Diffractive Optical Elements

Lens Design OPTI 517
Diffractive Lenses

- What they are
- How they work
- Zone spacing and blaze profile roles
- First order properties
- Dispersion
- Two point construction model
- Phase model
- Sweatt model
- Efficiency
- Diffractive landscape lens
Terminology

- Diffractive optical element: generic term
- Fresnel lens: Scale of zones and lack of organized phasing
- Kinoform: Phased Fresnel lens. Phase modulation from surface relief
- Holographic optical element: Produced by interfering two or more beams
- Binary optics: Made by staircases that approximate the ideal surface relief
- Fresnel zone plate: A particular pattern that produces amplitude modulation.
- Hybrid lens: combined refractive and diffractive power
- Computer generated hologram: A hologram produced by calculations in a computer
The work of a diffractive optical element

Organized rearrangement of the wavefront
A Fresnel lens reduces the amount of bulk glass. Scale of zones is large and the wavefront segments are not rearranged to re-create a spherical wavefront. The ring-zone segments is not properly organized.
Two contexts for DOE: amplitude and phase

- Blaze determines amplitude of diffracted orders
- Geometry of zone boundary determines wavefront shape (phase)

- The wavefront deformation introduced by a DOE is equal to the wavefront deformation represented by the DOE when it is thought of as an interferogram

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Example

- Straight fringes represent tilt and so the beam is deviated
Example

- Circular fringes represent defocus and so a DOE with these zone boundaries will introduce optical power.
- Depending on the spacing, spherical aberration can also be introduced.

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An infrared DOE

From Michael Morris
A Fresnel lens cut-away
First-order properties

\[ \sqrt{f^2 + r_n^2} = f + n\lambda \]
\[ f^2 + r_n^2 = f^2 + 2nf\lambda + n^2\lambda^2 \]
\[ r_n \approx \sqrt{2nf\lambda} \]

Given a focal length the zone boundaries are defined. The optical path difference between zones is one wavelength.
Paraxial diffractive lens definition

\[ r_n = \sqrt{2nf\lambda} \]

Design of a wide field diffractive landscape lens

Dale A. Buralli and G. Michael Morris
Zone Spacing

\[ r_n^2 \approx 2nf \lambda \]

\[ r_n^2 - r_{n-1}^2 = (r_n + r_{n-1})(r_n - r_{n-1}) \approx 2r_n dr = 2f \lambda \]

Spacing = \( dr \approx \frac{f}{2r_n} 2\lambda \approx F / \# \text{micrometers} \)
Focal length for a given spacing

\[ f = \frac{r_n \cdot dr}{\lambda_{\text{construction}}} \times \frac{\lambda_{\text{construction}}}{\lambda_{\text{reconstruction}}} = f_0 \times \frac{\lambda_{\text{construction}}}{\lambda_{\text{reconstruction}}} \]

Designed for \( \lambda_{\text{construction}} \)

Used at \( \lambda_{\text{reconstruction}} \)
Abbe’s number for a refractive lens

\[ \phi_{\text{refractive}} = \frac{(n-1)}{R} \]

\[ \frac{\partial \phi}{\partial \lambda} = \frac{1}{R} \frac{\partial n}{\partial \lambda} \]

\[ \partial \phi = \frac{1}{R} \left( n_d - 1 \right) \frac{n_f - n_c}{n_d - 1} = \phi_d \frac{n_f - n_c}{n_d - 1} = \phi_d \frac{\phi}{\nu} \]

\[ \nu_{\text{refractive}} = \frac{\phi}{\partial \phi} \]
Diffractive V-number

\[
\frac{\Delta \varphi}{\varphi} = \frac{r}{n_d - 1} \frac{n_f - n_c}{r} = \frac{n_f - n_c}{n_d - 1} = \frac{1}{\nu_{\text{refractive}}}
\]

\[
n' \sin(\theta') - n \sin(\theta) = \frac{m \lambda}{d}
\]

\[
f = \frac{1}{\varphi} \approx \frac{y}{\sin(\theta')} = \frac{y}{m \lambda / d}
\]

\[
\frac{\Delta \varphi}{\varphi} = \frac{y}{m \lambda_d / d} = \frac{m \left( \lambda_f - \lambda_c \right) / d}{y} = \frac{\lambda_f - \lambda_c}{\lambda_d} = \frac{1}{\nu_{\text{diffractive}}} \approx -3.5
\]
Diffractive focal length from grating perspective

\[ f = \frac{1}{\varphi} \equiv \frac{y}{\sin(\theta')} = \frac{y}{m\lambda / d} \]

\[ = \frac{y}{m\lambda_{\text{construction}} / d} \times \frac{\lambda_{\text{construction}}}{\lambda_{\text{reconstruction}}} \]

\[ = f_0 \times \frac{\lambda_{\text{construction}}}{\lambda_{\text{reconstruction}}} \]
Modeling Diffractive Optics

• Two point construction model
• Phase function
• Sweatt model
Two point construction model

\[ m, \lambda_{\text{construction}}, \lambda_{\text{reconstruction}} \]

\[ B(X,Y,Z) \]

\[ A(X,Y,Z) \]

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Phase model

\[ \phi(\rho) = 2\pi \cdot \left( a\rho^2 + b\rho^4 + c\rho^6 + d\rho^8 + \ldots \right) \]

\[ \rho = \sqrt{x^2 + y^2} \]
Phase model

\[ n' \sin(I') \cdot \Delta y = n \sin(I) \cdot \Delta y \]

\[ n' \sin(I') \cdot \Delta y - n \sin(I) \cdot \Delta y = \Delta \varphi(y) \]

\[ n' \sin(I') - n \sin(I) = \frac{\Delta \varphi(y)}{\Delta y} \rightarrow \frac{\partial \varphi(y)}{\partial y} \]

\[ \frac{\partial \varphi(y)}{\partial y} = n' \sin(I') - n \sin(I) \]
Sweatt’s model

\[ \delta = -\alpha (n - 1) \]

For \( n \approx 10,000 \) alpha must be very small to maintain the same deviation.

\[ \varphi = \frac{n - 1}{r} \]

For a plano convex lens with \( n \approx 10,000 \) the radius must be very long to maintain the same optical power.
Sweatt Model justification

Start with the diffraction grating equation

\[
n'\sin(I') - n\sin(I) = \left[n'\cos(I') - n\cos(I)\right] \cdot \frac{m\lambda}{n'\cos(I') - n\cos(I)}^{(1/d)}
\]

\[
n'\sin(I') - n\sin(I) = \left[n'\cos(I') - n\cos(I)\right] \cdot \tan(\alpha)
\]

\[
n'\{\sin(I') - \cos(I')\tan(\alpha)\} = n\{\sin(I) - \cos(I)\tan(\alpha)\}
\]

\[
n'\{\cos(\alpha)\sin(I') - \cos(I')\sin(\alpha)\} = n\{\cos(\alpha)\sin(I) - \cos(I)\sin(\alpha)\}
\]

\[
n'\{\sin(I'-\alpha)\} = n\{\sin(I - \alpha)\}
\]
Sweatt’s Model

\[ n'\{\sin(I'-\alpha)\} = n\{\sin(I-\alpha)\} \]

\[
\tan(\alpha) = \frac{m\lambda}{n'\cos(I') - n \cos(I)} \quad (1/d)
\]

For large \( n \)'s then \( \alpha \) is negligible and we have:

\[ n'\sin(I) = n \sin(I) \]

Thus for high index diffraction becomes like refraction!
Dispersion in Sweatt’s model

\[ \delta = -\alpha (n - 1) \]
\[ \sin (I') \approx \sin (I) + (n_d - 1) \alpha \]
\[ \Delta \approx \sin (I_F') - \sin (I_C') \approx (n_F - n_C) \alpha \]
\[
\frac{\delta}{\Delta} = v_{\text{refractive}} = \frac{(n_d - 1) \alpha}{(n_F - n_C) \alpha} = \frac{\lambda_d (10,000)}{\lambda_F (10,000) - \lambda_C (10,000)} = \frac{\lambda_d}{\lambda_F - \lambda_C} \approx -3.5
\]
Dispersion in Sweatt’s model

Consistent with diffraction case

\[
\sin(I'_d) - \sin(I_d) = \frac{m \lambda_d}{d} \approx \delta
\]

\[
\Delta \approx \sin(I'_F) - \sin(I'_C) = m \frac{\lambda_F - \lambda_C}{d}
\]

\[
\frac{\delta}{\Delta} = \nu_{\text{refractive}} \approx \frac{m \frac{\lambda_d}{d}}{m \frac{\lambda_F - \lambda_C}{d}} = \frac{\lambda_d}{\lambda_F - \lambda_C}
\]

In conclusion:
To include dispersion in the Sweatt model make the index of refraction equal to the wavelength times 10,000

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Schott: \[ n(\lambda)^2 = A + B\lambda^2 + \ldots \]
Structural coefficients: Thin lens (stop at lens)

\[ S_I = \frac{1}{4} y^4 \phi^3 [AX^2 - BXY + CY^2 + D] \]

\[ S_{II} = \frac{1}{2} \mathcal{K} y^2 \phi^2 [EX - FY] \]

\[ S_{III} = \mathcal{K}^2 \phi \]

\[ S_{IV} = \mathcal{K}^2 \phi \frac{1}{n} \]

\[ S_V = 0 \]

\[ C_L = y^2 \phi \frac{1}{\nu} \]

\[ C_T = 0 \]

\[ A = \frac{n + 2}{n(n-1)^2} \]

\[ B = \frac{4(n+1)}{n(n-1)} \]

\[ C = \frac{3n+2}{n} \]

\[ D = \frac{n^2}{(n-1)^2} \]

\[ E = \frac{n+1}{n(n-1)} \]

\[ F = \frac{2n+1}{n} \]
Diffractive lens
(n very large @ X=0)

\[
\begin{align*}
\sigma_I &= 3Y^2 + 1 \\
\sigma_{II} &= -2Y \\
\sigma_{III} &= 1 \\
\sigma_{IV} &= 0 \\
\sigma_V &= 0 \\
\sigma_L &= \frac{1}{\nu_{\text{diffractive}}} \\
\sigma_T &= 0
\end{align*}
\]

C = 3; D = 1; F = 2
Aberration coefficients for Y=1; X=0

\[ S_I = \frac{y^4}{f^3} \left( \frac{\lambda}{\lambda_0} \right)^3 \]

\[ S_{III} = \frac{\gamma K^2}{f} \left( \frac{\lambda}{\lambda_0} \right) \]

\[ S_{II} = \frac{-y^2}{f^2} \gamma K \left( \frac{\lambda}{\lambda_0} \right)^2 \]

\[ S_{IV} = 0 \]

\[ S_V = 0 \]

For general case one needs to be careful as the shape depends on the index for a given power.
Structural coefficients for diffractive lens

Structural aberration coefficients of a thin lens (Stop at lens)

<table>
<thead>
<tr>
<th>Paraxial identities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = (n'-n) \cdot (c_1 - c_2) = \frac{1}{R_1} - \frac{1}{R_2} )</td>
</tr>
<tr>
<td>( X = \frac{c_1 + c_2}{c_1 - c_2} = \frac{R_1 + R_2}{R_1 - R_2} )</td>
</tr>
<tr>
<td>( Y = \frac{w' + w}{w'' + w} = \frac{1 + m}{1 - m} )</td>
</tr>
<tr>
<td>( c_1 = \frac{1}{2} \frac{\phi}{n - 1} (X + 1) )</td>
</tr>
<tr>
<td>( c_2 = \frac{1}{2} \frac{\phi}{n - 1} (X - 1) )</td>
</tr>
<tr>
<td>( w = u = -\frac{1}{2} (Y - 1) (\phi \cdot y) )</td>
</tr>
<tr>
<td>( w' = u' = -\frac{1}{2} (Y + 1) (\phi \cdot y) )</td>
</tr>
</tbody>
</table>

Structural aberration coefficients

\[ \sigma_I = AX^2 - BXY + CY^2 + D \]

\[ A = -\frac{n + 2}{n(n - 1)^2} \]

\[ B = \frac{4(n + 1)}{n(n - 1)} \]

\[ C = \frac{3n + 2}{n} \]

\[ D = \frac{n^2}{(n - 1)^2} \]

\[ E = \frac{n + 1}{n(n - 1)} \]

\[ F = \frac{2n + 1}{n} \]

\[ \sigma_I = \frac{4}{(\phi R_2)^2} - \frac{8Y}{\phi R_2} + 3Y^2 + 1 \]

\[ \sigma_{II} = \frac{2}{\phi R_2} - 2Y \]

\[ \sigma_{IV} = 0 \]

\[ \sigma_{V} = 0 \]

\[ \sigma_T = 0 \]

\[ \sigma_{\text{III}} = 1 \]

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diffractive

College of Optical Sciences
THE UNIVERSITY OF ARIZONA
Field curvature correction hybrid lens
Verification

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OPD Alternate view

- OPD has two parts. One is due to material dispersion, the other to due to diffraction

\[
\begin{align*}
\text{OPD}_F &= \frac{y^2}{2R} \left( (n_F - 1) + (n_d - 1) \frac{\lambda_F}{\lambda_d} \right) \\
\text{OPD}_F - \text{OPD}_C &= \frac{y^2}{2R} \left( (n_F - 1) + (n_d - 1) \frac{\lambda_F}{\lambda_d} \right) \\
&\quad - \frac{y^2}{2R} \left( (n_C - 1) + (n_d - 1) \frac{\lambda_C}{\lambda_d} \right) \\
&= \frac{y^2}{2R} \left( (n_F - n_C) + (n_d - 1) \frac{\lambda_F - \lambda_C}{\lambda_d} \right) \\
&= \frac{y^2}{2} \phi \left( \frac{1}{\nu_{\text{ref}}} + \frac{1}{\nu_{\text{diff}}} \right)
\end{align*}
\]

\[\text{OPD} = n \times t + N \times \lambda\]
Spherical aberration

• Depending on the zone boundary distribution, DOE axially symmetric DOE can introduce different orders of spherical aberration

\[ \phi(\rho) = 2\pi \cdot \left( a\rho^2 + b\rho^4 + c\rho^6 + d\rho^8 + \ldots \right) \]
Calculating order efficiency

• Simple case of an amplitude device with a square wave profile
• Duty cycle

\[ \psi(x, y) = A_p \text{Comb}(x - nx_0)^{**\text{rect}} \left( \frac{x}{d} \right) \]
Square wave

\[ F(\nu) \approx \frac{A}{2} \text{SINC} \left( \frac{\nu}{2\nu_0} \right) \sum_{-\infty}^{\infty} \partial (\nu - n\nu_0) = \frac{A}{2} \sum_{-\infty}^{\infty} \text{SINC} \left( \frac{n}{2} \right) \partial (\nu - n\nu_0) \]

\[ f(t) = \text{square wave} = \frac{A}{2} \sum_{-\infty}^{\infty} \text{SINC} \left( \frac{n}{2} \right) e^{i2\pi n\nu_0 t} \]

\[ = \frac{A}{2} + \frac{A}{\pi} \left[ e^{i2\pi n\nu_0 t} + e^{-i2\pi n\nu_0 t} \right] + \frac{A}{3\pi} \left[ e^{i2\pi n3\nu_0 t} + e^{-i2\pi n3\nu_0 t} \right] \]

\[ + \frac{A}{5\pi} \left[ e^{i2\pi n5\nu_0 t} + e^{-i2\pi n5\nu_0 t} \right] + \frac{A}{7\pi} \left[ e^{i2\pi n7\nu_0 t} + e^{-i2\pi n7\nu_0 t} \right] + \ldots \]

\[ \nu_0 = T^{-1} \]

50% duty cycle \[ \left( \frac{1}{\pi} \right)^2 \approx 0.1 \]

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Binary optics technology

2 PHASE LEVELS

4 PHASE LEVELS

8 PHASE LEVELS

U.V. LIGHT

COMPUTER GENERATED AMPLITUDE

PHOTORESIST

SUBSTRATE (INDEX = n)

PHOTORESIST DEVELOPMENT

REACTIVE ION ETCH TO A DEPTH d = \frac{1}{2(n-1)}

REMOVE RESIDUAL PHOTORESIST

2 LEVEL ELEMENT
Efficiency for binary optics

\[ \eta_1^N = \left( \frac{\sin(\pi/N)}{\pi/N} \right)^2 \]

<table>
<thead>
<tr>
<th>Number of Levels ( N )</th>
<th>First-Order Efficiency ( \eta_1^N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.41</td>
</tr>
<tr>
<td>3</td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td>0.81</td>
</tr>
<tr>
<td>5</td>
<td>0.87</td>
</tr>
<tr>
<td>6</td>
<td>0.91</td>
</tr>
<tr>
<td>8</td>
<td>0.95</td>
</tr>
<tr>
<td>12</td>
<td>0.98</td>
</tr>
<tr>
<td>16</td>
<td>0.99</td>
</tr>
</tbody>
</table>

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Efficiency

\[ \sigma^2 = (n-1)^2 \frac{1}{2} \int_{-1}^{1} \left( \frac{hx}{N} \right)^2 dx = (n-1)^2 \left( \frac{h}{N} \right)^2 \frac{1}{2} x^3 \mid_{-1}^{1} = \frac{1}{3} (n-1)^2 \left( \frac{h}{N} \right)^2 \]

\[ h = 1 \]
\[ (n-1)2h = \lambda \]
\[ \sigma^2 = \frac{1}{3} \frac{4}{4} (n-1)^2 \left( \frac{h}{N} \right)^2 = \frac{1}{12} \lambda^2 \left( \frac{1}{N} \right)^2 \]

\[ S \approx 1 - \frac{\pi^2}{3} \left( \frac{1}{N} \right)^2 \]

\[ N = 2 \ ; S = 0.17 \]
\[ N = 4 \ ; S = 0.794 \]
\[ N = 8 \ ; S = 0.948 \]
\[ N = 16 \ ; S = 0.987 \]
\( \varepsilon = \sin c^2 \left( \pi \left[ \frac{\lambda_{\text{construction}}}{\lambda_{\text{reconstruction}}} \frac{n(\lambda_{\text{reconstruction}}) - 1}{n(\lambda_{\text{construction}}) - 1} - m \right] \right) \)
Efficiency

\[ \varepsilon \approx 1 - \left( \frac{2\pi}{\lambda \sigma} \right)^2 \approx 1 - \left( \frac{2\pi}{\lambda_{\text{reconstruction}}} \frac{\lambda_{\text{reconstruction}} - \lambda_{\text{construction}}}{3} \right)^2 \]

\[ \Delta \approx \lambda_{\text{reconstruction}} - \lambda_{\text{construction}} \]
Comparison
Standard lens, Fresnel lens and DOE lens

Refracting lens
Fresnel lens
DOE lens

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Images of extended objects

Acrylic powerless lens

Other orders produce images at different magnifications
Like ghost images

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Canon’s multilayer DOE’s

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How does it work?

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How does it work?

$$\varepsilon(\lambda) = \sin^2 c^2 \left( \pi \left[ \frac{\lambda_{\text{construction}}}{\lambda_{\text{reconstruction}}} \frac{n(\lambda_{\text{reconstruction}})^{-1}}{n(\lambda_{\text{construction}})^{-1}} - m \right] \right)$$

$$\varepsilon(\lambda) = \sin^2 c^2 \left( \pi \left[ d \frac{n(\lambda_{\text{reconstruction}})^{-1}}{\lambda_{\text{reconstruction}}} - m \right] \right) = \sin^2 c^2 \left( \pi \left[ d_{\text{construction}} \frac{d}{d_{\text{reconstruction}}} - m \right] \right)$$

$$\varepsilon(\lambda) = \sin^2 c^2 \left( \pi \left[ d_2 \frac{n_2(\lambda_{\text{reconstruction}})^{-1}}{\lambda_{\text{reconstruction}}} \pm d_1 \frac{n_1(\lambda_{\text{reconstruction}})^{-1}}{\lambda_{\text{reconstruction}}} - m \right] \right)$$

or $$d_2 \left( n_2(\lambda_{\text{reconstruction}})^{-1} \right) \pm d_1 \left( n_1(\lambda_{\text{reconstruction}})^{-1} \right) = \lambda_{\text{reconstruction}}$$
100% at two wavelengths
Alternate view
100% efficiency at $2\lambda$ (no ripple)

$$\lambda_2 = 2\lambda_1 = 2(450nm)$$
An actual lens application for controlling chromatic change of magnification

Note lack of lens symmetry about the stop

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Some Fresnel lens and DOE photographs

Plastic Fresnel lens; Diamond turned and replicated

Gray scale; note binary edge

Binary 8 levels

Binary 16 levels
Measurement of a DOE

3-Dimensional Interactive Display

Surface Stats:
Ra: 75.60 nm
Rq: 78.15 nm
Rt: 334.53 nm

Measurement Info:
Magnification: 10.24
Measurement Mode: PSI
Sampling: 820.31 nm
Array Size: 736 X 480

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Beware

- Modeling assumes DOEs having no physical structure
- Real modeling faces sampling issues
- Scalar treatment
- Zones are about ~7\(\lambda\) or more
- Light scattered at boundaries and zone shadowing effects
- Fabrication: Diamond turning, microlithography printing techniques, Grey scale techniques.
Examples

- Diffractive landscape lens
- Correction of chromatic change in the landscape lens, eyepieces, fish-eye lenses, unsymmetrical lenses
- Null-corrector Certifier
- Modeling a few zones