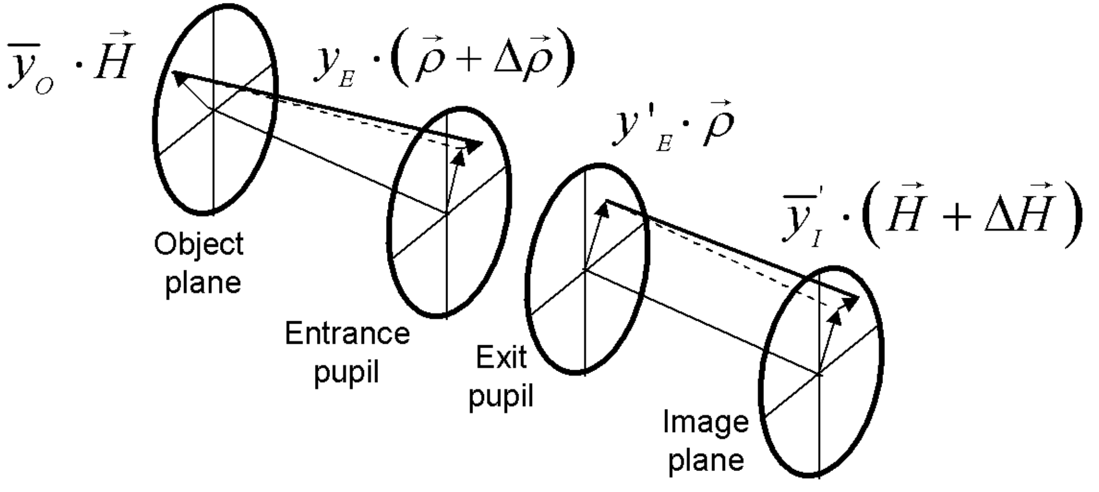
Introduction to aberrations

OPTI 518

Lecture 13



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Topics

- Aspheric surfaces
- Stop shifting
- Field curve concept

Aspheric Surfaces

- Meaning not spherical
- Conic surfaces: Sphere, prolate ellipsoid, hyperboloid, paraboloid, oblate ellipsoid or spheroid
- Cartesian Ovals
- Polynomial surfaces
- Infinite possibilities for an aspheric surface
- Ray tracing for quadric surfaces uses closed formulas; for other surfaces iterative algorithms are used

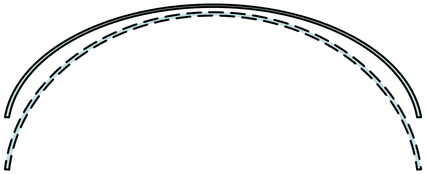
Aspheric surfaces

The concept of the sag of a surface

$$Z(S) = \frac{cS^2}{1+\sqrt{\left[1-(K+1)c^2S^2\right]}} + A_4S^4 + A_6S^6 + A_8S^8 + A_{10}S^{10} + \ldots$$

$$S^2 = x^2 + y^2$$

$$K = -\varepsilon^2$$



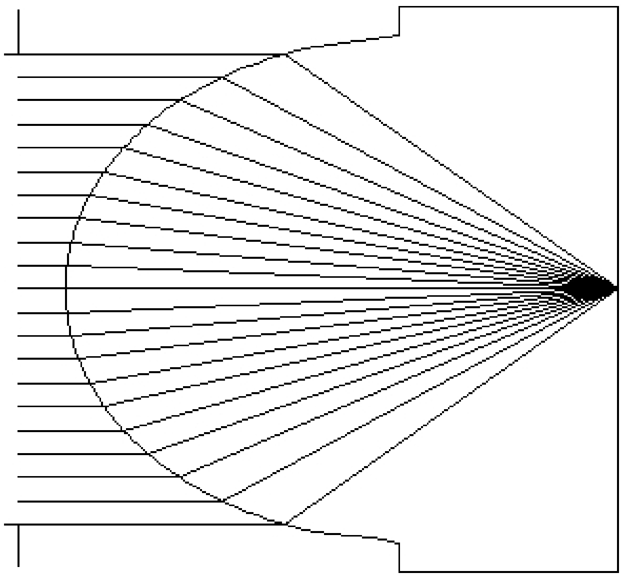
K is the conic constant
K=0, sphere
K=-1, parabola
K<-1, hyperola
-1<K<0, prolate ellipsoid
K>0, oblate ellipsoid

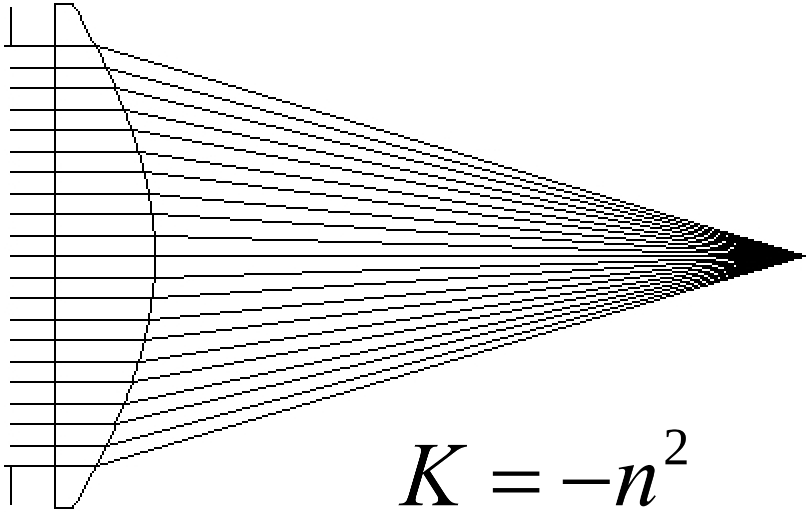
C is 1/r where r is the radius of curvature; K is the conic constant (the eccentricity squared);
A's are aspheric coefficients

Conic surfaces focal properties

- Focal points for mirrors
- Focal points of lenses

Ellipsoid case
Hyperboloid case
Oblate ellipsoid





$$K = -n^2$$

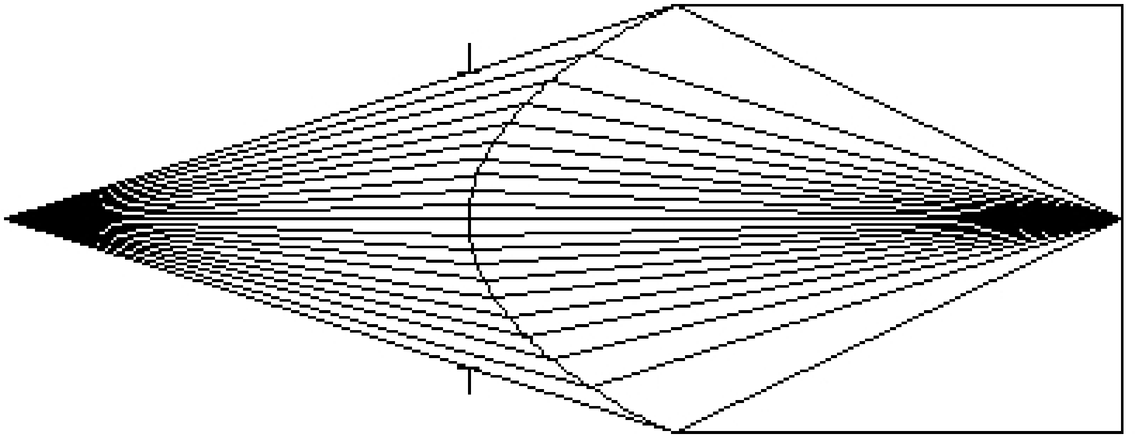
Hecht-Zajac Optics
Refraction at a spherical surface

Aspheric surface description

$$Z(S)=\frac{cS^2}{1+\sqrt{\left[1-(K+1)c^2S^2\right]}}+A_4S^4+A_6S^6+A_8S^8+A_{10}S^{10}+\ldots$$

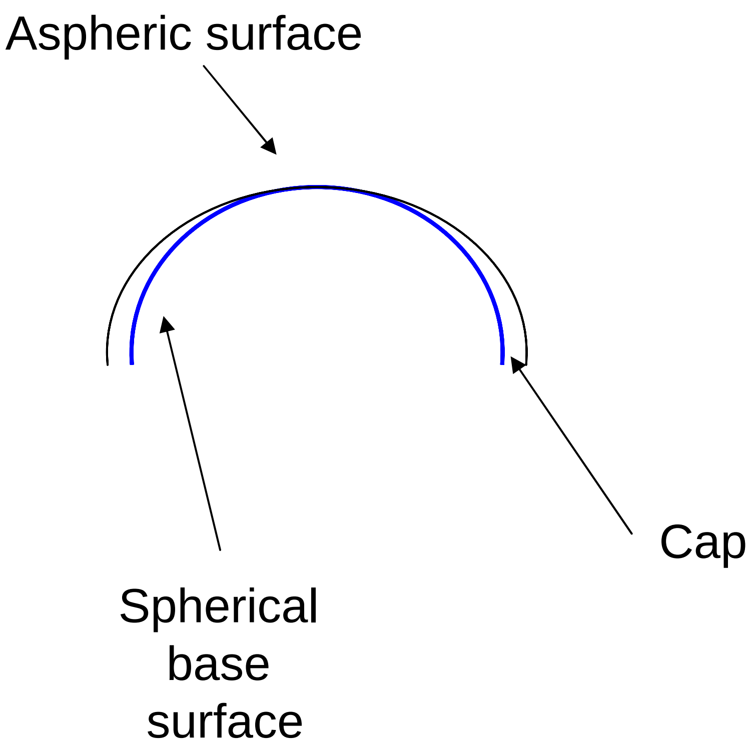
$$Z_{aspheric}=\frac{1}{2r}\left(x^2+y^2\right)+\frac{1}{8r}\left(1+K\right)\left(x^2+y^2\right)^2+A_4\left(x^2+y^2\right)^2$$

Cartesian Ovals



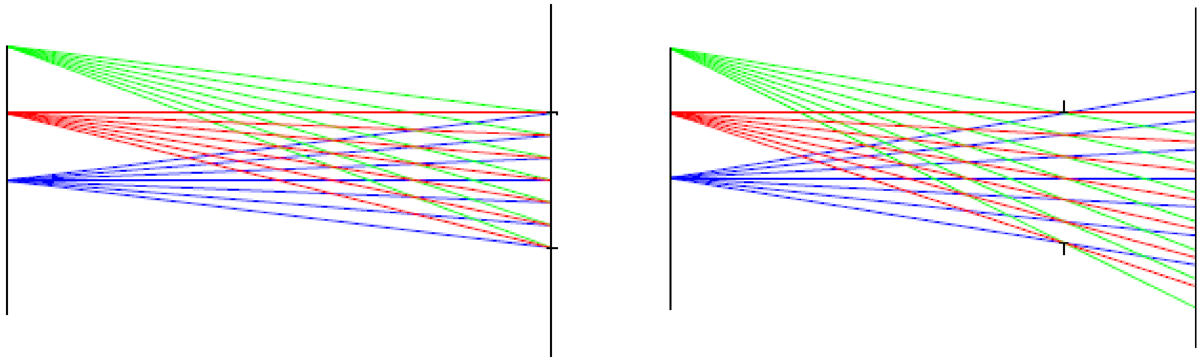
$$l \cdot n + l' \cdot n' = Cte.$$

Aspheric cap



The aspheric surface can be thought of as comprising a base sphere and an aspheric cap

Aspheric surface contributions



•When the stop is at the aspheric surface only spherical aberration is contributed given that all the beams see the same portion of the surface
•When the stop is away from the surface, different field beams pass through different parts of the aspheric surface and other aberrations are contributed

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Aspheric contributions I

$$\delta W_{040} = \frac{1}{8} a$$

$$\delta W_{131} = \frac{1}{2} \frac{\bar{y}}{y} a$$

$$\delta W_{222} = \frac{1}{2} \left(\frac{\bar{y}}{y} \right)^2 a$$

$$\delta W_{220} = \frac{1}{4} \left(\frac{\bar{y}}{y} \right)^2 a$$

$$\delta W_{311} = \frac{1}{2} \left(\frac{\bar{y}}{y} \right)^3 a$$

$$\delta C_L = 0$$

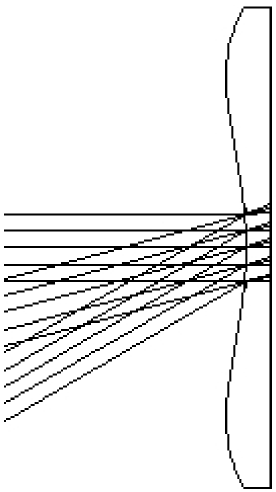
$$\delta C_T = 0$$

$$a = -\varepsilon^2 c^3 y^4 \Delta n$$

$$a = 8 A_4 y^4 \Delta n$$

Stop at aspheric surface

$$W_{cap}\left(\vec{H},\vec{\rho}\right)=A_4 y^4 \Delta\left(n\right)\left(\vec{\rho}\cdot\vec{\rho}\right)^2$$



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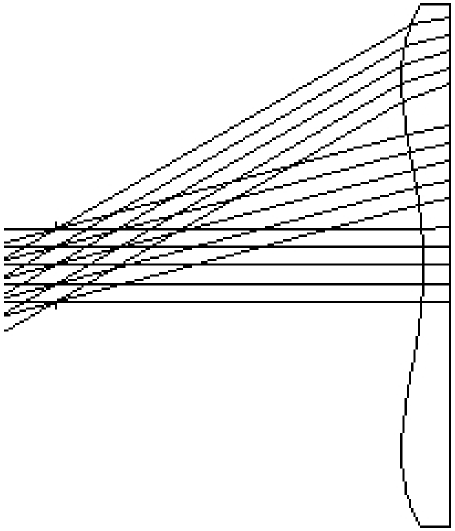
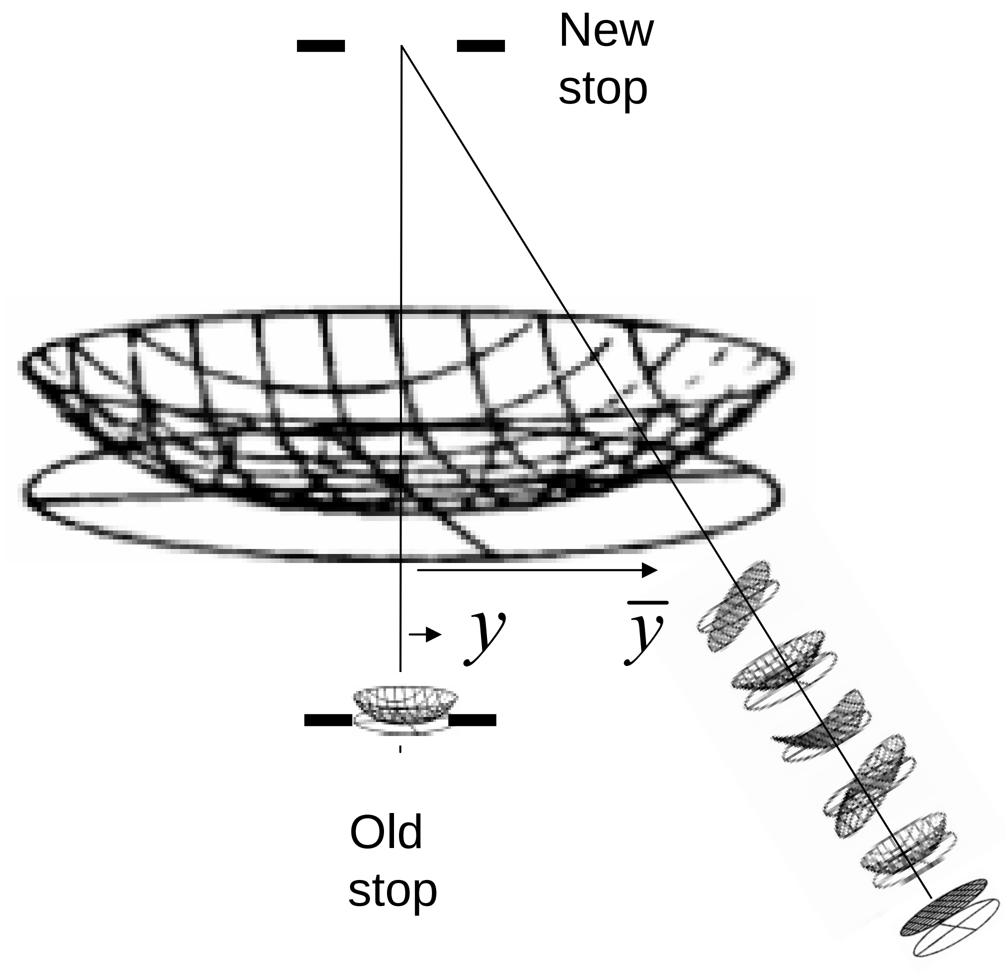
Upon stop shifting

$$W_{cap}\left(\vec{H},\vec{\rho}\right)=A_4 y^4 \Delta(n)\left(\vec{\rho}_{shift}\cdot\vec{\rho}_{shift}\right)^2$$



Aspheric cap contributions



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Stop shifting

$$W_{cap}\left(\vec{H},\vec{\rho}\right)=A_4 y^4 \Delta\left(n\right)\left(\vec{\rho}_{shift}\cdot\vec{\rho}_{shift}\right)^2$$

$$\vec{\rho}_{shift}=\vec{\rho}+\overline{S}\vec{H}$$

$$\overline{S}=\frac{\overline{y}_{new}-\overline{y}_{old}}{y}=\frac{\overline{y}_{new}-0}{y}=\frac{\overline{y}}{y}$$

Expansion of $\left(\vec{\rho}_{shift}\cdot\vec{\rho}_{shift}\right)^2$

$$\left(\vec{\rho}_{shift}\cdot\vec{\rho}_{shift}\right)^2=\left[\vec{\rho}\cdot\vec{\rho}+2\overline{S}\vec{H}\cdot\vec{\rho}+\left(\overline{S}\right)^2\vec{H}\cdot\vec{H}\right]\times$$

$$\left[\vec{\rho}\cdot\vec{\rho}+2\overline{S}\vec{H}\cdot\vec{\rho}+\left(\overline{S}\right)^2\vec{H}\cdot\vec{H}\right]=$$

$$=\left(\vec{\rho}\cdot\vec{\rho}\right)^2+4\overline{S}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+4\overline{S}^2\left(\vec{H}\cdot\vec{\rho}\right)^2$$

$$+2\overline{S}^2\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)+4\overline{S}^3\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\overline{S}^4\left(\vec{H}\cdot\vec{H}\right)^2$$

Aberration function upon stop shifting for an aspheric cap

$$W_{cap}\left(\vec{H},\vec{\rho}\right)=A_4 y^4\Delta(n)\left(\vec{\rho}_{shift}\cdot\vec{\rho}_{shift}\right)^2$$
$$=A_4 y^4\Delta(n)(\vec{\rho}\cdot\vec{\rho})^2+4A_4 y^4\Delta(n)\bar{S}\left(\vec{H}\cdot\vec{\rho}\right)(\vec{\rho}\cdot\vec{\rho})$$
$$+4A_4 y^4\Delta(n)\bar{S}^2\left(\vec{H}\cdot\vec{\rho}\right)^2+2A_4 y^4\Delta(n)\bar{S}^2\left(\vec{H}\cdot\vec{H}\right)(\vec{\rho}\cdot\vec{\rho})$$
$$+4A_4 y^4\Delta(n)\bar{S}^3\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+A_4 y^4\Delta(n)\bar{S}^4\left(\vec{H}\cdot\vec{H}\right)^2$$
$$=\frac{1}{8}a(\vec{\rho}\cdot\vec{\rho})^2+\frac{1}{2}a\bar{S}\left(\vec{H}\cdot\vec{\rho}\right)(\vec{\rho}\cdot\vec{\rho})+\frac{1}{2}a\bar{S}^2\left(\vec{H}\cdot\vec{\rho}\right)^2$$
$$+\frac{1}{4}a\bar{S}^2\left(\vec{H}\cdot\vec{H}\right)(\vec{\rho}\cdot\vec{\rho})+\frac{1}{2}a\bar{S}^3\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)+\frac{1}{8}a\bar{S}^4\left(\vec{H}\cdot\vec{H}\right)^2$$

$$a=8A_4 y^4\Delta(n)$$

Aspheric contributions

$\delta S_I = a$
$\delta S_{II} = \left(\frac{\bar{y}}{y}\right) a$
$\delta S_{III} = \left(\frac{\bar{y}}{y}\right)^2 a$
$\delta S_{IV} = 0$
$\delta S_V = \left(\frac{\bar{y}}{y}\right)^3 a$
$\delta S_{VI} = \left(\frac{\bar{y}}{y}\right)^4 a$
$\delta C_L = 0$
$\delta C_T = 0$
$a = -\varepsilon^2 c^3 y^4 \Delta(n)$ For a conic surface of eccentricity ε
$a = 8 A_4 y^4 \Delta(n)$ For an aspheric surface with fourth order coefficient A_4

Aberration function of base sphere and aspheric cap

$$W\left(\vec{H},\vec{\rho}\right)=W_{spherical}\left(\vec{H},\vec{\rho}\right)+W_{cap}\left(\vec{H},\vec{\rho}\right)$$

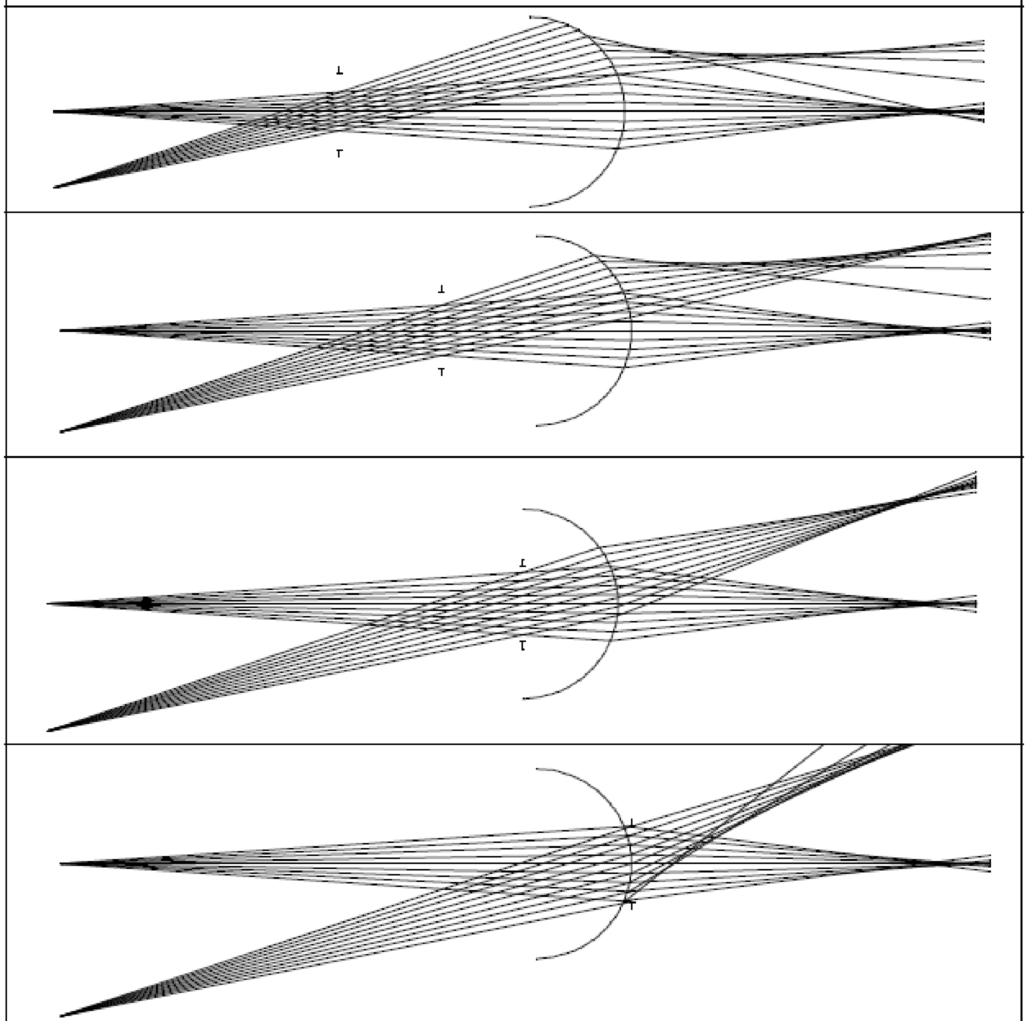
Case of an aspherical surface with the stop at the surface:

$$W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2=-\frac{1}{8}A^2y\Delta\left(\frac{u}{n}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)^2+A_4y^4\Delta\left(n\right)\left(\vec{\rho}\cdot\vec{\rho}\right)^2$$

$$Sag=Sphere+A_4\left(x^2+y^2\right)^2$$

Stop shifting

As the stop shifts different off-axis rays are selected and the on-axis rays remain the same. The stop diameter changes to maintain the F/#.



Stop shifting

Stop-shift formulas

The process of moving the stop to a new location to obtain a different aberration content is known as stop-shift. The aberration description of an optical system depends on the stop position along the optical axis. As the stop changes position the chief ray is redefined and the Seidel coefficients change. There is no need to recompute the Seidel coefficients; the thus called stop-shift formulas provide the new Seidel coefficients from the old Seidel coefficients and the ratio $\delta\bar{y}/y$. These changes upon stop-shift are given in the following Table where the ratio $\delta\bar{y}/y$ is the change in chief ray height divided by the marginal ray height. The ratio $\delta\bar{y}/y$ is an invariant and can be computed at any plane of the optical system, and in particular it is easier to compute it at the plane of the old stop surface where $\delta\bar{y} = \bar{y}_{new} - \bar{y}_{old} = \bar{y}_{new} - 0 = \bar{y}_{new}$.

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Stop shifting

- How do the aberration coefficients change as we shift the stop?
- Used to ease calculations
- Most importantly, the formulas give insights
- The concept of object shifting

Stop shifting parameter

$$\bar{S} = \frac{\bar{u}_{new} - \bar{u}_{old}}{u} = \frac{\bar{y}_{new} - \bar{y}_{old}}{y} = \frac{\bar{A}_{new} - \bar{A}_{old}}{A}$$

$$\bar{A}_{new} = \bar{A}_{old} + \bar{S}A$$

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Stop shifting formulas

$S_I^* = S_I$
$S_{II}^* = S_{II} + \bar{S} S_I$
$S_{III}^* = S_{III} + 2 \cdot \bar{S} S_{II} + \bar{S}^2 S_I$
$S_{IV}^* = S_{IV}$
$S_V^* = S_V + \bar{S} \left(S_{IV} + 3 \cdot S_{III} \right) + 3 \cdot \bar{S}^2 S_{II} + \bar{S}^3 S_I$
$C_L^* = C_L$
$C_T^* = C_T + \bar{S} C_L$

Derivation for $\bar{A}^*$

$$Ж = \bar{A}_1 y - A\bar{y}_1$$

$$Ж = \bar{A}_2 y - A\bar{y}_2$$

$$0 = \left(\bar{A}_1 - \bar{A}_2\right) y - A\left(\bar{y}_1 - \bar{y}_2\right)$$

$$\Delta\bar{A} = A\frac{\bar{y}_2 - \bar{y}_1}{y} = A\frac{\Delta\bar{y}}{y} = A\bar{S}$$

$$\bar{A}^* = \bar{A} + A\bar{S}$$

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Derivation of new Seidel sums upon stop shifting

$$\bar{A}^* = \bar{A} + \bar{S}A$$

$$S_I = -\sum A^2 y\Delta\left(\frac{u}{n}\right)$$

$$S_I^* = S_I$$

$$S_{II}^* = -\sum A\bar{A}^* y\Delta\left(\frac{u}{n}\right) = -\sum A\left(\bar{A} + \bar{S}A\right) y\Delta\left(\frac{u}{n}\right)$$

$$= -\sum A\bar{A}y\Delta\left(\frac{u}{n}\right) - \bar{S}\sum A^2 y\Delta\left(\frac{u}{n}\right)$$

$$S_{II}^* = S_{II} + \bar{S}S_I$$

$$S_{III}^* = -\sum\left(\bar{A}^*\right)^2 y\Delta\left(\frac{u}{n}\right) = -\sum\left(\left(\bar{A} + \bar{S}A\right)\right)^2 y\Delta\left(\frac{u}{n}\right)$$

$$= -\sum\left(\bar{A}^2 + 2\bar{A}A\bar{S} + \bar{S}^2 A^2\right) y\Delta\left(\frac{u}{n}\right)$$

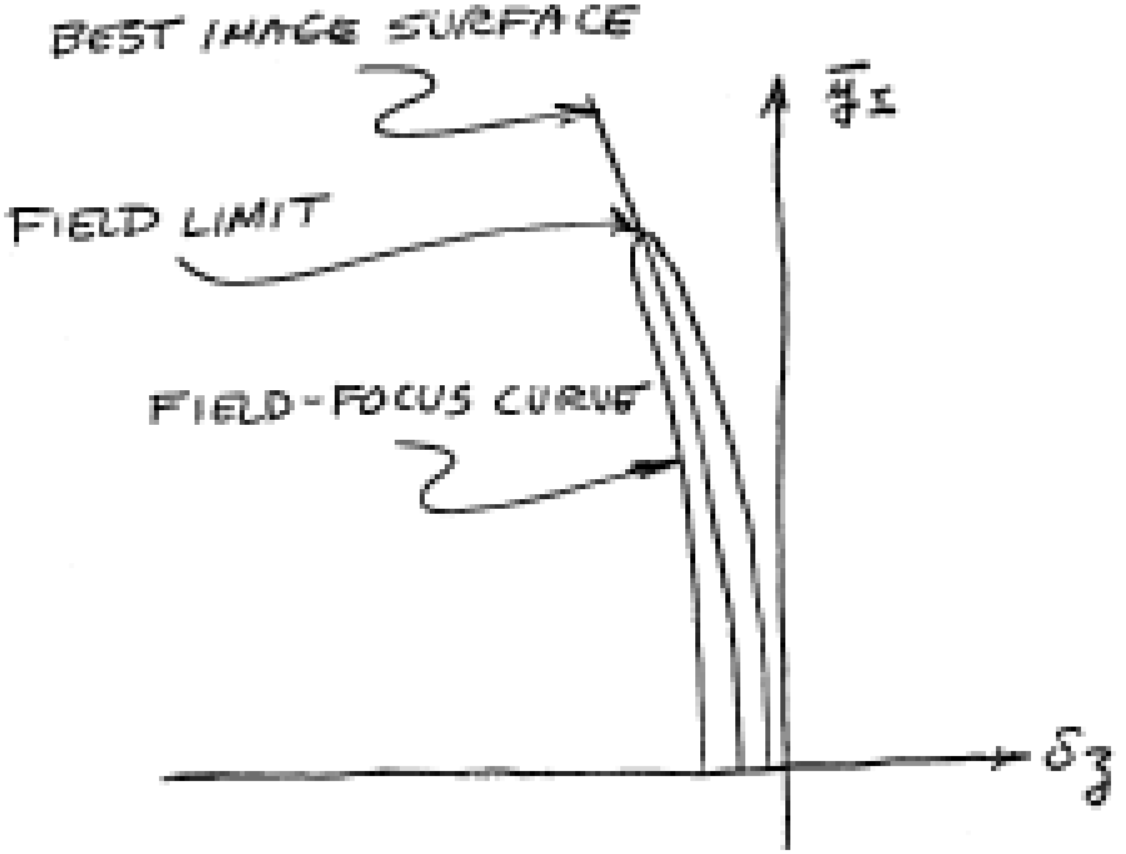
$$S_{III}^* = S_{III} + 2\bar{S}S_{II} + \bar{S}^2 S_I$$

And similarly for:

$$S_{IV}^* = S_{IV}$$

$$S_V^* = S_V + \bar{S}\left(S_{IV} + 3S_{III}\right) + 3\bar{S}^2 S_{II} + \bar{S}^3 S_I$$

Field focus curve concept



$$\sigma_w^2$$

Provides limit for a given wavefront variance.

Field focus curve concept

FIELD-FOCUS CURVE (FIXED VALUE OF σ_W^2)

$$\sigma_W^2 = \frac{1}{12}\left[\Delta W_{20} + W_{040} + \left(W_{220} + \frac{1}{2}W_{222}\right)H^2\right]^2 + \frac{1}{180}W_{040}^2 + \frac{1}{72}W_{131}^2 H^2 + \frac{1}{24}W_{222}^2 H^4$$

$$\Delta W_{20} = -W_{040} - \left(W_{220} + \frac{1}{2}W_{222}\right)H^2 \pm \sqrt{12\left\{\left(\sigma_W^2 - \frac{1}{180}W_{040}^2\right) - \frac{1}{72}W_{131}^2 H^2 - \frac{1}{24}W_{222}^2 H^4\right\}}$$

ON AXIS: $(H=0)$

$$\Delta W_{20} = -W_{040} \pm \sqrt{12\left(\sigma_W^2 - \frac{1}{180}W_{040}^2\right)}$$

FIELD VANISHES FOR $|W_{040}| \geqq 6\sqrt{5}\sqrt{\sigma_W^2}$

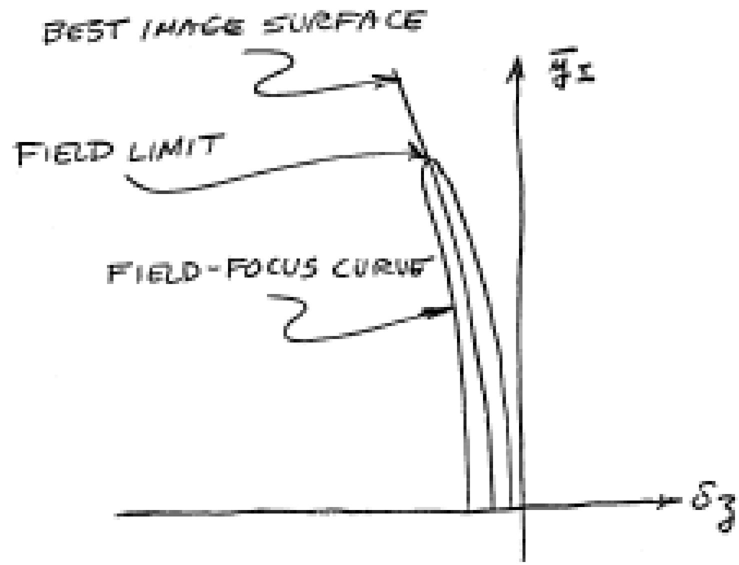
Field focus curve concept

FIELD LIMIT OCCURS WHEN RADICAND = 0

$$H_{MAX}^2 = \frac{1}{\frac{1}{12} W_{222}^2} \left\{ \sqrt{\left(\frac{1}{72} W_{131}^2\right)^2 + \frac{1}{6} W_{222}^2 \left(\sigma_W^2 - \frac{1}{180} W_{040}^2\right)} - \frac{1}{72} W_{131}^2 \right]$$

UNLESS $W_{222} = 0$, IN WHICH CASE

$$H_{MAX}^2 = \frac{\left(\sigma_W^2 - \frac{1}{180} W_{040}^2\right)}{\frac{1}{72} W_{131}^2}$$



- Must preserve DOF

Summary

- Variety in aspheric surfaces
- Aspheric surface contributions to fourth-order aberrations
- Stop shifting and how aberrations change
- The concept of field focus curve