Summary of formulas
Summary of optical systems

Lens Design OPTI 517

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Imaging: central projection

$$X' = \frac{a_1 X + b_1 Y + c_1 Z + d_1}{a_0 X + b_0 Y + c_0 Z + d_0}$$

$$Y' = \frac{a_2 X + b_2 Y + c_2 Z + d_2}{a_0 X + b_0 Y + c_0 Z + d_0}$$

$$Z' = \frac{a_3 X + b_3 Y + c_3 Z + d_3}{a_0 X + b_0 Y + c_0 Z + d_0}$$

$$m = \frac{1}{1 - Z \backslash f}$$

$$m = 1 - \frac{Z'}{f'}$$

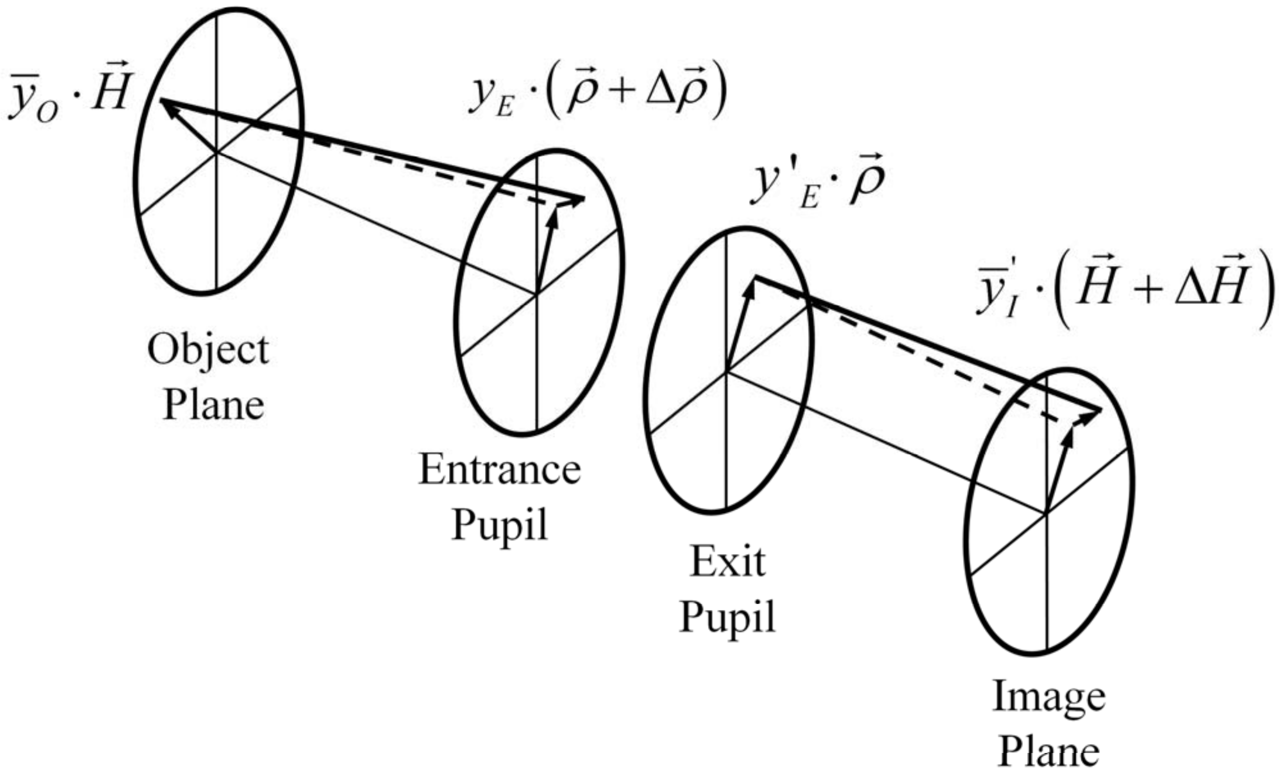
$$\frac{f'}{Z'} + \frac{f}{Z} = 1$$

$$\frac{Z'}{f'} = 1 - m$$

$$\frac{Z}{f} = 1 - \frac{1}{m}$$

Collinear transformation
Newtonian equations
Gaussian equations
Cardinal points

Optical system conceptual model



Wave aberration function

$$W(H,\rho) = W_{000} + W_{111} H \rho \cos(\theta) + W_{020} \rho^2 + W_{200} H^2 +$$
$$+ W_{040} \rho^4 + W_{131} H \rho^3 \cos(\theta) + W_{222} H^2 \rho^2 \cos^2(\theta) +$$
$$W_{220} H^2 \rho^2 + W_{311} H^3 \rho \cos(\theta) + W_{400} H^4$$

Aberration Coefficients in terms of Seidel sums	
Coefficient	Seidel sum
$W_{040}=\frac{1}{8}S_I$	$S_I=-\sum_{i=1}^{j}\left(A^2 y\Delta\left(\frac{u}{n}\right)\right)_i$
$W_{131}=\frac{1}{2}S_{II}$	$S_{II}=-\sum_{i=1}^{j}\left(A\bar{A}y\Delta\left(\frac{u}{n}\right)\right)_i$
$W_{222}=\frac{1}{2}S_{III}$	$S_{III}=-\sum_{i=1}^{j}\left(\bar{A}^2 y\Delta\left(\frac{u}{n}\right)\right)_i$
$W_{220}=\frac{1}{4}(S_{IV}+S_{III})$	$S_{IV}=-Ж^2\sum_{i=1}^{j}P_i$
$W_{311}=\frac{1}{2}S_V$	$S_V=-\sum_{i=1}^{j}\left(\frac{\bar{A}}{A}\left[Ж^2 P+\bar{A}^2 y\Delta\left(\frac{u}{n}\right)\right]\right)_i$
$W_{311}=\frac{1}{2}S_V$	$S_V=-\sum_{i=1}^{j}\left(\bar{A}\left[\bar{A}^2\Delta\left(\frac{1}{n^2}\right)y-\left(Ж+\bar{A}y\right)\bar{y}P\right]\right)_i$
$\delta_\lambda W_{020}=\frac{1}{2}C_L$	$C_L=\sum_{i=1}^{j}\left(Ay\Delta\left(\frac{\delta n}{n}\right)\right)_i$
$\delta_\lambda W_{111}=C_T$	$C_T=\sum_{i=1}^{j}\left(\bar{A}y\Delta\left(\frac{\delta n}{n}\right)\right)_i$

Chromatic coefficients

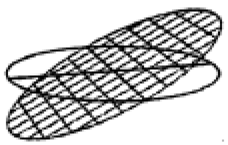
For a system of j surfaces

$$\partial_\lambda W_{111} = \sum_{i=1}^{j} \bar{A}_j \Delta_j \left(\partial n / n\right) y_j$$

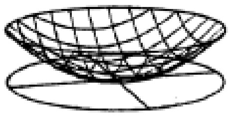
$$\partial_\lambda W_{020} = \frac{1}{2} \sum_{i=1}^{j} A_j \Delta_j \left(\partial n / n\right) y_j$$

For a system of thin lenses in air

$$\partial_\lambda W_{111} = \sum_{i=1}^{j} \left[\frac{\phi}{\nu} \bar{y} y \right]_i$$



$$\partial_\lambda W_{020} = \frac{1}{2} \sum_{i=1}^{j} \left[\frac{\phi}{\nu} y^2 \right]_i$$



Aspheric contributions

$$\delta W_{040} = \frac{1}{8} a$$

$$\delta W_{131} = \frac{1}{2} \frac{\bar{y}}{y} a$$

$$\delta W_{222} = \frac{1}{2} \left(\frac{\bar{y}}{y} \right)^2 a$$

$$\delta W_{220} = \frac{1}{4} \left(\frac{\bar{y}}{y} \right)^2 a$$

$$\delta W_{311} = \frac{1}{2} \left(\frac{\bar{y}}{y} \right)^3 a$$

$$\delta C_L = 0$$

$$\delta C_T = 0$$

$$a = -\varepsilon^2 c^3 y^4 \Delta n$$

$$a = 8 A_4 y^4 \Delta n$$

Stop Shifting

$$\delta S_I = 0$$

$$\delta S_{II} = \frac{\delta \bar{y}}{y} S_I$$

$$\delta S_{III} = 2\frac{\partial \bar{y}}{y} S_{II} + \left(\frac{\delta \bar{y}}{y}\right)^2 S_I$$

$$\delta S_{IV} = 0$$

$$\delta S_V = \frac{\delta \bar{y}}{y}\left\{S_{IV} + 3S_{III}\right\} + 3\left(\frac{\delta \bar{y}}{y}\right)^2 S_{II} + \left(\frac{\delta \bar{y}}{y}\right)^3 S_I$$

$$\delta C_L = 0$$

$$\delta C_T = \frac{\partial \bar{y}}{y} C_L$$

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Structural coefficients

Seidel sums in terms of structural aberration coefficients
Pupils located at principal planes
$S_I = \frac{1}{4} y_P^4 \Phi^3 \sigma_I$
$S_{II} = \frac{1}{2} Ж y_P^2 \Phi^2 \sigma_{II}$
$S_{III} = Ж^2 \Phi \sigma_{III}$
$S_{IV} = Ж^2 \Phi \sigma_{IV}$
$S_V = \frac{2Ж^3 \sigma_V}{y_P^2}$
$C_L = y_P^2 \Phi \sigma_L$
$C_T = 2Ж \sigma_T$

Stop-shift from principal planes
$\sigma_I^* = \sigma_I$
$\sigma_{II}^* = \sigma_{II} + \bar{S}_\sigma \sigma_I$
$\sigma_{III}^* = \sigma_{III} + 2\bar{S}_\sigma \sigma_{II} + \bar{S}_\sigma^2 \sigma_I$
$\sigma_{IV}^* = \sigma_{IV}$
$\sigma_V^* = \sigma_V + \bar{S}_\sigma \left(\sigma_{IV} + 3\sigma_{III}\right) + 3\bar{S}_\sigma^2 \sigma_{II} + \bar{S}_\sigma^3 \sigma_I$
$\sigma_L^* = \sigma_L$
$\sigma_T^* = \sigma_T + \bar{S}_\sigma \sigma_L$
$\bar{S}_\sigma = \frac{y_P \bar{y}_P \Phi}{2Ж}$
$\Delta\bar{S}_\sigma = \frac{y_P \Delta\bar{y}_P \Phi}{2Ж} = \frac{y_P^2 \Phi}{2Ж} \bar{S}$

Structural coefficients: Thin lens (stop at lens)

$$S_I = \frac{1}{4} y^4 \phi^3 \left[AX^2 - BXY + CY^2 + D \right]$$

$$S_{II} = \frac{1}{2} \text{Ж} y^2 \phi^2 \left[EX - FY \right]$$

$$S_{III} = \text{Ж}^2 \phi$$

$$S_{IV} = \text{Ж}^2 \phi \frac{1}{n}$$

$$S_V = 0$$

$$C_L = y^2 \phi \frac{1}{\nu}$$

$$C_T = 0$$

$$X = \frac{c_1 + c_2}{c_1 - c_2} = \frac{r_2 + r_1}{r_2 - r_1}$$

$$Y = \frac{1+m}{1-m} = \frac{u'+u}{u'-u}$$

$$\phi = \Delta n \Delta c = (n-1)(c_1 - c_x)$$

$$A = \frac{n+2}{n(n-1)^2}$$

$$B = \frac{4(n+1)}{n(n-1)}$$

$$C = \frac{3n+2}{n}$$

$$D = \frac{n^2}{(n-1)^2}$$

$$E = \frac{n+1}{n(n-1)}$$

$$F = \frac{2n+1}{n}$$

Miscellaneous

Petzval sum:

$$\frac{1}{n'_k\ \rho'_k} - \frac{1}{n_1\rho_1} = \sum \frac{n'-n}{n'nr}$$

For a system of thin lenses in air:

$$\frac{1}{\rho'_k} = \sum \frac{\phi}{n}$$

$$Distortion = \frac{H-h}{h} \bullet 100$$

Miscellaneous

Coddington equations

$$\frac{n'}{s'} - \frac{n}{s} = \frac{n'\cos I' - n\cos I}{R_s}$$

$$\frac{n'\cos^2 I'}{t'} - \frac{n\cos^2 I}{t} = \frac{n'\cos I' - n\cos I}{R_t}$$

Sine condition

$$\frac{u}{u'} = \frac{\sin(U)}{\sin(U')}$$

Some angular quantities

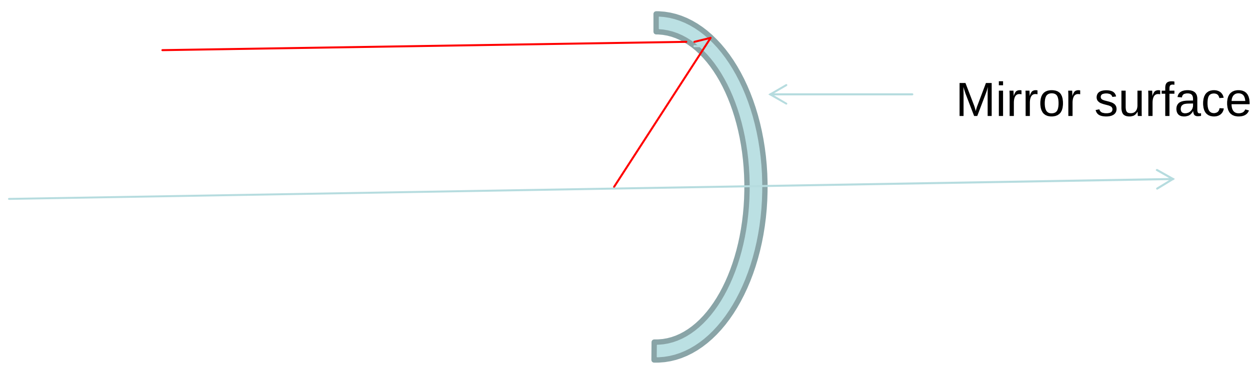
The mnemonic 12345

- Index of air ~1.0003
- One arc-minute ~0.0003
- One arc second ~0.000005
- 15 degrees ~1/4
- One arc minute ~healthy eye resolution
- Half degree ~Sun, Moon angular subtend
- +/- 23.4 degrees FOV of 35 mm camera with 50 mm lens.

Mangin mirror

Invented in 1876 by Alphonse Mangin in France

Actually described by Newton in his description about making the mirrors for his celebrated telescope



A Mangin mirror uses a lens with one surface coated to be a mirror.

Mangin mirrors are used to reflect light from the light sources in lighthouses without the problem of tarnishing due to the salty environment.

Mangin mirrors are variable aberration generators for a given optical power. They have very useful aberration correcting properties. Both mono-chromatic and chromatic.

Telescopes Objectives

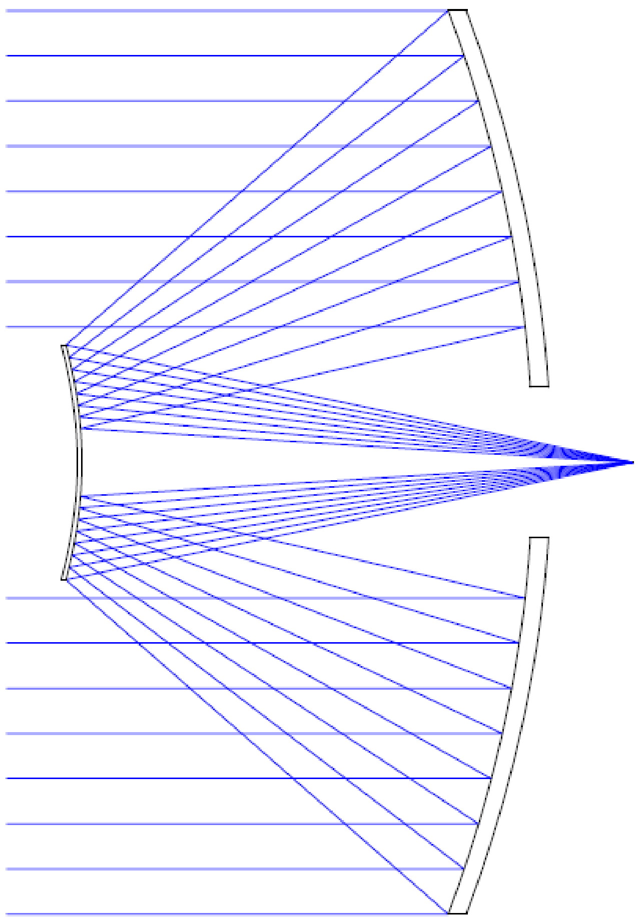
- Achromatic and aplanatic doublet
- Apochromatic and aplanatic doublet/triplet
- Schupmann medial single glass objective

Review of telescopes

- Cassegrain
- Mersenne
- Schmidt camera
- Maksutov
- Houghton

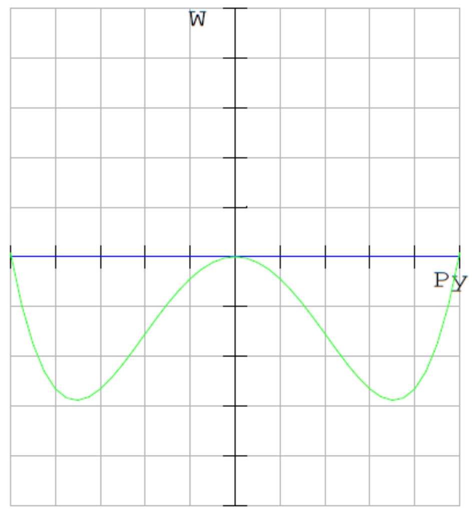
- Paul-Baker
- Offner relay
- Meinel's two stage optics

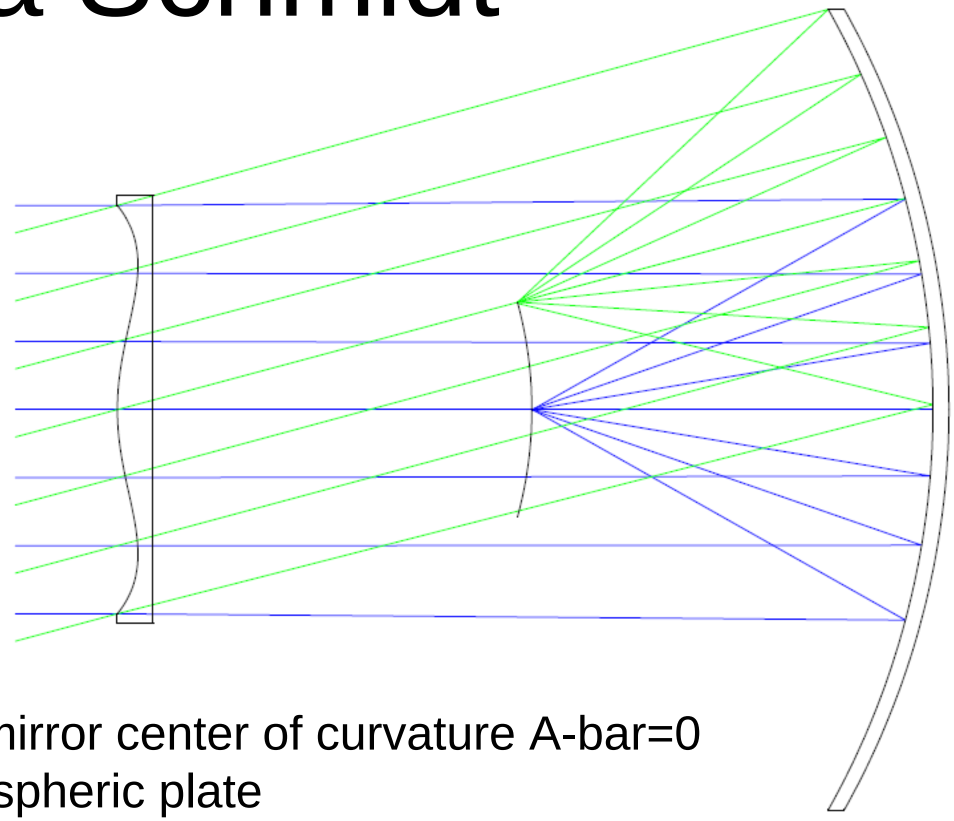
Cassegrain type



•True Cassegrain
•Ritchey-Chretien: aplanatic
•Dall-Kirkham: spherical secondary
•Pressman-Carmichel; spherical primary
•Olivier Guyon (no diffraction rings)

Camera Schmidt





Aspheric plate at mirror center of curvature A-bar=0
Stop aperture at aspheric plate
Note symmetry about mirror CC
No spherical aberration
No coma
No astigmatism.
Anastigmatic over a wide field of view!
Satisfies Conrady's D-d sum

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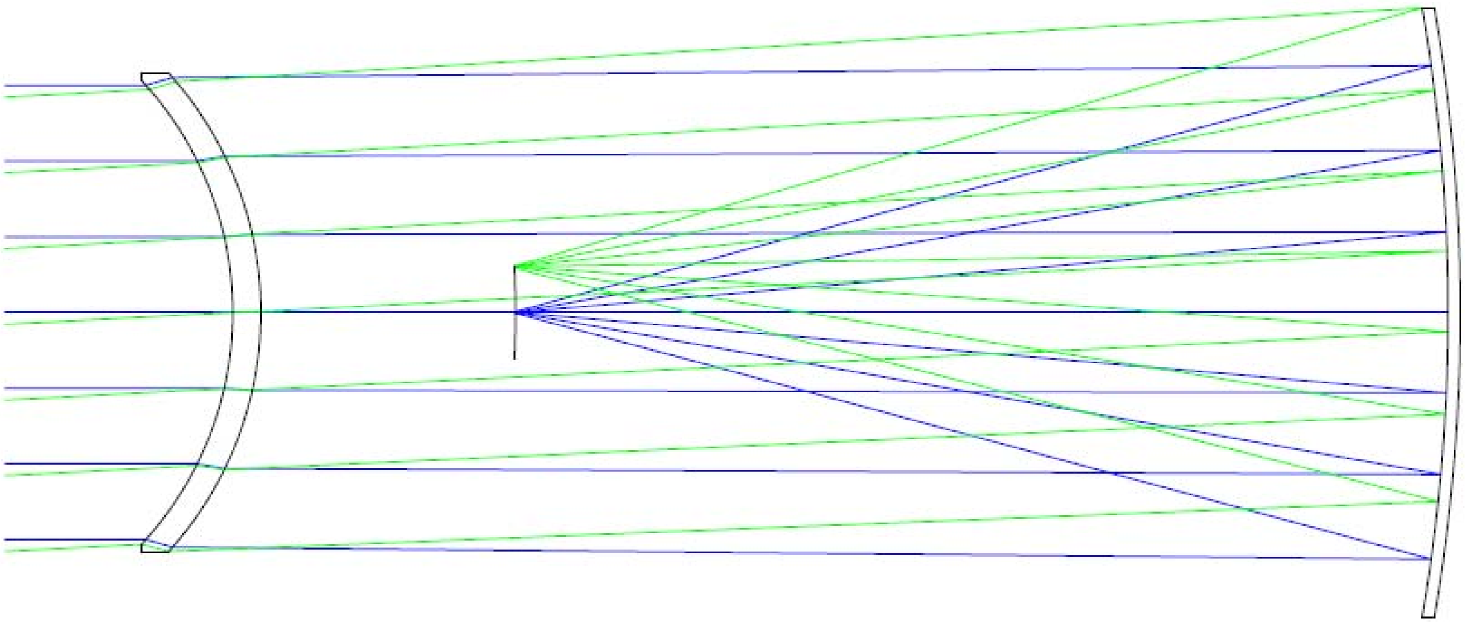
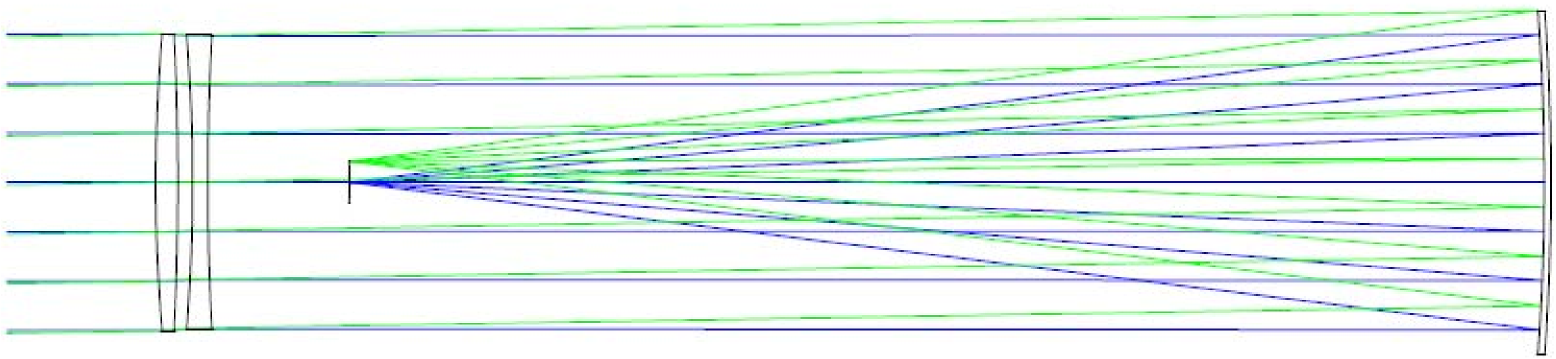
Maksutov: Meniscus lens



- Strong meniscus shape corrects spherical aberration
- Coma by the position of the stop and meniscus lens
- Chromatic change of focus by the meniscus lens thickness
- Single glass achromat

$$\partial_\lambda W_{020} = \frac{1}{2}\sum_{i=1}^{j}\left[\frac{\phi}{\nu}y^2\right]_i$$

Houghton: nearly afocal doublet



Same glass doublet
Several solutions
Correction for spherical aberration, coma, and chromatic aberrations
Correction for astigmatism
All spherical surfaces

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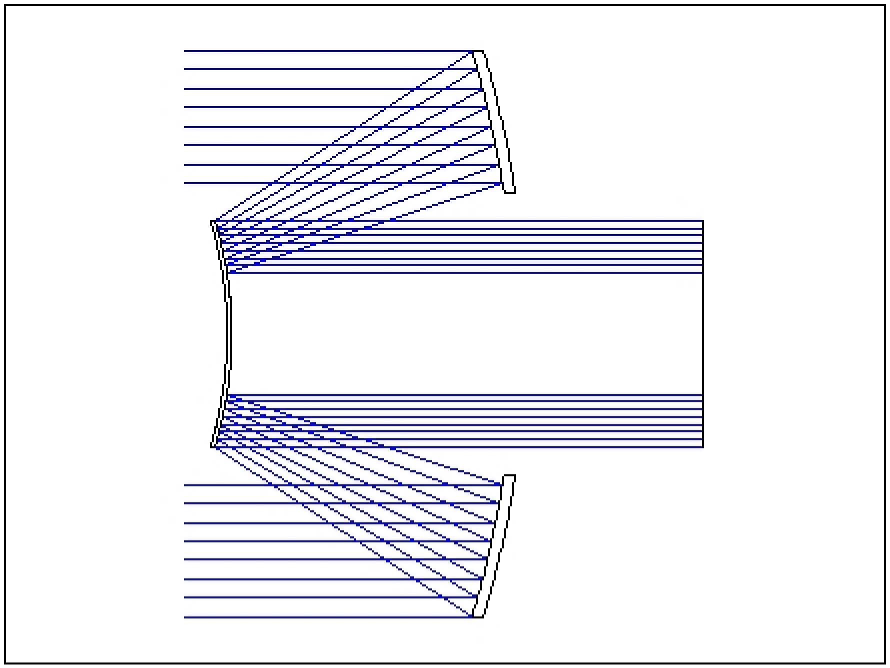
Merssene afocal system
Anastigmatic
Confocal paraboloids



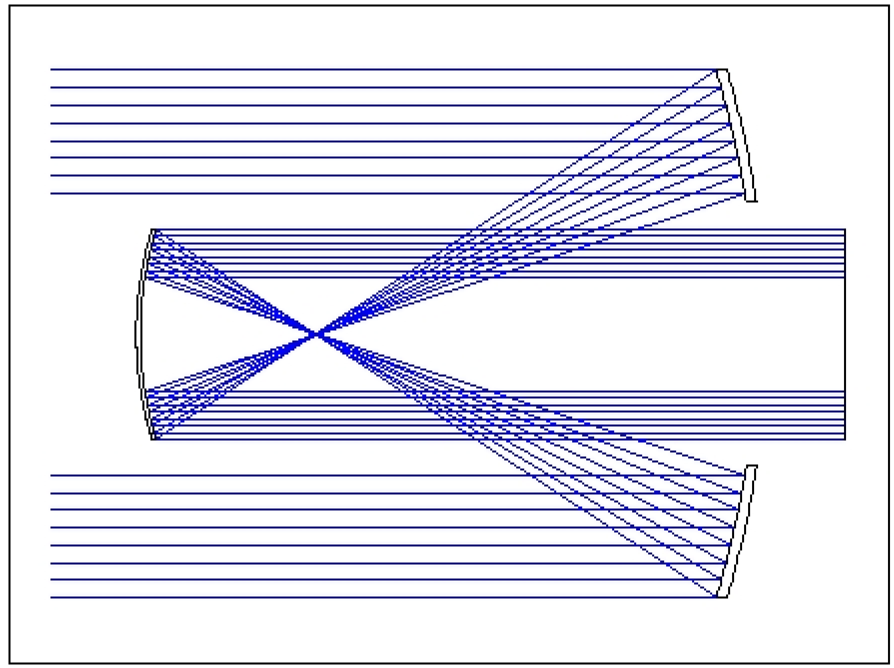
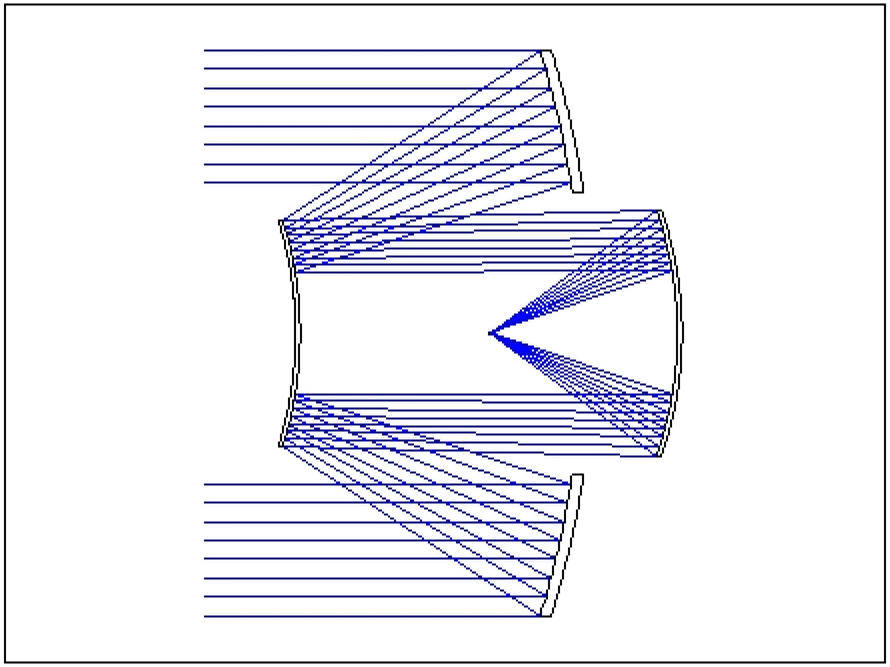
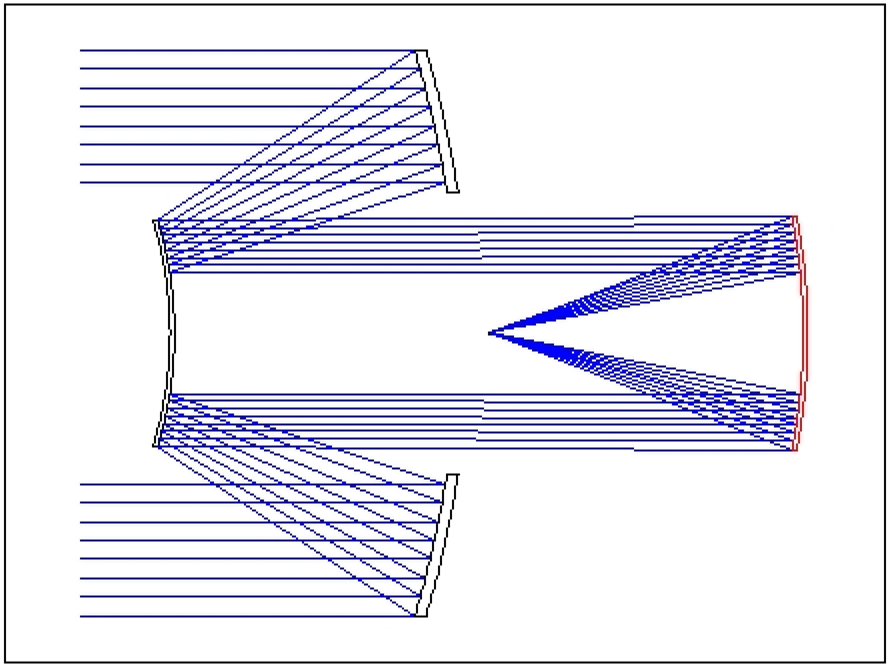


Image and Pupil Aberration relationships

Identity between pupil and image aberration coefficients	
$\bar{W}_{040} = W_{400}$	
$\bar{W}_{131} = W_{311} + \frac{1}{2} Ж \cdot \Delta\left\{\bar{u}^2\right\}$	
$\bar{W}_{222} = W_{222} + \frac{1}{2} Ж \cdot \Delta\left\{u\bar{u}\right\}$	
$\bar{W}_{220} = W_{220} + \frac{1}{4} Ж \cdot \Delta\left\{u\bar{u}\right\}$	
$\bar{W}_{311} = W_{131} + \frac{1}{2} Ж \cdot \Delta\left\{u^2\right\}$	
$\bar{W}_{400} = W_{040}$	

Paul-Baker system
Anastigmatic-Flat field

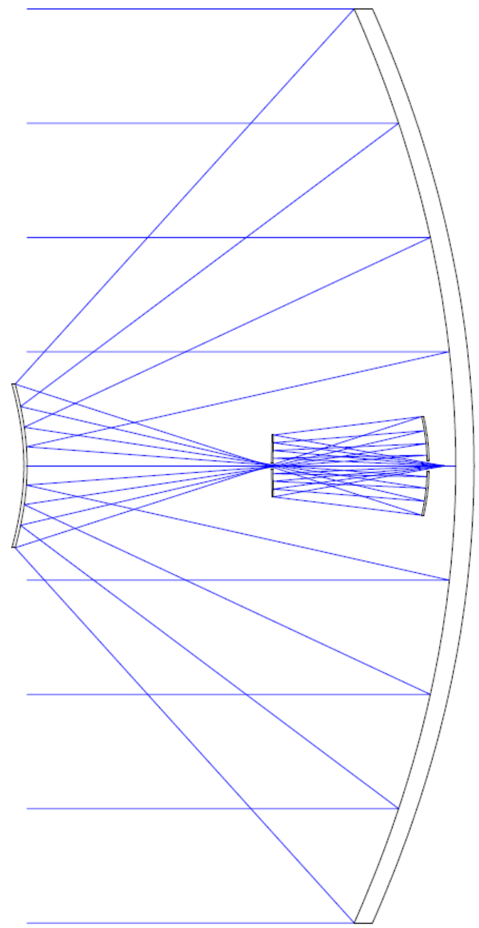




Anastigmatic
Parabolic primary
Spherical secondary and tertiary
Curved field
Tertiary CC at secondary

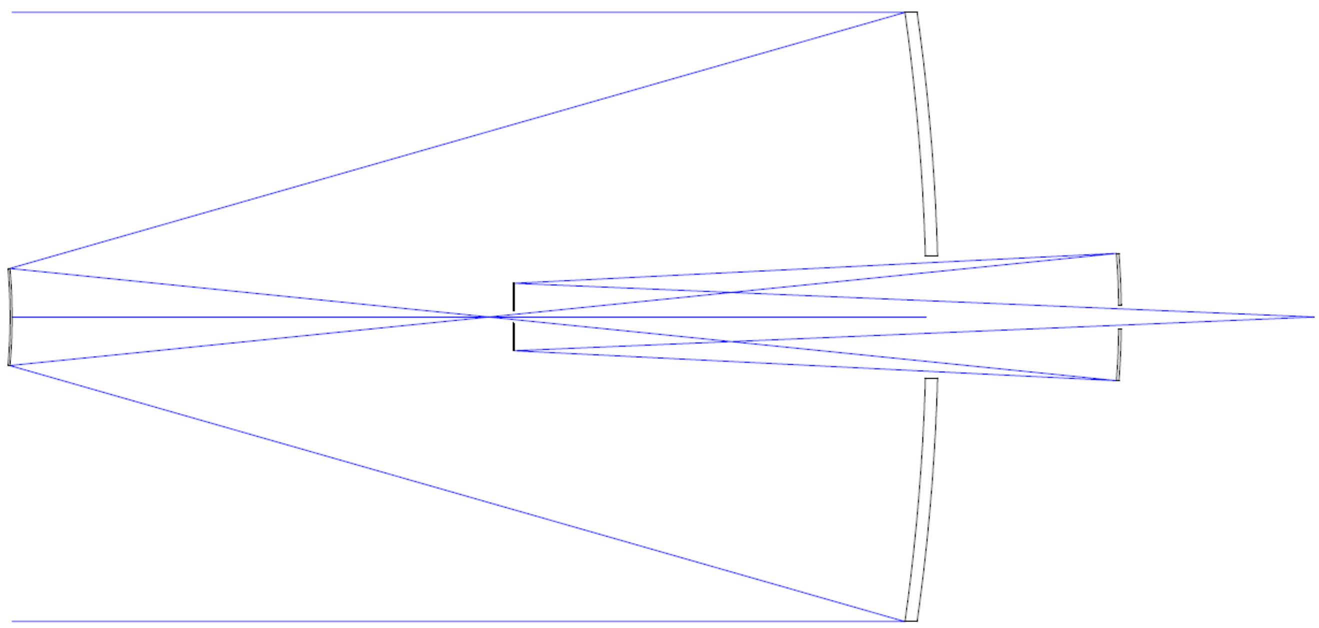
Anastigmatic, Flat field
Parabolic primary
Elliptical secondary
Spherical tertiary
Tertiary CC at secondary

Meinel's two stage optics concept (1985)



Large Deployable Reflector
Second stage corrects for errors of first stage; fourth mirror is at the exit pupil.

Aplanatic, Anastigmatic, Flat-field, Orthoscopic (free from distortion, rectilinear, JS 1987)



Spherical primary telescope.
The quaternary mirror is near the exit pupil. Spherical aberration and Coma are then corrected with a single aspheric surface. The Petzval sum is zero. If more aspheric surfaces are allowed then more aberrations can be corrected.