Chromatic Aberrations

Lens Design OPTI 517

Prof. Jose Sasian

College of Optical Sciences
The University of Arizona

Second-order chromatic aberrations

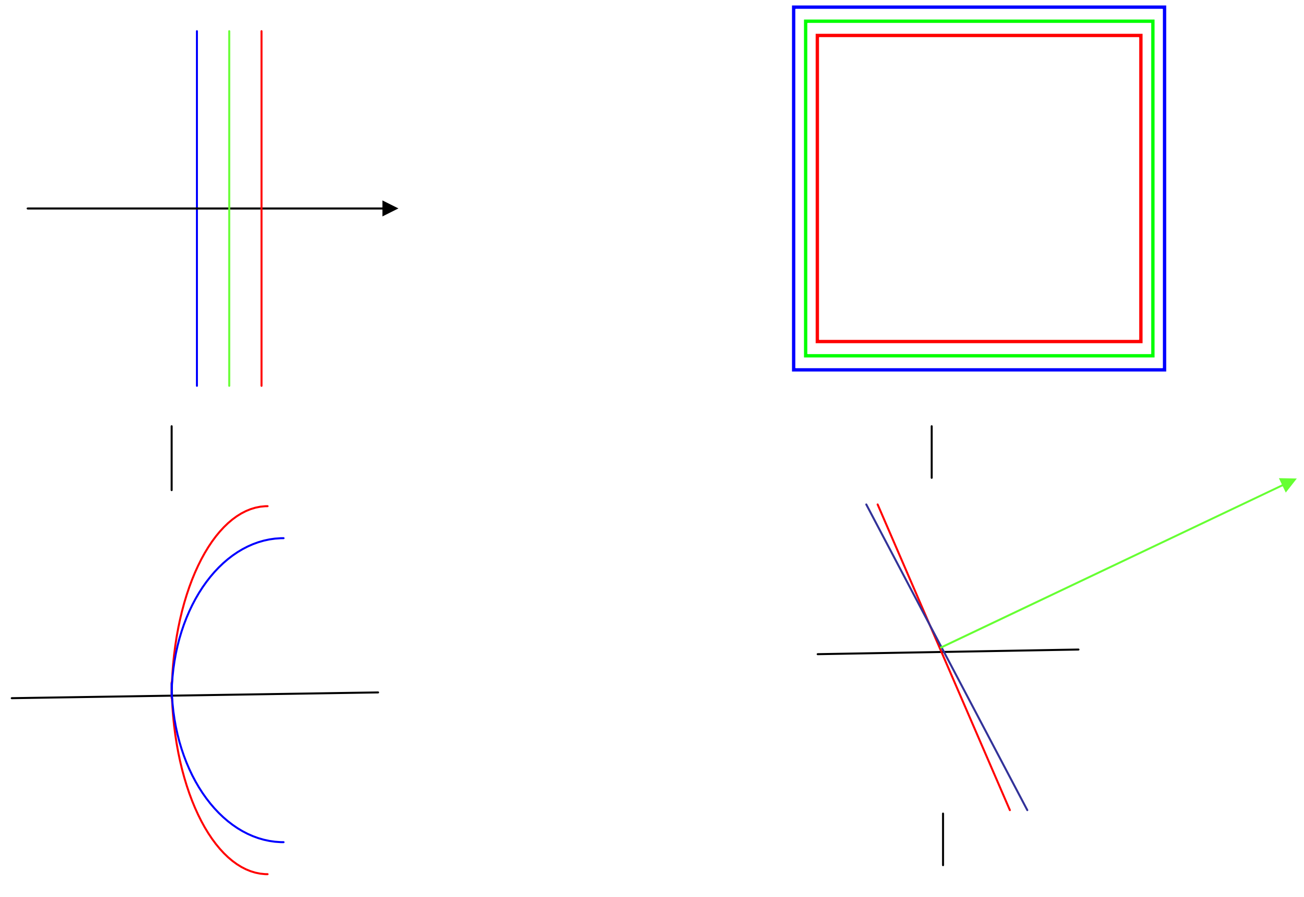
$$\partial_\lambda W\left(\vec{H},\rho\right)=\partial_\lambda W_{000}+\partial_\lambda W_{200}H^2+\partial_\lambda W_{111}H\rho\cos\left(\varphi\right)+\partial_\lambda W_{020}\rho^2$$

- Change of image location with λ (axial or longitudinal chromatic aberration)
- Change of magnification with λ (transverse or lateral chromatic aberration

Chromatic Aberrations

- Variation of lens aberrations as a function of wavelength
- Chromatic change of focus: W_{020}
- Chromatic change of magnification W_{111}
- Fourth-order: W_{040} and other
- Spherochromatism

Chromatic Aberrations



$$\partial_\lambda W_{020}\rho^2 \qquad \partial_\lambda W_{111} H \rho \cos(\varphi)$$

Topics

- Chromatic coefficients
- Optical glass and selection
- Index interpolation
- Achromats: crown and flint: different solutions
- Achromats: dialyte; single glass
- Mangin lens
- Third-order behavior
- Spherochromatism
- Secondary spectrum
- Tertiary spectrum
- Apochromats
- Super-apochromats
- Buried surface
- Monochromatic design: one task at a time
- Lateral color correction as an odd aberration
- Color correction in the presence of axial color
- Field lens to control lateral color: field lenses in general
- Conrady's D-d sum

Chromatic aberration coefficients

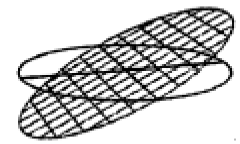
For a system of j surfaces

$$\partial_\lambda W_{111} = \sum_{i=1}^{j} \bar{A}_j \Delta_j \left(\partial n / n\right) y_j$$

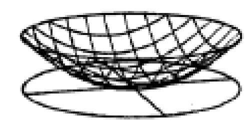
$$\partial_\lambda W_{020} = \frac{1}{2} \sum_{i=1}^{j} A_j \Delta_j \left(\partial n / n\right) y_j$$

For a system of thin lenses

$$\partial_\lambda W_{111} = \sum_{i=1}^{j} \left[\frac{\phi}{\nu} \bar{y} y \right]_i$$



$$\partial_\lambda W_{020} = \frac{1}{2} \sum_{i=1}^{j} \left[\frac{\phi}{\nu} y^2 \right]_i$$



Prof. Jose Sasian

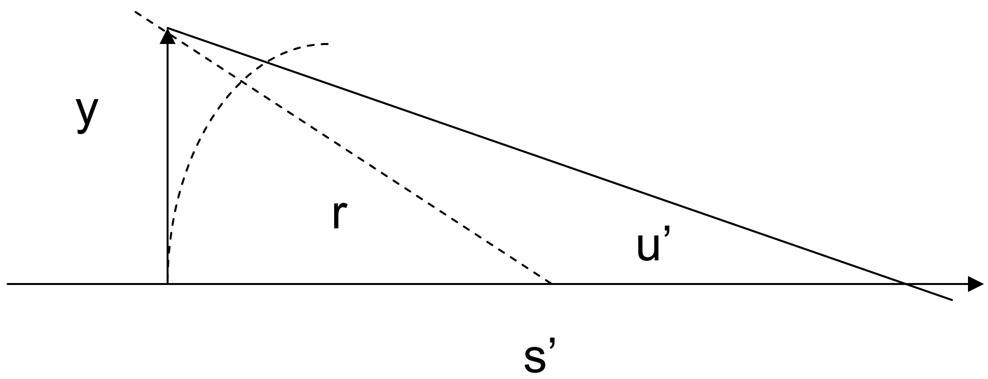
College of Optical Sciences
THE UNIVERSITY OF ARIZONA®

With stop shift

$$\Delta\partial_{\lambda}W_{020} = 0$$

$$\Delta\partial_{\lambda}W_{111} = 2\frac{\Delta\bar{y}}{y}\partial_{\lambda}W_{020}$$

Review of paraxial quantities



Quantities derived from first-order ray data used in computing the aberration coefficients				
Refraction invariant marginal ray	Refraction invariant chief ray	Lagrange invariant	Surface curvature	Petzval sum term
$A = ni = nu + nyc$	$\bar{A} = n\bar{i} = n\bar{u} + n\bar{y}c$	$Ж = n\bar{u}y - nu\bar{y}$ $= \bar{A}y - A\bar{y}$	$c = \frac{1}{r}$	$P = c \cdot \Delta\left(\frac{1}{n}\right)$

$$\partial n / n = \frac{n-1}{\nu n}$$

Prof. Jose Sasian

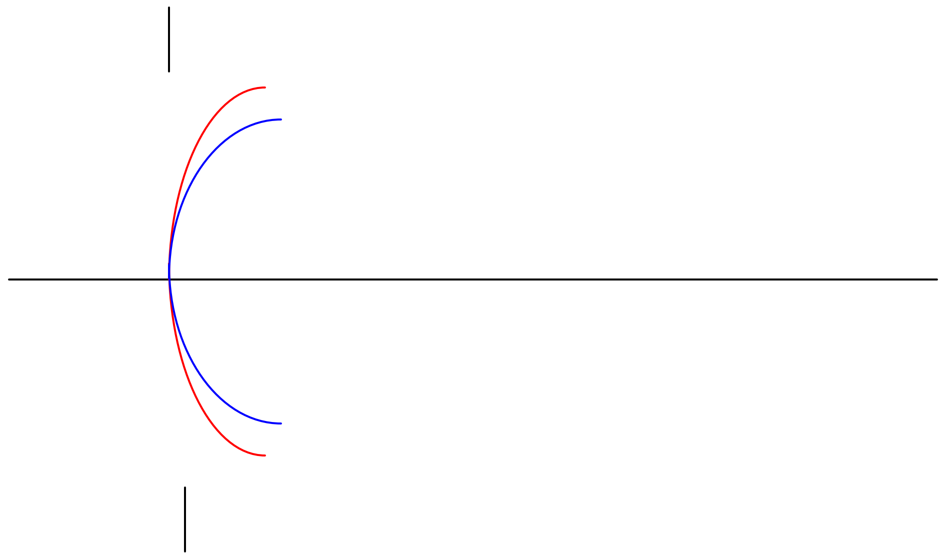
College of Optical Sciences
The University of Arizona

Chromatic coefficients

$$W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)=-\frac{y^2}{2}\left\{n'\left(\frac{1}{s'}-\frac{1}{r}\right)-n\left(\frac{1}{s}-\frac{1}{r}\right)\right\}\left(\vec{\rho}\cdot\vec{\rho}\right)$$

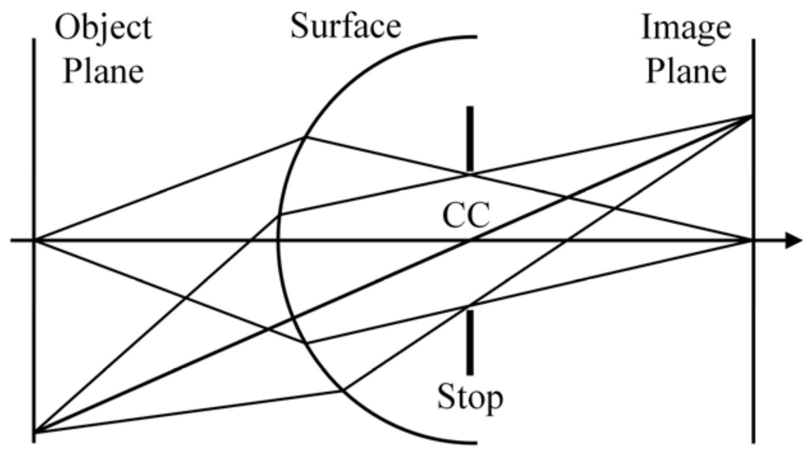
$$\partial_\lambda W_{020}=-\frac{y^2}{2}\left\{\left(n'+\partial n'\right)\cdot\left(\frac{1}{s'}-\frac{1}{r}\right)-\left(n+\partial n\right)\cdot\left(\frac{1}{s}-\frac{1}{r}\right)\right\}$$

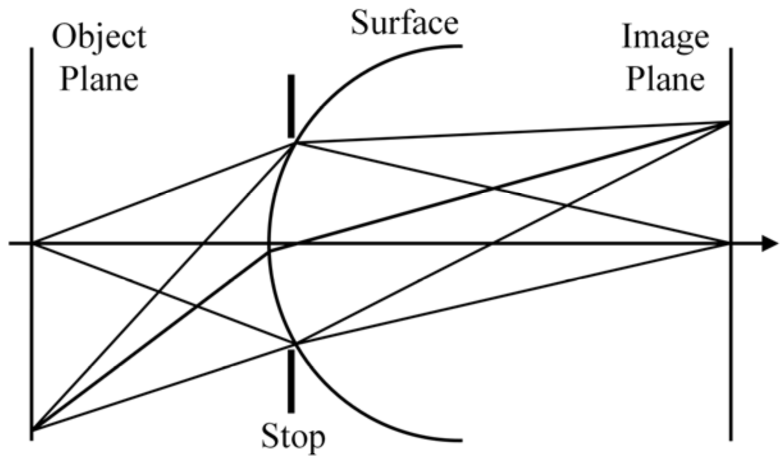
$$\partial_\lambda W_{020}=-\frac{y^2}{2}\left\{\partial n'\cdot\left(\frac{1}{s'}-\frac{1}{r}\right)-\partial n\cdot\left(\frac{1}{s}-\frac{1}{r}\right)\right\}=\frac{y}{2}\cdot A\cdot\Delta\left\{\frac{\partial n}{n}\right\}$$



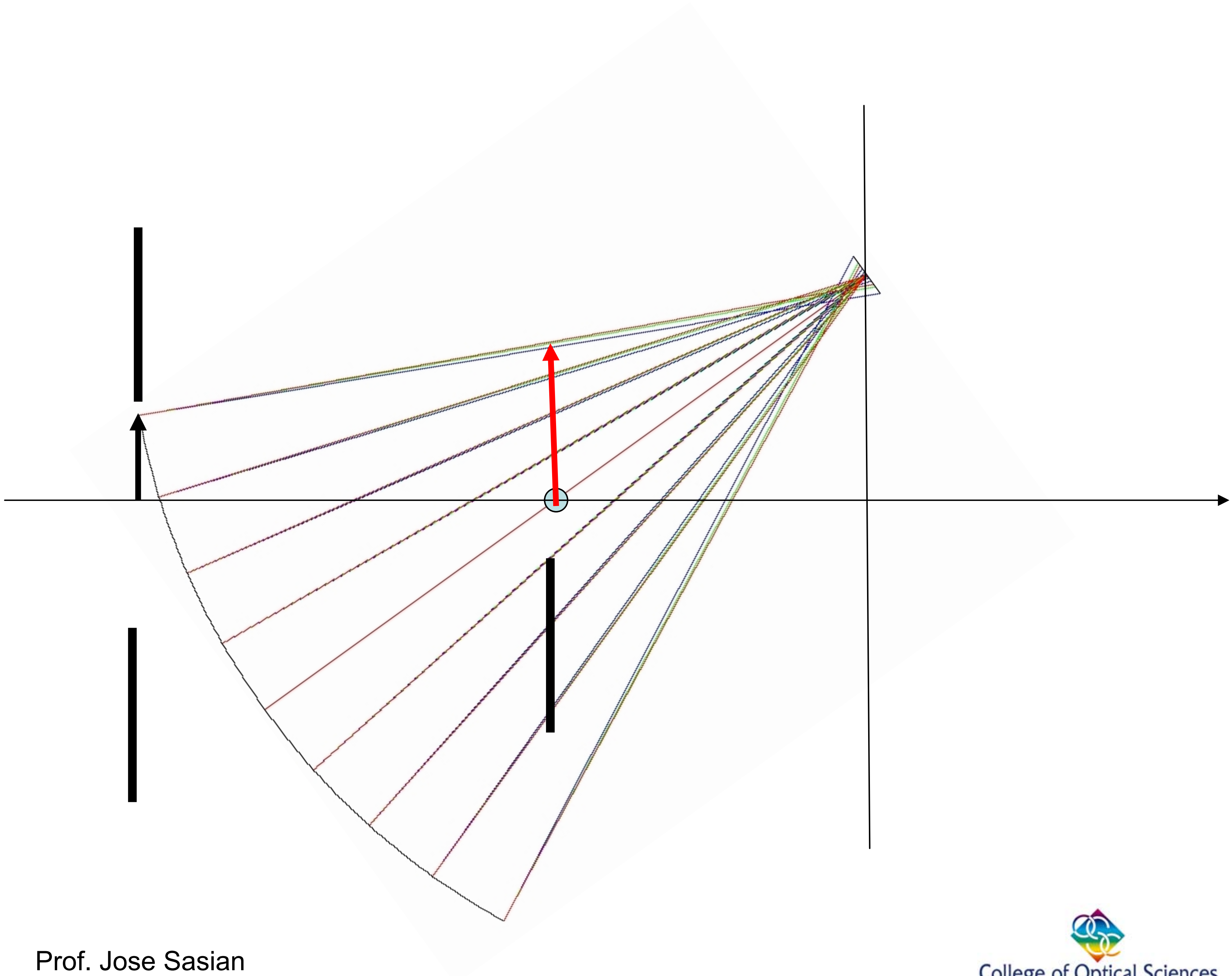
Prof. Jose Sasian

Stop shifting





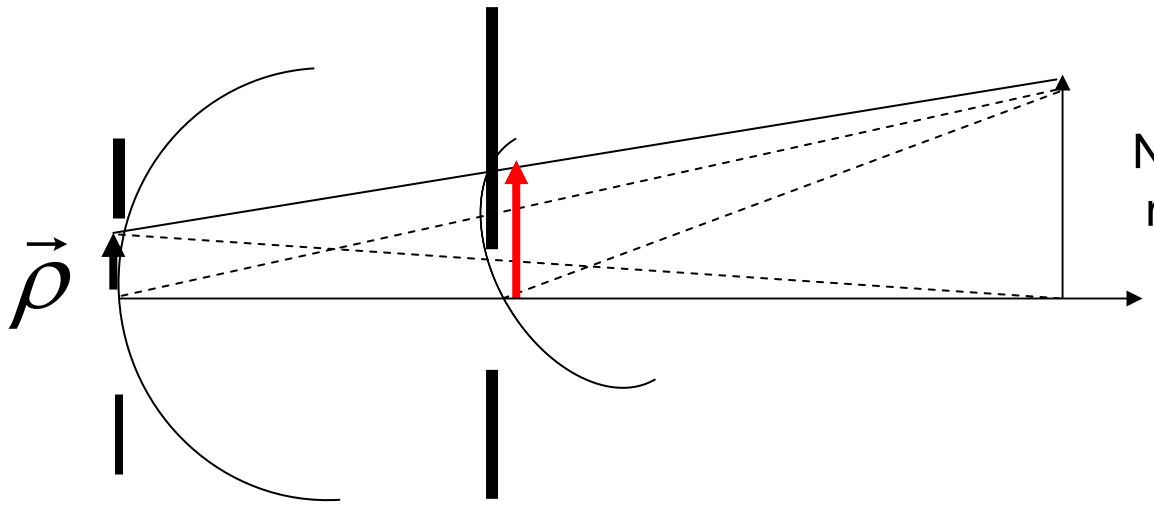
Prof. Jose Sasian



Prof. Jose Sasian

College of Optical Sciences
THE UNIVERSITY OF ARIZONA

Stop shifting



New chief ray height at old pupil $\bar{y}_E$

Marginal ray height at old pupil y_E

$$y_E \vec{\rho}_{shift} = y_E \vec{\rho} + \bar{y}_E \vec{H}$$

$$\vec{\rho}_{shift} = \vec{\rho} + \frac{\bar{y}_E}{y_E} \vec{H} = \vec{\rho} + \frac{\bar{A}}{A} \vec{H}$$

Chromatic coefficients

$$\partial_\lambda W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)=\frac{y}{2}\cdot A\cdot\Delta\left\{\frac{\partial n}{n}\right\}\left(\vec{\rho}\cdot\vec{\rho}\right)$$

$$\vec{\rho}_{shift}=\vec{\rho}+\frac{\bar{y}_E}{y_E}\vec{H}=\vec{\rho}+\frac{\bar{A}}{A}\vec{H}$$

$$\vec{\rho}_{shift}\cdot\vec{\rho}_{shift}=\vec{\rho}\cdot\vec{\rho}+2\frac{\bar{A}}{A}\vec{H}\cdot\vec{\rho}+\left(\frac{\bar{A}}{A}\right)^2\vec{H}\cdot\vec{H}$$

$$\partial_\lambda W_{111}=\bar{A}\cdot\Delta\left\{\frac{\partial n}{n}\right\}\cdot y$$

The ratio $\frac{\delta \bar{y}}{y}$

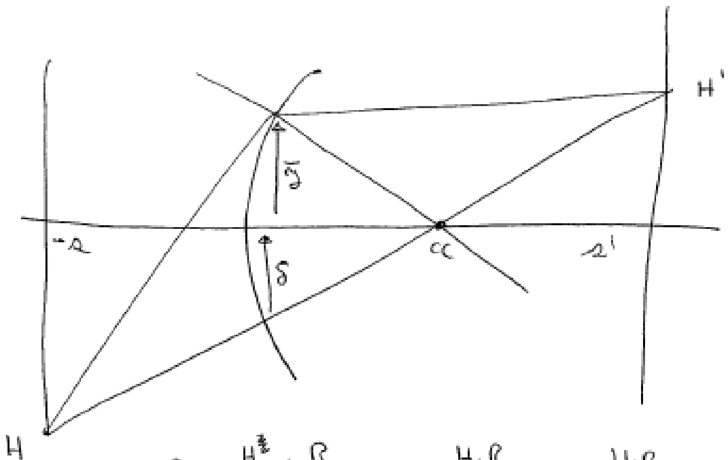
Can be calculated at any plane in the optical system

$$\bar{S} = \frac{\bar{u}_{new} - \bar{u}_{old}}{u} = \frac{\bar{y}_{new} - \bar{y}_{old}}{y} = \frac{\bar{A}_{new} - \bar{A}_{old}}{A}$$

$\bar{S}$ is the stop shifting parameter

Can show equality using the Lagrange invariant

The ratio $\frac{\bar{y}_E}{y_E} = \frac{\delta + \bar{y}}{y} = \frac{\bar{A}}{A}$



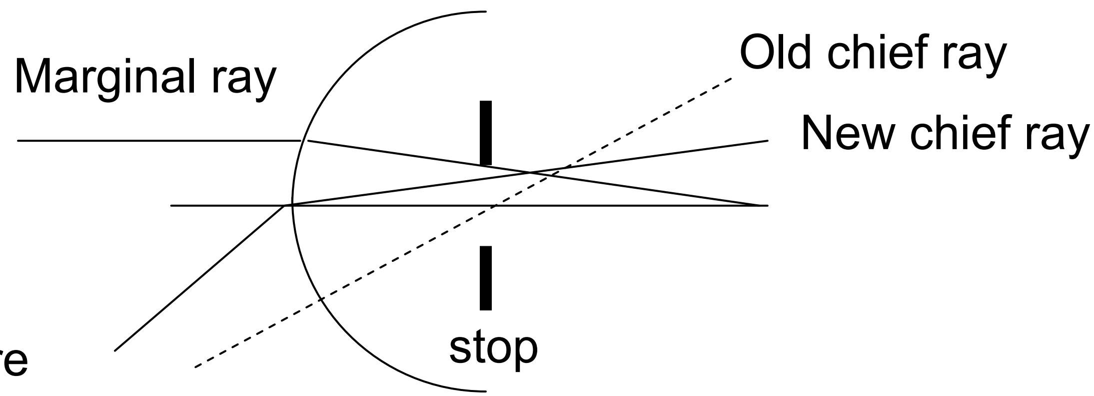
$$\delta = \frac{H \cdot R}{R - s} = \frac{H \cdot R}{s - R} = \frac{H \cdot R}{R \cdot s}\,\frac{1}{\frac{s-R}{Rs}} = \frac{H}{s}\,\frac{1}{\frac{1}{R}-\frac{1}{s}}\,\frac{y}{y}$$

$$A = ni = n\left\{\frac{-y}{s} + \frac{y}{R}\right\} = n(u + \alpha)$$

$$\delta = -\frac{H u n}{A} = \frac{\mathcal{M}}{A} = \frac{B y - A\bar{y}}{A}$$

$$\boxed{\delta + \bar{y} = \frac{B}{A} y}$$

The ratio $\bar{A} / A$



Parameters are at the surface

$$Ж = \bar{A}_1 y - A\bar{y}_1$$

$$Ж = \bar{A}_2 y - A\bar{y}_2$$

$$-A\left(\bar{y}_2 - \bar{y}_1\right) = y\left(\bar{A}_2 - \bar{A}_1\right)$$

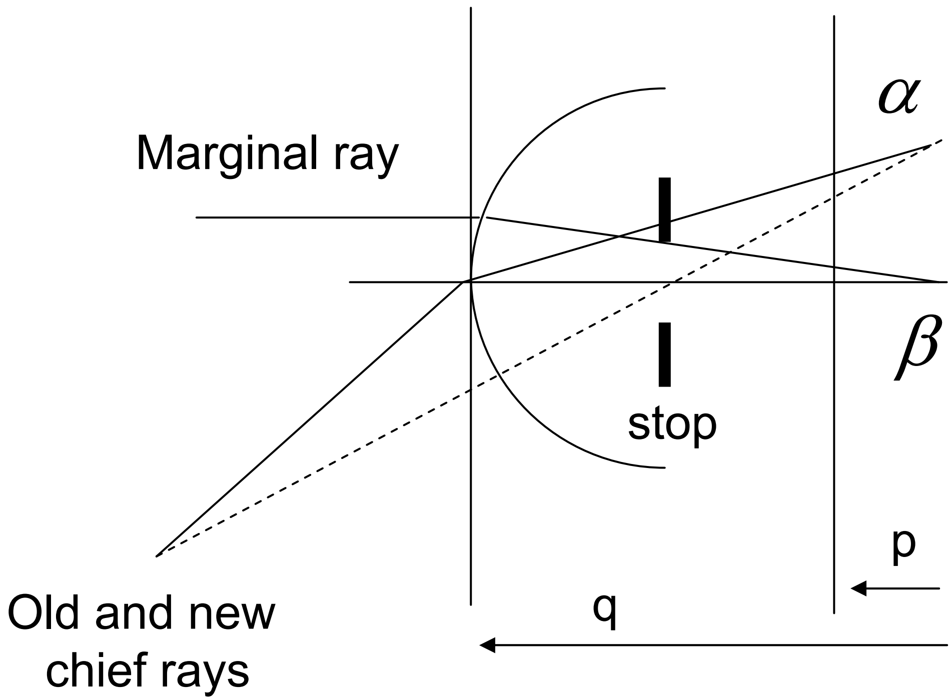
$$-\frac{\bar{y}_2 - \bar{y}_1}{y} = \frac{\bar{A}_2 - \bar{A}_1}{A}$$

When the stop is shifted at the cc

$$\bar{A}_1 = 0$$

$$\frac{\bar{y}_2 - \bar{y}_1}{y} = \frac{\bar{y}_{cc}}{y_{cc}} = \frac{\bar{A}}{A}$$

The ratio $\frac{\bar{y}_2 - \bar{y}_1}{y} = \frac{\bar{y}_{cc}}{y_{cc}}$



$$\frac{\bar{y}_{2q} - \bar{y}_{1q}}{q} = \alpha = \frac{\bar{y}_{2p} - \bar{y}_{1p}}{p}$$

$$\frac{y_q}{q} = \beta = \frac{y_p}{p}$$

Does not depend on plane where it is calculated given similar triangles

For a system of thin lenses

$$\partial_\lambda W_{020} = \frac{1}{2}\sum_{i=1}^{j}\left[\frac{\phi}{\nu}y^2\right]_i$$

$$\partial_\lambda W_{111} = \sum_{i=1}^{j}\left[\frac{\phi}{\nu}\bar{y}y\right]_i$$

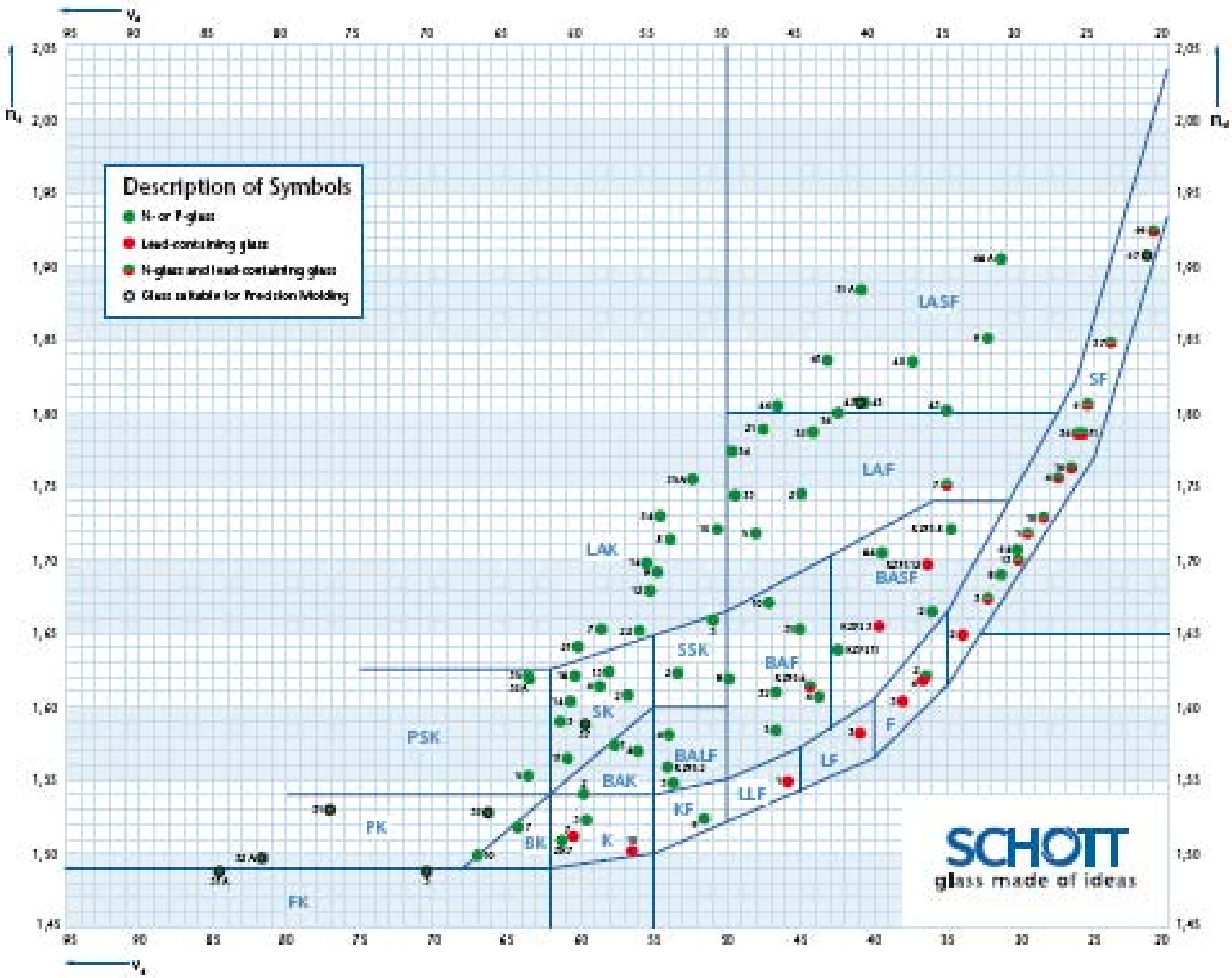
V is the glass V-number
Φ is the optical power
y is the marginal ray height
y-bar is the chief ray height

Glass

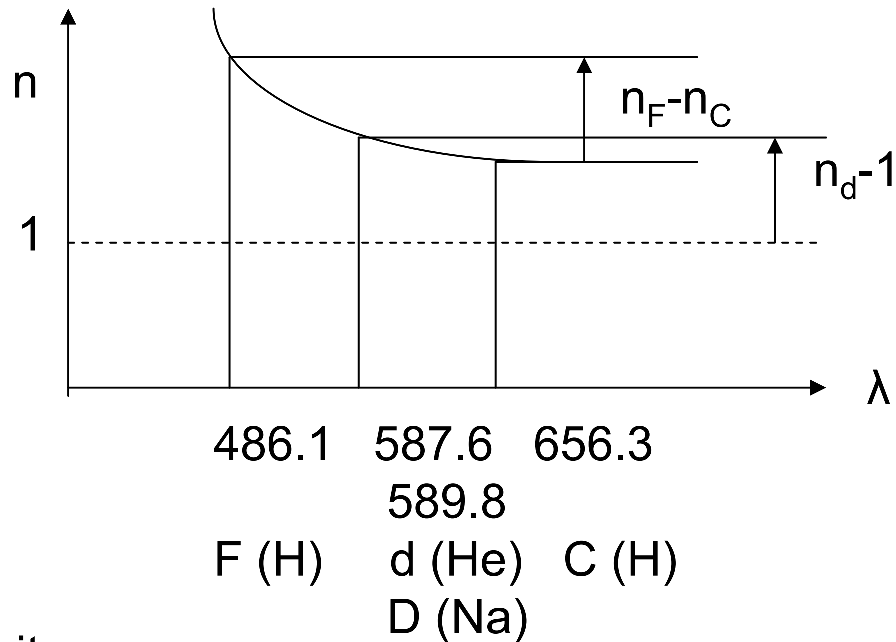
- Schott, Hoya, Ohara glass catalogues (A wealth of information; must peruse glass catalogue)
- Crowns and flints are divided at V=50
- Normal glasses:
- Soda-lime, silica, lead (older glasses)
- Crowns, light flints, flints, dense flints
- Barium glasses (~1938)
- Lanthanum or rare-earth glasses
- Titanium
- Fluorites and phosphate
- Environmental and health issues in the production of glass. Lead replaced with Titanium and Zirconium.

Other materials

- For the UV
- For the IR
- Plastics
- Advances come usually with new materials that extend or have new properties.
- The design is limited by the material


Description of Symbols
N- or P-glass
Lead-containing glass
N-glass and lead-containing glass
Glass suitable for Precision Molding
LASF
SF
LAF
LAK
BASF
SSK
BAF
SK
PSK
BALF
BAK
LF
F
LLF
KF
PK
BK
K
FK
SCHOTT
glass made of ideas

Glass properties



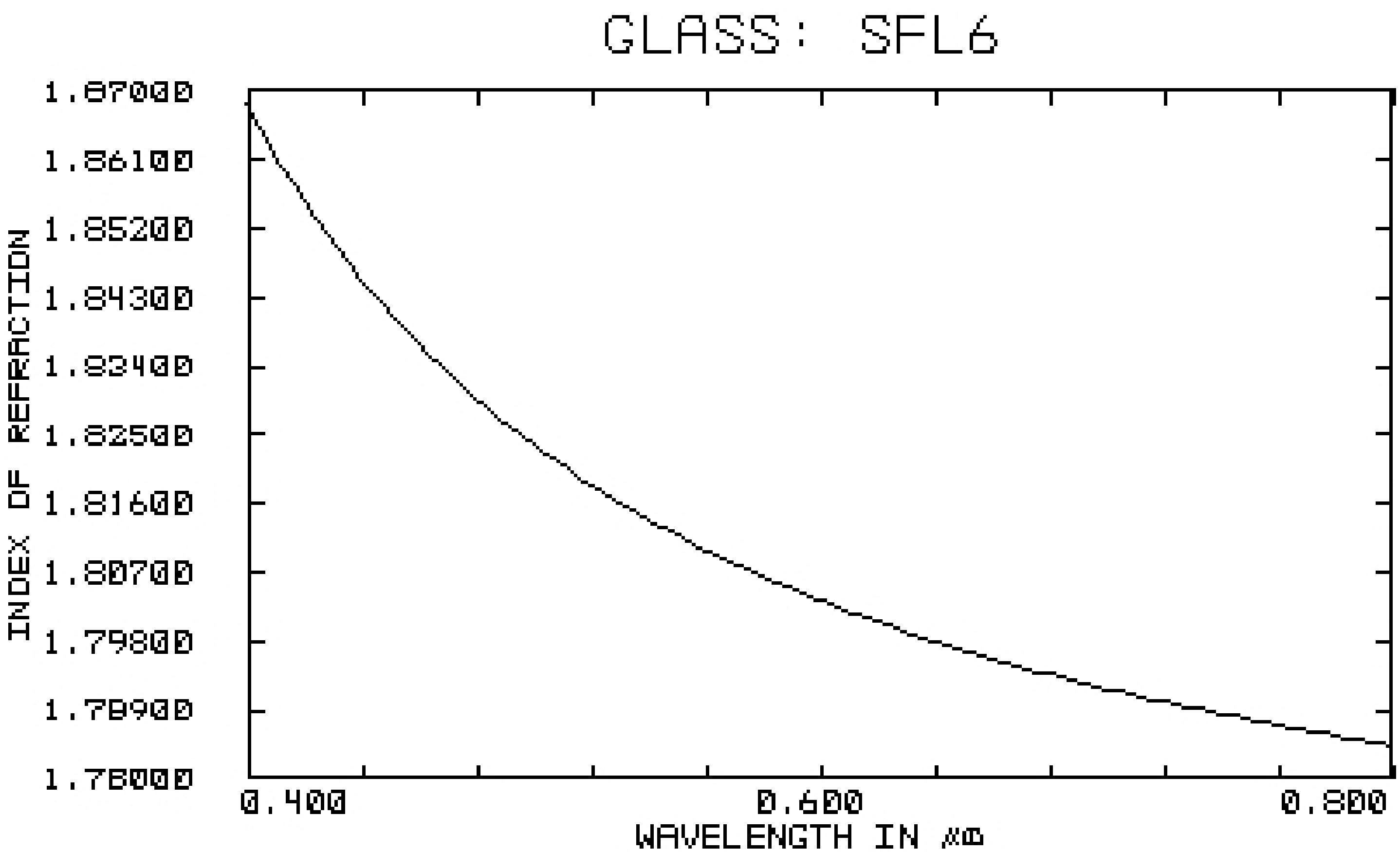
$n_d - 1$ Refractivity
$n_F - n_C$ Mean dispersion
$n_d - n_C$ Partial dispersion

$v = (n_d - 1)/(n_F - n_C)$ v-value, reciprocal dispersive power, Abbe number
$P = (n_d - n_C)/(n_F - n_C)$ Partial dispersion ratio

Glass properties

- Homogeneity
- Transmission
- Stria
- Bubbles
- Ease of fabrication; soft glasses
- Coefficient of thermal expansion
- Opto-thermal coefficient
- Birefringence

Index of refraction variation



Rate of slope change in the blue makes it more difficult to correct for color

Prof. Jose Sasian

Index interpolation

Sellmeier

$$n^2 = a + \frac{b\lambda^2}{c-\lambda^2} + \frac{d\lambda^2}{e-\lambda^2} + \ldots$$

Schott

$$n^2 - 1 = A + A_1\lambda^2 + A_2\lambda^{-2} + A_4\lambda^{-4} + A_6\lambda^{-6} + A_8\lambda^{-8}\ldots$$

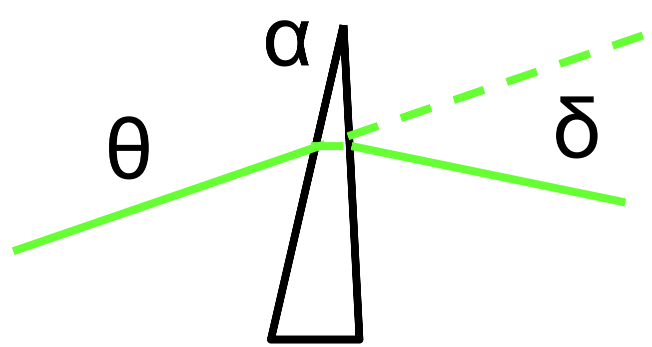
Hartmann
Conrady
Kettler-Drude

Must verify index of refraction

Prof. Jose Sasian

College of Optical Sciences
The University of Arizona®

The optical wedge



$$\theta_1^{'} = \frac{1}{n}\theta_1$$

$$\theta_2 = \theta_1^{'} - \alpha$$

$$\theta_2^{'} = n\theta_2$$

$$\partial = \alpha - \theta_1 + \theta_2^{'}$$

$$\partial = -(n-1)\alpha$$

The deviation is independent of the angle of incidence for small θ
(First order approximation)

Wedge

$$\partial = -\alpha\left(n_d - 1\right)$$

$$\Delta = \left(n_F - 1\right)\left(-\alpha\right) - \left(n_C - 1\right)\left(-\alpha\right) = \left(n_F - n_C\right)\left(-\alpha\right)$$

$$\varepsilon = \left(n_d - 1\right)\left(-\alpha\right) - \left(n_C - 1\right)\left(-\alpha\right) = \left(n_d - n_C\right)\left(-\alpha\right)$$

$$\frac{\partial}{\Delta} = \frac{\left(n_d - 1\right)}{\left(n_F - n_C\right)} = v$$

$$\frac{\varepsilon}{\Delta} = \frac{\left(n_d - n_C\right)}{\left(n_F - n_C\right)} = P$$

$$\Delta = \frac{\partial}{v}$$

$$\varepsilon = P\frac{\partial}{v}$$

δ Deviation
Δ Dispersion
ε Secondary dispersion

Achromatic wedge pair

$$\Delta = \Delta_1 + \Delta_2 = \frac{\partial_1}{v_1} + \frac{\partial_2}{v_2} = 0$$

Deviation without dispersion

$$\partial_2 = -\frac{v_2}{v_1}\partial_1$$

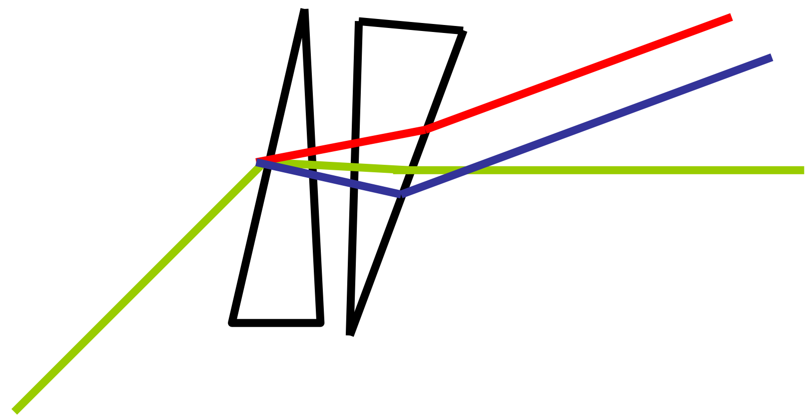
$$\partial = \partial_1 + \partial_2 = \partial_1 - \frac{v_2}{v_1}\partial_1 = \left(v_1 - v_2\right)\frac{\partial_1}{v_1} = -\left(v_1 - v_2\right)\frac{\partial_2}{v_2}$$

$$\frac{\alpha_1}{\partial} = -\left(\frac{1}{v_1 - v_2}\right)\left(\frac{v_1}{n_{d1} - 1}\right)$$

$$\frac{\alpha_2}{\partial} = \left(\frac{1}{v_1 - v_2}\right)\left(\frac{v_2}{n_{d2} - 1}\right)$$

$$\frac{\varepsilon}{\partial} = -\left(\frac{1}{v_1 - v_2}\right)\left(P_1 - P_2\right)$$

Achromatic wedge

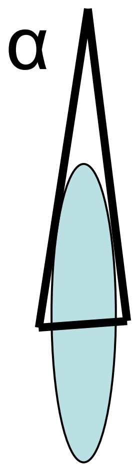


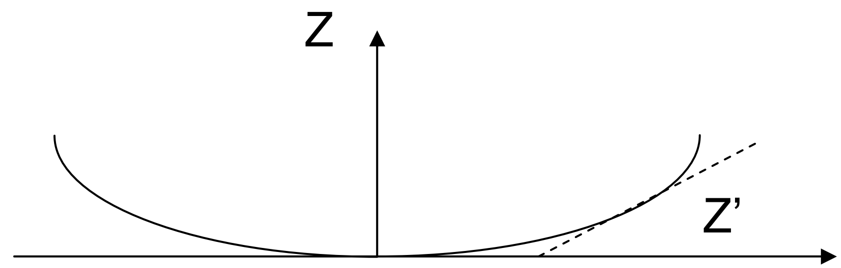
- There is deviation
- There is no dispersion
- Red and blue rays are parallel
- Independent of theta to first order

Schematic drawing

Achromatic doublet

(Treated as two wedges)



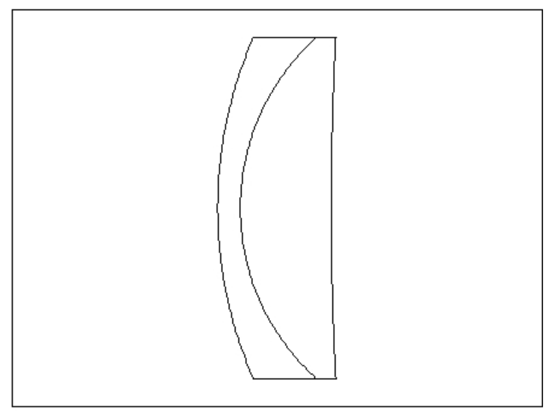


$$Z = sag = \frac{Y^2}{2r}; \quad Z' = \frac{Y}{r}$$

$$\alpha = Z_1^{'} + Z_2^{'} = Y\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = \frac{Y}{n_d - 1}\phi_1$$

$$\frac{\partial_1}{v_1} + \frac{\partial_2}{v_2} = 0$$

$$\frac{Y\phi_1}{v_1} + \frac{Y\phi_2}{v_2} = 0$$



Prof. Jose Sasian

College of Optical Sciences
THE UNIVERSITY OF ARIZONA®

Achromatic doublet

$$\frac{Y\phi_1}{v_1} + \frac{Y\phi_2}{v_2} = 0$$

$$\phi = \phi_1 + \phi_2$$

$$\frac{\phi_1}{\phi} = \frac{v_1}{v_1 - v_2}$$

$$\frac{\phi_2}{\phi} = -\frac{v_2}{v_1 - v_2}$$

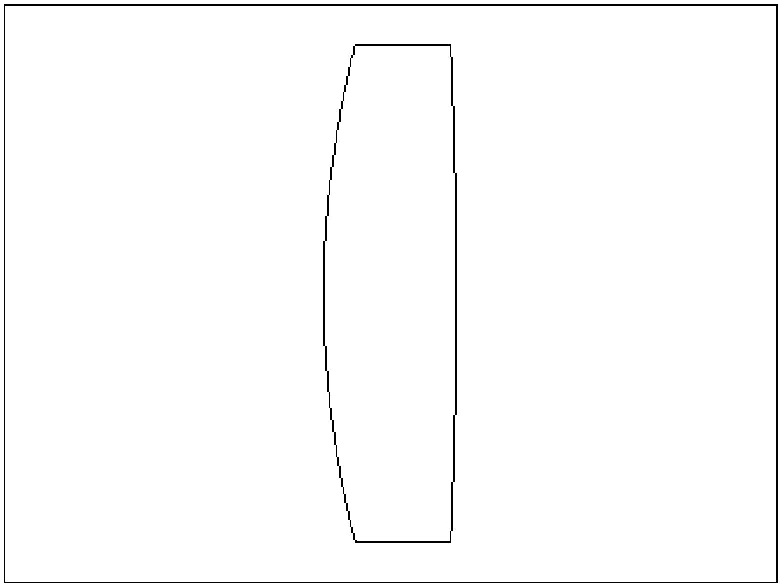
Independent of conjugate
Requires finite difference

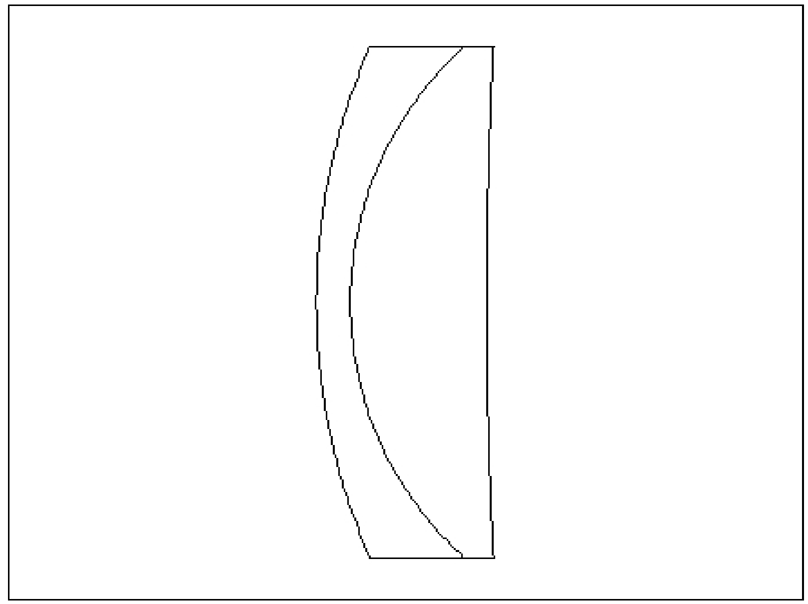
$$v_1 - v_2$$

Can lead to strong optical powers

Prof. Jose Sasian

Relative sag
(for 100 mm focal length)





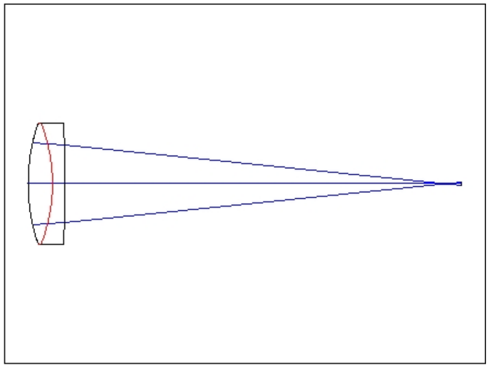
Zonal spherical aberration
Critical airspace

Prof. Jose Sasian

College of Optical Sciences
The University of Arizona

Achromatic doublet

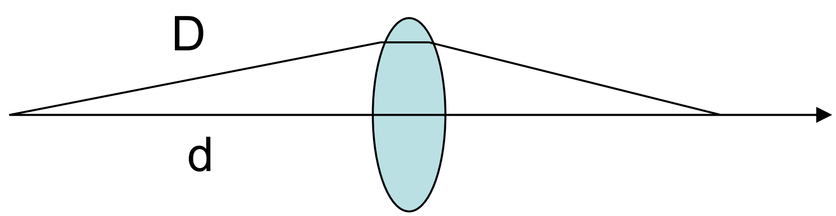
- Must have opposite power (Glass)
- Strong positive and weaker negative lens
- Cemented doublet
- Crown in front
- Flint in front
- Corrected for spherical aberration
- Degrees of freedom
- Large achromats and cementing
- Conrady D-d sum
- Zonal spherical aberration



Conrady's D-d sum

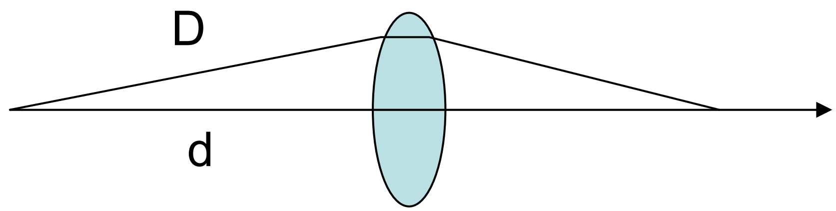
- In the presence of sphero-chromatism the best state of correction is achieved when:

$$\sum \left(D - d \right) \Delta n = 0$$

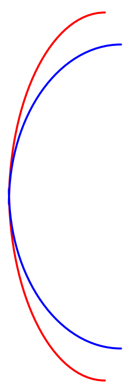


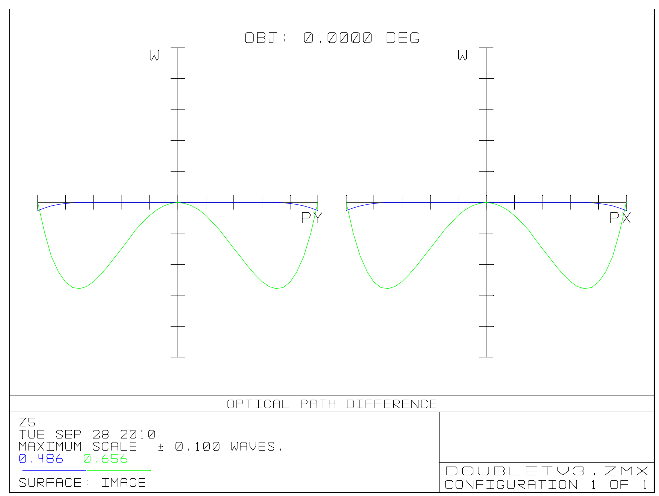
Is the difference of optical path between the marginal F and C rays.

Conrady's D-d sum



$$Optical\_path\_difference = \left(\sum Dn_f - \sum dn_f\right) - \left(\sum Dn_c - \sum dn_c\right) = \sum (D-d)\Delta n$$



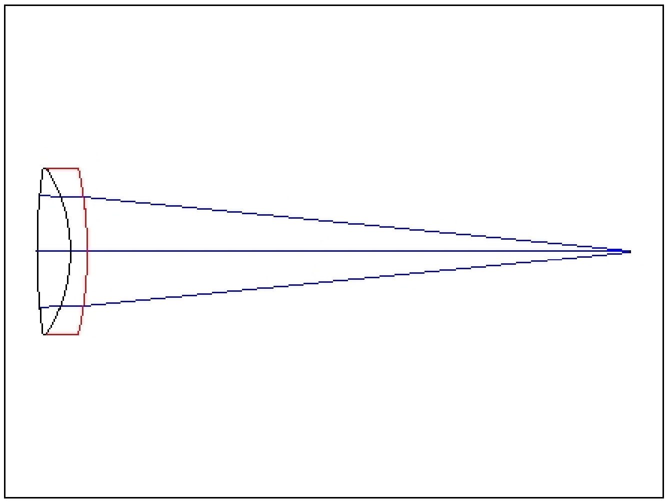


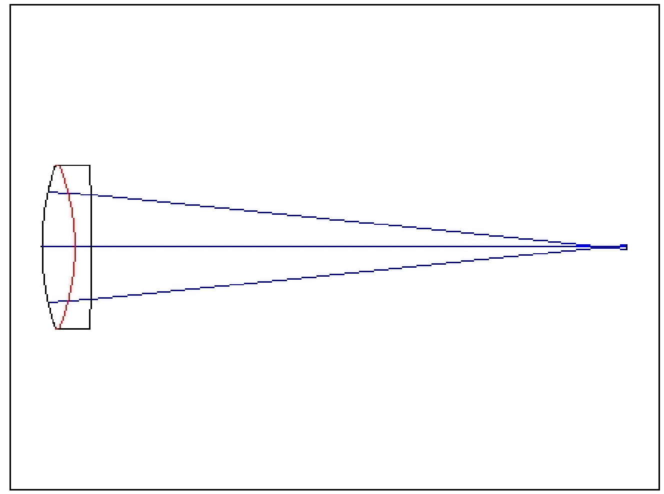
Minimizes the rms OPD difference by joining the opd curves at the edge of The aperture. Valid for fourth order sphero-chromatism.

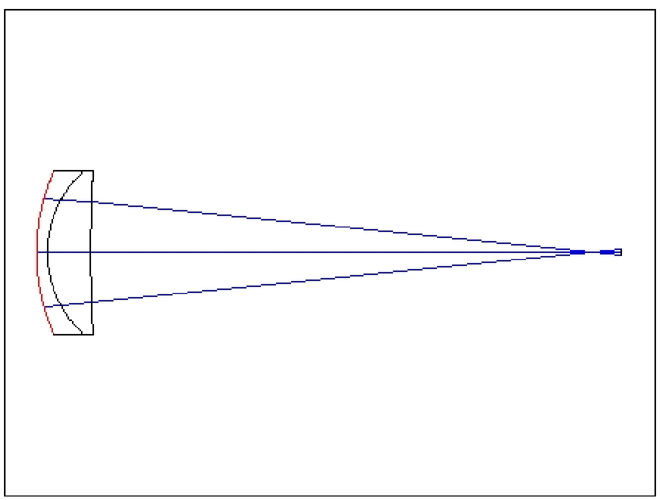
Cemented doublet solutions

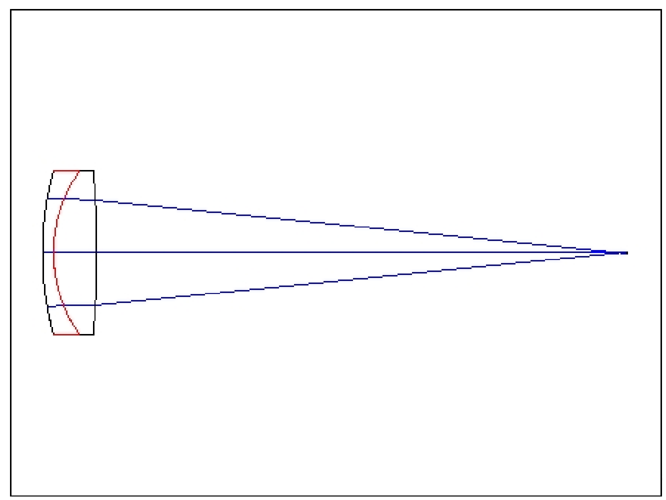
- Correction for chromatic change of focus
- Correction for spherical aberration
- Degrees of freedom: relative powers for a set of glasses; shapes
- Crown in front: two solutions
- Flint in front: two solutions
- Note multiple solutions

Crown in front and flint in front doublet solutions (BK7 and F2)









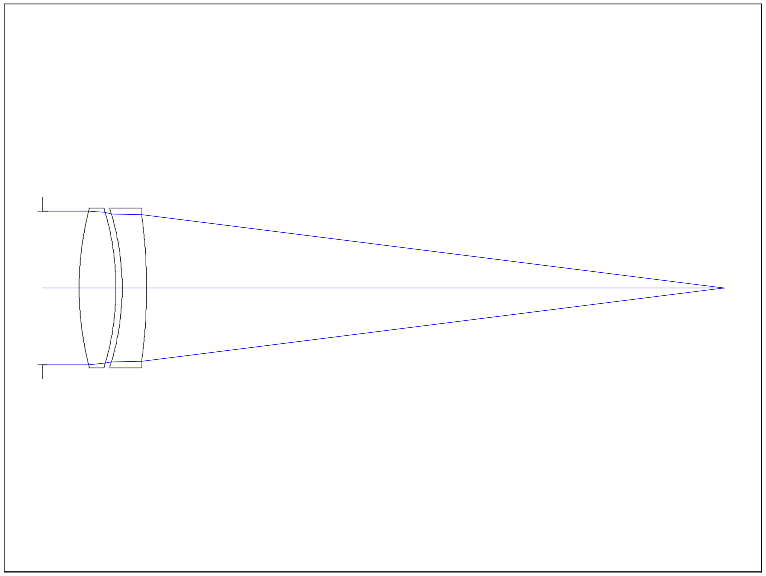
Contact options for doublets

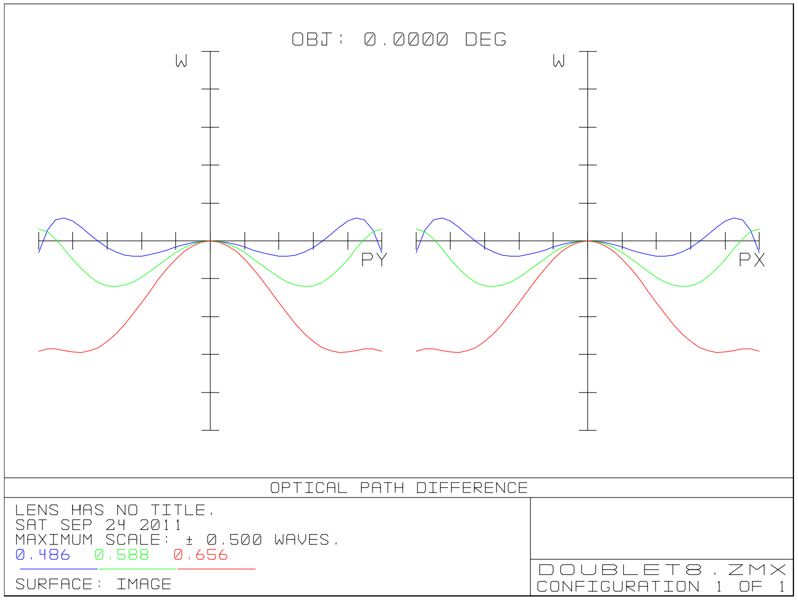
Full contact (cemented)
Air spaced

Edge contacted
Center contacted

Limitations

Secondary spectrum, spherochromatism and zonal spherical aberration set limits



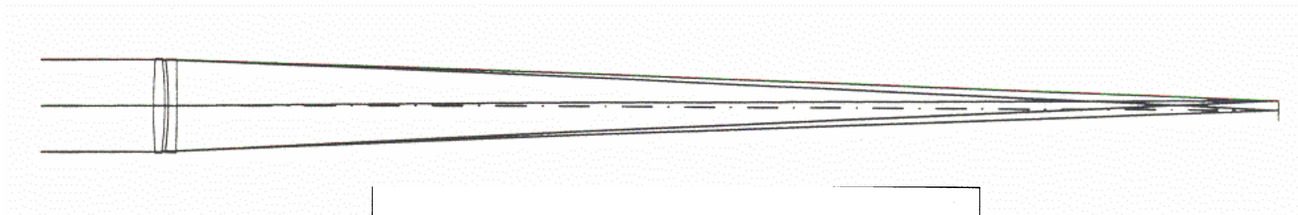


F=100 mm, f/4, 0.5 wave scale

Prof. Jose Sasian

College of Optical Sciences
The University of Arizona

Achromatic doublet

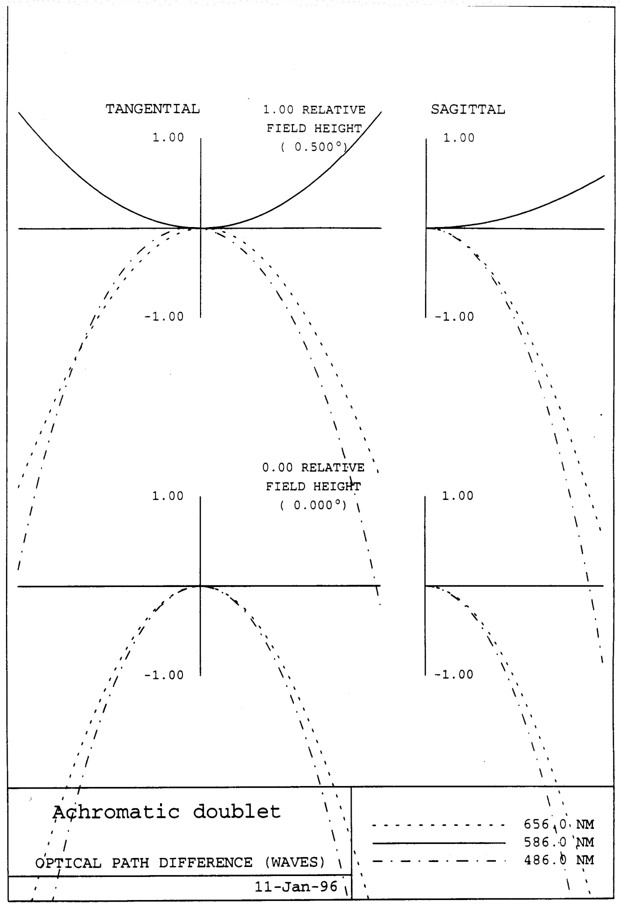


20 inch diameter

F/12

BK7

F4



Prof. Jose Sasian

College of Optical Sciences
The University of Arizona

In this lecture

- Chromatic coefficients
- Basic glass properties
- Achromatic wedge-pair and lens doublets
- Examples
- D-d method
- Achromatic doublet
- Diversity of solutions