Astigmatism
Field Curvature
Distortion

Lens Design OPTI 517

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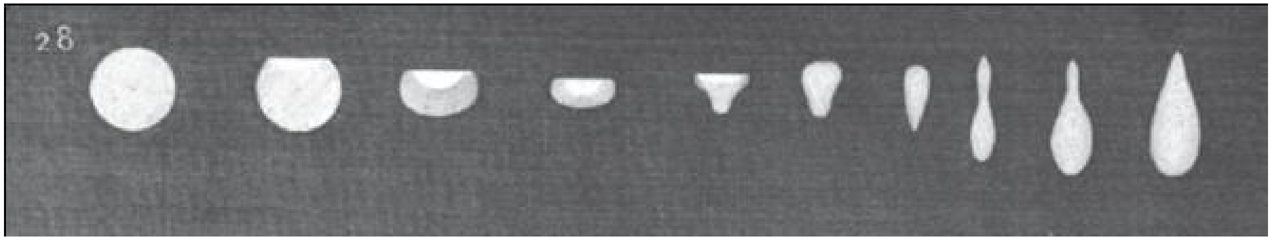
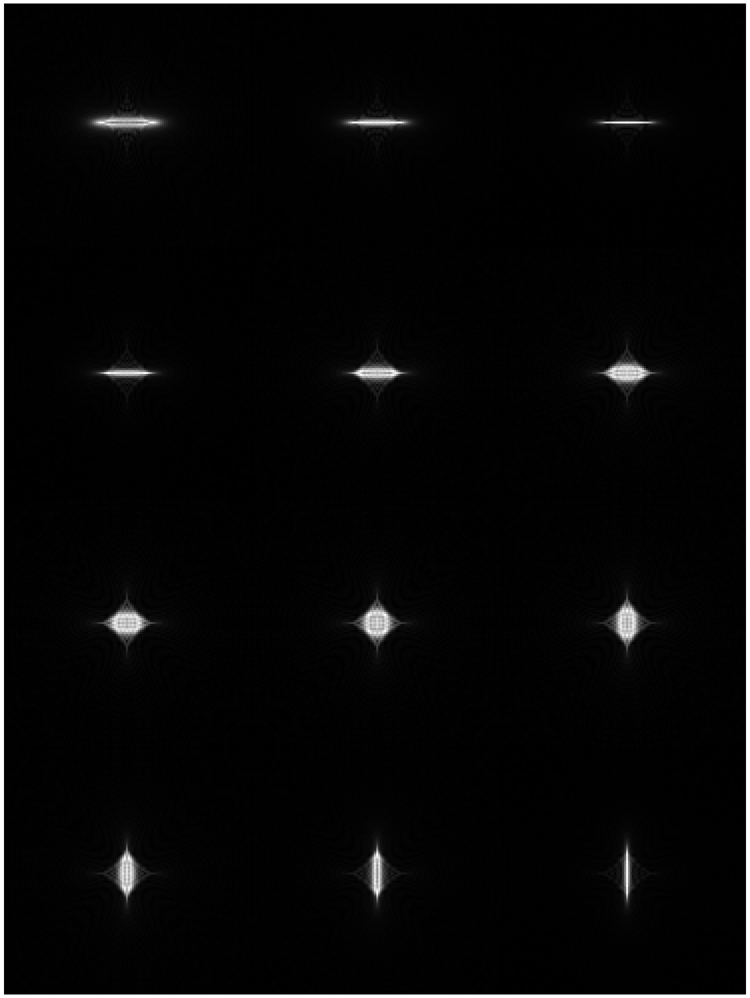
Earliest through focus images



T. Young, “On the mechanism of the eye,”
Phil Trans Royal Soc Lond 1801; 91: 23–88 and plates.

Astigmatism through focus



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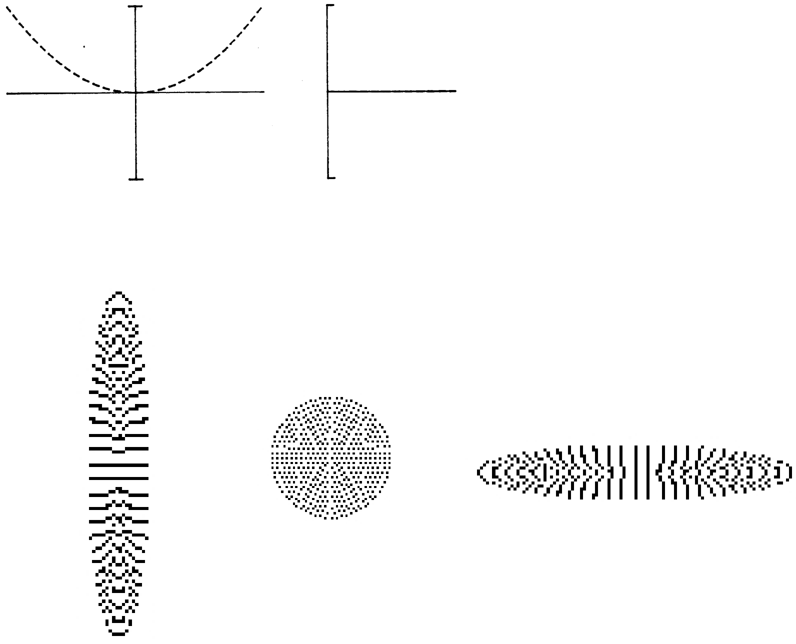
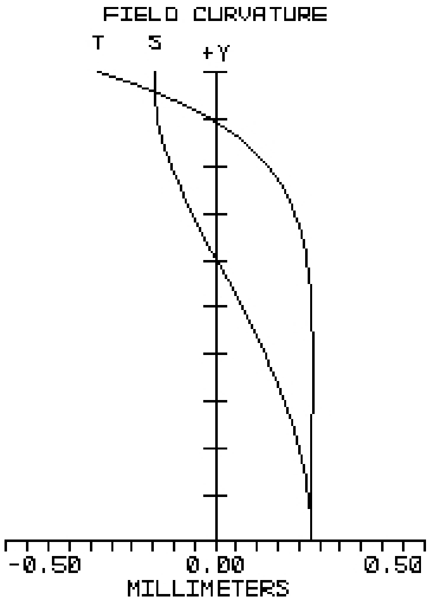
Astigmatism

$$W(H,\rho) = W_{111} H\rho\cos(\theta) + W_{020}\rho^2 + W_{200}H^2 +$$
$$+ W_{040}\rho^4 + W_{131}H\rho^3\cos(\theta) + W_{222}H^2\rho^2\cos^2(\theta) + W_{220}H^2\rho^2 + W_{311}H^3\rho\cos(\theta) + W_{400}H^4$$

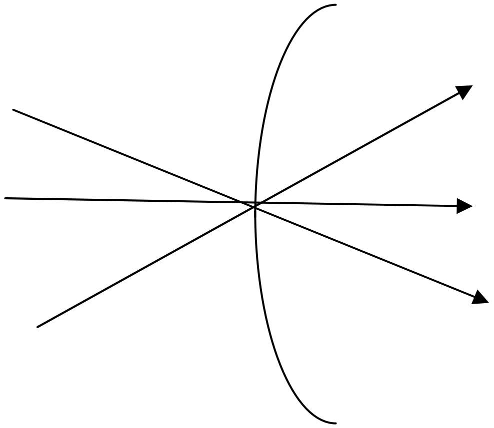




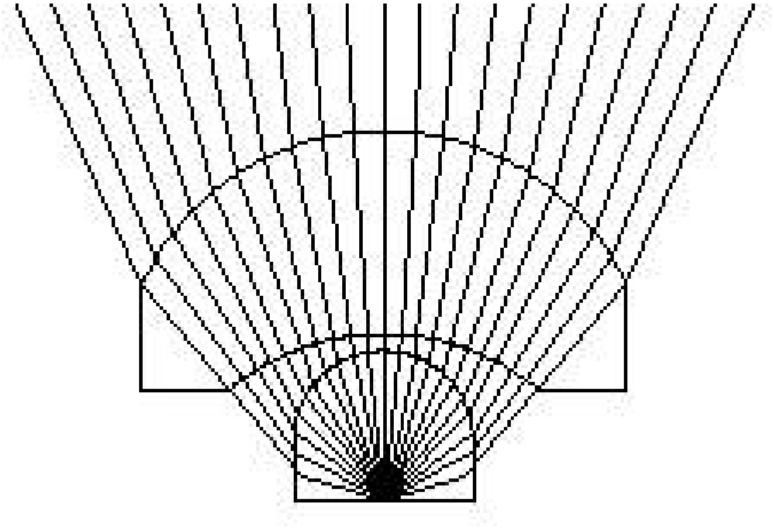
Anastigmatic

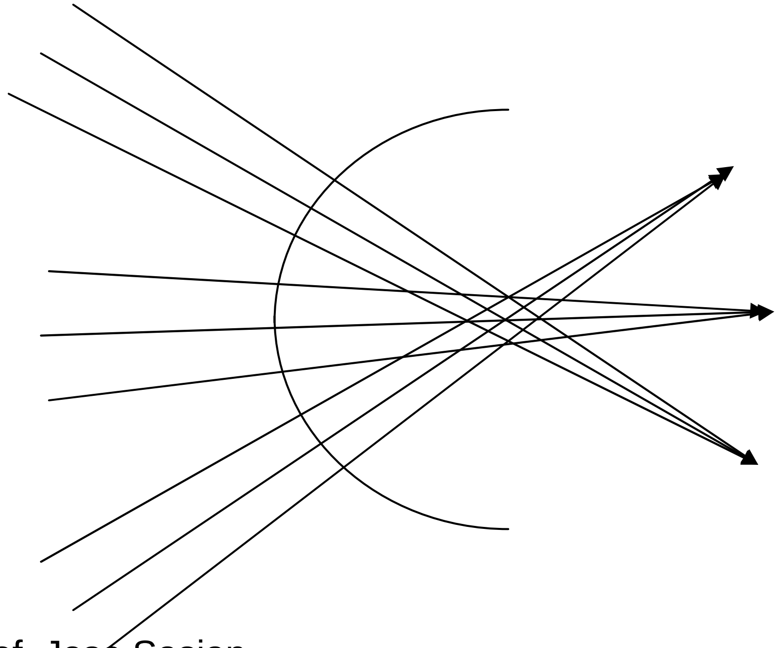
- Aplanatic: free from spherical aberration and coma.
- Stigmatic ~ pointy
- Astigmatism: No pointy
- Anastigmatic: No-No pointy = pointy
- Anastigmatic: free from spherical aberration, coma, and astigmatism
- Aplanatic: coined by John Herschel
- Astigmatism: coined by George Airy

Cases of zero astigmatism



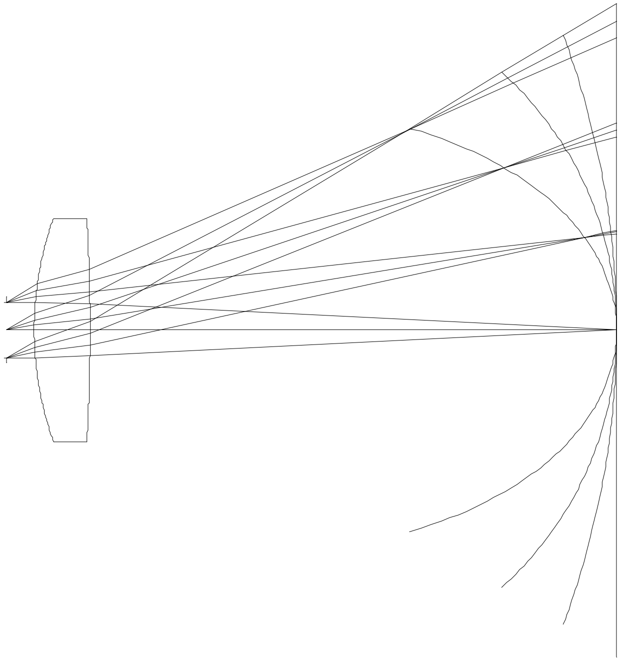
$$W_{222} = -\frac{1}{2}\bar{A}^{2}\Delta\left\{\frac{u}{n}\right\}y$$





Field behavior

$$W(H,\rho)=W_{222}H^{2}\rho^{2}\cos^{2}(\theta)+W_{220}H^{2}\rho^{2}$$



$$W_{222}=-\frac{1}{2}\bar{A}^{2}\Delta\left\{\frac{u}{n}\right\}y \qquad W_{220}=-\frac{1}{4}\bar{A}^{2}\Delta\left\{\frac{u}{n}\right\}y-\frac{1}{2}\text{Ж}^{2}P$$

Review of aberrations coefficients

$$W_{040} = \frac{1}{8} S_I$$

$$W_{131} = \frac{1}{2} S_{II}$$

$$W_{222} = \frac{1}{2} S_{III}$$

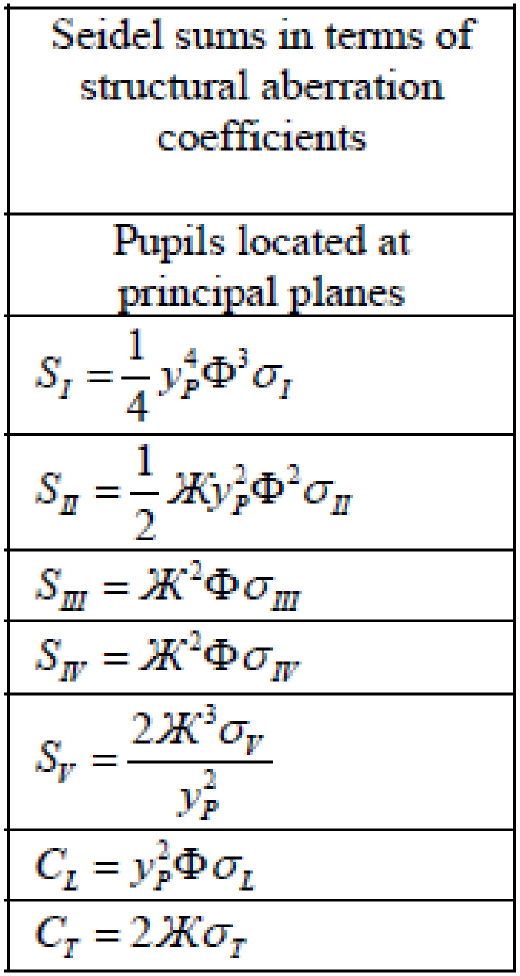
$$W_{220P} = \frac{1}{4} S_{IV}$$

$$W_{311} = \frac{1}{2} S_V$$

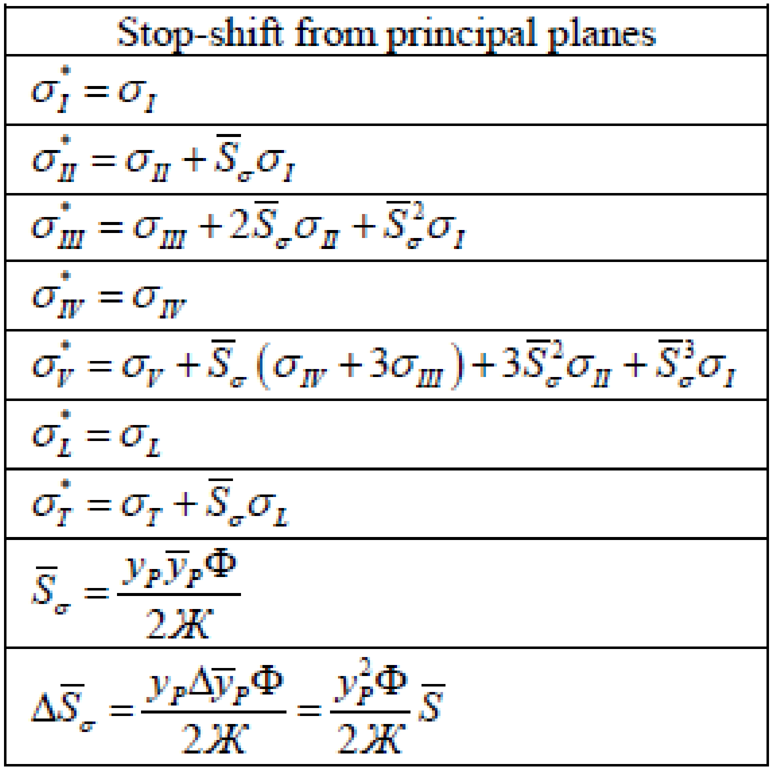
$$\partial_\lambda W_{020} = \frac{1}{2} C_L$$

$$\partial_\lambda W_{111} = C_T$$

Structural coefficients



Seidel sums in terms of structural aberration coefficients
Pupils located at principal planes
$S_I = \frac{1}{4} y_P^4 \Phi^3 \sigma_I$
$S_{II} = \frac{1}{2} \text{Ж} y_P^2 \Phi^2 \sigma_{II}$
$S_{III} = \text{Ж}^2 \Phi \sigma_{III}$
$S_{IV} = \text{Ж}^2 \Phi \sigma_{IV}$
$S_V = \frac{2\text{Ж}^3 \sigma_V}{y_P^2}$
$C_L = y_P^2 \Phi \sigma_L$
$C_T = 2\text{Ж} \sigma_T$



Stop-shift from principal planes
$\overset{\bullet}{\sigma}_I = \sigma_I$
$\overset{\bullet}{\sigma}_{II} = \sigma_{II} + \bar{S}_\sigma \sigma_I$
$\overset{\bullet}{\sigma}_{III} = \sigma_{III} + 2\bar{S}_\sigma \sigma_{II} + \bar{S}_\sigma^2 \sigma_I$
$\overset{\bullet}{\sigma}_{IV} = \sigma_{IV}$
$\overset{\bullet}{\sigma}_V = \sigma_V + \bar{S}_\sigma \left(\sigma_{IV} + 3\sigma_{III}\right) + 3\bar{S}_\sigma^2 \sigma_{II} + \bar{S}_\sigma^3 \sigma_I$
$\overset{\bullet}{\sigma}_L = \sigma_L$
$\overset{\bullet}{\sigma}_T = \sigma_T + \bar{S}_\sigma \sigma_L$
$\bar{S}_\sigma = \frac{y_P \bar{y}_P \Phi}{2\text{Ж}}$
$\Delta\bar{S}_\sigma = \frac{y_P \Delta\bar{y}_P \Phi}{2\text{Ж}} = \frac{y_P^2 \Phi}{2\text{Ж}} \bar{S}$

Seidel sum for thin lens (stop at lens)

$$S_I = \frac{1}{4} y^4 \phi^3 \left[AX^2 - BXY + CY^2 + D \right]$$

$$S_{II} = \frac{1}{2} \text{Ж} y^2 \phi^2 \left[EX - FY \right]$$

$$S_{III} = \text{Ж}^2 \phi$$

$$S_{IV} = \text{Ж}^2 \phi \frac{1}{n}$$

$$S_V = 0$$

$$C_L = y^2 \phi \frac{1}{\nu}$$

$$C_T = 0$$

$$X = \frac{c_1 + c_2}{c_1 - c_2} = \frac{r_2 + r_1}{r_2 - r_1}$$

$$Y = \frac{1 + m}{1 - m} = \frac{u' + u}{u' - u}$$

$$\phi = \Delta n \Delta c = (n-1)(c_1 - c_x)$$

$$A = \frac{n+2}{n(n-1)^2}$$

$$B = \frac{4(n+1)}{n(n-1)}$$

$$C = \frac{3n+2}{n}$$

$$D = \frac{n^2}{(n-1)^2}$$

$$E = \frac{n+1}{n(n-1)}$$

$$F = \frac{2n+1}{n}$$

Thin lens astigmatism

$$S_{III} = Ж^2 \phi$$

When the stop is a the thin lens astigmatism is fixed.

Shifting the stop in the presence of spherical aberration or coma
Allows changing astigmatism

$$\sigma_{III}^{*} = \sigma_{III} + 2\bar{S}_{\sigma}\sigma_{II} + \bar{S}_{\sigma}^{2}\sigma_{I}$$

Controlling astigmatism

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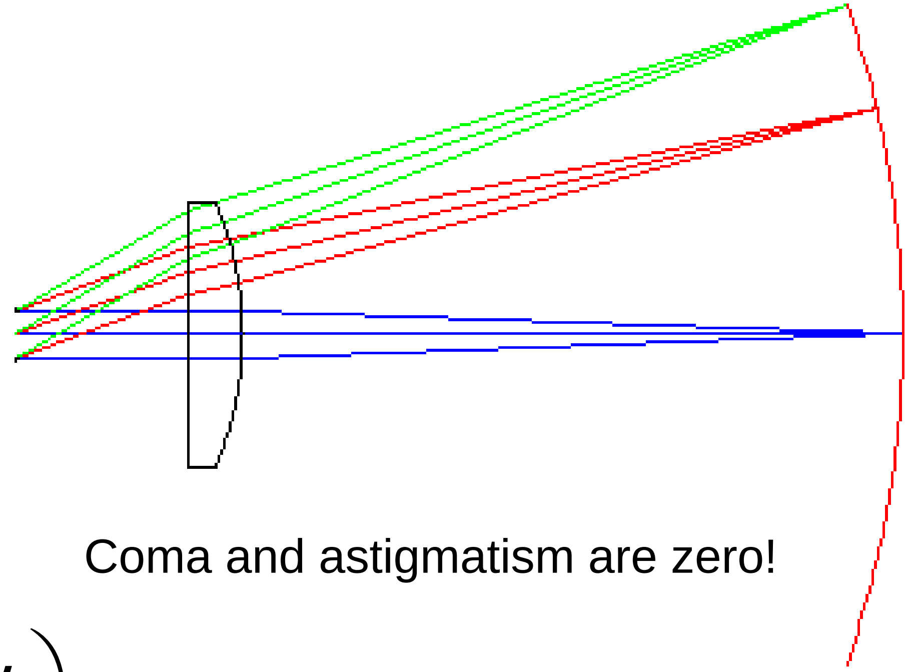
1) Stop position: singlet lens



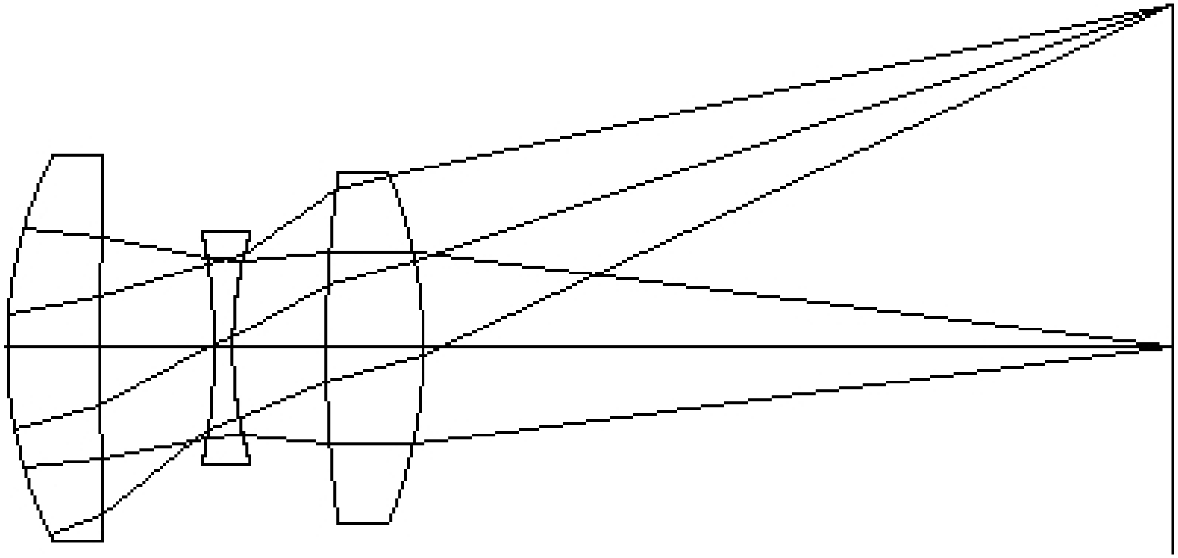
Coma and astigmatism are zero!

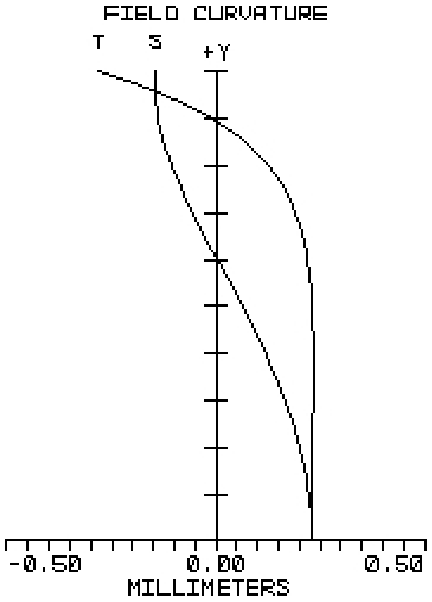
$$\Delta\left(\frac{u}{n}\right)_1 = 0$$

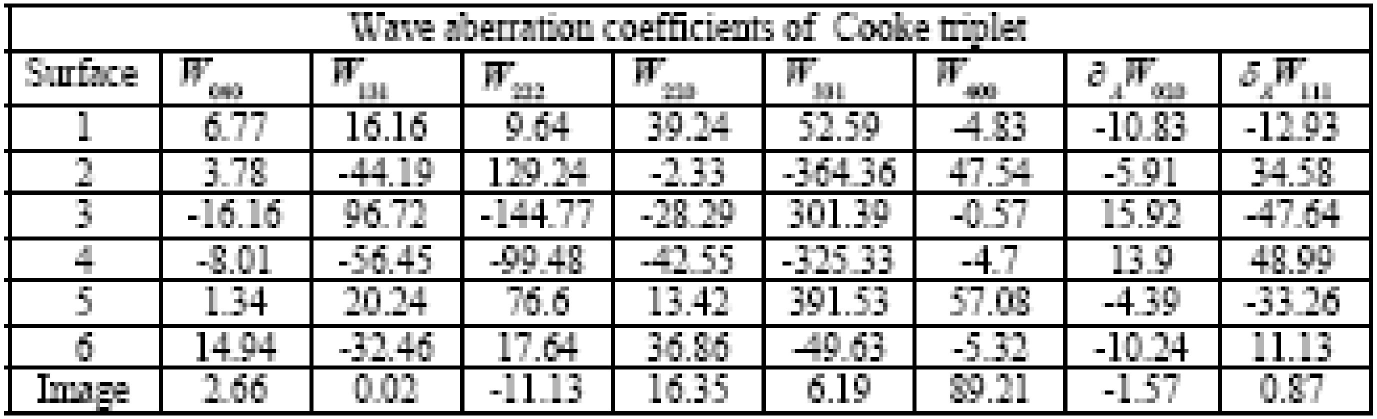
$$S_{III}^{*} = S_{III} + 2\cdot\bar{S}S_{II} + \bar{S}^2 S_I$$

$$\bar{A}_2 = 0$$

2) Canceling/balancing negative and positive astigmatism







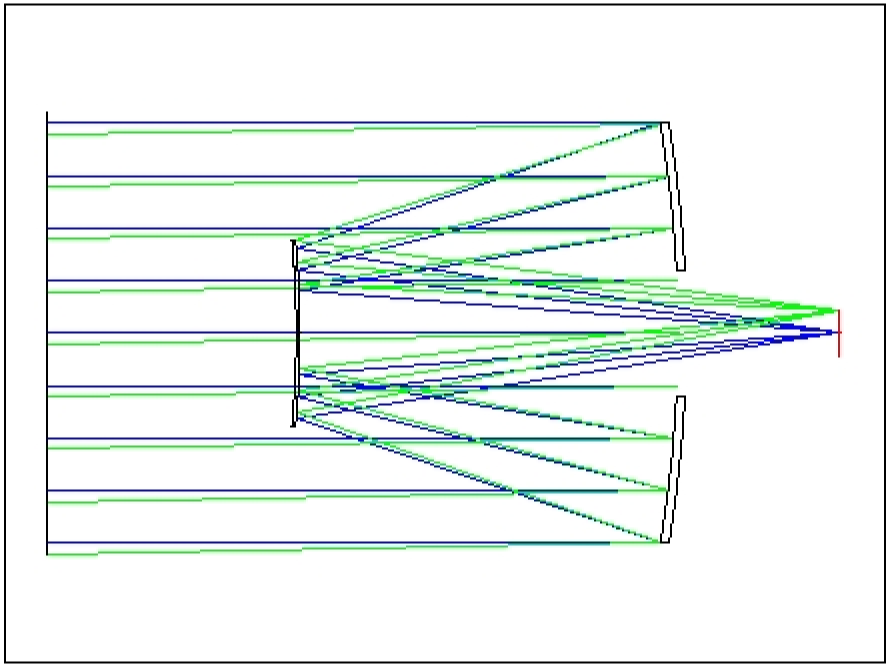
Wave aberration coefficients of Cooke triplet								
Surface	W_{040}	W_{131}	W_{222}	W_{220}	W_{311}	W_{400}	$\partial_{\lambda}W_{020}$	$\delta_{\lambda}W_{111}$
1	6.77	16.16	9.64	39.24	52.59	-4.83	-10.83	-12.93
2	3.78	-44.19	129.24	-2.33	-364.36	47.54	-5.91	34.58
3	-16.16	96.72	-144.77	-28.29	301.39	-0.57	15.92	-47.64
4	-8.01	-56.45	-99.48	-42.55	-325.33	-4.7	13.9	48.99
5	1.34	20.24	76.6	13.42	391.53	57.08	-4.39	-33.26
6	14.94	-32.46	17.64	36.86	-49.63	-5.32	-10.24	11.13
Image	2.66	0.02	-11.13	16.35	6.19	89.21	-1.57	0.87

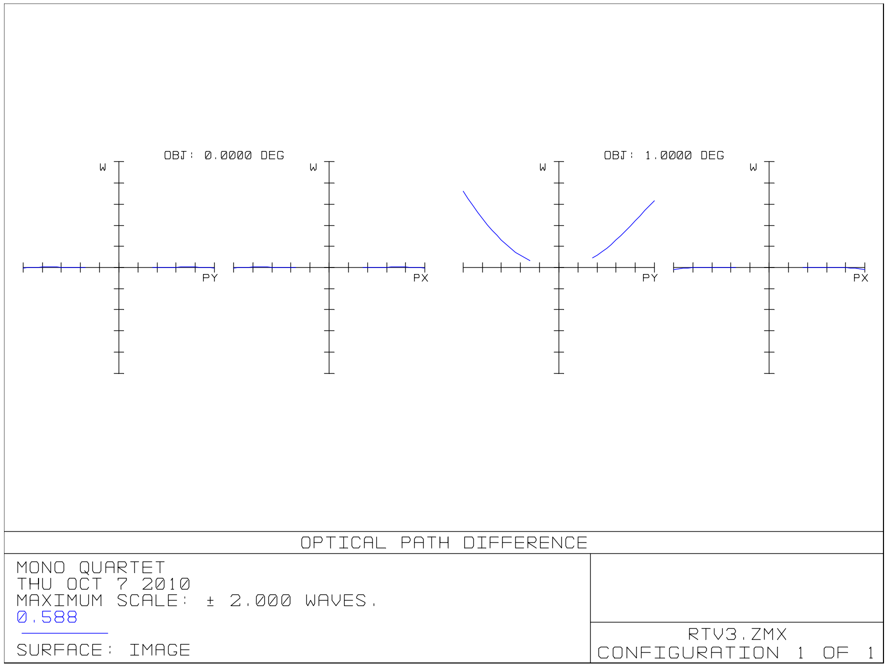
3-a) Adding a degree of freedom

- In this case one adds a lens which contributes the opposite amount of astigmatism.
- The spherical aberration and coma of the new lens are corrected by the system that has the degrees of freedom for such.
- New lens hopefully contributes little coma and spherical aberration.

3-b) Adding a degree of freedom Ritchey-Chretien I

1.7 waves of astigmatism @ f.3.3





At best surface (Sagittal field surface)

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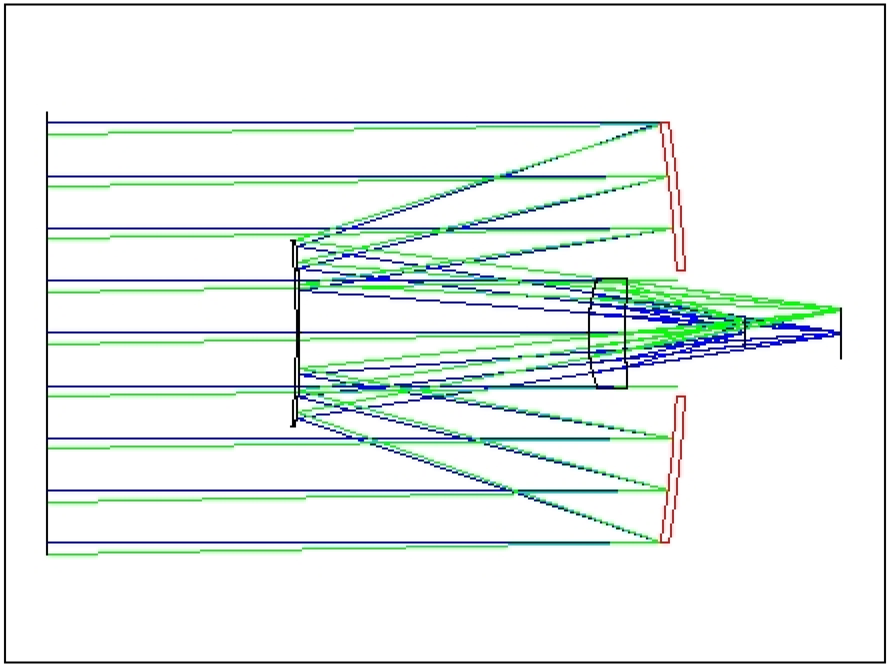
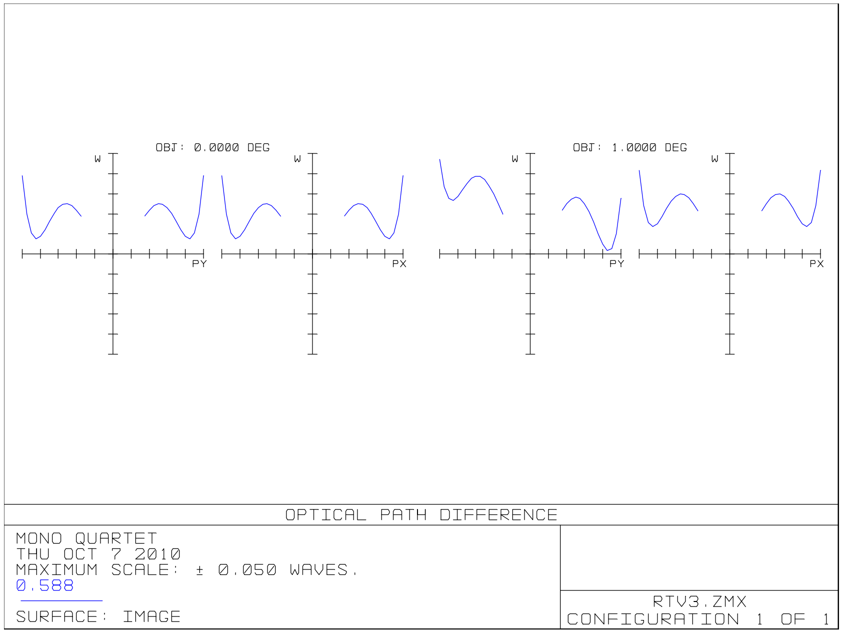
3-c) Adding a degree of freedom
Ritchey-Chretien II

0.0 waves of astigmatism @ f/1.9 after conic tweak

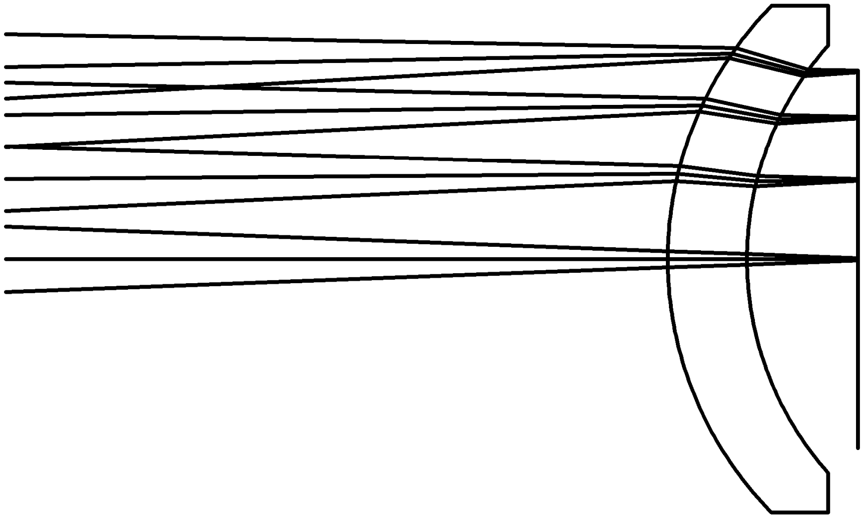




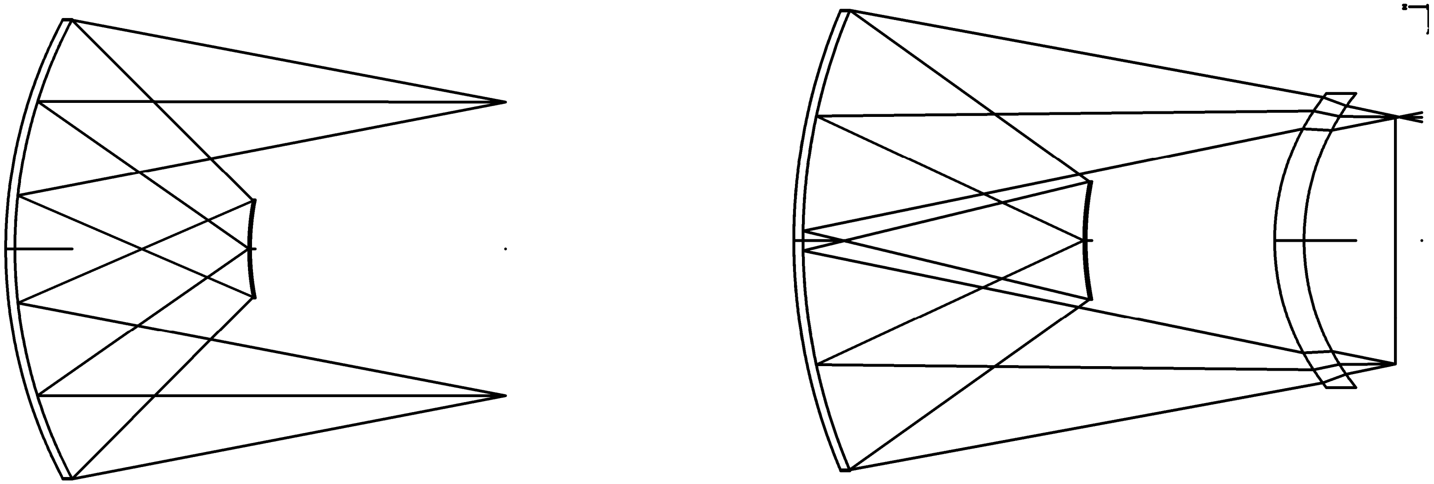
MONO QUARTET
THU OCT 7 2010
MAXIMUM SCALE: ± 0.050 WAVES.
0.588
SURFACE: IMAGE

RTV3.ZMX
CONFIGURATION 1 OF 1

4) Shells near the image plane (or aspheric plate)

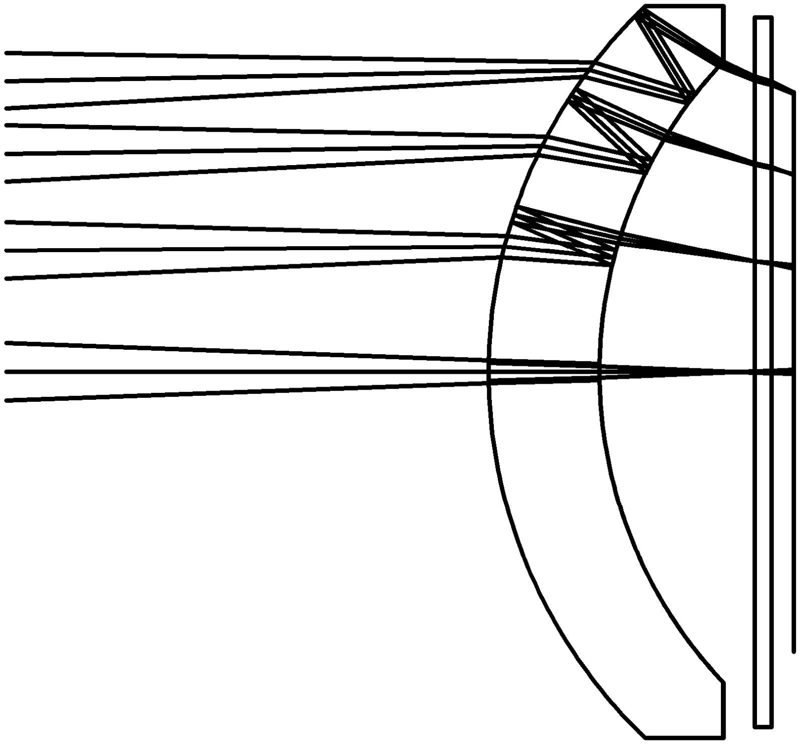


Offner unit magnification relay



- Offner relay system:
- Three spherical mirrors
- Negative unit magnification
- No primary aberrations
- Ring field concept
- Improvement of field with shell

However; beware of ghosts



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Field curvature

$$W_{220} = -\frac{1}{4}\sum\left(\text{Ж}^2 P - \overline{A}^2 \Delta\left\{\frac{u}{n}\right\} y \right) \qquad P = C \cdot \Delta\left(\frac{1}{n}\right)$$

Petzval sum:

$$\frac{1}{n'_k\ \rho'_k} - \frac{1}{n_1 \rho_1} = \sum \frac{n' - n}{n' n r}$$

For a system of thin lenses:

$$\frac{1}{\rho'_k} = \sum \frac{\phi}{n}$$

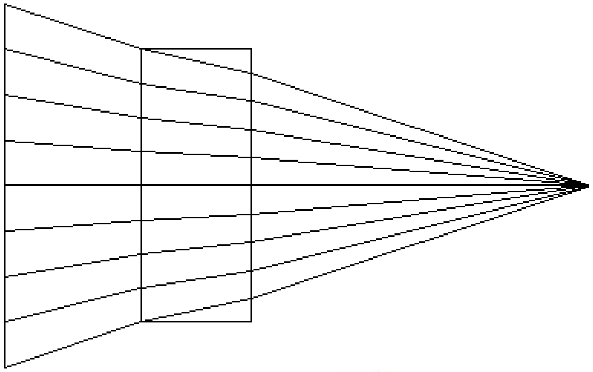
Field curvature interpretation

- Assume same glass and consider sag of Petzval surface at a height *y*:

$$\frac{y^2}{2\rho'_k} = \sum \frac{n'-n}{n}\frac{y^2}{2r}$$

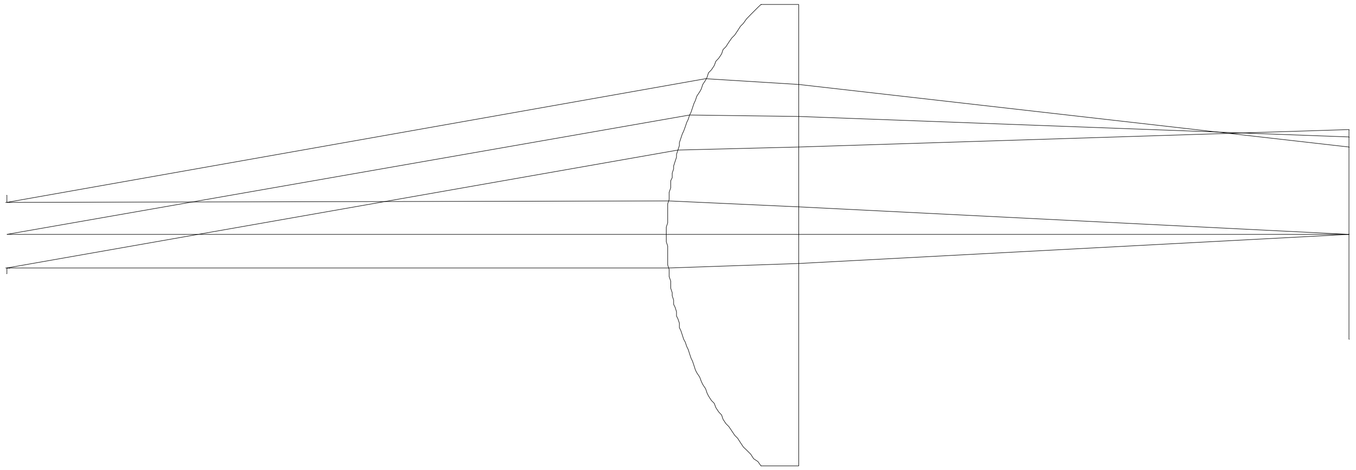
- If the Petzval sum is zero then the lens has constant thickness across the aperture or across the field.
- Compare with the image displacement *S* caused by a plano parallel plate:

$$S = \frac{n-1}{n}t$$



- The conclusion is that Petzval field curvature arises because the overall lens thickness variation across the aperture (in the general case the index of refraction enters as a weight).

Thickness variation in a telecentric lens



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Four classical ways

- 1) A thick meniscus lens can contribute optical power but no field curvature if both surfaces have the same radius. Consider double Gasuss lens. Note the correction for color.

- 2) Separated thin lenses: Bulges and constrictions

  Consider the Cooke triplet and lenses for microlithography.

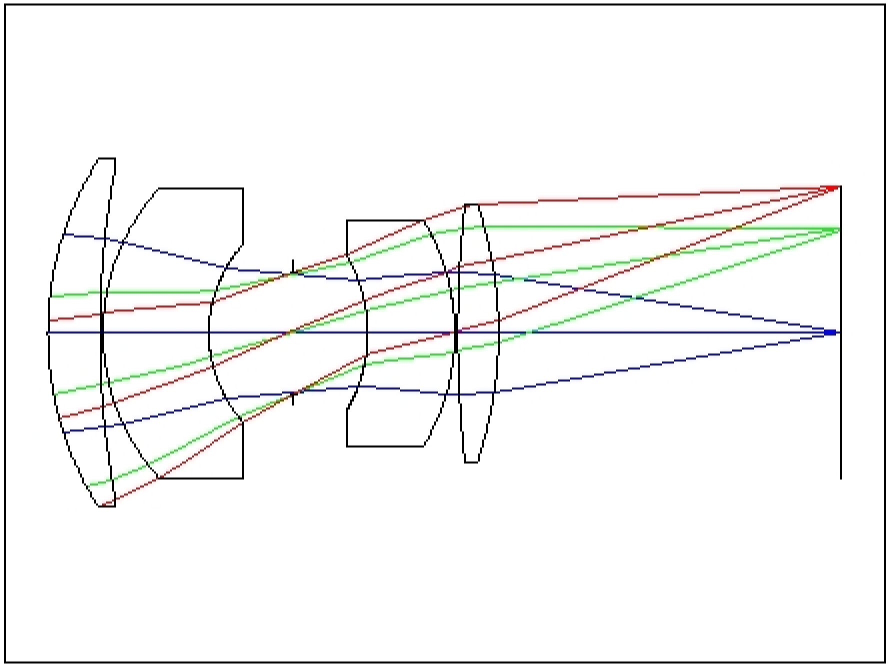
- 3) A field flattener: Fully contributes to Petzval but not to spherical, coma, or astigmatism. Also there is little contribution to optical power.

  Consider Petzval lens with a field flattener.

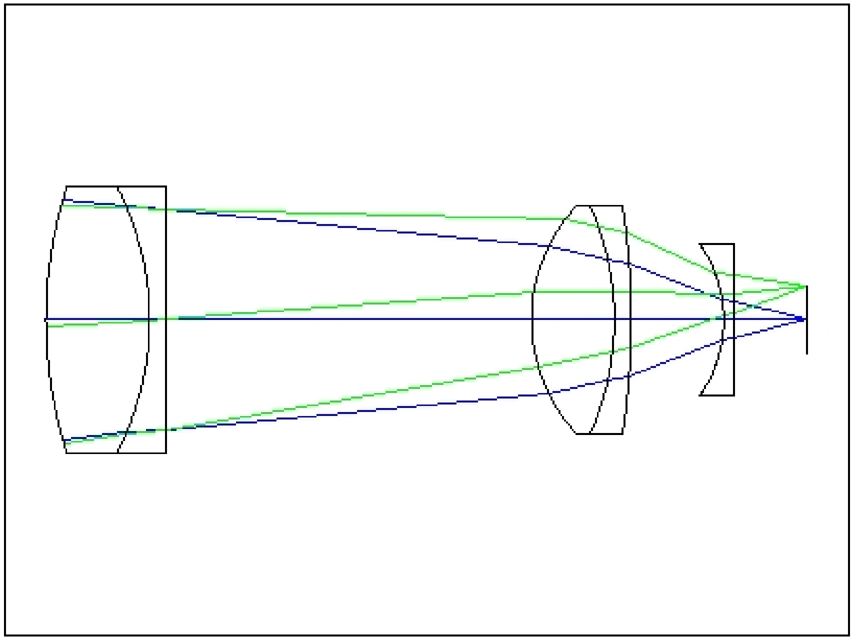
- 4) New achromat: use to advantage new glass types.

$$\frac{1}{\rho'_k} = \sum \frac{\phi}{n}$$

Four classical ways

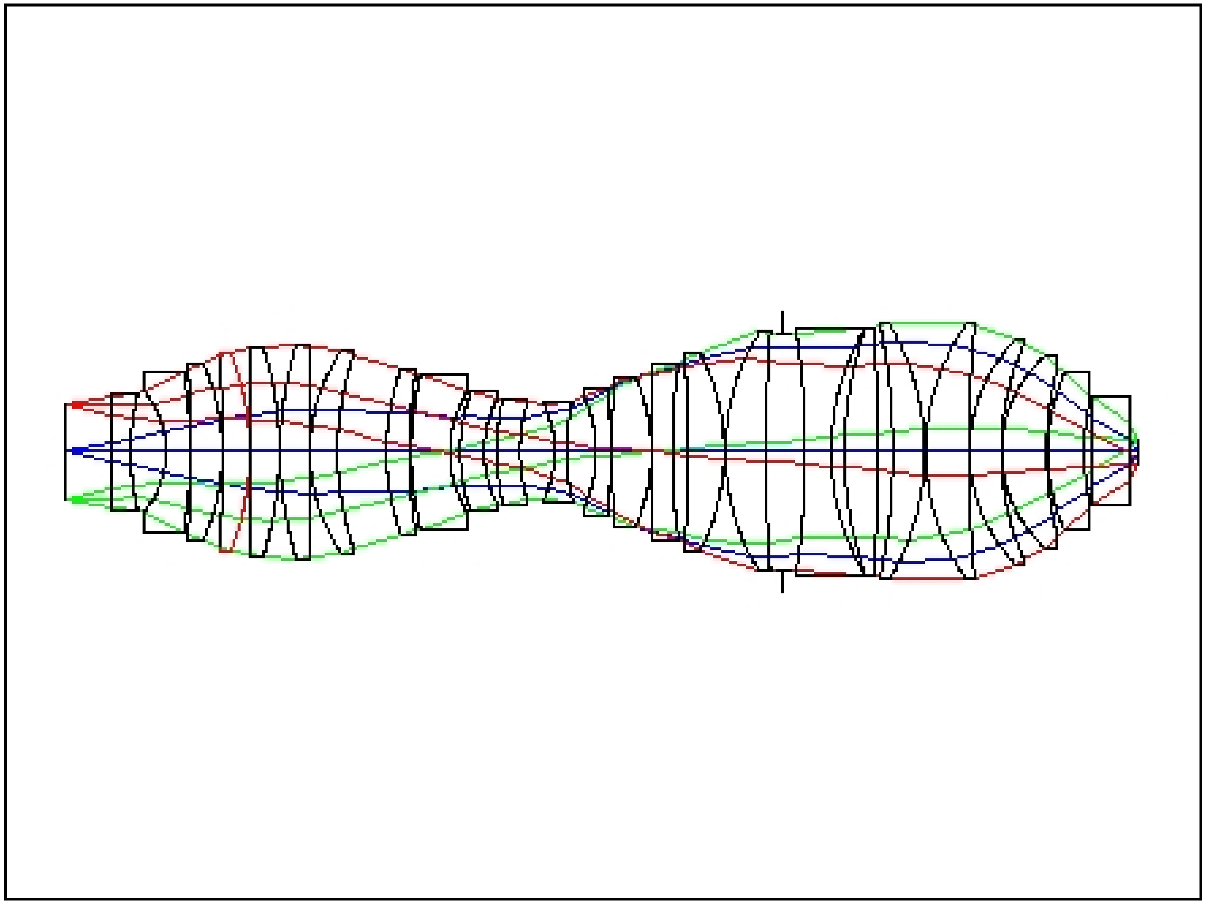


Use of a thick meniscus lens



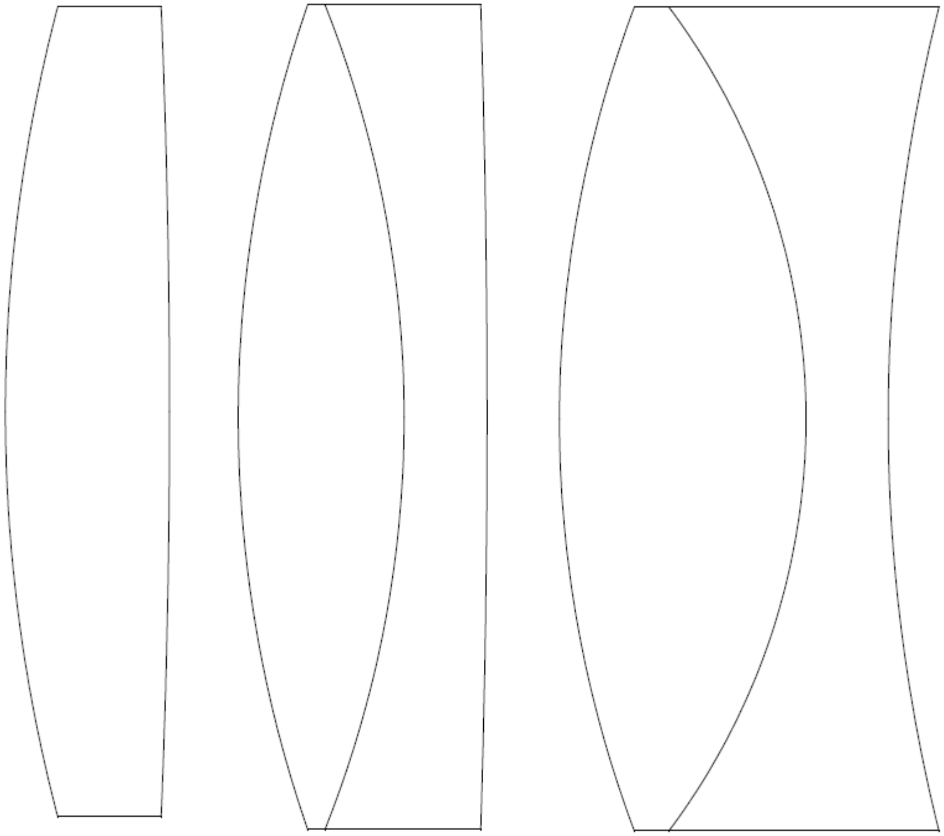
Use of a field flattener lens

Four classical ways



Creating beam bulges and constrictions

Four classical ways: Use of glass



BK7
P=-152 mm

BK7-F2
P=-139 mm

SSKN5-LF5
P=-219 mm

V-number for flint increases
V-number for crown decreases

N for crown increases
N for flint decreases

$$f_a \cdot \nu_a = f_b \cdot \nu_b = F \cdot (\nu_a - \nu_b)$$

F=100 mm

$$\frac{1}{\rho'_k} = \sum \frac{\phi}{n}$$

Distortion

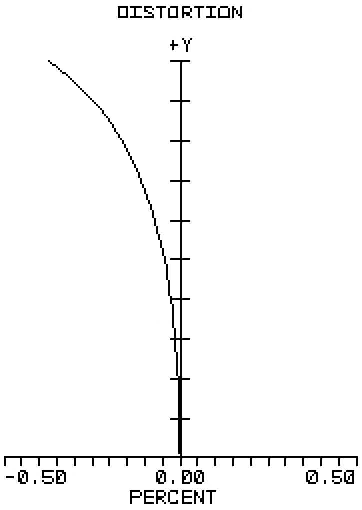
$$W(H,\rho) = W_{111} H\rho\cos(\theta) + W_{020}\rho^2 + W_{200}H^2 +$$

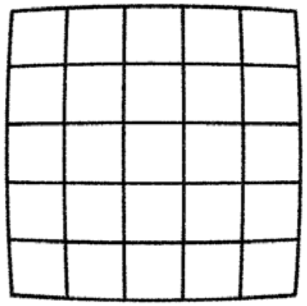
$$+ W_{040}\rho^4 + W_{131}H\rho^3\cos(\theta) + W_{222}H^2\rho^2\cos^2(\theta) + W_{220}H^2\rho^2 + W_{311}H^3\rho\cos(\theta) + W_{400}H^4$$

With respect to chief ray, geometrical or physical centroid

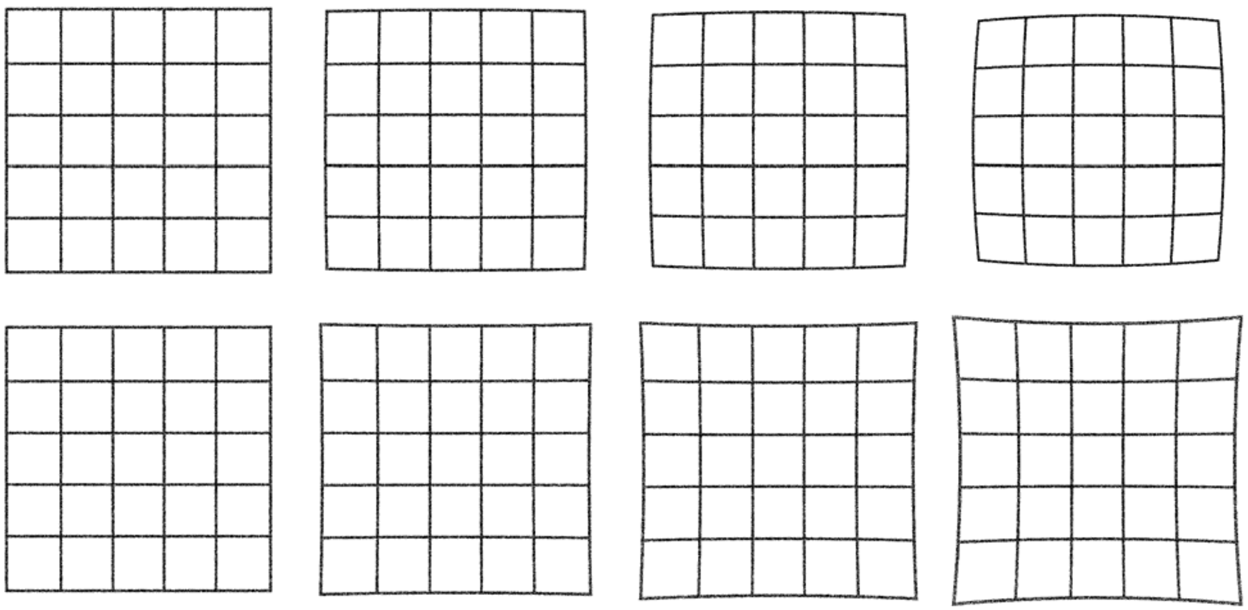
$W311\ H3\rho\cos(\theta)$
$W511\ H5\rho\cos(\theta)$

$$Distortion = \frac{H-h}{h} \bullet 100$$





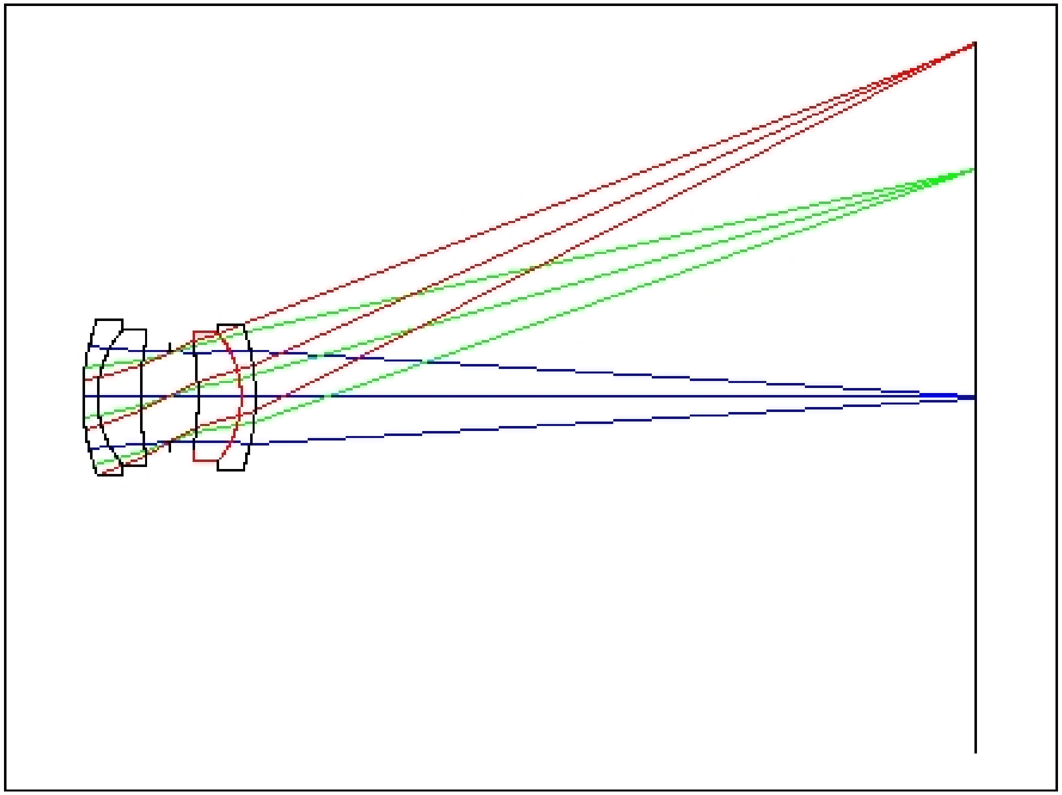
Distortion



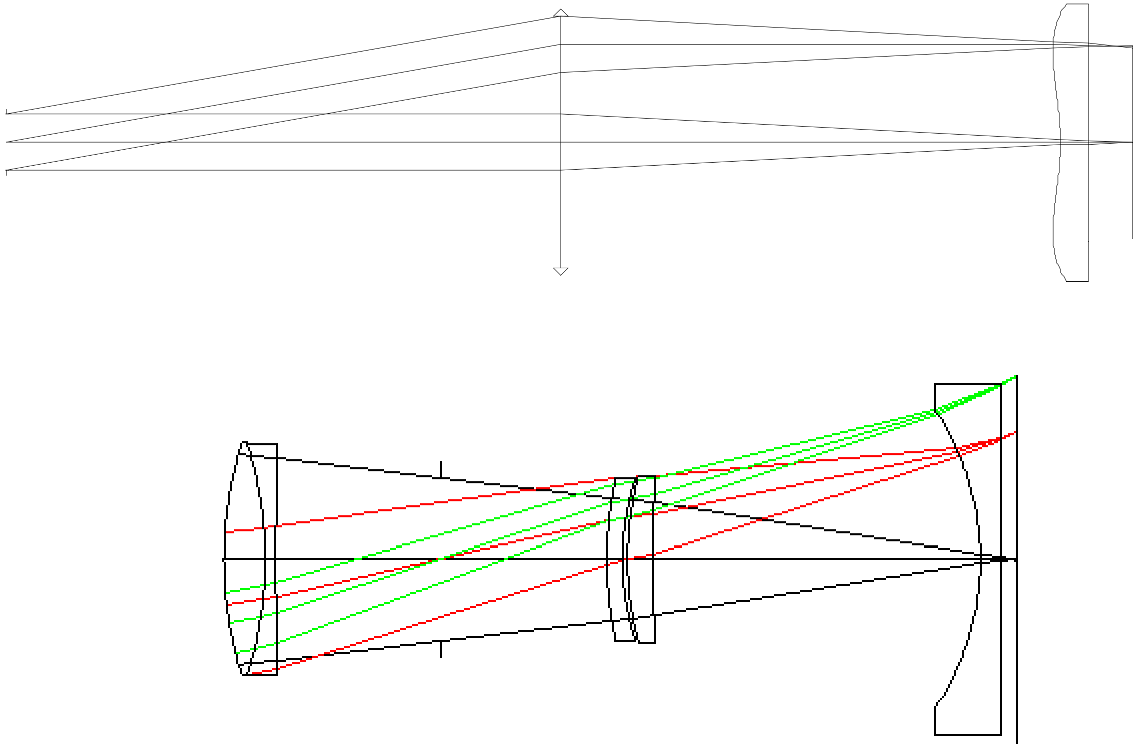
Top row, (barrel) distortion:0%, 2.5%, 5% and 10%. Bottom row, (pincushion) distortion 0%, 2.5%, 5% and 10%.

1) By Symmetry about the stop or phantom stop

Distortion is an odd aberration: It can be cancelled by symmetry
About the stop



2) Aspheric plate or bending a field flattener



Exercise: Galilean telescope

A plano-convex lens objective with a focal length of about 750-1000 mm.
A plano-concave lens for the eyepiece (ocular) with a focal length
of about 50 mm. The objective lens was stopped down
to an aperture of 12.5 to 25 mm. The field of view is about 15 arc-minutes.
The instrument's magnifying power is 15-20.

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