First-order optics review

Lens Design OPTI 517

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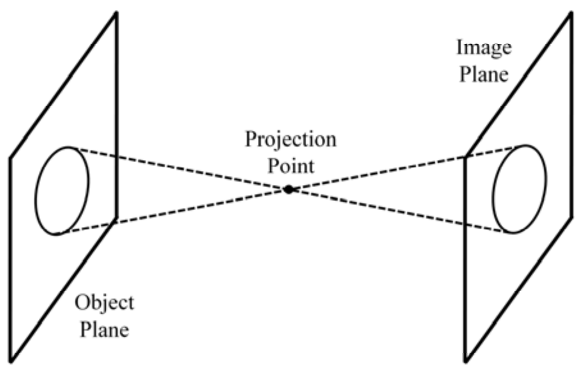
Lecture overview

- Collinear transformation review
- Review of first–order optics concepts
- The stop aperture
- Rays vs. Wavefronts
- How a lens design program works

- Kingslake/Johnson: chapters 1, 2, and 3
- Shannon: chapters 1,2
- J. Greivenkamp, “Field Guide to Geometrical Optics,” SPIE Press, 2004.

Collinear transformation

- Collinear transformation is not specific to optics
- Camera obscura and lens focusing/imaging link the collinear transformation
- Constants defining ideal imaging
- Gaussian or Newtonian equations represent the collinear transformation



First-order optics

- First-order ray-tracing is a powerful method to analyze an optical system
- Obtain substantial lens system information from tracing two first-order rays (paraxial)
- Reflects early designers cleverness
- Real rays were difficult to trace
- Many lens design tasks are still done using first-order rays.
- Lens design software uses some significant amount of first-order ray tracing
- Need to understand well first-order optics

Ray tracing

- Real rays are traced to within the computer precision using Snell's law and the actual surface shape
- First-order rays are traced using the first-order refraction invariant: $n'i'=ni$ and a flat surface with optical power
- Paraxial rays are very, very close to the optical axis and are traced like first-order rays with a very small factor 10^{-250} for heights and angles which is not explicitly written

First-order optics

- A first-order layout is like the skeleton or foundation of a lens. It helps to create a useful structure.
- The language of first-order optics

Three levels of learning 101

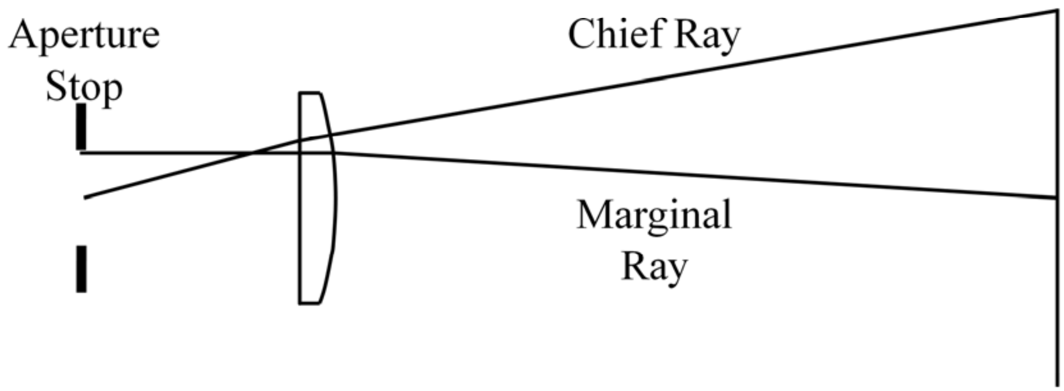
- I have heard/read and understand the concept
- I can apply the concept
- I can think of different ways to do the same and I practice the concept

First-order review

1. Gaussian and Newtonian equations
2. F/#, NA, aperture size
3. Magnification, magnifying power, optical power
4. Image location and size
5. The stop and pupils
6. Afocal, telecentric, doubly telecentric
7. Cardinal points and planes
8. Chief and marginal rays
9. Paraxial/first-order ray tracing
10. Lagrange invariant
11. Basic lens configurations: m=-1, m=0, 4f system
12. The eye and the sphere

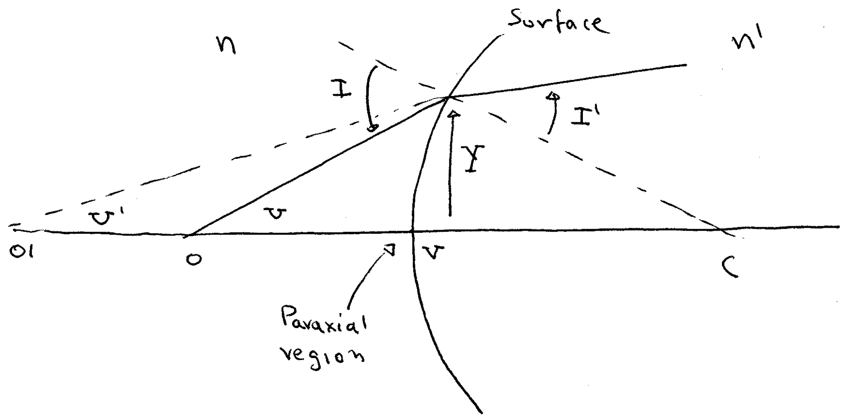
101 Exercise: Review the 12 first-order points and determine you level of Knowledge about each item

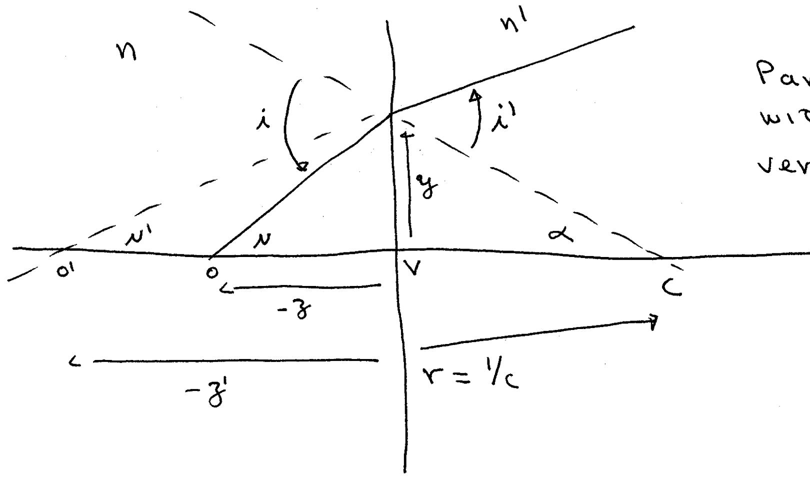
Chief and marginal first-order rays



A meridional ray that passes through the axial object point and the edge of the stop is known as a marginal ray. A meridional ray that passes through the edge of the field of view and the stop center is known as a chief ray.

Paraxial image formation





Paraxial region with expanded vertical scale

Constructions to show paraxial ray refraction and associated quantities.

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Refraction of a paraxial ray

$$\alpha = -y\,c \;;\; i = -\alpha + u \;;\; i' = -\alpha + u'$$

$$u = -y/z \qquad u' = -y/z'$$

In the paraxial region Snell's law

becomes: $n' i' = n\, i$

Therefore: $-n'\alpha' - n'u' = -n\alpha + n u$

or: $n'u' = n u + (n'-n)\alpha = n u - y(n'-n)\,c$

$\phi = (n'-n)\,c = \dfrac{(n'-n)}{r}$ is the power

which is measured in diopters

Thus, refraction of a paraxial ray leads to:

$$\boxed{n'u' = n u - y\,\phi}$$ refraction

$$\boxed{y' = y + u'\,t'}$$ Transfer

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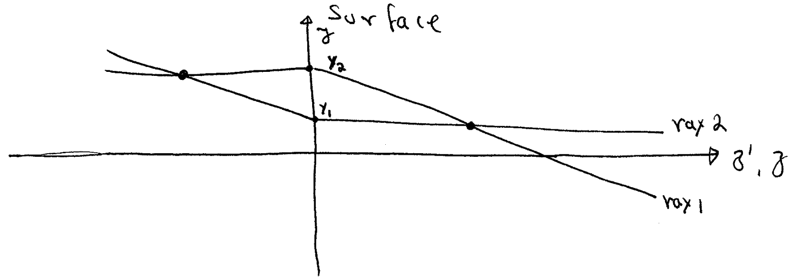
t is the thicknes to the next surface

z, z' coordinates for arbitrary point



In object space:

ray 1 : $y = u_1 z + y_1$

ray 2 : $y = u_2 z + y_2$ $\rightarrow$ $z(u_2 - u_1) = -(y_2 - y_1)$

or: $z = -\dfrac{y_2 - y_1}{u_2 - u_1}$

In Image space: $z' = -\dfrac{y_2 - y_1}{u_2' - u_1'}$

but $u_2' = \dfrac{n u_2 - y_2 \phi}{n'}$ and $u_1' = \dfrac{n u_1 - y_1 \phi}{n'}$

$$z' = \frac{-\,(y_2 - y_1)}{\dfrac{n u_2 - y_2 \phi}{n'} - \dfrac{n u_1 - y_1 \phi}{n'}} = -\frac{n' (y_2 - y_1)}{n(u_2 - u_1) - \phi (y_2 - y_1)}$$

$$z' = \frac{-n' \dfrac{y_2 - y_1}{u_2 - u_1}}{n - \phi \dfrac{y_2 - y_1}{u_2 - u_1}} = \frac{+n' z}{n + \phi z}$$

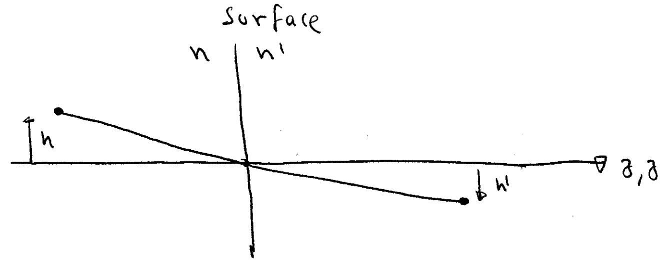
or: $\dfrac{n}{z} + \phi = +\dfrac{n'}{z'}$

$$\boxed{\frac{n'}{z'} - \frac{n}{z} = \phi}$$

for arbitrary y_1, y_2, u_1, u_2 !

$f' = \dfrac{n'}{\phi}$ $\quad f = -\dfrac{n}{\phi}$

Magnification



when $y = 0$ we have $n' u' = n u$

$$n' \frac{h'}{z'} = \frac{n h}{z}$$

$$\boxed{m = \frac{h'}{h} = \frac{n}{n'} \frac{z'}{z}}$$

but $\frac{h'}{h} = -\frac{f'}{f}$

Thus: $\boxed{\frac{z'}{z} = -\frac{f'}{f} m}$ and $\boxed{\frac{f'}{z'} + \frac{f}{z} = 1}$

Conclusion: paraxial image formation is congruent with the collinear transformation

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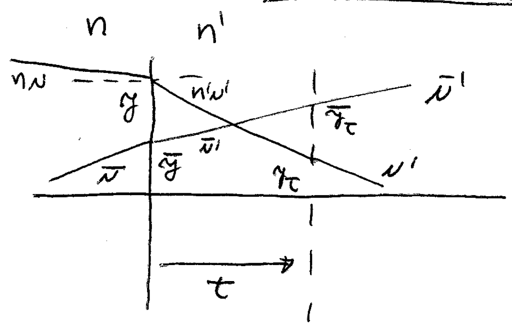
THE OPTICAL INVARIANT AND THE LAGRANGE INVARIANT



REFRACTION :

$$n'u' = nu - y\phi$$
$$n'\bar{u}' = n\bar{u} - \bar{y}\phi \qquad \phi = \frac{nu - n'u'}{y} = \frac{n\bar{u} - n'\bar{u}'}{\bar{y}}$$

$$nu\bar{y} - n'u'\bar{y} = n\bar{u}y - n'\bar{u}'y$$

$$n'\bar{u}'y - n'u'\bar{y} = n\bar{u}y - nu\bar{y}$$ INVARIANT ON REFRACTION

TRANSFER :

$$y_\tau = y + \frac{t}{n'}(n'u')$$
$$\bar{y}_\tau = \bar{y} + \frac{t}{n'}(n'\bar{u}') \qquad \frac{t}{n'} = \frac{y_\tau - y}{n'u'} = \frac{\bar{y}_\tau - \bar{y}}{n'\bar{u}'}$$

$$n'\bar{u}y_\tau - n'\bar{u}'y = n'u'\bar{y}_\tau - n'u'\bar{y}$$

$$n'\bar{u}y_\tau - n'u'\bar{y}_\tau = n'\bar{u}'y - n'u'\bar{y}$$

INVARIANT on transfer

∴ therefore we have an optical invariant

IF THE RAYS ARE THE MARGINAL AND CHIEF RAYS, WE HAVE THE LAGRANGE INVARIANT:

$$\mathcal{H} = n\bar{u}y - nu\bar{y}$$

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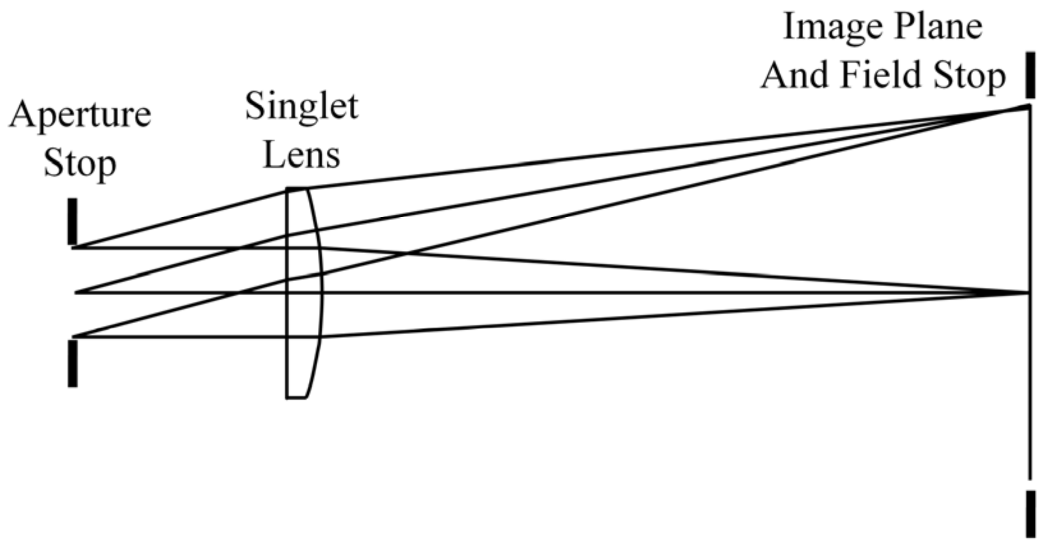
Aperture stop and a singlet lens



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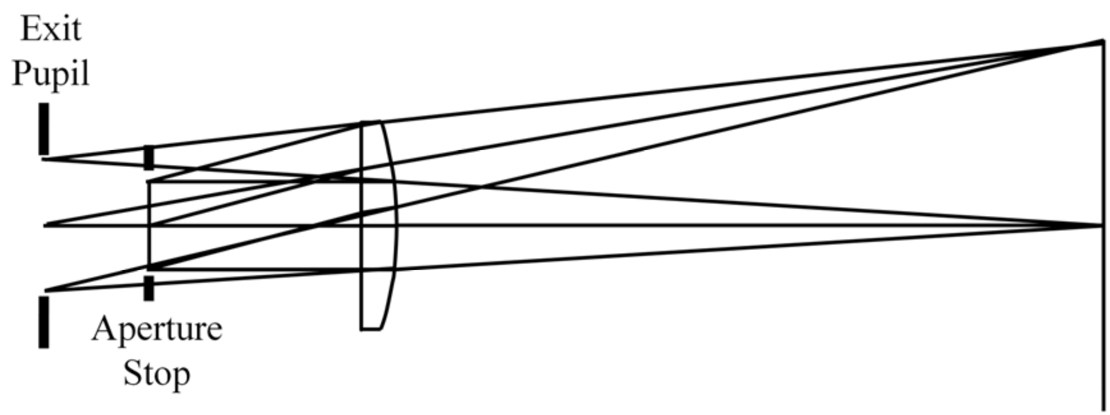
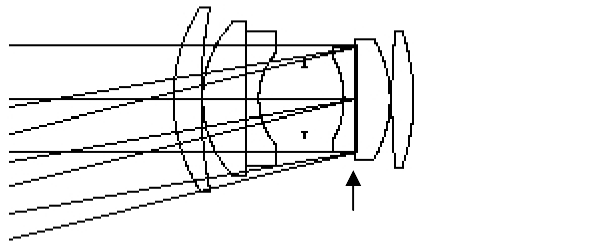
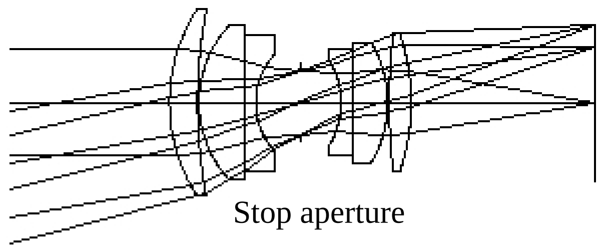
A virtual exit pupil



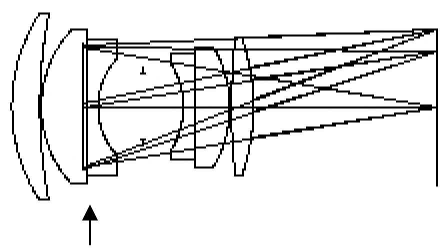
The Aperture Stop and the pupils



Entrance pupil

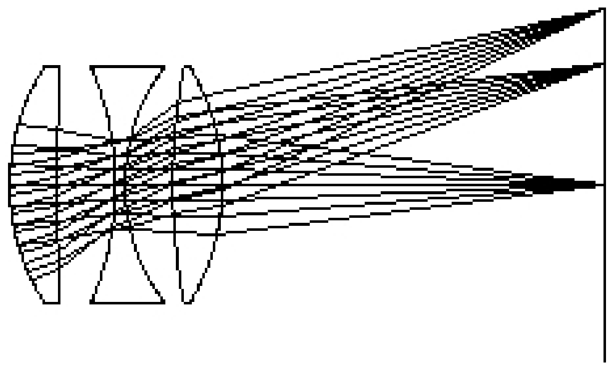


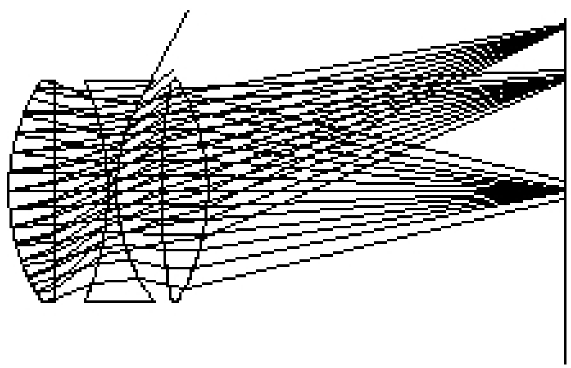
Stop aperture



Exit pupil

Aperture Stop





•The stop helps to well define a lens system.
•Without a stop aperture light that propagates through an optical system is not organized into well defined beams.

Aperture Stop

- The stop helps to well define a lens system. Without a well-defined stop it is not clear how light propagates through an optical system
- The stop limits the amount of light through a lens system
- The stop brings order into how the beams of light from different field points propagate towards the image plane
- The stop helps to control stray light
- The stop defines the entrance and exit pupils
- Beams from different field points spatially coincide and appear to pivot about the pupils.
- Diffraction effects from beam clipping at the stop are minimized at the exit pupil
- The light beam amplitude variation and wavefront deformation can be calculated at the exit pupil
- Identifying the aperture stop should become second nature

Cardinal points and planes

- Observation of the behavior of an optical imaging system leads one to conclude that there are a number of relevant points and planes; the points are called cardinal points. The significance of the cardinal points is that they can represent any focal system regardless of how complex the system is. Cardinal points also establish a convenient reference from which to measure distances in an optical system.

- The conjugate to the point at infinity in the object space is the rear focal point (F'). The conjugate to the point at infinity in the image space is the front focal point (F). The planes intersecting the focal points are called correspondingly the front and rear focal planes.

- The front and rear nodal points N and N' are defined as the centers of perspective; they are conjugate. Any ray passing by one nodal point will pass by the other and will preserve its direction. The planes intersecting the nodal points are called correspondingly the front and rear nodal planes.

- The front (P) and rear (P') principal points define the rear and front principal planes. The principal planes are the planes of unit magnification; they are conjugate.

- The distances from the principal points to the focal points are called the front and rear focal lengths respectively.

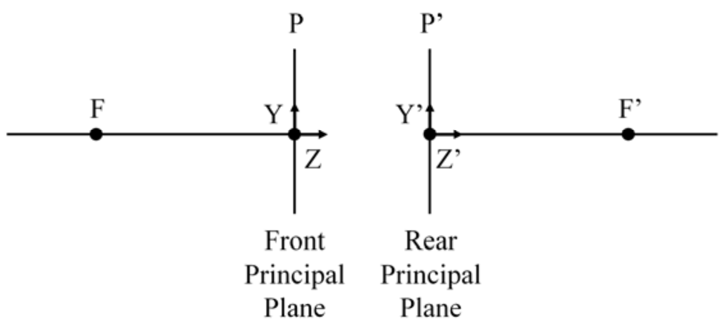
Geometrical construction

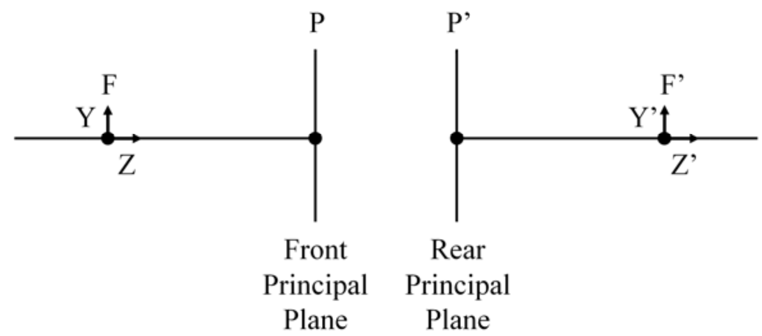
- The cardinal points of a lens system provide a graphical method to find the image position and size.

- The cardinal points establish references to define distances and angles

Gaussian and Newtonian equations:

- Gaussian equations use the principal points to locate the coordinate origins in the object space and image space. Newtonian equations use the focal points to locate the coordinate origins in the object space and image space.





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Gaussian and Newtonian imaging equations

$$m = 1 - \frac{Z'}{f'} \qquad m = \frac{1}{1 - \frac{Z}{f}} \qquad \frac{Z'}{f'} - \frac{Z}{f} = 1$$

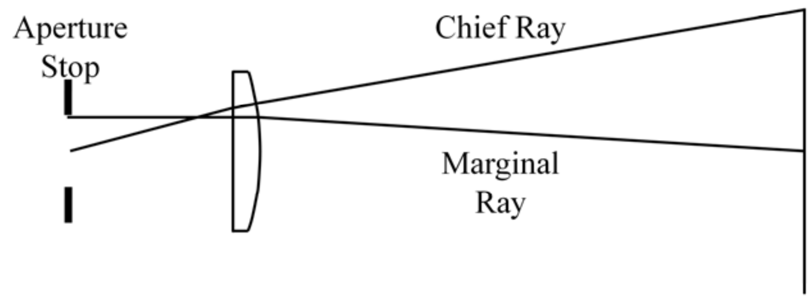
$$\frac{Z'}{f'} = m \qquad \frac{Z}{f} = \frac{1}{m} \qquad ZZ' = ff'$$

Summary of significant points and distances:

Significant points on axis	Object space	Image space
• Object/image points (conjugate)	O	O'
• Focal points (not conjugate)	F	F'
• Nodal point (conjugate)	N	N'
• Principal points (conjugate)	P	P'
• Entrance/exit pupil (conjugate)	E	E'

Significant distances	Object space	Image space
• Front/rear focal lengths	f=PF	f’=P'F'
• Object/image distances	s=PO	s’=P'O'
• Principal point separation	PP'	

Lens specifications



Singlet constructional parameters			
Surface	Radius of curvature	Thickness to next surface	Glass
Stop		30.775 mm	Air
2	∞	5 mm	Bk7 (n=1.5168)
3	-51.680 mm	100 mm	Air
Image			

First-order ray trace

First-order ray trace. $\mathcal{K} = 1.67$						
Surface	y	u	ni	$\bar{y}$	$\bar{u}$	$n\bar{i}$
Stop	6.2500	0.0000	0.0000	0.0000	0.2679	0.2679
2.0000	6.2500	0.0000	0.0000	8.2462	0.1767	0.2679
3.0000	6.2500	-0.0625	-0.1834	9.1295	0.1767	0.0000
Image	0.0000	-0.0625	-0.0625	26.7949	0.1767	0.1767

Real Ray-tracing

To face page 42.

CALCULATION No. 13

$Y = 0{\cdot}3536$ $r_1 = +6{\cdot}083''$ $r_2 \equiv r_3 = -4{\cdot}444''$ $r_4 = -12{\cdot}610''$ $d_1' = 0{\cdot}200''$ $d_3' = 0{\cdot}150''$

	Crown	*Flint*
$N_C =$	1·51500	1·69054
$N_F =$	1·52356	1·71341

	First Surface C ray	*First Surface* F ray	*Second Surface* C ray	*Second Surface* F ray	*Third Surface* C ray	*Third Surface* F ray
L $-r$			17·6797 + 4·444	17·4889 + 4·444	36·6239 +12·610	38·6775 12·610
$(L-r)$	$Y = 0{\cdot}3536$		22·1237	21·9329	49·2339	51·2875
log sin U + log $(L-r)$			8·29631 1·34486	8·30097 1·34110	7·97849 1·69226	7·95485 1·71001
log $(L-r)$ sin U − log r	9·54851 0·78412	9·54851 0·78412	9·64117 0·64777n	9·64207 0·64777n	9·67075 1·10072n	9·66486 1·10072n
log sin I + log $\frac{N}{N'}$	8·76439 9·81959	8·76439 9·81714	8·99340n 9·95239	8·99430n 9·94900	8·57003n 0·22802	8·56414n 0·23386
log sin I' + log r	8·58398 0·78412	8·58153 0·78412	8·94579n 0·64777n	8·94330n 0·64777n	8·79805n 1·10072n	8·79800n 1·10072n
log r . sin I' − log sin U'	9·36810 8·29631	9·36565 8·30097	9·59356 7·97849	9·59107 7·95485	9·89877 8·54654	9·89872 8·54642
log $(L'-r)$	1·07179	1·06468	1·61507	1·63622	1·35223	1·35230
U $+I$	0– 0– 0 3–19–57	0– 0– 0 3–19–57	1– 8– 1 −5–39– 8	1– 8–45 −5–39–51	0–32–43 −2– 7–46	0–30–59 −2– 6– 2
$U+I$ $-I'$	3–19–57 2–11–56	3–19–57 2–11–12	−4–31– 7 5– 3–50	−4–31 –6 5– 2– 5	−1–35– 3 3–36– 5	−1–35– 3 3–36– 3
U'	1– 8– 1	1– 8–45	0–32–43	0–30–59	2– 1– 2	2– 1– 0
$L'-r$ $+r$	11·7975 6·083	11·6059 6·083	41·2164 −4·444	43·2733 −4·444	22·5025 −12·610	22·5061 −12·610
L' $-d'$	17·8805 0·200	17·6889 0·200	36·7724 0·150	38·8293 0·150	9·8925	9·8961
new L	17·6805	17·4889	36·6224	38·6793		
$-\frac{1}{2}U$ $+\frac{1}{2}I$	0– 0– 0 1–39–58	0– 0– 0 1–39–58	−0–34– 0 −2–49–34	−0–34–22 −2–49–56	−0–16–22 −1– 3–53	−0–15–30 −1– 3– 1
$\frac{1}{2}(I-U)$	1–39–58	1–39–58	−3–23–34	−3–24–18	−1–20–15	−1–18–31
$\frac{1}{2}I'$ $-\frac{1}{2}U'$	1– 5–58 −0–34– 0	1– 5–36 −0–34–22	−2–31–55 −0–16–22	−2–31– 2 −0–15–30	−1–48– 2 −1– 0–31	−1–48– 2 −1– 0–30
$\frac{1}{2}(I'-U')$	0–31–58	0–31–14	−2–48–17	−2–46–32	−2–48–33	−2–48–32
log L + log sin U or log Y (for $\parallel$ light) + log sec $\frac{1}{2}(I-U)$	9·54851 0·00018	9·54851 0·00018	1·24748 8·29631 0·00076	1·24276 8·30097 0·00077	1·56376 7·97849 0·00012	1·58746 7·95485 0·00011
log PA + log cosec U' + log cos $\frac{1}{2}(I'-U')$	9·54869 1·70369 9·99998	9·54869 1·69903 9·99998	9·54455 2·02151 9·99948	9·54450 2·04515 9·99949	9·54237 1·45346 9·99948	9·54242 1·45358 9·99948
log L'	1·25236	1·24770	1·56554	1·58914	0·99531	0·99548
L' (by check) $-d'$	17·8797 0·200	17·6889 0·200	36·7739 0·150	38·8275 0·150	9·8926	9·8965
new L	17·6797	17·4889	36·6239	38·6775		

CALCULATION No. 14

Ray-trace through last surface corrected by Chr. (1), *i.e.*, $r_4 = -12{\cdot}999$ inches.

LAST SURFACE

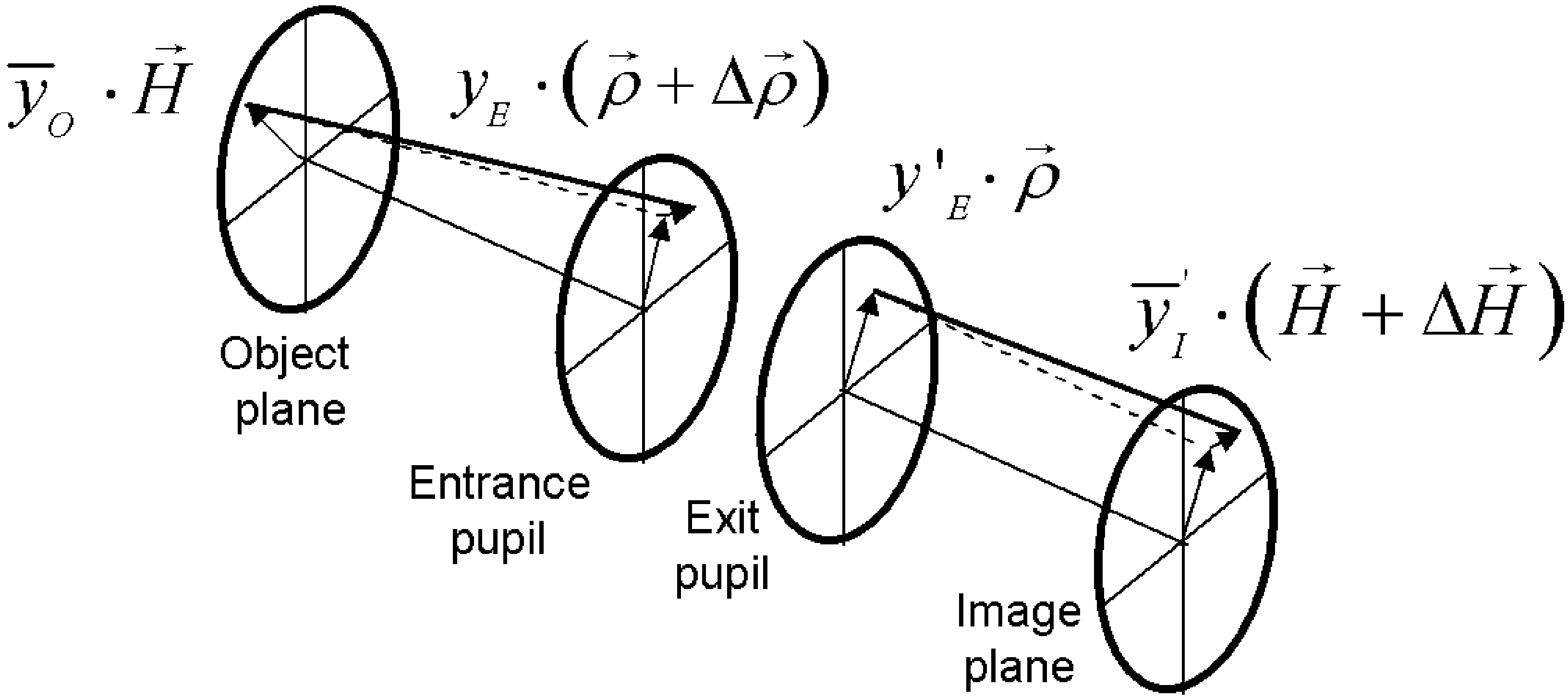
	C ray	F ray
L $-r$	36·6239 +12·299	38·6775 +12·299
$(L-r)$	48·9229	50·9765
log sin U + log $(L-r)$	7·97849 1·68951	7·95485 1·70737
log $(L-r)$ sin U − log r	9·66800 1·08987n	9·66222 1·08987n
log sin I + log $\frac{N}{N'}$	8·57813n 0·22802	8·57235n 0·23386
log sin I' + log r	8·80615n 1·08987n	8·80621n 1·08987n
log r . sin I' − log sin U'	9·89602 8·55248	9·89608 8·55254
log $(L'-r)$	1·34354	1·34354
U $+I$	0–32–43 − 2–10–10	0–30–59 − 2– 8–27
$U+I$ $-I'$	− 1–37–27 3–40– 9	− 1–37–28 3–40–11
U'	2– 2–42	2– 2–43
$L'-r$ $+r$	22·0567 −12·299	22·0567 −12·299
final L'	9·7577	9·7577
$-\frac{1}{2}U$ $+\frac{1}{2}I$	− 0–16–22 − 1– 5– 5	− 0–15–30 − 1– 4–14
$\frac{1}{2}(I-U)$	− 1–21–27	− 1–19–44
$\frac{1}{2}I'$ $-\frac{1}{2}U'$	− 1–50– 4 − 1– 1–21	− 1–50– 6 − 1– 1–22
$\frac{1}{2}(I'-U')$	− 2–51–25	− 2–51–28
log L + log sin U + log sec $\frac{1}{2}(I-U)$	1·56376 7·97849 0·00012	1·58746 7·95485 0·00012
log PA + log cosec U' + log cos $\frac{1}{2}(I'-U')$	9·54237 1·44752 9·99946	9·54243 1·44746 9·99946
log L'	0·98935	0·98935
final L' (by check)	9·7578	9·7578

Chromatic Aberration $(L_C' - L_F') = 0{\cdot}0000$ inches.

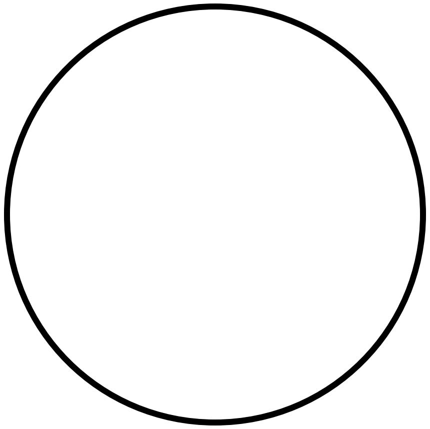
Chrom. Ab. Tolerance $= \pm \dfrac{0{\cdot}5\ \lambda}{N' \sin^2 U'_{\text{marg}}} = \pm\ 0{\cdot}0089$ inches.

From B. K. Johnson 1948
Optical design and lens computation

Real and first-order rays



The sphere



$$\frac{1}{f}=\frac{n-1}{r}+\frac{1-n}{-r}-\frac{2r}{n}\frac{n-1}{r}\frac{1-n}{-r}=\frac{n-1}{r}\left(2+\frac{2r}{n}\frac{1-n}{r}\right)$$

$$=\frac{n-1}{r}\left(2+\frac{2}{n}\frac{1-n}{1}\right)=\frac{n-1}{r}\left(\frac{2n}{n}+\frac{2}{n}\frac{1-n}{1}\right)=$$

$$=\frac{n-1}{r}\frac{2}{n}=\frac{1.5-1}{0.3M}\frac{2}{1.5}=\frac{1}{0.3}\frac{2}{3}\frac{1}{M}$$

$$f=0.45M$$

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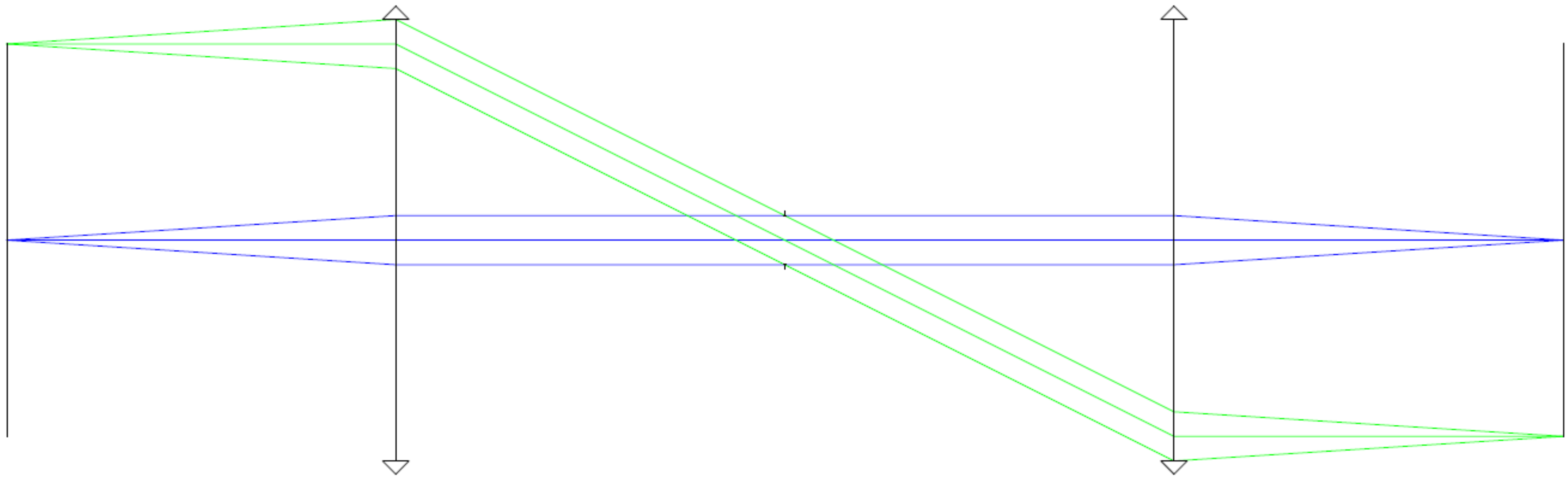
4f optical relay system



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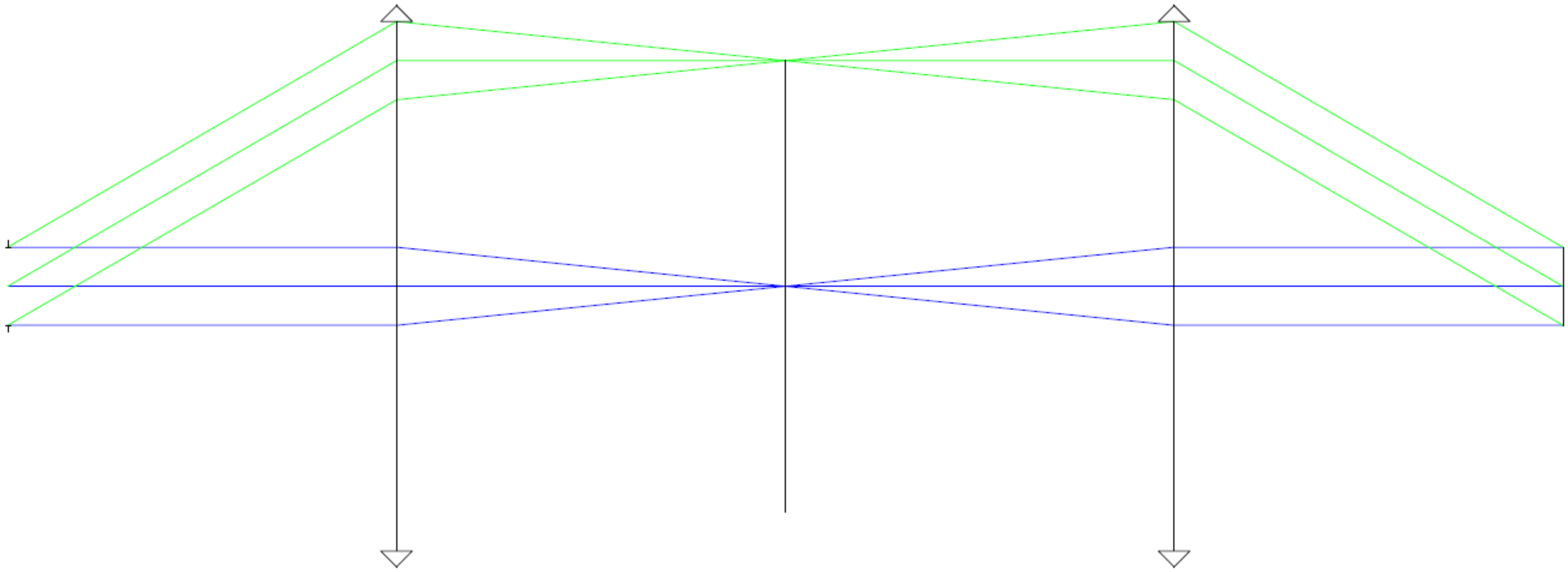
4f optical system



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Solves in a lens design program

- First-order calculations
- Distance to the ideal image plane
- Surface curvature for a given ray angle

Pick ups

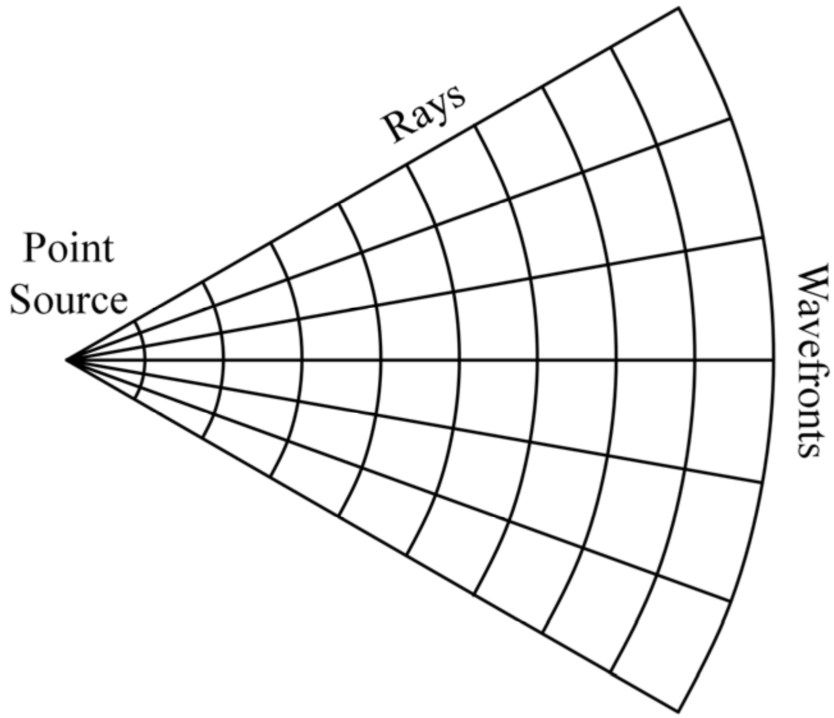
- Radii
- Index
- Thickness
- Aspheric coefficient
- Many more

The ‘complete optical’ system

- Optical elements/components
- Lenses, mirrors, gratings, filters, prisms
- Apertures
- Light baffles
- Detectors
- Coatings
- Opto-mechanics

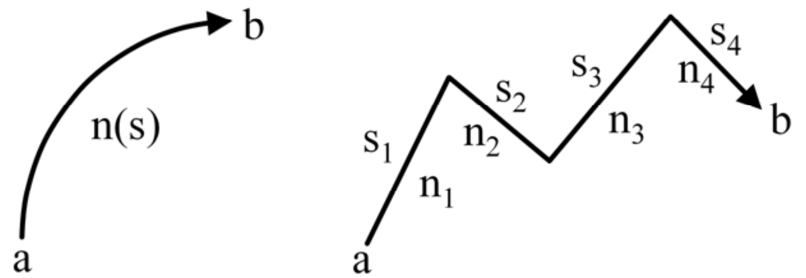
Point in case: Am I missing something?

Rays and wavefronts: two approaches for understanding light propagation



- Ray approach
- Wave approach
- The unphysical point source concept

Optical path length/difference



$$OPL = \int_a^b n(s)\,ds \qquad OPL = \sum_i n_i s_i$$

Main lecture topic

- First-order optics review
- Check in your knowledge
- Go one step further toward mastering the basics of first-order optics

Summary

- Review of first-order optics
- The aperture stop

- Next class: review of aberrations