

Introduction to aberrations

OPTI518

Lecture 2

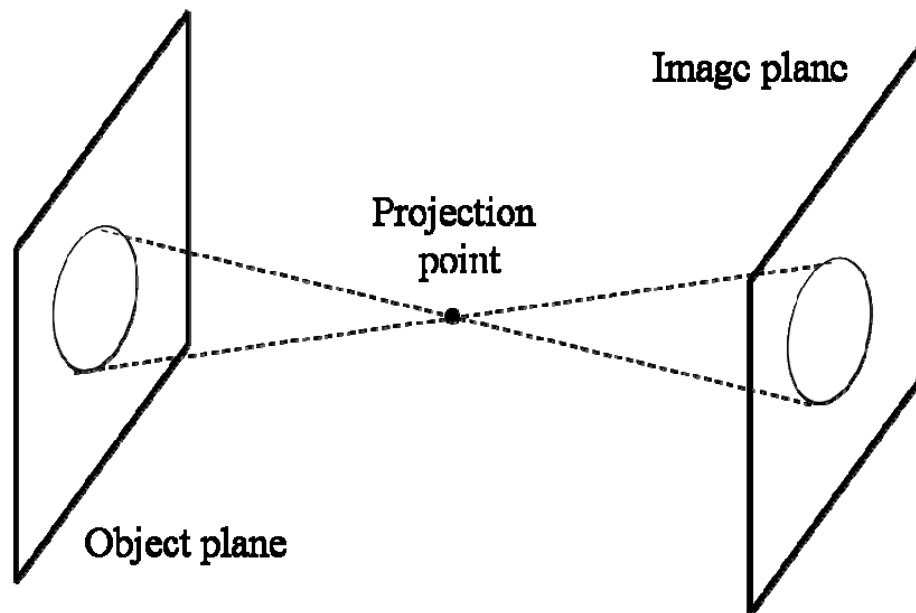
Imaging with rays and with light
waves

Clarifying the context

Aberrations and images

- Several topics of interest
 - Aberration theory
 - Aberration evaluation
 - Image formation
 - Image evaluation
- Geometrical and physical theories

Imaging with rays



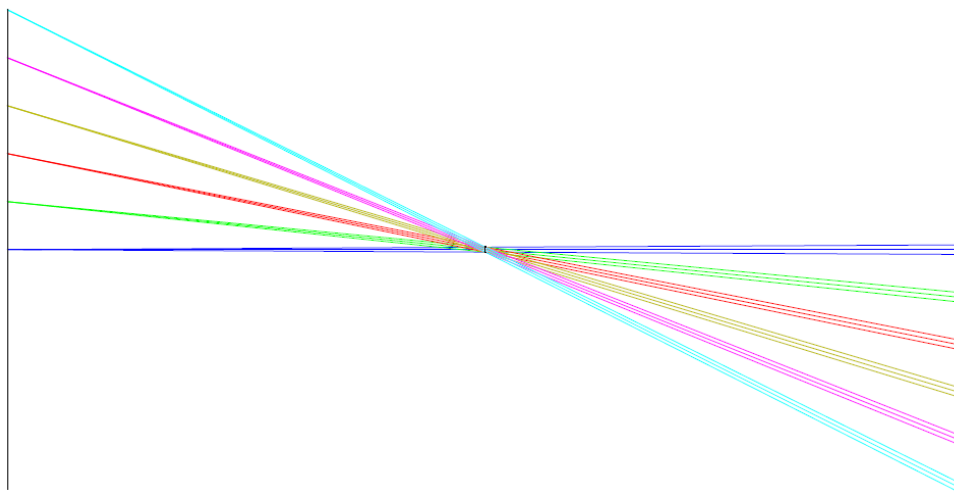
Collinear transformation

$$X' = \frac{a_1X + b_1Y + c_1Z + d_1}{a_0X + b_0Y + c_0Z + d_0}$$

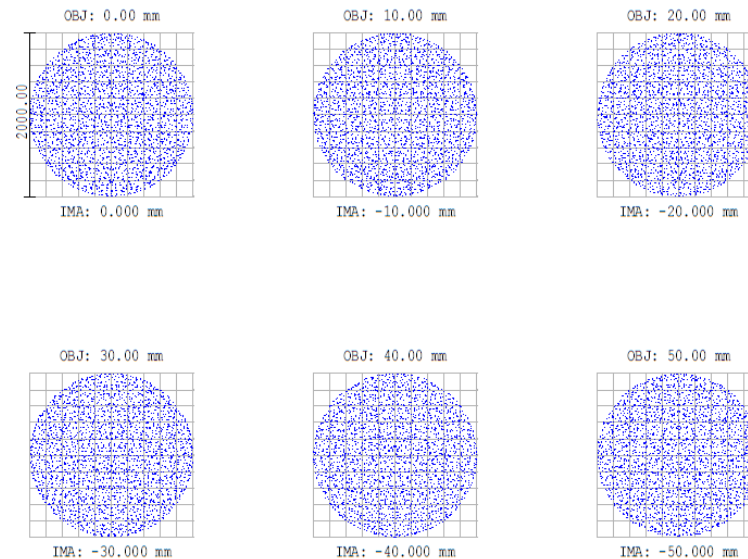
$$Y' = \frac{a_2X + b_2Y + c_2Z + d_2}{a_0X + b_0Y + c_0Z + d_0}$$

$$Z' = \frac{a_3X + b_3Y + c_3Z + d_3}{a_0X + b_0Y + c_0Z + d_0}$$

Camera obscura



Use a finite size
aperture



How an image is formed?

Object points into image points

Object points into image spots

Extended objects into extended images

Theory

- Assume that the imaging system behaves as a linear shift invariant system
- Is this plausible for the camera obscura?

Linear shift invariant system

$$L\{f_1(x, y)\} = g_1(x, y)$$

$$L\{f_2(x, y)\} = g_2(x, y)$$

$$\begin{aligned} L\{a_1 f_1(x, y) + a_2 f_2(x, y)\} &= L\{a_1 f_1(x, y)\} + L\{a_2 f_2(x, y)\} \\ &= a_1 L\{f_1(x, y)\} + a_2 L\{f_2(x, y)\} = a_1 g_1(x, y) + a_2 g_2(x, y) \end{aligned}$$

$$L\{f(x - x_0, y - y_0)\} = g(x - x_0, y - y_0)$$

Complex exponentials are eigen-functions of LSI systems

$$L\{e^{-i2\pi\xi_0 x}\} = H(\xi_0) e^{-i2\pi\xi_0 x}$$

LSIS Properties

- Characterized by an impulse function $h(x,y)$
- The output is the convolution of the object function with the impulse function
- In the spatial frequency domain the image spectrum is the product of the transfer function with the object spectrum.

Convolution

$$g(x', y') = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) h(x' - x, y' - y) dx dy$$

In the frequency domain

$$G(\eta, \xi) = F(\eta, \xi) H(\eta, \xi)$$

G, F, and H are the FT (plane wave representation) of g, f, and h

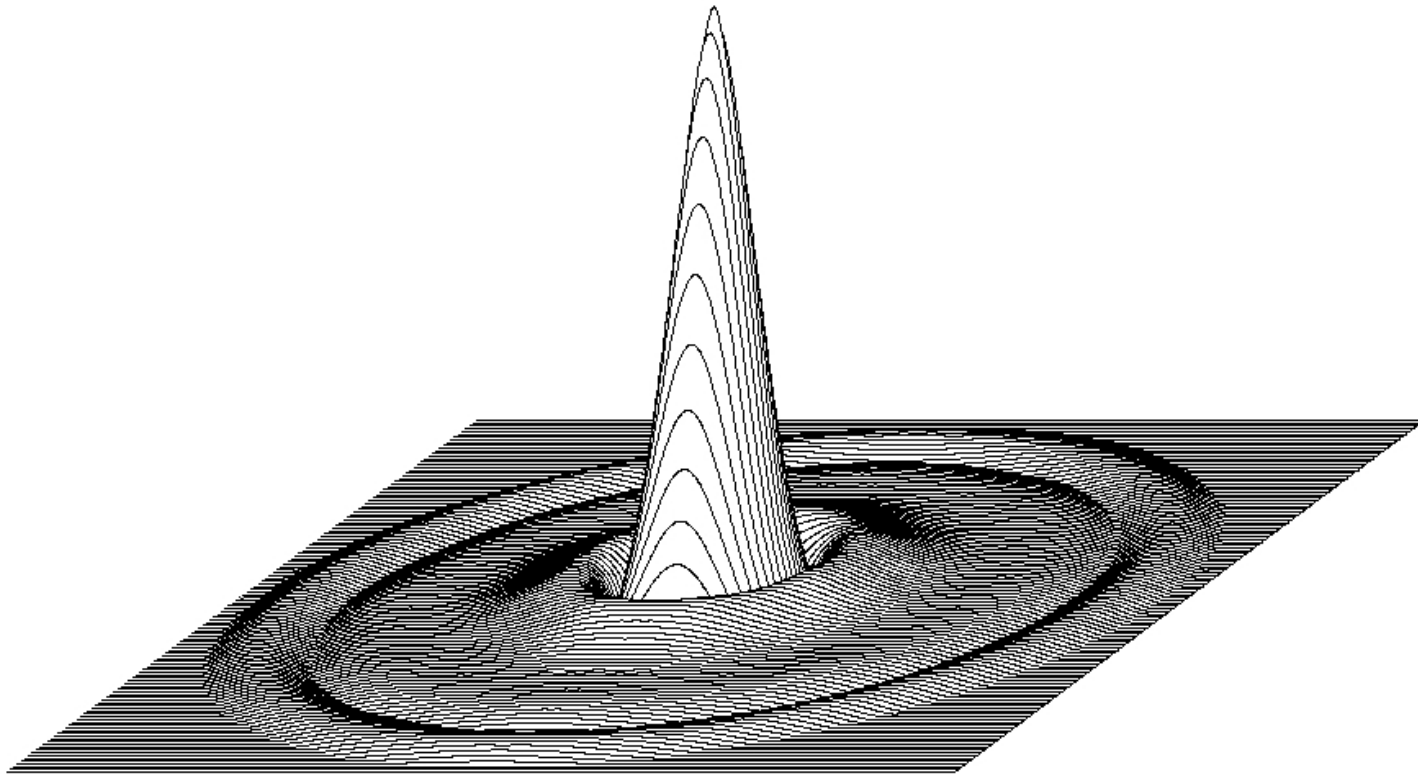
OTF and MTF

- H is actually called the optical transfer function (OTF)
- The modulus of the OTF is the modulation transfer function MTF
- The images of complex exponentials (sines and cosines) in a LSI system are complex exponentials
- The contrast is given by the MTF

MTF

Two dimensional

(not necessarily axially symmetric)



$$\text{MTF} = \frac{\text{Max} - \text{Min}}{\text{Max} + \text{Min}}$$

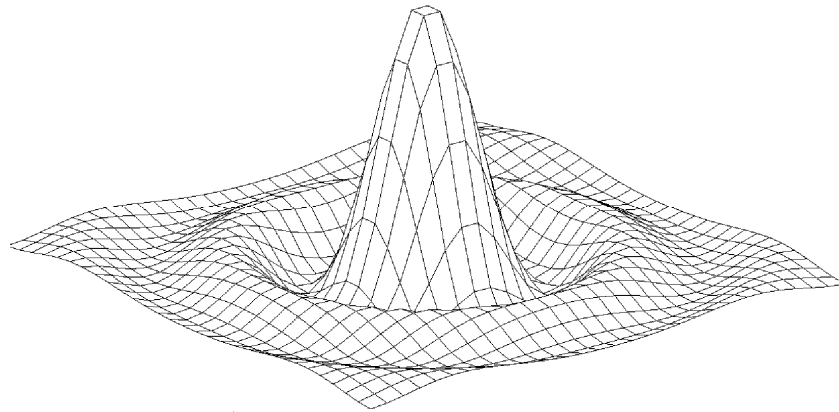


MTF provides the contrast change as a function of spatial frequency

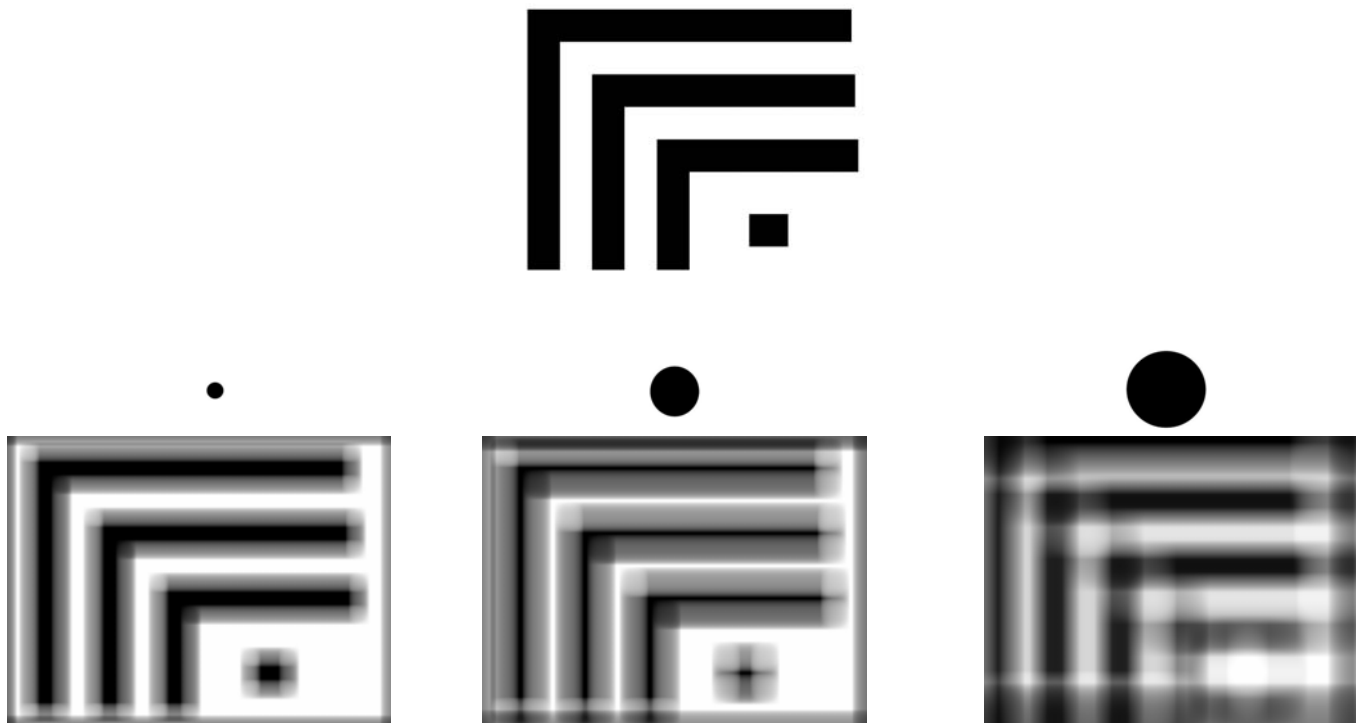
Camera obscura

$$\text{cyl}\left(\frac{r}{d}\right) = \begin{cases} 1, & 0 \leq r \leq \frac{d}{2} \\ \frac{1}{2}, & r = \frac{d}{2} \\ 0, & r > \frac{d}{2} \end{cases}$$

$$H(\eta, \zeta) = FT \left\{ \text{Cyl}\left(\frac{r}{d}\right) \right\} = \frac{\pi}{4} d^2 \text{sombbrero}(d\rho)$$



Images of corners under convolution with circles



Point spread function

$$h(x, y)$$

- Because in a linear shift invariant system the impulse response is key to determine the image, we are concerned about how optical aberrations change the point spread function.

Imaging with light waves

- Physical image formation
- Depends on coherent, incoherent, or partial coherence illumination
- H. H. Hopkins, “On the diffraction theory of optical images,” Proc. Roy. Soc. A. 217, 408-431, 1952.

HAROLD HORACE HOPKINS

6 December 1918–22 October 1994

Elected F.R.S. 1973



H. H. Hopkins.

The concept of partial coherence in optics

BY H. H. HOPKINS, PH.D.

$$I(x', y') = \int_{S_1} \int_{S_2} \gamma_{12}(x_1 - x_2, y_1 - y_2) A(x_1, y_1) A(x_2, y_2) E(x_1, y_1) \times E^*(x_2, y_2) f(x' - x_1, y' - y_1) f^*(x' - x_2, y' - y_2) dS_1 dS_2, \quad (44)$$

where γ_{12} is the phase-coherence factor.

If the object is coherently illuminated, $\gamma_{12} = 1$ and $I(x', y')$ is then found to be

$$I(x', y') = \left| \int_S A(x, y) E(x, y) f(x' - x, y' - y) dS \right|^2. \quad (45)$$

On the other hand, if the object is incoherent, $\gamma_{12} = 0$ except when $x_1 = x_2, y_1 = y_2$, for which $\gamma_{12} = 1$, (44) then gives

$$I(x', y') = \int_S I(x, y) |E(x, y)|^2 |f(x' - x, y' - y)|^2 dS, \quad (46)$$

where $I(x, y)$ is the intensity falling on (x, y) . Only the modulus of $E(x, y)$ now appears, showing that the image is unaffected by the argument of $E(x, y)$. Since a transparent

The Concept of Partial Coherence in Optics

Author(s): H. H. Hopkins

Source: *Proceedings of the Royal Society of London. Series A, Mathematical and Physical Sciences*, Vol. 208, No. 1093 (Aug. 22, 1951), pp. 263-277

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H. H. Hopkins' equation

$$I(x, y) = \left(\frac{1}{f\lambda} \right)^2 \iint_{\infty} \sigma(x_0, y_0) \left| s(x, y) t(x, y) ** psf(x, y) \right|^2 dx_0 dy_0$$

(it can be cast in different forms)

Two routes to obtain the OTF

$p(x, y)$ Pupil function, complex amplitude
of the optical field at the exit pupil

$h(\eta, \zeta) = FT \{ p(x, y) \}$ Coherent impulse response of system

$h_{incoherent}(\eta, \zeta) = |h_{coherent}(\eta, \zeta)|^2 = |FT \{ p(x, y) \}|^2$
Incoherent impulse response

$OTF(x, y) = FF \{ |h_{coherent}(\eta, \zeta)|^2 \}$ Optical transfer function

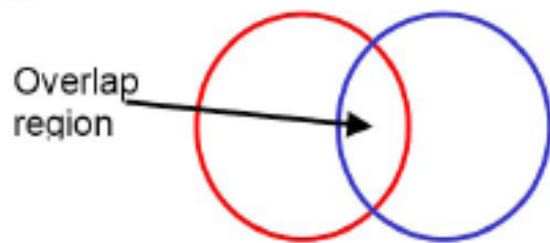
$OTF(x, y) = p(x, y) \otimes \otimes p(x, y)$ Pupil autocorrelation

$MTF = |OTF(x, y)|$
Prof. Jose Sasian
OPTI 518

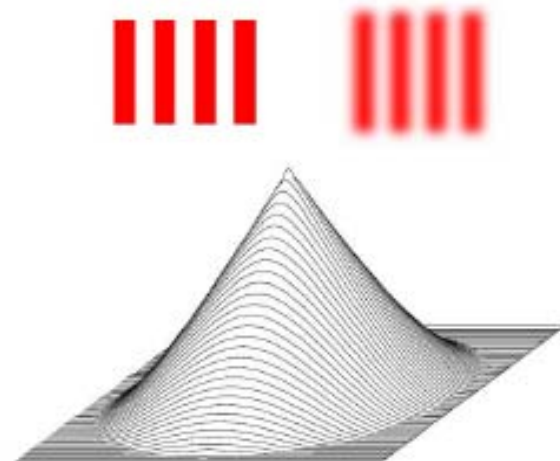
Modulation transfer function

Modulation transfer function for a circular aperture

- Extended image evaluation vs image of point object evaluation
- System is modeled as a LSI system: Superposition and response is invariant upon input variable change
- Two routes to calculate the MTF: 1) Via Pupil autocorrelation;
2) Via Fourier transform of PSF
- OTF is a 2D complex function
- MTF is modulus of OTF
- Key concept: MTF gives the image contrast as a function spatial frequency
- Cut-off frequency $2NA/\lambda$
- Incoherent illumination



$$MTF = \frac{Area_{\text{overlap}}}{Area_{\text{used}}}$$



Cut-off frequency $2NA/\lambda$

Moon under Galileo's telescope



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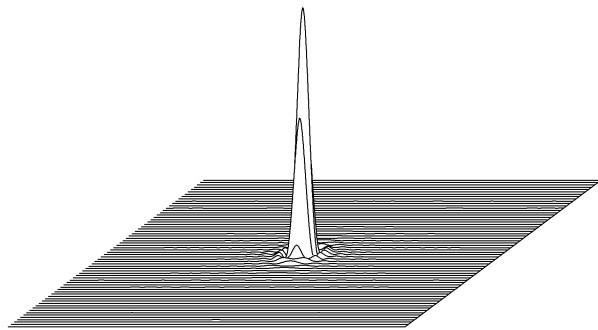


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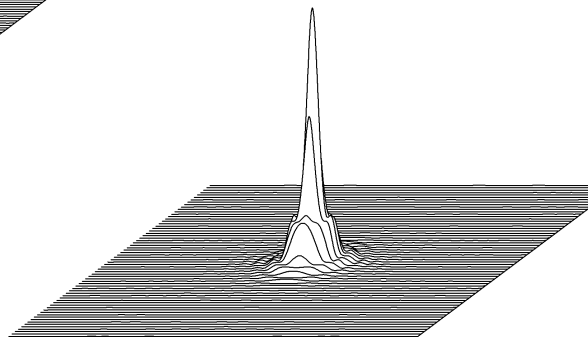


PSF for a lens under defocus

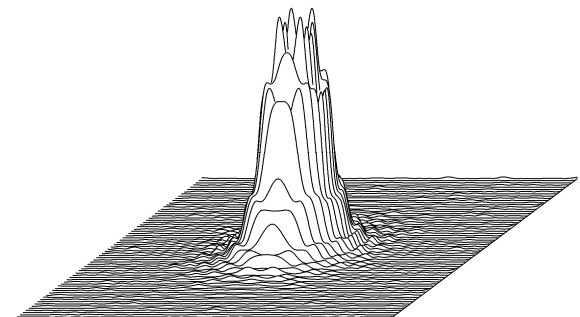
$F/10$; $f=100$ mm; $\text{Wave}=0.0005$ mm



0.00

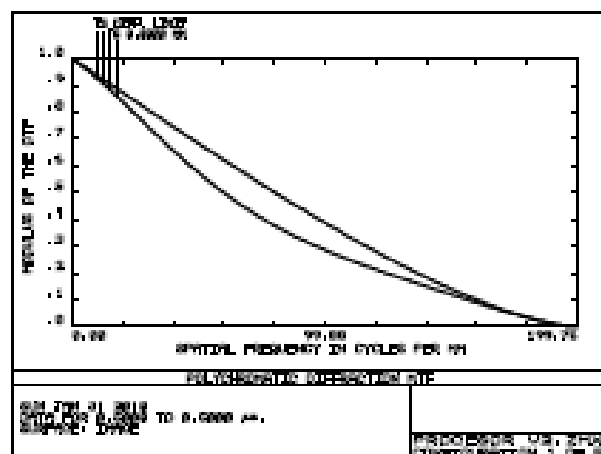


0.2 mm

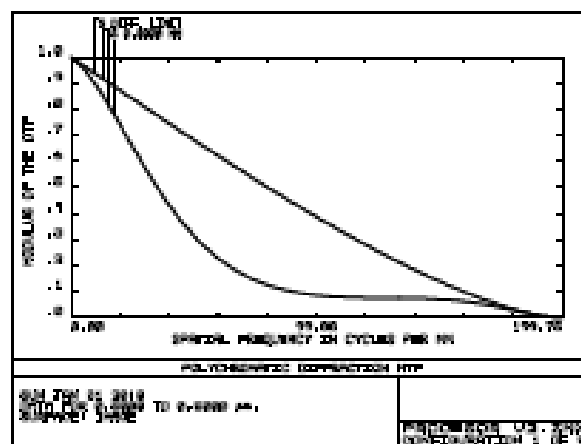


0.4 mm

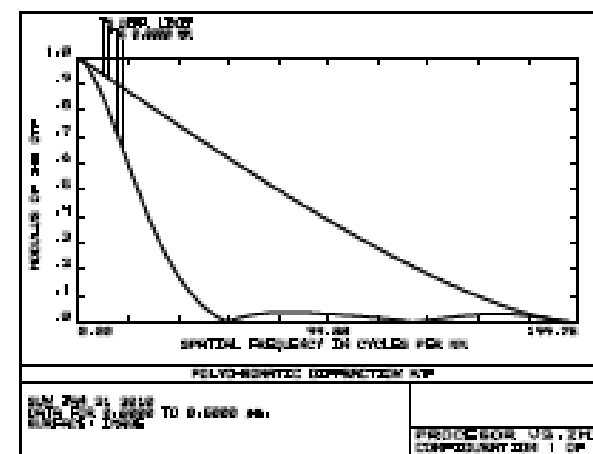
MTF drop, $f/10$, $f=100$, $\lambda=0.5$



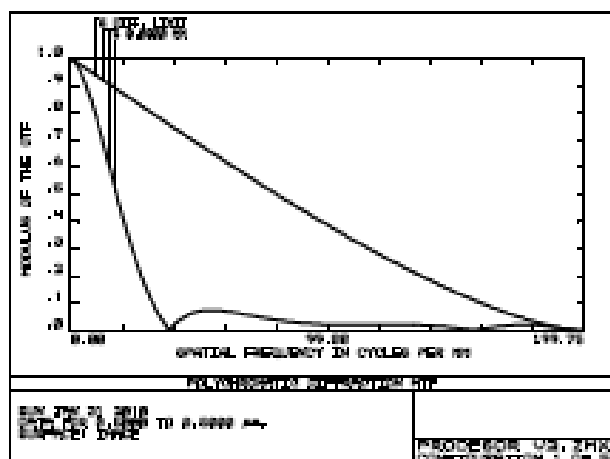
0.1 mm



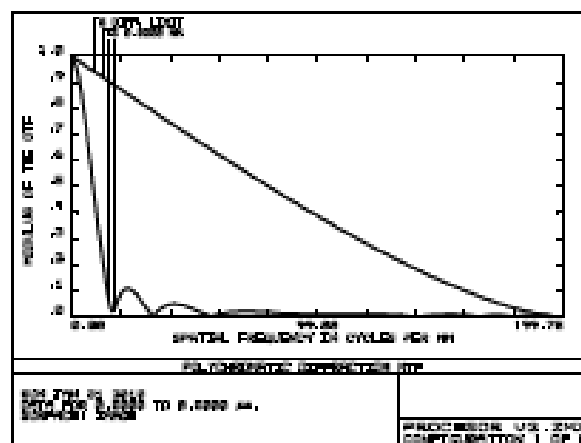
0.2 mm



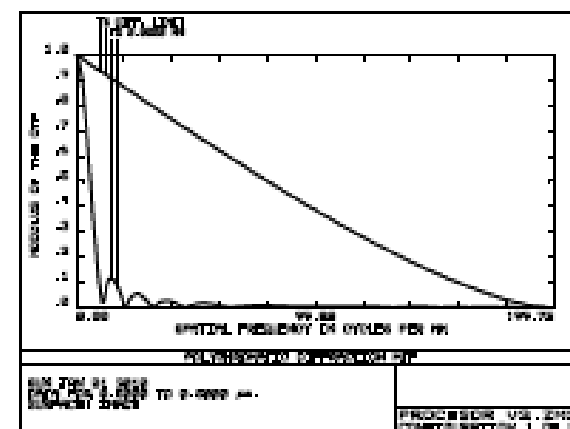
0.3 mm



0.4 mm

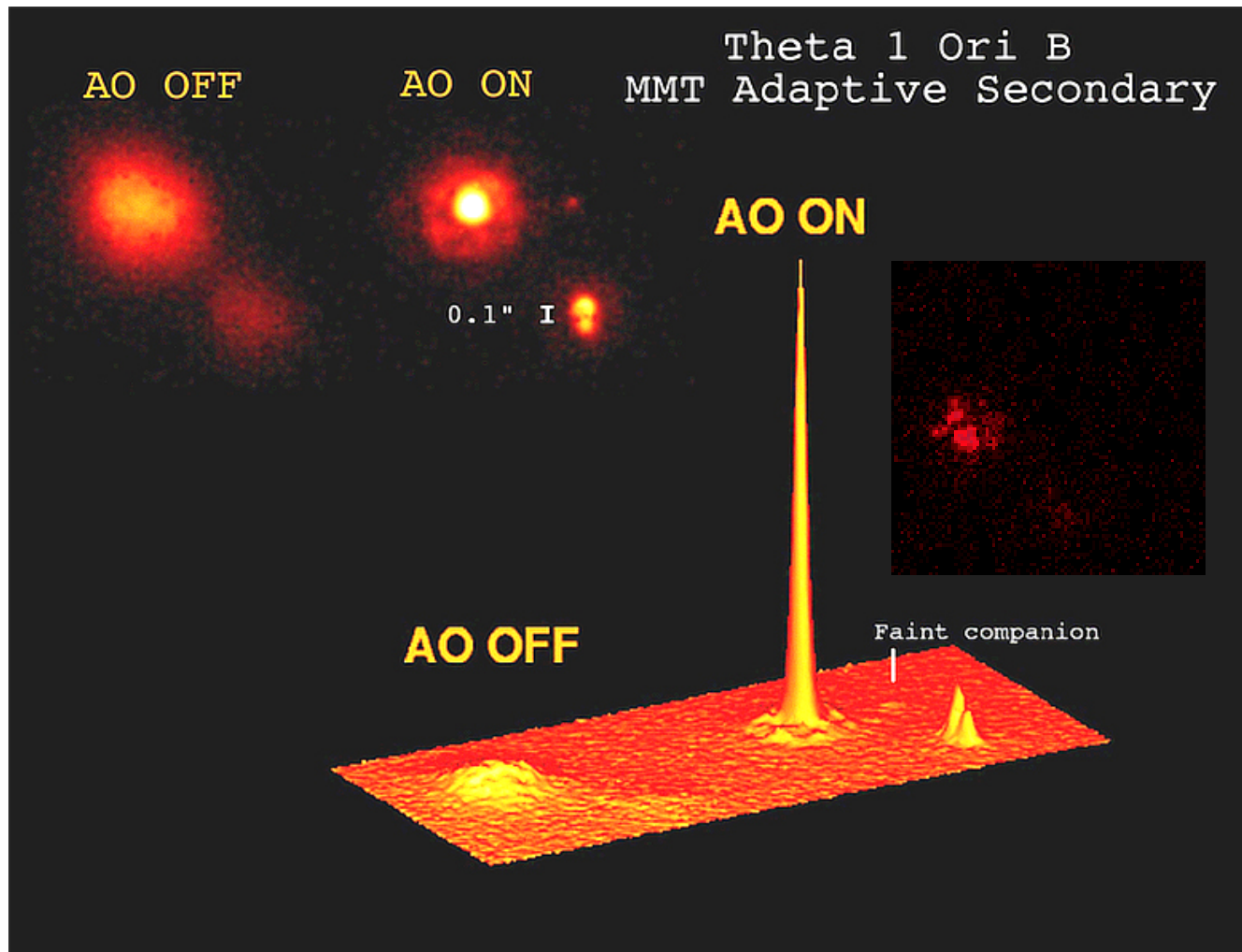


0.8 mm



1.2 mm

Random aberrations



Summary

- Imaging with rays
- Imaging with light waves
- Linear shift invariant systems
- The impulse function
- H. H. Hopkins pioneering work