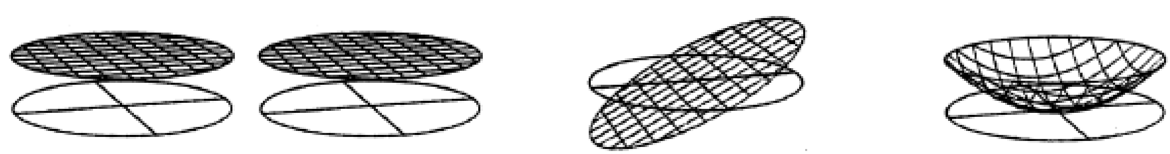
Summary I



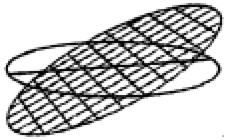


$$W\left(\vec{H},\rho\right) = W_{000} + W_{200}H^2 + W_{111}H\rho\cos\left(\varphi\right) + W_{020}\rho^2$$

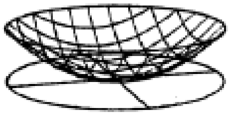


Summary II

$$\partial_\lambda W_{111} = \sum_{i=1}^{j} \bar{A}_j \Delta_j \left(\partial n / n\right) y_j \qquad \partial_\lambda W_{111} = \sum_{i=1}^{j} \left[\frac{\phi}{\nu} \bar{y} y \right]_i$$



$$\partial_\lambda W_{020} = \frac{1}{2} \sum_{i=1}^{j} A_j \Delta_j \left(\partial n / n\right) y_j \qquad \partial_\lambda W_{020} = \frac{1}{2} \sum_{i=1}^{j} \left[\frac{\phi}{\nu} y^2 \right]_i$$



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Summary III

$$\Delta s' = -2\frac{\Delta W_{020}}{n'}\frac{1}{u'^2}$$

$$\Delta s' = -8\frac{\Delta W_{020}}{n'}\left(f/\#\right)^2$$

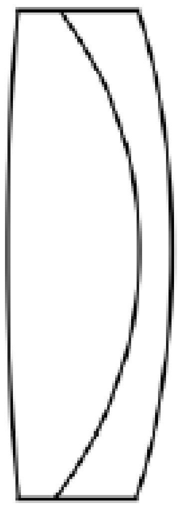
$$\Delta s' \approx \left(f/\#\right)^2 \ in\ micrometers\ \ for\ \Delta W_{020} = \frac{\lambda}{4}$$

$$\lambda = 0.0005\ mm$$

$$\nu_c = \frac{2 \cdot NA}{\lambda}$$

Cut-off spatial frequency for a LSIS with incoherent illumination

Achromatic doublet



$$\Phi = \Phi_a + \Phi_b \qquad \frac{\Phi_a}{\nu_a} = -\frac{\Phi_b}{\nu_b}$$

$$\Phi_a = \Phi \frac{\nu_a}{\nu_a - \nu_b}$$

$$\Phi_b = -\Phi \frac{\nu_b}{\nu_a - \nu_b}$$

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Chromatic coordinates

$$n^2(\lambda) = 1 + \frac{B_1\lambda^2}{\lambda^2 - C_1} + \frac{B_2\lambda^2}{\lambda^2 - C_2} + \cdots + \frac{B_i\lambda^2}{\lambda^2 - C_i},$$

Sellmeier

$$n(\omega) = n_0 + \nu_1\omega^1 + \nu_2\omega^2 + \cdots + \nu_j\omega^j$$

$$\omega = \frac{\lambda - \lambda_0}{1 + \alpha(\lambda - \lambda_0)}$$

Buchdahl

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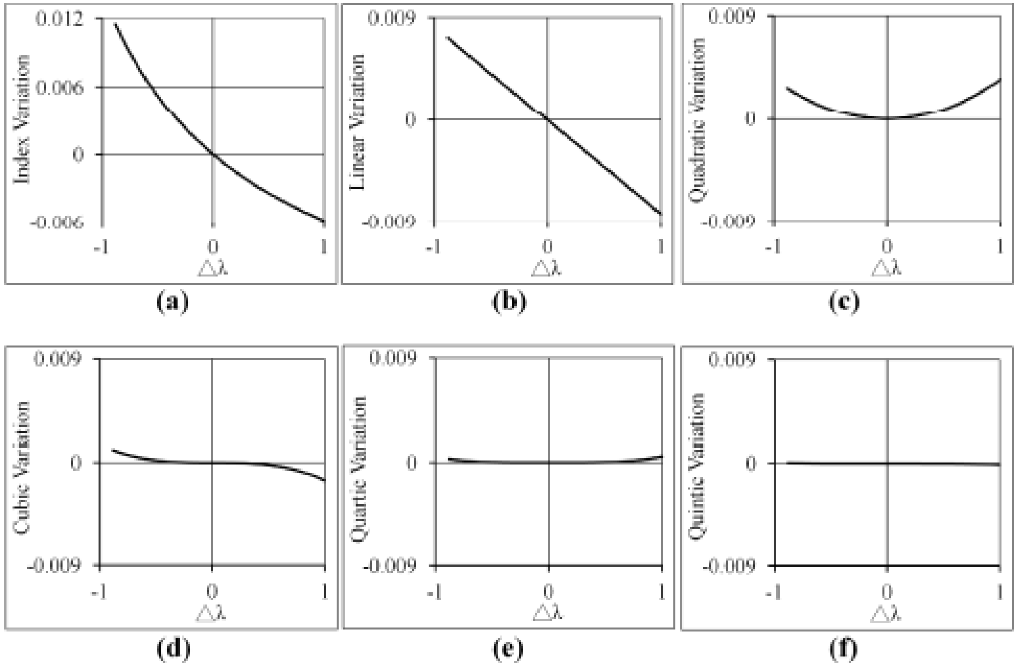
Adaptive formula

$$n = n_0 + A_1(\Delta\lambda) + A_2(\Delta\lambda)^2 + \cdots + A_q(\Delta\lambda)^q + \frac{A_{q+1}(\Delta\lambda)^{q+1}}{1 + K(\Delta\lambda)}$$

$$\Delta\lambda = \frac{\lambda - \lambda_0}{\Lambda}$$



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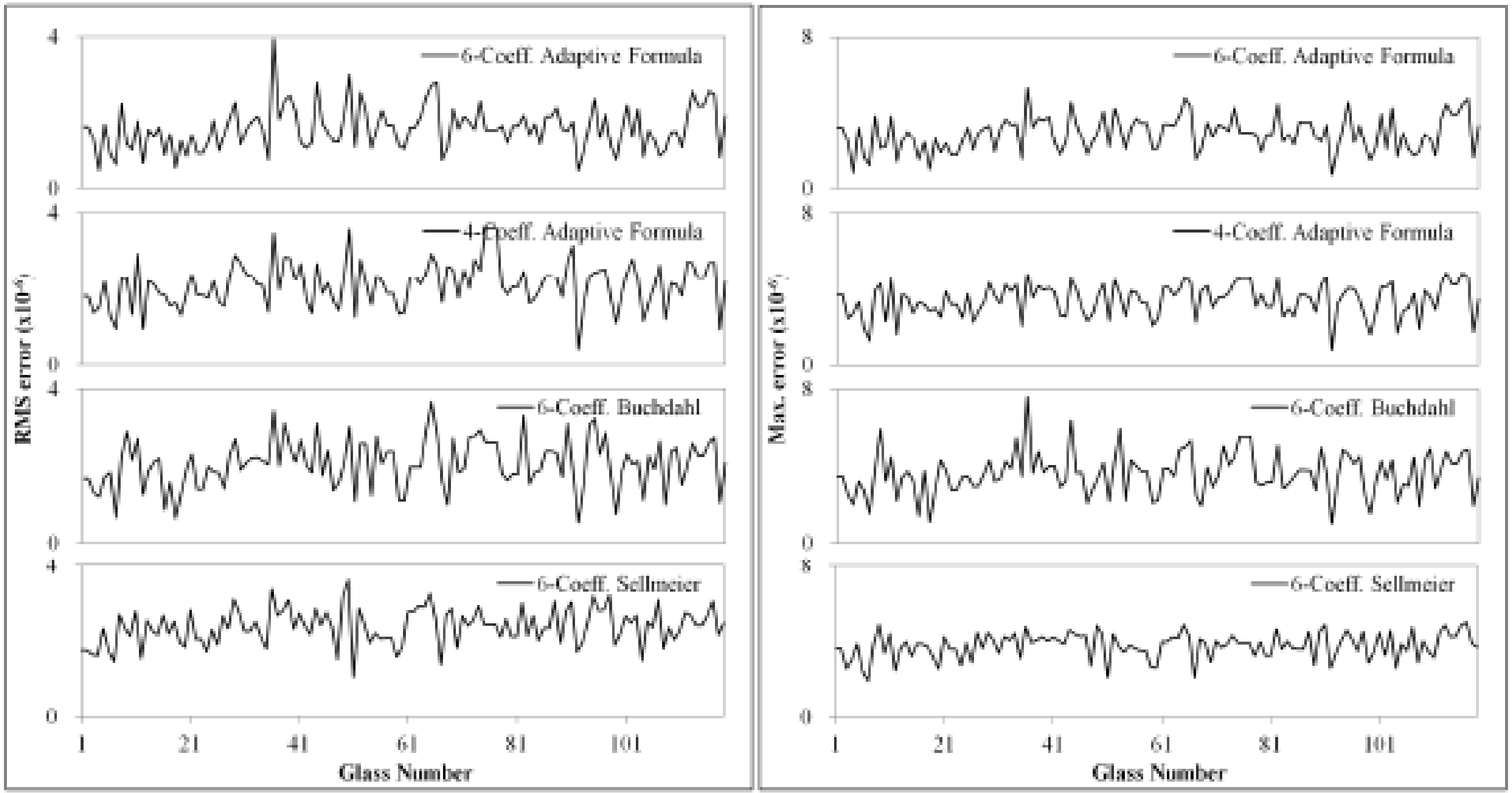
Index fit



(a) RMS Fitting Error (b) Maximum Fitting Error

Schott glass catalogue

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Chromatic change of focus

$$\partial_{\lambda} W_{020} = \sum_{i=1}^{N}\left[\frac{y_i^2}{2}(c_{1i}-c_{2i})(n_i-n_{0i})\right]$$

$$= \sum_{i=1}^{N}\left[S_i\left(A_{1i}(\Delta\lambda)+A_{2i}(\Delta\lambda)^2+\cdots+A_{qi}(\Delta\lambda)^q+\frac{A_{q+1,i}(\Delta\lambda)^{q+1}}{1+K_i(\Delta\lambda)}\right)\right],$$

$$S=\frac{y^2}{2}(c_1-c_2)$$

$$0=\sum_{i=1}^{N}S_iA_{1i}=\sum_{i=1}^{N}S_iA_{2i}=\sum_{i=1}^{N}S_iA_{3i}$$

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Example

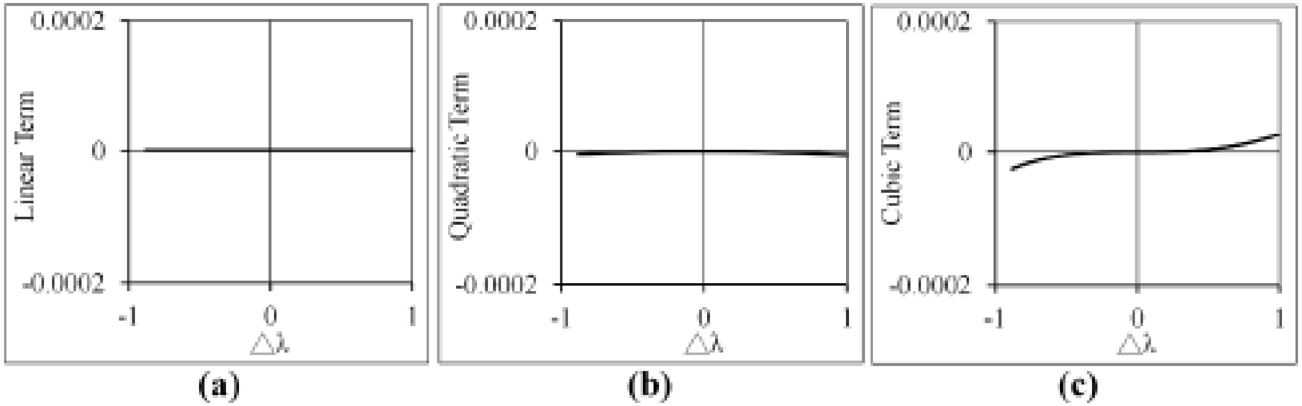
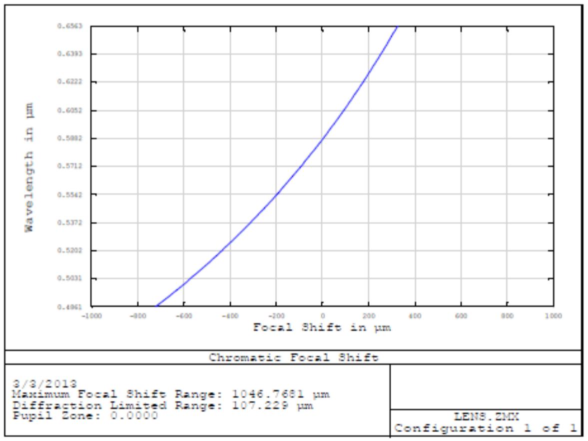
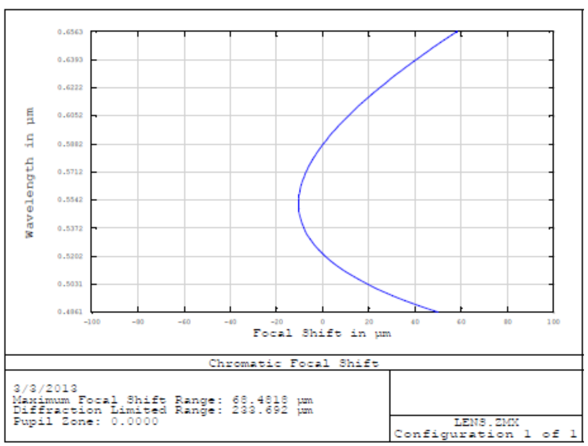


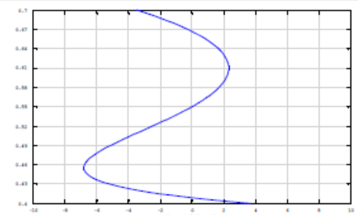
Fig. 4. Cancelation of primary and secondary spectrum in a doublet by using N-FK51A and N-PSK3 glasses: (a) linear term, (b) quadratic term, (c) residual cubic term.

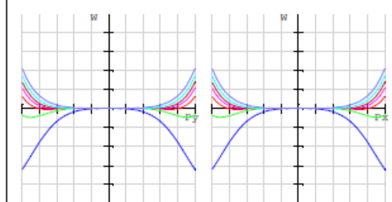
Chromatic focal shift: singlet, achromat, apochromat, superapochromat

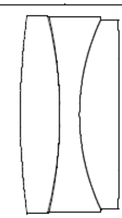




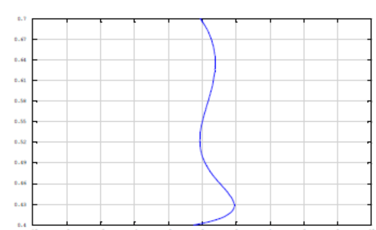
Lens	Glass	R1	R2
L1	N-FK51A	32.191	-18.72
L2	N-PSK3	-19.161	228.474

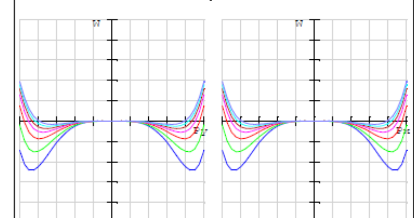






Lens	Glass	R1	R2
L 1	LF5	37.470	-20.360
L2	N-KZFS4	-22.692	11.141
L3	N-BK7	11.141	-355.792





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Adaptive dispersion formula for index interpolation and chromatic aberration correction

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Table 3. Coefficients of the Adaptive Dispersion Formula

Glass	Glascode	RMS error	Max error	n_0	A_1	A_2	A_3	K
F2	620364.360	1.78×10^{-6}	3.66×10^{-6}	1.6240797991	−0.0175823138	0.0084939540	−0.0039437159	0.4482924521
F2HT	620364.360	1.78×10^{-6}	3.66×10^{-6}	1.6240797991	−0.0175823138	0.0084939540	−0.0039437159	0.4482924521
F5	603380.347	1.34×10^{-6}	2.41×10^{-6}	1.6071803097	−0.0163564579	0.0078466009	−0.0036212272	0.4427577747
K10	501564.252	1.50×10^{-6}	2.68×10^{-6}	1.5034895779	−0.0091235644	0.0039963730	−0.0017361587	0.4078457752
K7	511604.253	2.17×10^{-6}	3.29×10^{-6}	1.5131399068	−0.0086740087	0.0037429354	−0.0016023252	0.4056074727
KZFS12	696363.384	1.29×10^{-6}	1.90×10^{-6}	1.7005486953	−0.0197559775	0.0093812806	−0.0043473655	0.4481076563
LAFN7	750350.438	9.14×10^{-7}	1.27×10^{-6}	1.7545809783	−0.0221108140	0.0106482744	−0.0049607472	0.4508494797
LF5	581409.322	2.28×10^{-6}	3.88×10^{-6}	1.5848200074	−0.0146682637	0.0069250085	−0.0031571546	0.4438275348
LLF1	548458.294	2.25×10^{-6}	4.24×10^{-6}	1.5509865723	−0.0123292334	0.0057038704	−0.0025694851	0.4278574641
N-BAF10	670471.375	1.30×10^{-6}	2.27×10^{-6}	1.6734077339	−0.0146314049	0.0067350023	−0.0030083033	0.4184418426
N-BAF4	606437.289	2.93×10^{-6}	4.50×10^{-6}	1.6089700000	−0.0142513307	0.0066758774	−0.0030899162	0.4489441850
N-BAF51	652450.333	9.32×10^{-7}	1.60×10^{-6}	1.6556884323	−0.0149422042	0.0069230748	−0.0030925600	0.4325816755
N-BAF52	609466.305	2.18×10^{-6}	3.65×10^{-6}	1.6117303475	−0.0134213158	0.0062205045	−0.0028584367	0.4353441662
N-BAK1	573576.319	2.10×10^{-6}	3.53×10^{-6}	1.5748684175	−0.0102142823	0.0045090173	−0.0019318776	0.3974743819
N-BAK2	540597.286	1.86×10^{-6}	2.68×10^{-6}	1.5421182517	−0.0092770487	0.0040325181	−0.0017255081	0.3991595342
N-BAK4	569560.305	1.82×10^{-6}	3.30×10^{-6}	1.5712484846	−0.0104272578	0.0046121424	−0.0020063385	0.3988090023
N-BALF4	580539.311	1.53×10^{-6}	3.12×10^{-6}	1.5821197901	−0.0110506897	0.0049349611	−0.0021552280	0.4052109538
N-BALF5	547536.261	1.61×10^{-6}	2.77×10^{-6}	1.5498197672	−0.0104818527	0.0046568148	−0.0020542052	0.4205205555
N-BASF2	664360.315	1.28×10^{-6}	2.96×10^{-6}	1.6688298093	−0.0190288069	0.0092672742	−0.0043988202	0.4668742354
N-BASF64	704394.320	1.86×10^{-6}	2.48×10^{-6}	1.7082375202	−0.0184101344	0.0087455915	−0.0040350268	0.4456748237
N-BK10	498670.239	2.37×10^{-6}	3.83×10^{-6}	1.4995991678	−0.0076080500	0.0031294407	−0.0013115008	0.4099663546
N-BK7	517642.251	1.81×10^{-6}	3.14×10^{-6}	1.5187231399	−0.0082423581	0.0034670871	−0.0014780390	0.3974299435