Introduction to aberrations

OPTI 518

Lecture 15

Pupil aberrations

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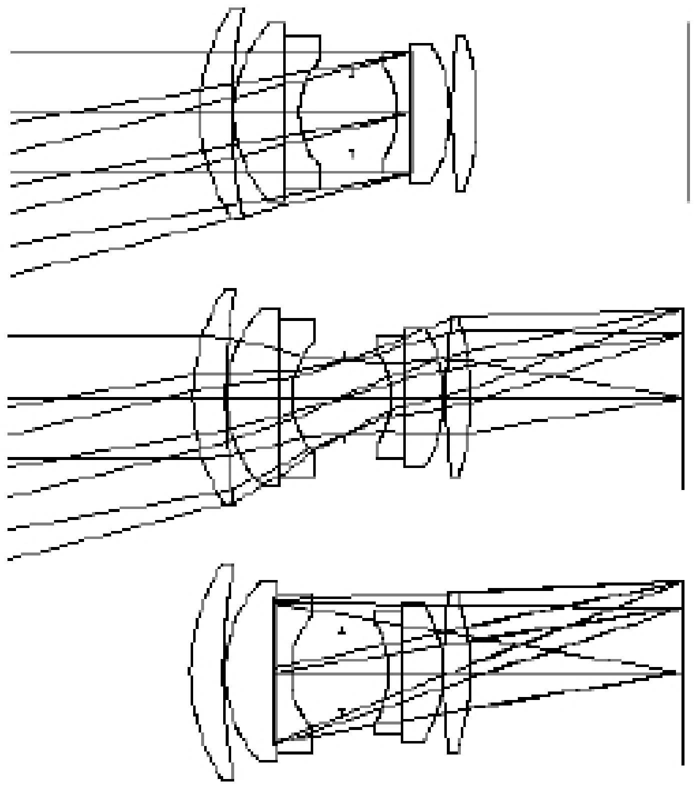
Pupils



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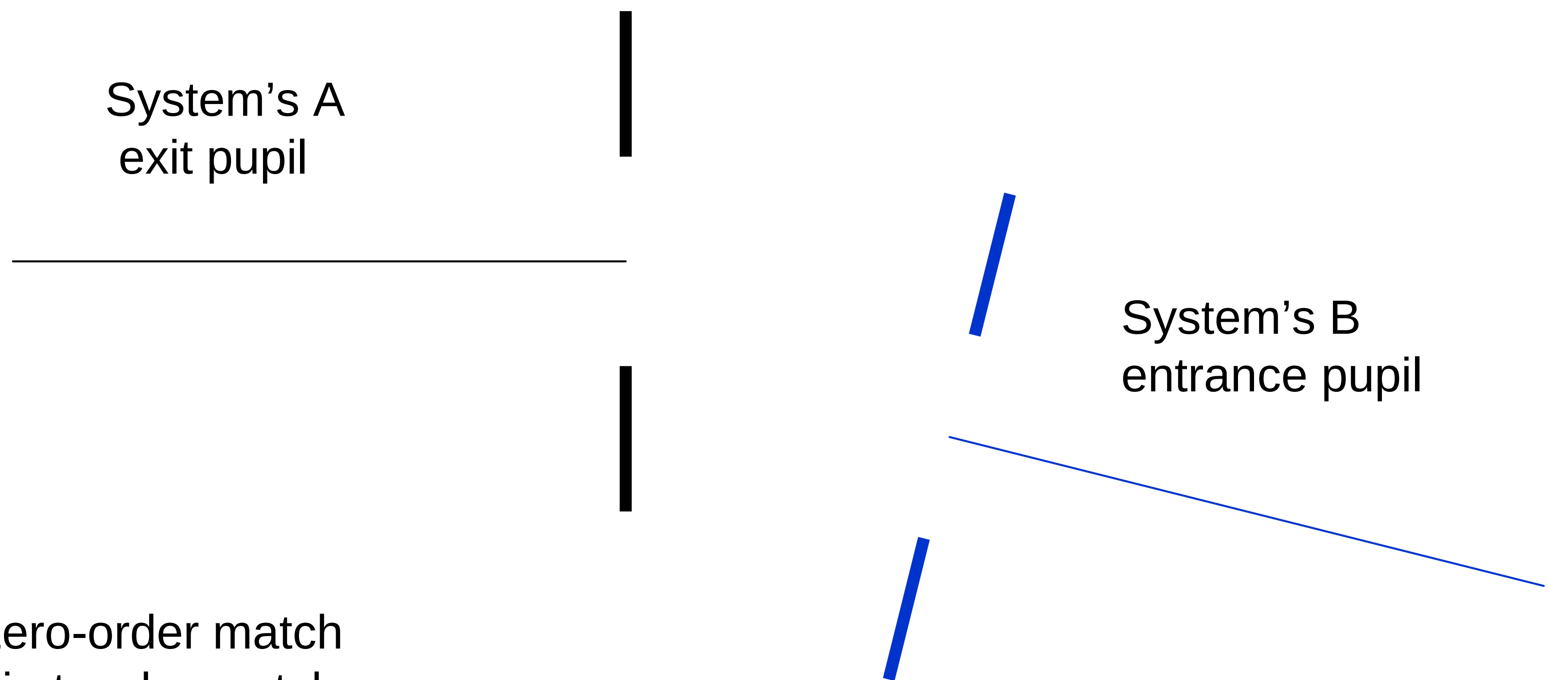
Pupil mismatch

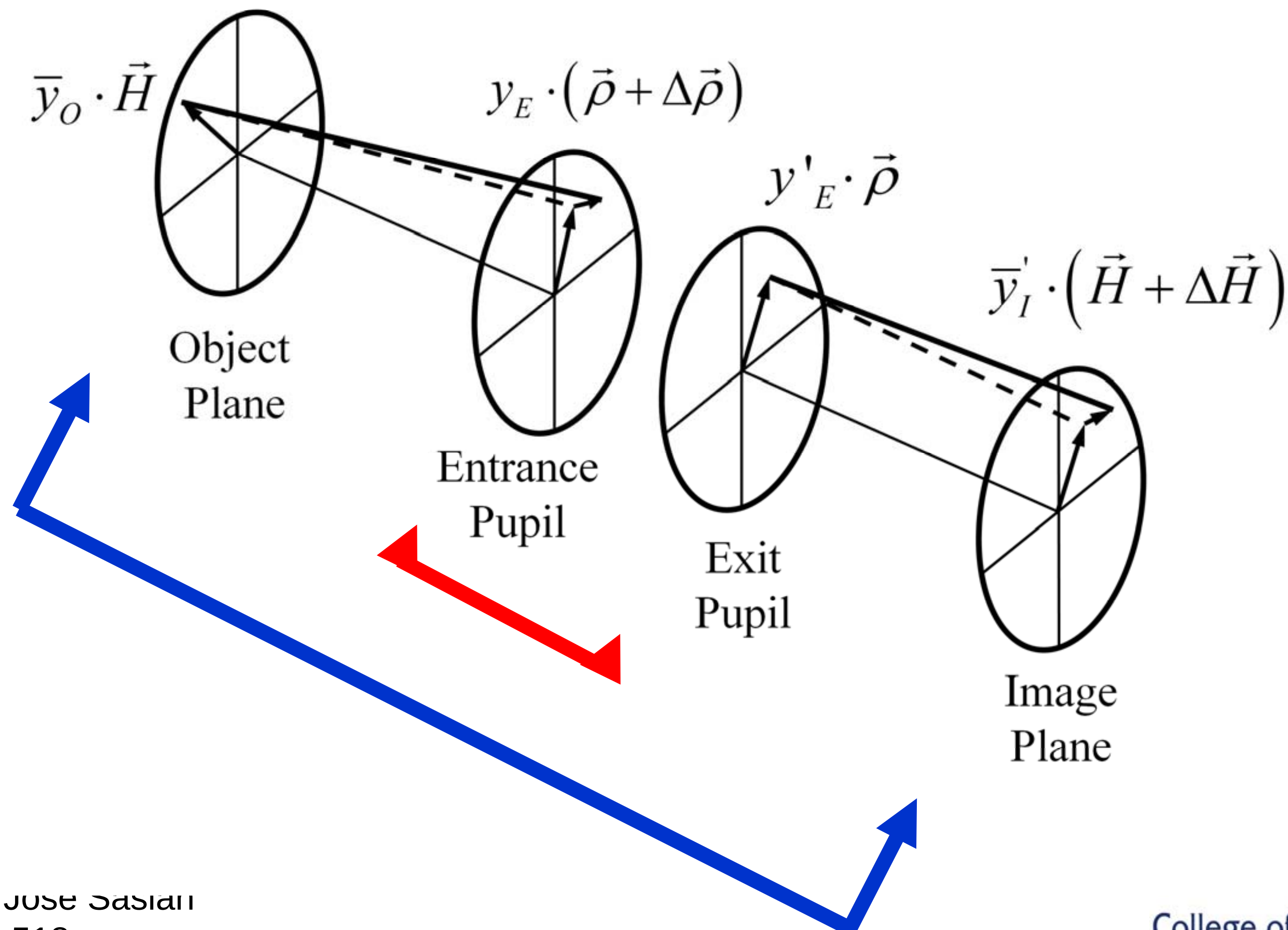


Zero-order match
First-order match
Third-order mismatch ←

Some references

- T. Smith, “The changes in aberrations when the object and stop are moved,” Trans. Opt. Soc. 23, 139-153, 1921/1922
- C. C. Wynne, “Primary aberrations and conjugate change,” Proc. Phys. Soc. Lond. 65 b, 429-437 (1952)
- J. Sasian, Interpretation of pupil aberrations in imaging systems, SPIE V. 6342-634206 (2006)

Object-image system
Pupil-pupil system



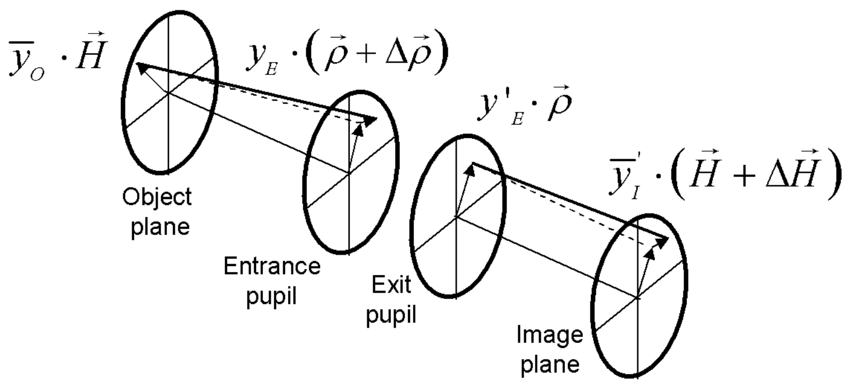
Pupil aberration function

$$W\left(\vec{H},\vec{\rho}\right)=W_{000}+W_{200}\left(\vec{H}\cdot\vec{H}\right)+W_{111}\left(\vec{H}\cdot\vec{\rho}\right)+W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)$$
$$+W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2+W_{131}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$$
$$+W_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+W_{311}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+W_{400}\left(\vec{H}\cdot\vec{H}\right)^2$$

$$\overline{W}\left(\vec{H},\vec{\rho}\right)=\overline{W}_{000}+\overline{W}_{200}\left(\vec{\rho}\cdot\vec{\rho}\right)+\overline{W}_{111}\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{020}\left(\vec{H}\cdot\vec{H}\right)$$
$$+\overline{W}_{040}\left(\vec{H}\cdot\vec{H}\right)^2+\overline{W}_{131}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$$
$$+\overline{W}_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+\overline{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\overline{W}_{400}\left(\vec{\rho}\cdot\vec{\rho}\right)^2$$

Pupil aberrations

- Object-image interchange role with entrance-and exit pupils
- The chief ray becomes the marginal ray, and the marginal ray becomes the chief ray. Lagrange invariant changes sign
- Image and pupil aberrations are connected



Identity between pupil and image aberration coefficients	
$\bar{W}_{040} = W_{400}$	
$\bar{W}_{131} = W_{311} + \frac{1}{2} Ж \cdot \Delta\left\{\bar{u}^2\right\}$	
$\bar{W}_{222} = W_{222} + \frac{1}{2} Ж \cdot \Delta\left\{u\bar{u}\right\}$	
$\bar{W}_{220} = W_{220} + \frac{1}{4} Ж \cdot \Delta\left\{u\bar{u}\right\}$	
$\bar{W}_{311} = W_{131} + \frac{1}{2} Ж \cdot \Delta\left\{u^2\right\}$	
$\bar{W}_{400} = W_{040}$	

Table 12.2 Image and pupil coefficient relationships for a spherical surface			
$4W_{040}\dfrac{\bar{y}}{y} = \bar{W}_{311} - \bar{W}^{0}_{311}$	(12.9)	$4\bar{W}_{040}\dfrac{y}{\bar{y}} = W_{311} - W^{0}_{311}$	(12.10)
$W_{131}\dfrac{\bar{y}}{y} = \bar{W}_{222} - \bar{W}^{0}_{311}\dfrac{\bar{y}}{y}$	(12.11)	$\bar{W}_{131}\dfrac{y}{\bar{y}} = W_{222} - W^{0}_{311}\dfrac{y}{\bar{y}}$	(12.12)
$W_{222}\dfrac{\bar{y}}{y} = \bar{W}_{131} + W^{0}_{311}$	(12.13)	$\bar{W}_{222}\dfrac{y}{\bar{y}} = W_{131} + \bar{W}^{0}_{311}$	(12.14)
$W_{220}\dfrac{\bar{y}}{y} = \dfrac{1}{2}\bar{W}_{131} - \dfrac{1}{2}\bar{W}^{0}_{311}\left(\dfrac{\bar{y}}{y}\right)^2$	(12.15)	$\bar{W}_{220}\dfrac{y}{\bar{y}} = \dfrac{1}{2}W_{131} - \dfrac{1}{2}W^{0}_{311}\left(\dfrac{y}{\bar{y}}\right)^2$	(12.16)
$W_{311}\dfrac{\bar{y}}{y} = 4\bar{W}_{040} + W^{0}_{311}\dfrac{\bar{y}}{y}$	(12.17)	$\bar{W}_{311}\dfrac{y}{\bar{y}} = 4W_{040} + \bar{W}^{0}_{311}\dfrac{y}{\bar{y}}$	(12.18)
$W^{0}_{311} = W^{y=0}_{311} = \dfrac{1}{2}\dfrac{1}{r}\text{Ж}\bar{A}\bar{y}\Delta\left\{\dfrac{1}{n}\right\}$ $= \dfrac{1}{2}\text{Ж}\bar{\alpha}\left(\bar{u}' - \bar{u}\right)$	(12.19)	$\bar{W}^{0}_{311} = \bar{W}^{\bar{y}=0}_{311} = -\dfrac{1}{2}\dfrac{1}{r}\text{Ж}Ay\Delta\left\{\dfrac{1}{n}\right\}$ $= -\dfrac{1}{2}\text{Ж}\alpha\left(u' - u\right)$	(12.20)
$\alpha = \dfrac{y}{r}$	(12.21)	$\bar{\alpha} = \dfrac{\bar{y}}{r}$	(12.22)

Some interesting "shift" relationships

$$\nabla_H \bar{W}\left(\vec{H},\vec{\rho}\right) = \nabla_H \bar{W}_{PP}\left(\vec{H},\vec{\rho}\right) + \bar{S}\nabla_\rho W\left(\vec{H},\vec{\rho}\right)$$

$$\nabla_\rho W\left(\vec{H},\vec{\rho}\right) = \nabla_\rho W_{PP}\left(\vec{H},\vec{\rho}\right) + S\nabla_H \bar{W}\left(\vec{H},\vec{\rho}\right)$$

$$\begin{aligned}\Delta\vec{\Omega} &= \frac{\bar{y}_{PP}}{y_{PP}}\Delta\vec{H}' - \Delta\vec{\rho} \\ &= \frac{1}{Ж}\left[\nabla_H \bar{W}(\vec{H},\vec{\rho}) - \frac{\bar{y}_{PP}}{y_{PP}}\nabla_\rho W(\vec{H},\vec{\rho})\right].\end{aligned}$$

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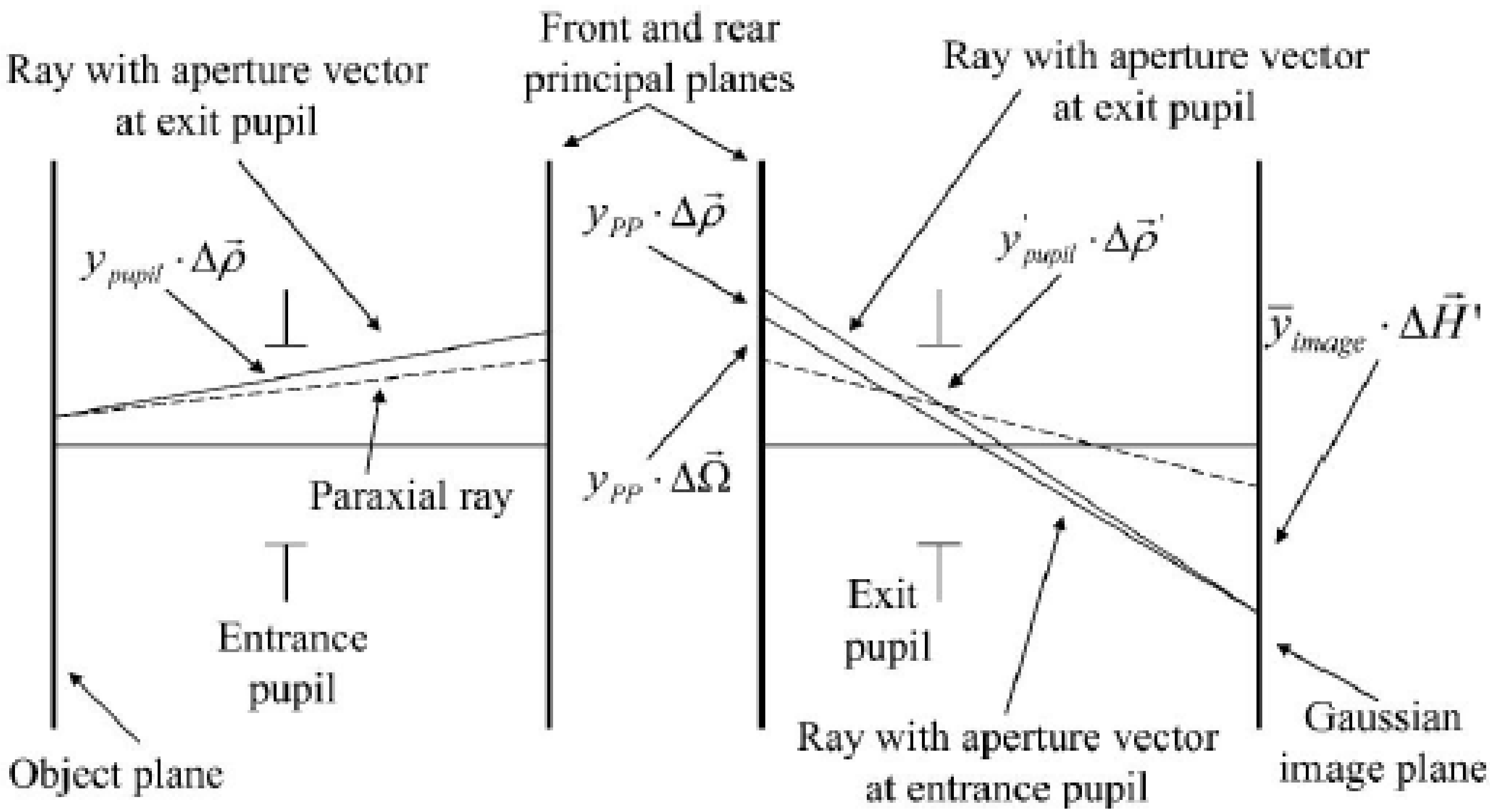
Geometry for derivation



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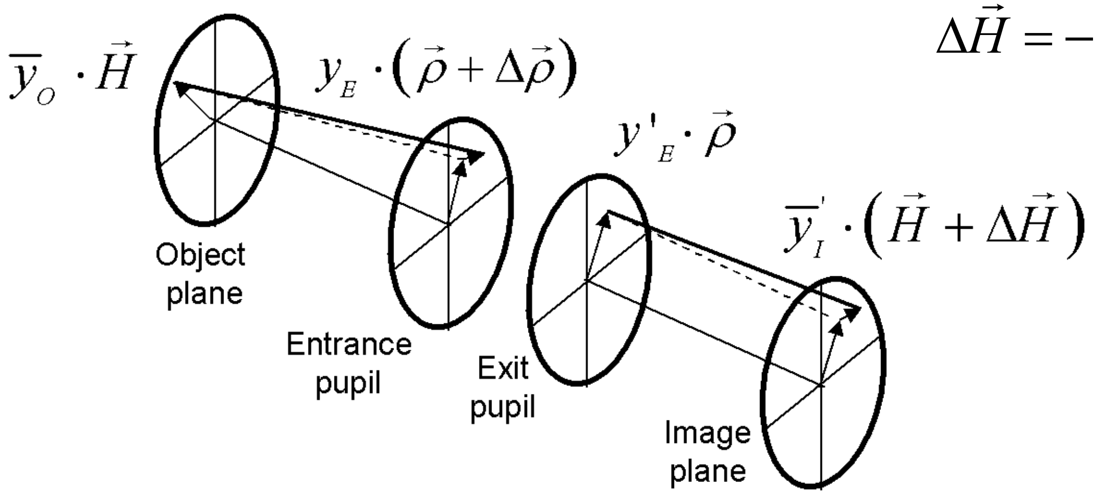
The displacement vector at the entrance pupil

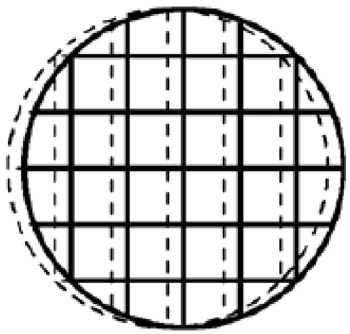


$$\Delta\vec{H} = -\frac{1}{Ж}\nabla_\rho W\left(\vec{H},\vec{\rho}\right) + O^{(5)}$$

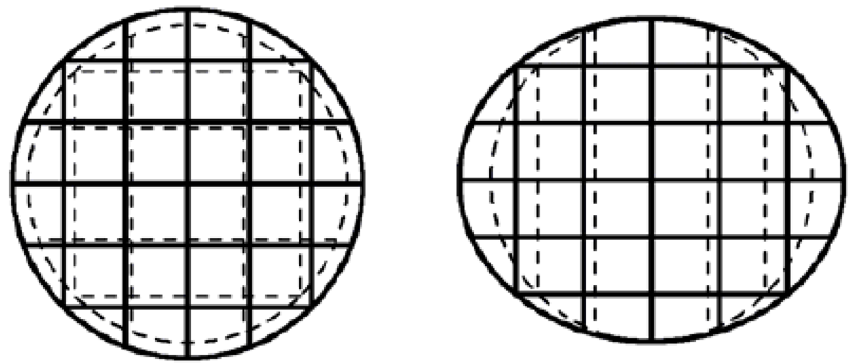
$$\Delta\vec{\rho} = -\frac{1}{Ж}\nabla_H \bar{W}\left(\vec{H},\vec{\rho}\right) = -\frac{1}{Ж}\cdot\begin{Bmatrix} 4\cdot\bar{W}_{040}\left(\vec{H}\cdot\vec{H}\right)\vec{H} + \bar{W}_{131}\left\{\left(\vec{H}\cdot\vec{H}\right)\vec{\rho} + 2\cdot\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}\right\} + \\ 2\cdot\bar{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho} + 2\cdot\bar{W}_{220}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H} + \bar{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} \end{Bmatrix}$$

Beam deformation at pupil

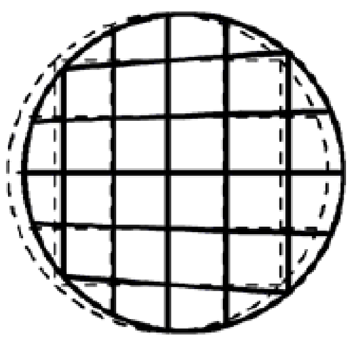
$$\Delta\vec{\rho} = -\frac{1}{Ж}\nabla_H \overline{W}\left(\vec{H},\vec{\rho}\right) = -\frac{1}{Ж}\cdot\left\{\begin{array}{l} 4\cdot\overline{W}_{040}\left(\vec{H}\cdot\vec{H}\right)\vec{H} + \overline{W}_{131}\left\{\left(\vec{H}\cdot\vec{H}\right)\vec{\rho} + 2\cdot\left(\vec{H}\cdot\vec{\rho}\right)\vec{H}\right\} + \\ 2\cdot\overline{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)\vec{\rho} + 2\cdot\overline{W}_{220}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{H} + \overline{W}_{311}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} \end{array}\right\}$$



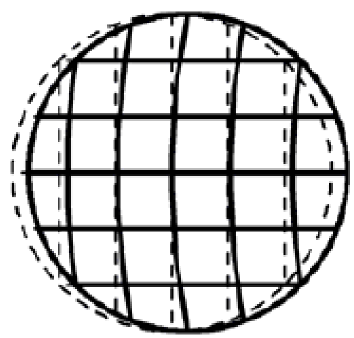
$\overline{W}_{040}\left(\vec{H}\cdot\vec{H}\right)^2$



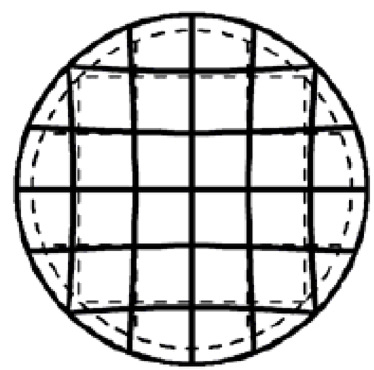
$\overline{W}_{131}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)$



$\overline{W}_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2$



$\overline{W}_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$



$\overline{W}_{311}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$

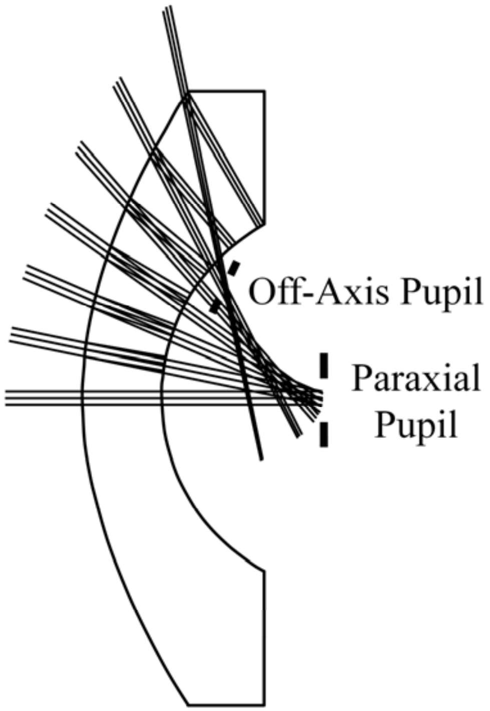
Pupil aberration consequences

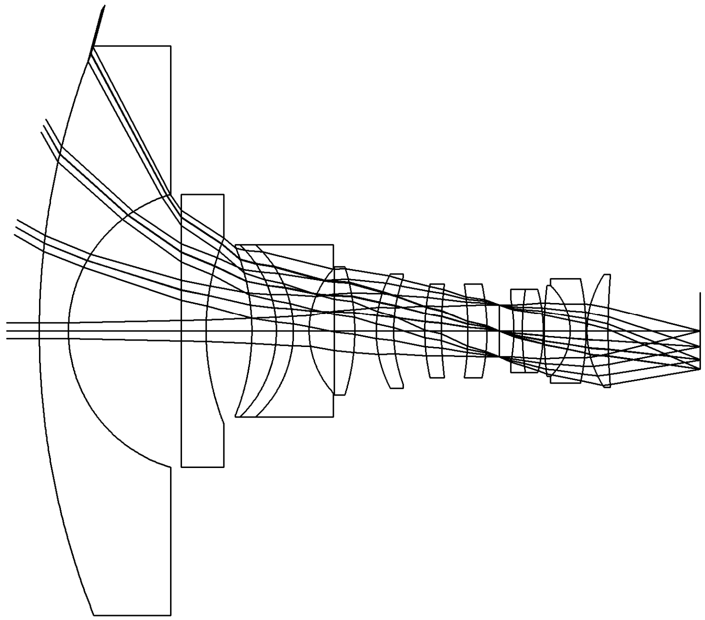
- Can now determine aberration change upon object shift
- Spherical aberration of the pupil
- Coma of the pupil
- Astigmatism and field curvature of the pupil
- Extrinsic sixth-order aberrations
- Bow-Sutton conditions

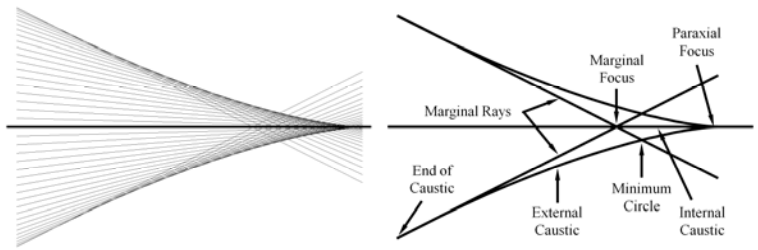
Effects from pupil aberrations

- Kidney bean effect. This is a partial obscuration in the form of a kidney bean caused by pupil spherical aberration
- Loss of telecentricity in relay systems. The chief slope varies as a function of the field of view.
- Vignetting. Spherical aberration of the pupil can lead to light vignetting.
- Pupil walking. Notably in fish eye lenses.
- Slyusarev effects. Due to pupil coma, the exit pupil changes size impacting the relative illumination
- Pupil Apodization

Pupil walking







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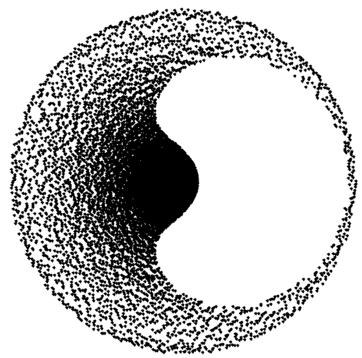
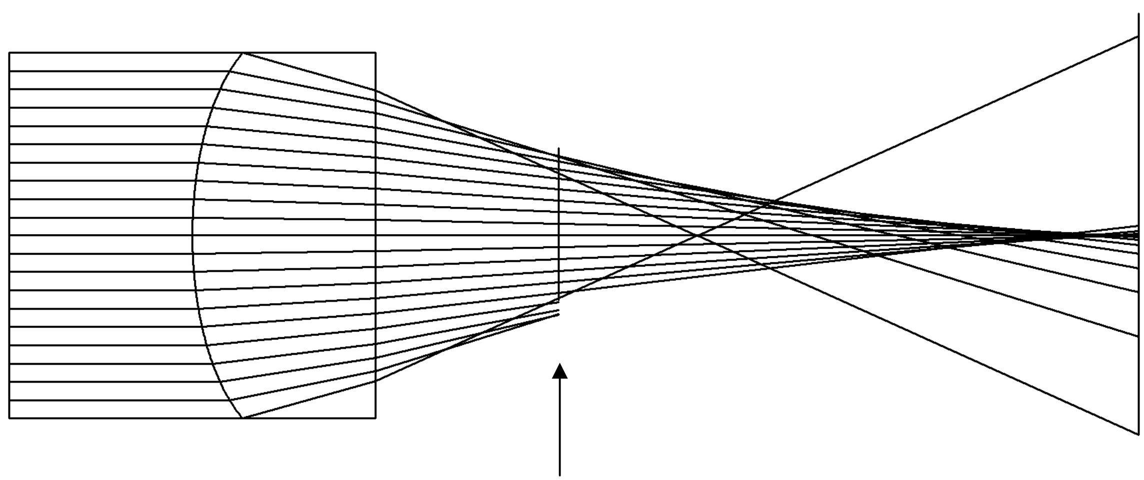
Kidney bean effect

From pupil spherical aberration

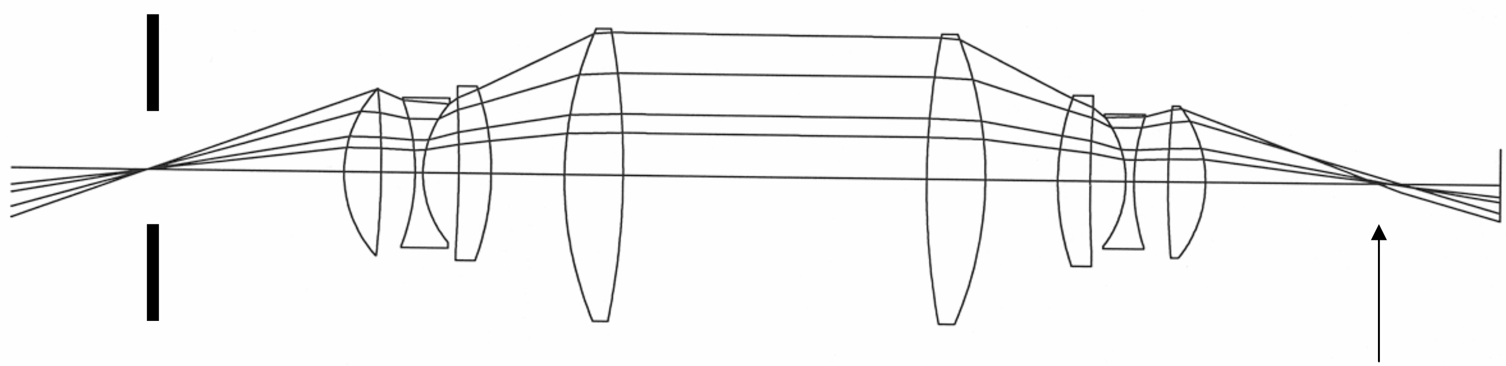


Chief rays



Off-axis clipping aperture (eye iris)

Loss of telecentricity



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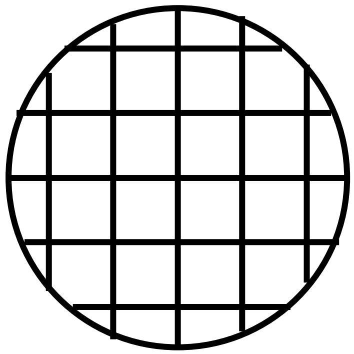
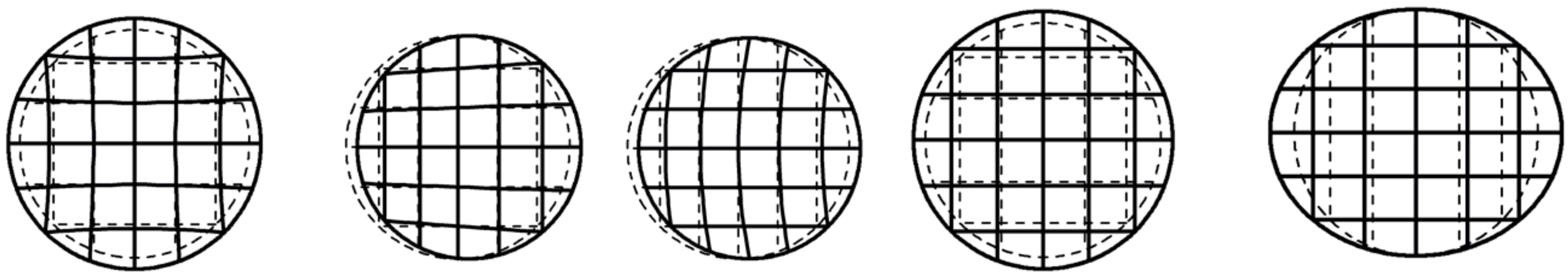
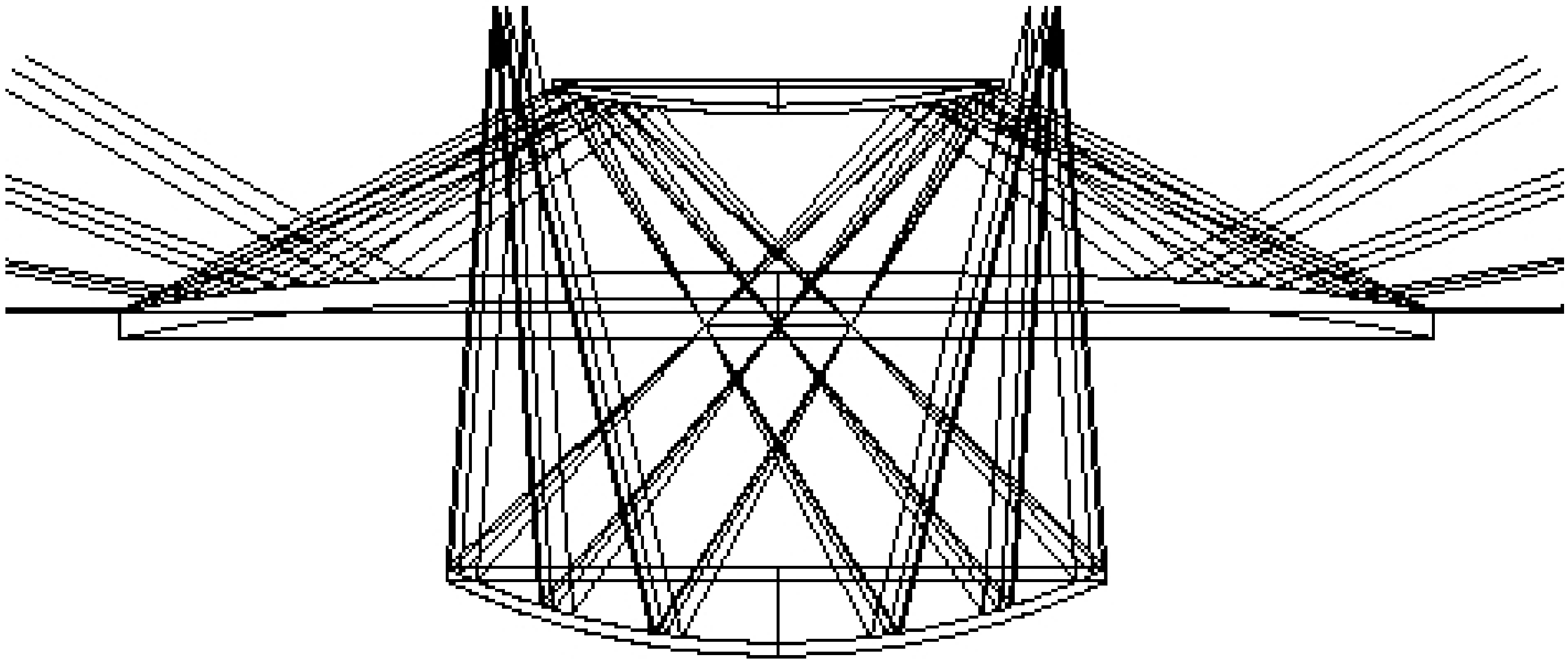
Change in relative illumination

- Vignetting
- Image distortion
- Cosine to the fourth law
- Pupil distortion
- Coatings





Effects from pupil coma



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Bow-Sutton conditions

- If the lens is symmetrical but the conjugates are not equal, then distortion will be corrected only if the entrance and exit pupils are free from pupil spherical aberration.
- Similarly, lateral color will be absent if the entrance and exit pupils are free from axial chromatic aberration.

See Kingslake’s lens design fundamentals
New edition with Barry Johnson

Bow-Sutton Conditions

- Since the lens is symmetrical ubar'=ubar
- Thus Coma pupil = image distortion
- A unit magnification the system is fully symmetrical for the stop and for the image systems and so, Coma pupil = image distortion=0
- When the object moves at other conjugate coma of the pupil according to stop shift equations is,
- New pupil coma=old pupil coma + pupil spherical aberration * y/ybar.
- The old coma is zero, thus,
- New pupil coma=pupil spherical aberration * y/ybar.
- Therefore if pupil spherical aberration is zero then the new pupil coma is zero.
- And therefore image distortion is zero.
- ubar'=ubar, still holds for the second conjugate

$$\bar{W}_{131} = W_{311} + \frac{1}{2} \text{Ж} \cdot \Delta\left\{\bar{u}^2\right\} \qquad \bar{W}^{*}_{131} = \bar{W}_{131} + \frac{\Delta y}{\bar{y}} \bar{W}_{040}$$

Object shift equations

Table 12.3 Object shift equations according to the parameter S	
$W^*_{040} = W_{040} + \left(W_{131} + \frac{1}{8}\text{Ж}\Delta\left(u^2\right)\right)S + \left(\frac{3}{2}W_{222} + \frac{3}{8}\text{Ж}\Delta\left(u\bar{u}\right) + W_{220P}\right)S^2 + \left(W_{311} + \frac{3}{8}\text{Ж}\Delta\left(\bar{u}^2\right)\right)S^3 + \bar{W}_{040}S^4$	(12.26)
$W^*_{131} = W_{131} + \left(3W_{222} + \frac{1}{2}\text{Ж}\Delta\left(u\bar{u}\right) + 2W_{220P}\right)S + \left(3W_{311} + \text{Ж}\Delta\left(\bar{u}^2\right)\right)S^2 + 4\bar{W}_{040}S^3$	(12.27)
$W^*_{220P} = W_{220P}$	(12.28)
$W^*_{222} = W_{222} + \left(2W_{311} + \text{Ж}\Delta\left(\bar{u}^2\right)/2\right)S + 4\bar{W}_{040}S^2$	(12.29)
$W^*_{311} = W_{311} + 4\bar{W}_{040}S$	(12.30)
$\partial_\lambda W^*_{111} = \partial_\lambda W_{111} + 2\partial_\lambda \bar{W}_{020}S$	(12.31)
$\partial_\lambda W^*_{020} = \partial_\lambda W_{020} + \frac{1}{2}\left(\partial_\lambda W_{111} + \partial_\lambda \bar{W}_{111}\right)S + \partial_\lambda \bar{W}_{020}S^2$	(12.32)

Formulas

$$S = \frac{u^* - u}{\overline{u}} = \frac{y^* - y}{\overline{y}} = \frac{A^* - A}{\overline{A}}$$

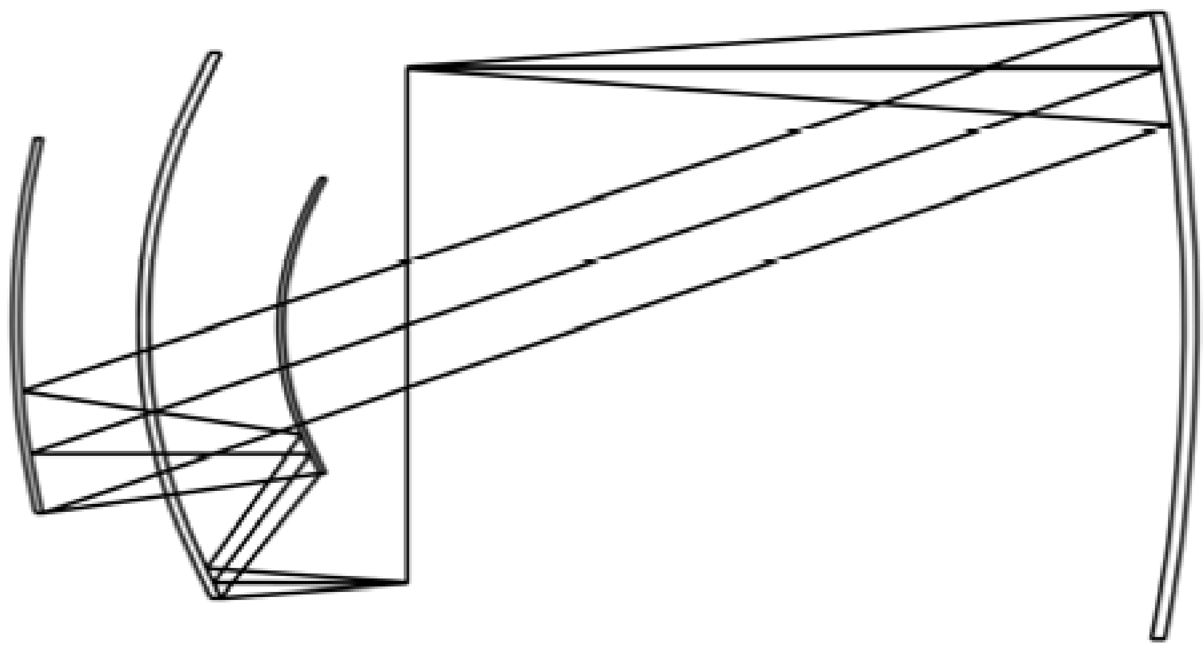
$$y^* = y + S \cdot \overline{y} \qquad u^* = u + S \cdot \overline{u}$$

$$A^* = A + S \cdot \overline{A} \qquad \Delta\left(\frac{u^*}{n}\right) = \Delta\left(\frac{u}{n}\right) + S\Delta\left(\frac{\overline{u}}{n}\right)$$

$$W_{040}^* = -\frac{1}{8}\left(A^*\right)^2 \Delta\left(\frac{u^*}{n}\right) y^*$$

$$= -\frac{1}{8}\left(A^2 + 2SA\overline{A} + S^2\overline{A}^2\right)\left(\Delta\left(\frac{u}{n}\right) + S\Delta\left(\frac{\overline{u}}{n}\right)\right)\left(y + S\overline{y}\right)$$

Invariance of aberrations



No primary aberrations independently of stop position or conjugate!
Negative unit magnification system

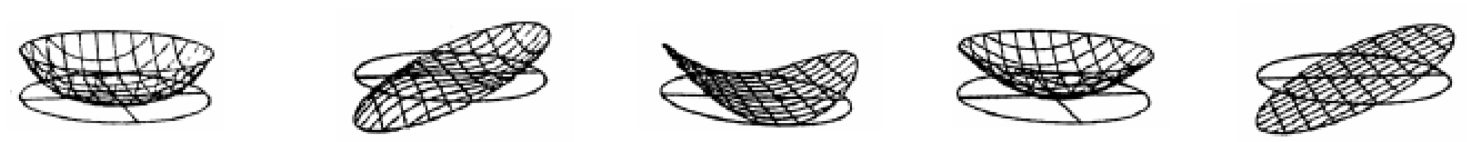
Chromatic pupil aberrations

Table 12.4
Image and pupil chromatic coefficients

Image		Pupil	
$\partial_\lambda W_{020} = \frac{1}{2} A\Delta\left(\frac{\partial n}{n}\right) y$	(12.39)	$\partial_\lambda \bar{W}_{020} = \frac{1}{2} \bar{A}\Delta\left(\frac{\partial n}{n}\right) \bar{y}$	(12.40)
$\partial_\lambda W_{111} = \bar{A}\Delta\left(\frac{\partial n}{n}\right) y$	(12.41)	$\partial_\lambda \bar{W}_{111} = A\Delta\left(\frac{\partial n}{n}\right) \bar{y}$	(12.42)
$\partial_\lambda W_{200} = \frac{1}{2} A\left(\frac{\bar{A}}{A}\right)^2 \Delta\left(\frac{\partial n}{n}\right) y$	(12.43)	$\partial_\lambda \bar{W}_{200} = \frac{1}{2} \bar{A}\left(\frac{A}{\bar{A}}\right)^2 \Delta\left(\frac{\partial n}{n}\right) \bar{y}$	(12.44)

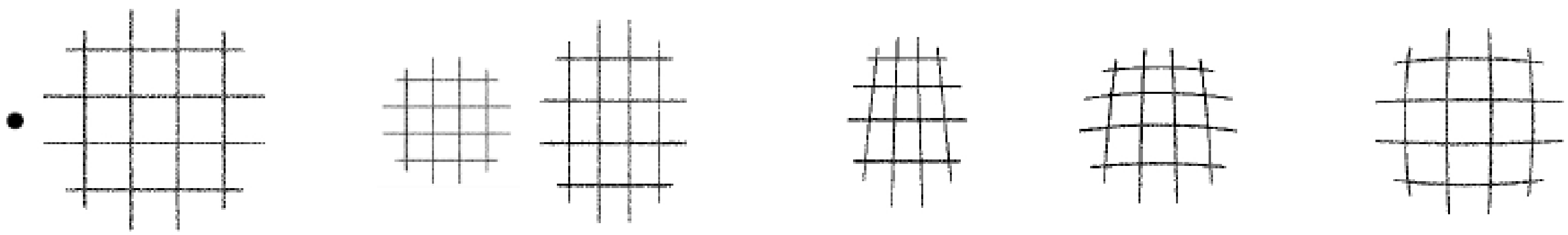
Image vs. Pupil aberrations

Basic wavefront deformation shapes



W_{040} W_{131} W_{222} W_{220} W_{311}

Basic cross-section deformation shapes



$\overline{W}_{040}$ $\overline{W}_{131}$ $\overline{W}_{222}$ $\overline{W}_{220}$ $\overline{W}_{311}$

Summary

- Optical systems are formed by concatenating one system to the next
- The exit pupil of one system becomes the entrance pupil for the next system
- Every pupil mismatch leads to light loss, aberration, or other effect.
- Pupil aberrations are also important