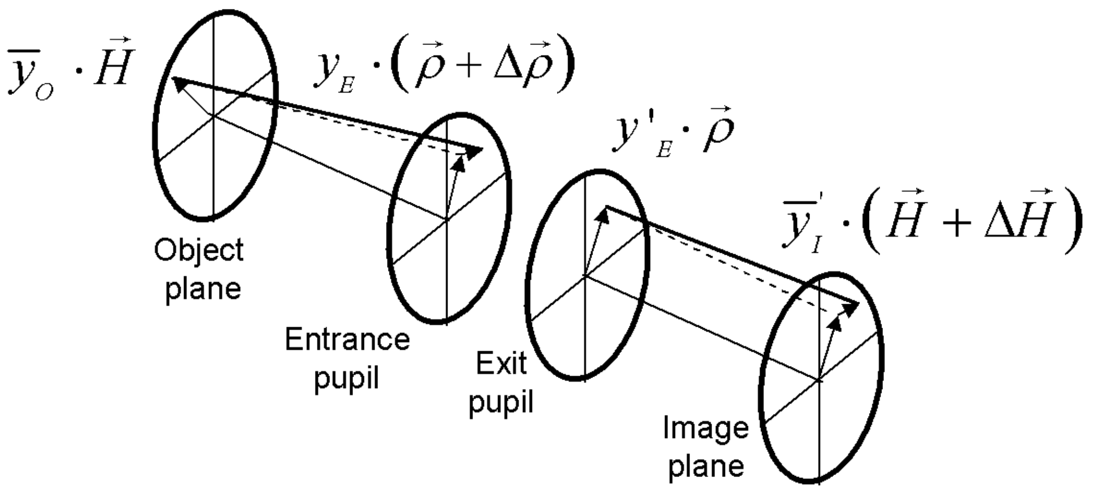
Introduction to aberrations

OPTI 518

Lecture 12



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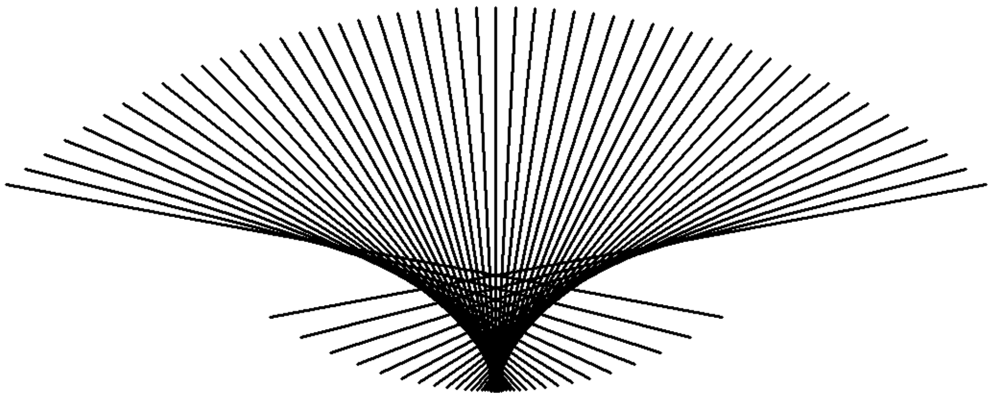
Caustics



Caustic ~ burning

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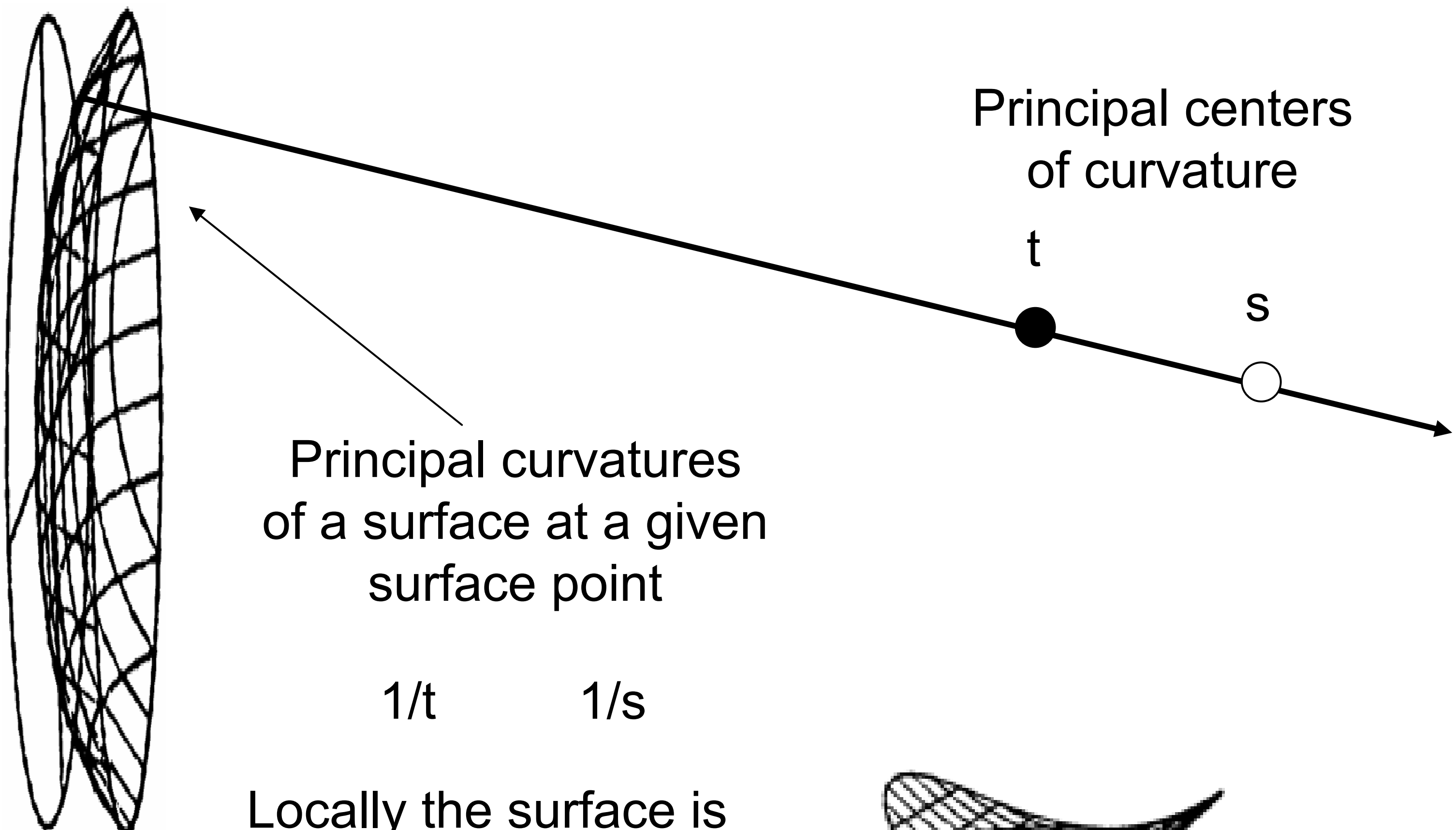
Caustic



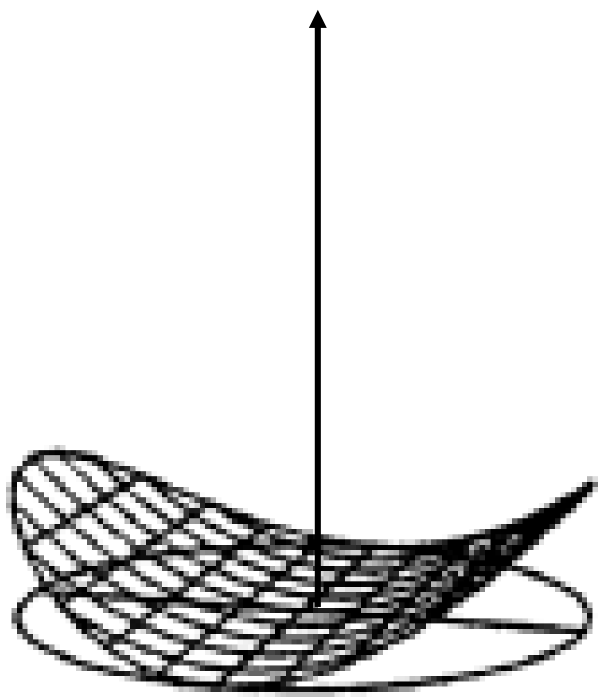
Locally the surface is a cylindrical parabolic

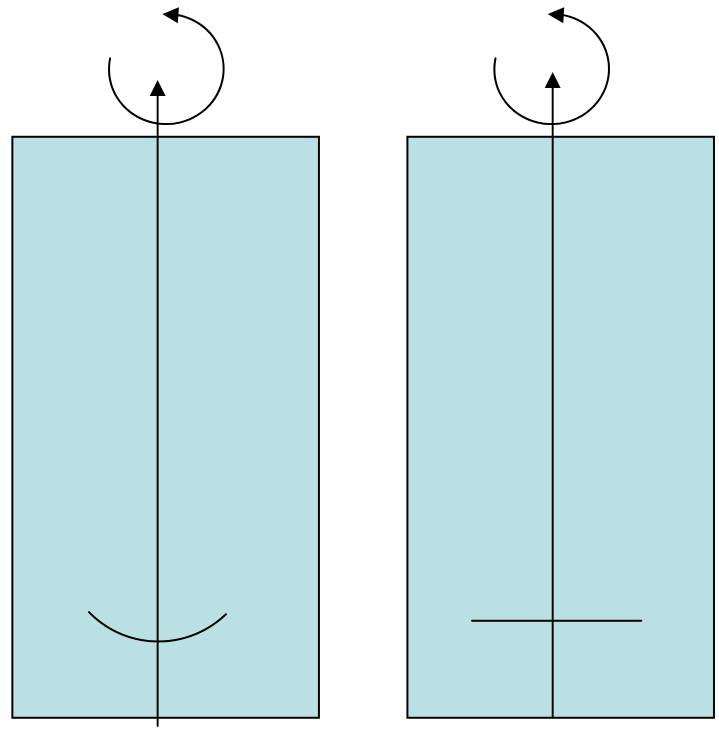
Principal curvatures

For any smoothly continuous surface, at each point in the surface there is a normal. A plane containing the normal intersects the surface in a plane curve. With a finite curvature at the point in question. The center of this curvature lies on the normal. If the plane is rotated about the normal, the resultant curvature in general fluctuates continuously between two extreme values. These extreme values are the *principal curvatures* of the surface at the point in question, and they lie in planes which are necessarily perpendicular to each other. Their centers of curvature are the *principal centers of curvature* and the intermediate centers of curvature lie between them.

Principal curvatures

$$Z = \frac{x^2}{2R_t} + \frac{y^2}{2R_s}; \quad Z = \frac{1}{2R_s}\left(\vec{I}_{\perp} \cdot \vec{\rho}\right)^2 + \frac{1}{2R_t}\left(\vec{I}_{\parallel} \cdot \vec{\rho}\right)^2$$





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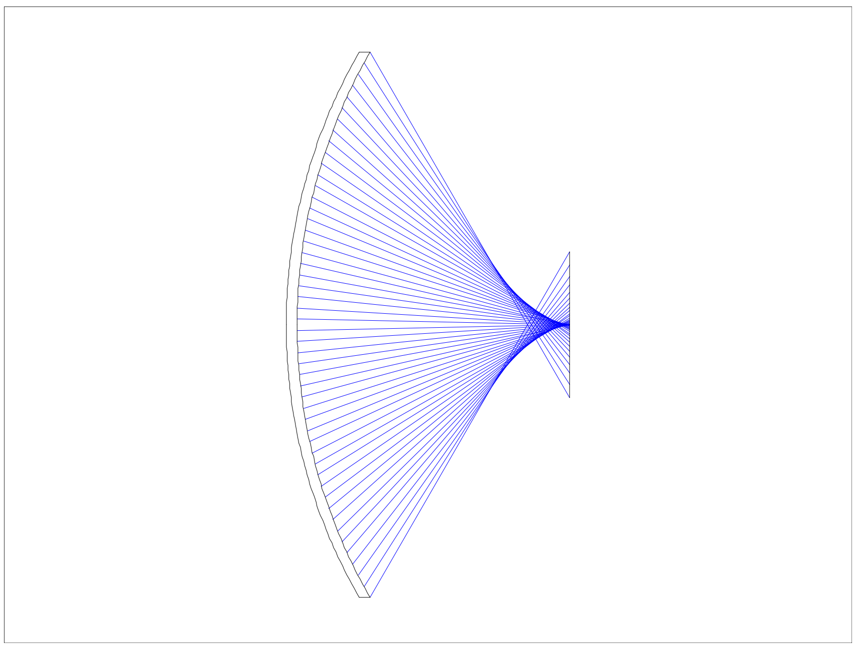
Caustic

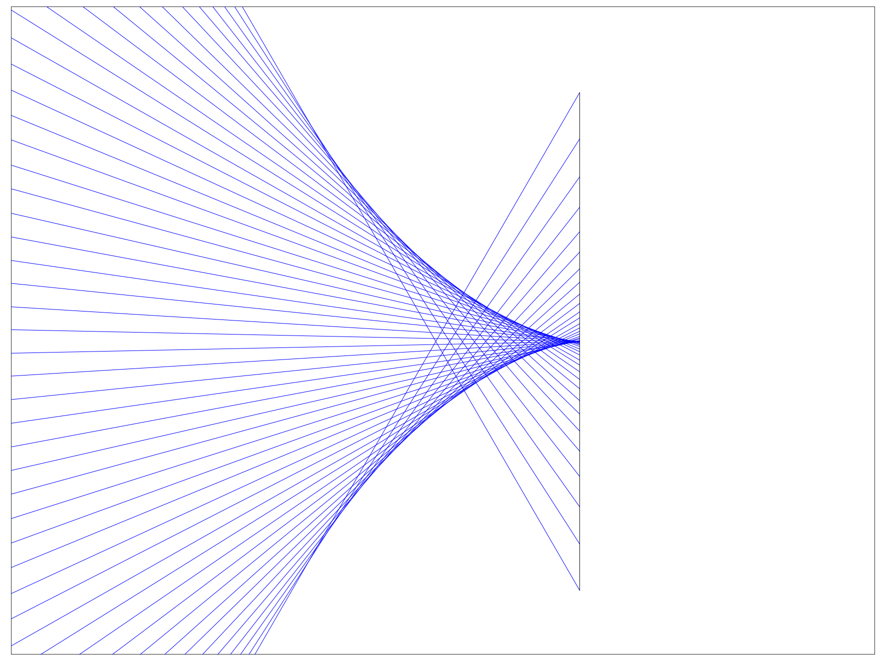
- For each point on the surface there are two principal centers of curvature, and for neighboring points on the surface, the principal centers of curvature are also pair-wise neighbors. Thus, for every given smoothly continuous surface there exits also in general a pair of surfaces containing the principal centers of curvature of the given surface. This pair is called the caustic (of two sheets) of the surface. Either or both sheets of the caustic may degenerate into a line, or, for a spherical surface, they degenerate to two coincident points.

Caustic

- If a sheet is not degenerate, then the normal to the given surface passing through the principal center of curvature lying in the caustic sheet is tangent to the sheet at that point, and all neighboring normal lines lie on the same side.

Caustic from a spherical mirror





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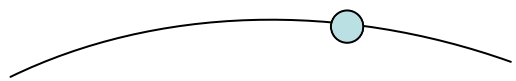
Principal curvatures

$$z(x, y) = ax + by + cx^2 + dxy + ey^2$$

$$z(x, y) = cx^2 + ey^2$$

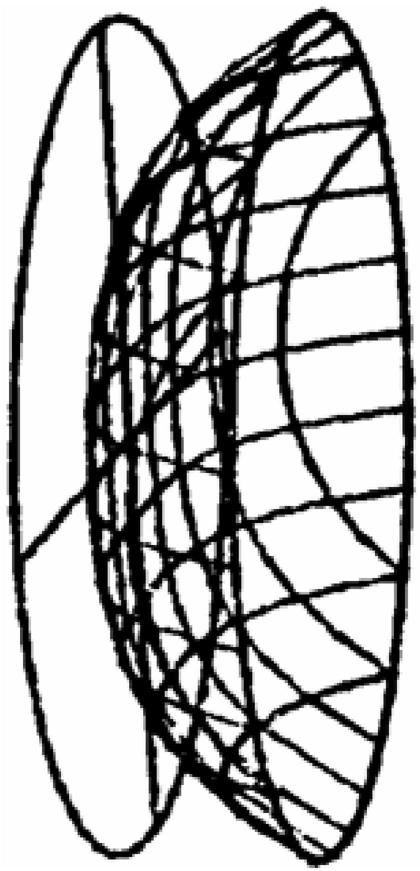
$$z(x, y) = \frac{x^2}{2R_s} + \frac{y^2}{2R_t}$$

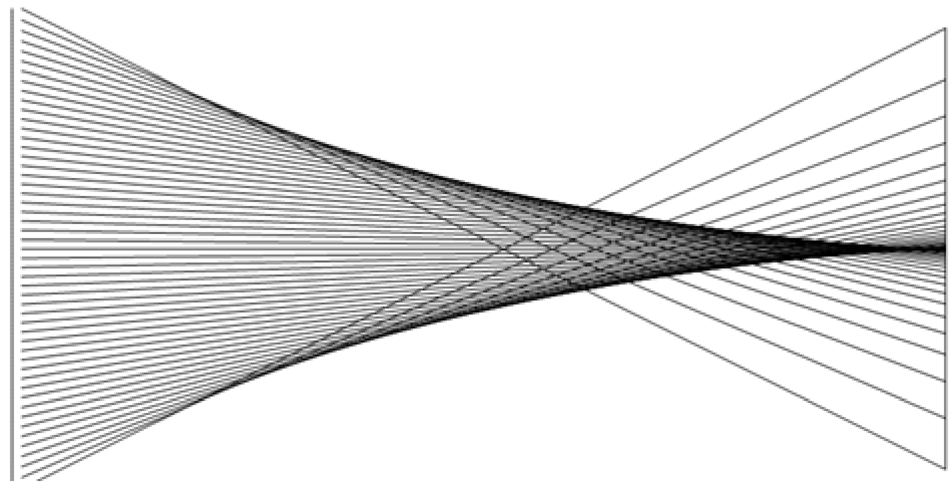
Curvature of a curve



$$c = \frac{\dfrac{\partial^2 y}{\partial x^2}}{\left(1 + \left(\dfrac{\partial y}{\partial x}\right)^2\right)^{3/2}} \approx \frac{\partial^2 y}{\partial x^2}$$

Spherical aberration case





Need to determine the caustic shape

The Laplacian

$$\nabla_\rho \left(\nabla_H \bar{W} \left(\vec{H}, \vec{\rho} \right) \right) = \frac{\partial}{\partial \rho_h} \left(\nabla_H \bar{W} \left(\vec{H}, \vec{\rho} \right) \cdot \vec{h} \right) + \frac{\partial}{\partial \rho_i} \left(\nabla_H \bar{W} \left(\vec{H}, \vec{\rho} \right) \cdot \vec{i} \right)$$

$$= \frac{\partial}{\partial \vec{\rho}} \frac{\partial}{\partial \vec{H}} W \left(\vec{H}, \vec{\rho} \right) + \frac{1}{\vec{\rho} \cdot \vec{i}} \frac{\partial}{\partial \vec{H}} W \left(\vec{H}, \vec{\rho} \right) \cdot \vec{i}$$

$$\nabla_\rho \left(\nabla_\rho W \left(\vec{H}, \vec{\rho} \right) \right) = \frac{\partial}{\partial \vec{\rho}} \frac{\partial}{\partial \vec{\rho}} W \left(\vec{H}, \vec{\rho} \right) + \frac{1}{\vec{\rho} \cdot \vec{i}} \frac{\partial}{\partial \vec{\rho}} W \left(\vec{H}, \vec{\rho} \right) \cdot \vec{i}$$

Spherical aberration external caustic

- The external caustic is the locus of meridional rays centers of curvature
- On axis the wavefront deformation has no curvature (no second-order terms)
- The caustic is found by adding focus so that for a given aperture zone, the wavefront has no curvature.

$$c = \frac{\dfrac{\partial^2 y}{\partial x^2}}{\left(1+\left(\dfrac{\partial y}{\partial x}\right)^2\right)^{3/2}} \approx \frac{\partial^2 y}{\partial x^2}$$

External caustic

$$0 = \left(12W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right) + 2W_{020}\right)$$

$$W_{020} = -6W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)$$

We add defocus to bring the caustic point to the Gaussian image plane for a given aperture zone ρ.

In longitudinal terms this is, $\Delta z' = -2\dfrac{\Delta W_{020}}{n'}\dfrac{1}{u'^2}$

$$\Delta z' = -\frac{n'}{\left(n'u'\right)^2}2W_{020} = \frac{n'}{\left(n'u'\right)^2}12W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)$$

External caustic

The transverse ray aberration is,

$$\bar{y}_I \Delta\vec{H} = \frac{1}{n'u'}\vec{\nabla}_\rho W\left(\vec{H},\vec{\rho}\right)$$

$$= \frac{1}{n'u'}\vec{\nabla}_\rho \left(W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2 + W_{020}\left(\vec{\rho}\cdot\vec{\rho}\right)\right)$$

$$= \frac{1}{n'u'}\left(4W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} + 2W_{020}\vec{\rho}\right)$$

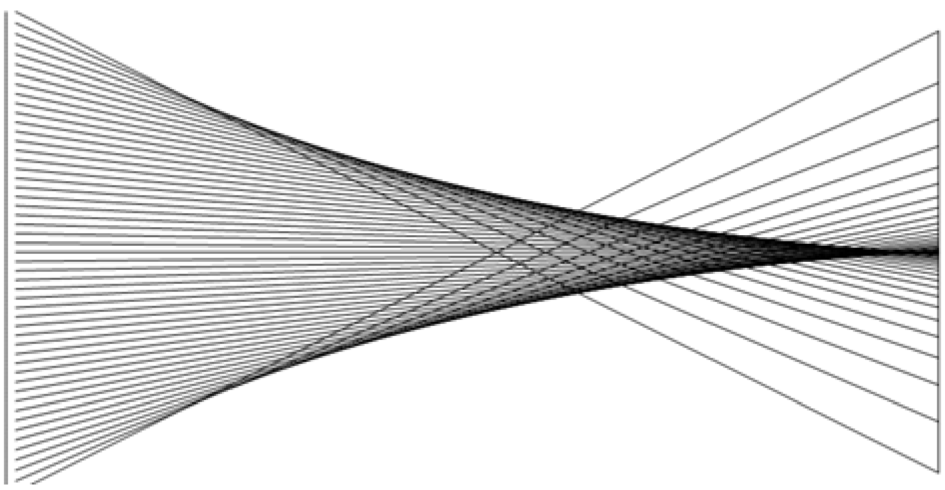
$$= \frac{1}{n'u'}\left(4W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho} - 12W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}\right)$$

$$= -\frac{1}{n'u'}8W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}$$

Caustic parametric equations

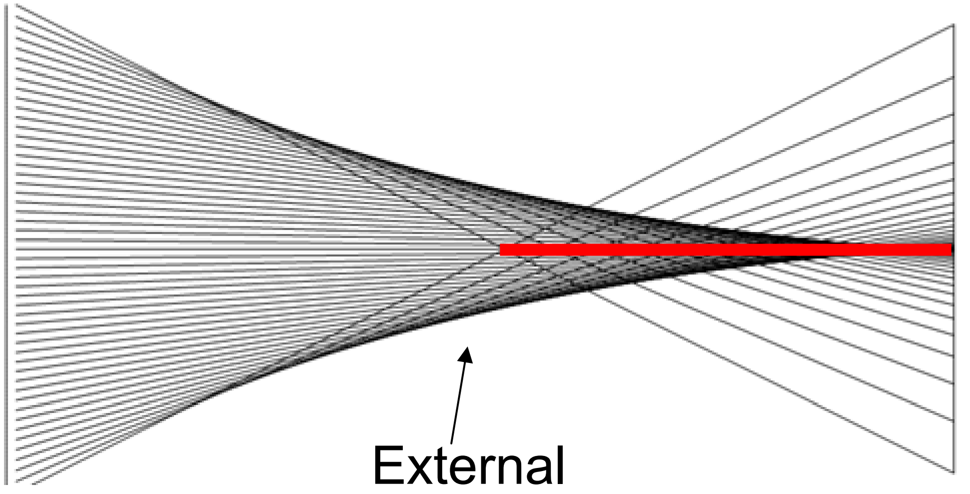
$$\Delta z' = -\frac{n'}{\left(n'u'\right)^2} 12 W_{040} \left(\vec{\rho}\cdot\vec{\rho}\right)$$

$$\bar{y}_I \Delta\vec{H} = -\frac{1}{n'u'} 8 W_{040} \left(\vec{\rho}\cdot\vec{\rho}\right)\vec{\rho}$$



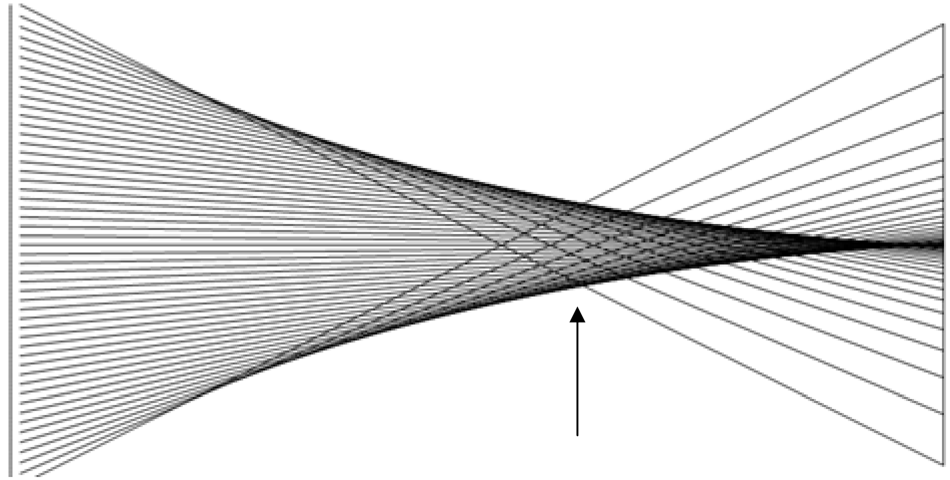
Internal caustic

The second sheet is the internal caustic and it is degenerated into a line that coincides with the optical axis



Minimum Circle

$$\vec{\rho} = \rho \vec{g}$$



The minimum circle is located
where the external caustic meets the marginal ray

$$\bar{y}_I \Delta\vec{H} = -\frac{1}{n'u'} 8W_{040}\rho_c^3 \vec{g} = -\left(4W_{040}\left(1^3\right) + 2W_{020}\left(1\right)\right)\vec{g}$$

Caustic

Marginal
ray

Minimum Circle

$$\bar{y}_I \Delta\vec{H} = -\frac{1}{n'u'} 8W_{040}\rho_c^3 \vec{g} = -\frac{1}{n'u'}\left(4W_{040}\left(1^3\right) + 2W_{020}\left(1\right)\right)\vec{g}$$

For caustic $\quad W_{020} = -6W_{040}\rho_c^2$

$$8W_{040}\rho_c^3 = 4W_{040} - 12W_{040}\rho_c^2$$

HW Fill details

$\longrightarrow$ $\rho_c = -1$, $\rho_c = \frac{1}{2}$ $\longrightarrow$ $W_{020} = -6W_{040}\left(\frac{1}{4}\right) = -\frac{3}{2}W_{040}$

Significant locations

	ΔW_{020}	$\frac{(n'u')^2 \Delta z'}{n'}$	$(n'u')\bar{y}_I \lvert\Delta\vec{H}\rvert$	
Paraxial focus	0	0	$-4W_{040}$	Marginal cone
Minimum circle	$-\frac{3}{2}W_{040}$	$-3W_{040}$	$-W_{040}$	Marginal cone=caustic
Marginal focus	$-2W_{040}$	$-4W_{040}$	$-\frac{8}{3\sqrt{3}}W_{040}$	Caustic $\rho_c^3 = \frac{1}{3}$

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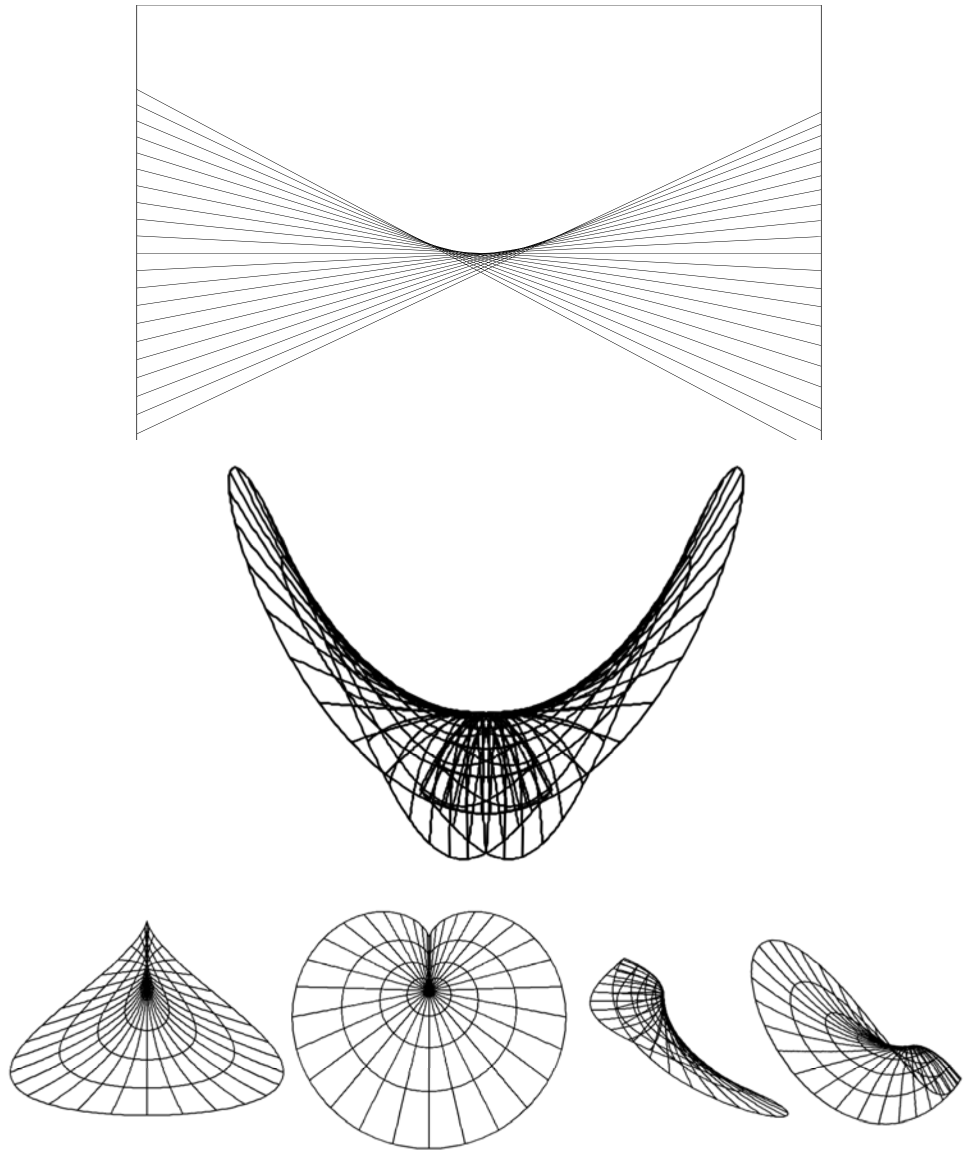
Coma caustic



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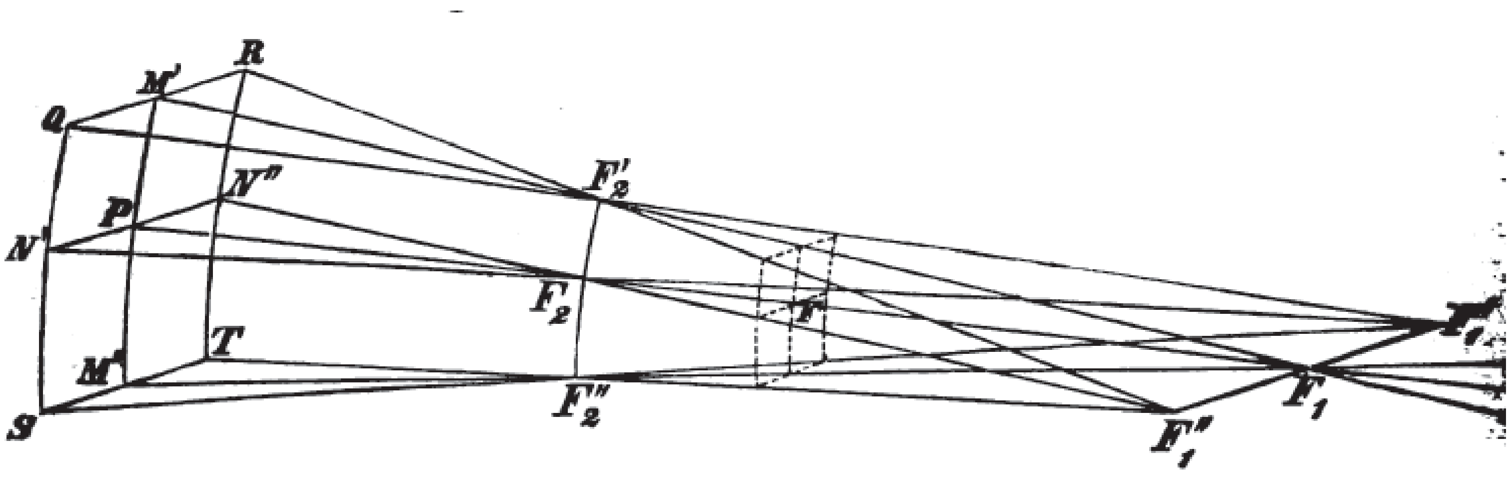
Astigmatism



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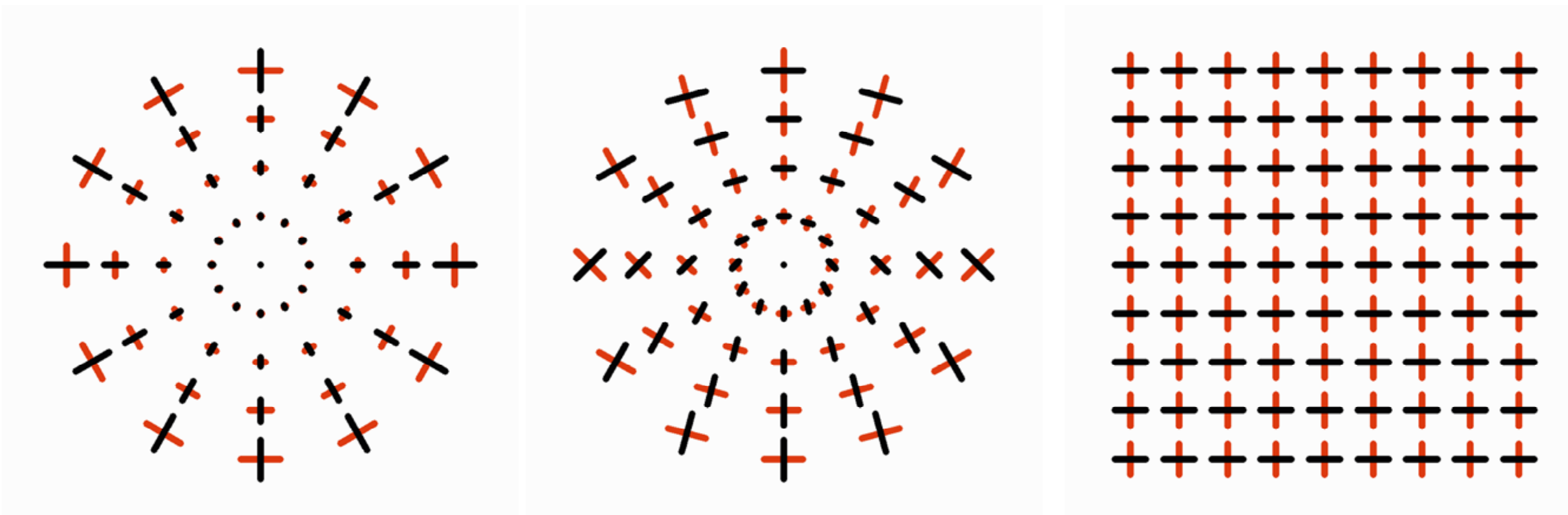
Wavefront principal curvatures orientation and magnitude



As a function of the exit pupil position

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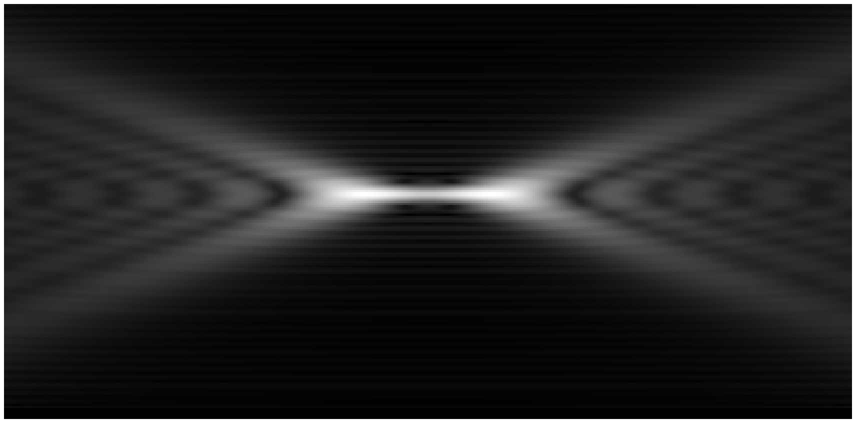
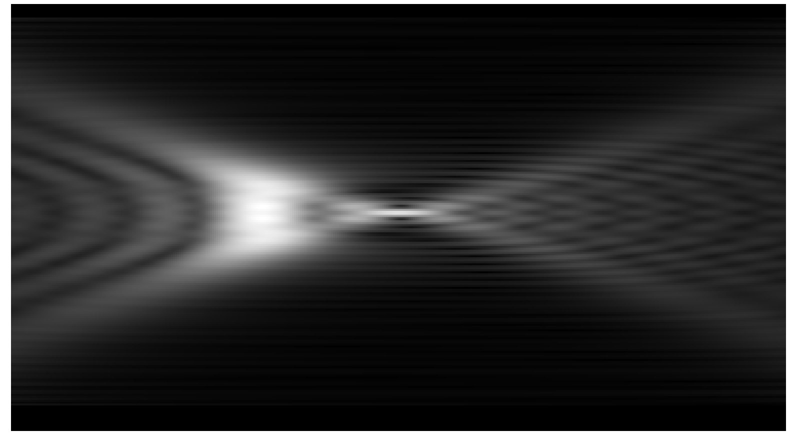
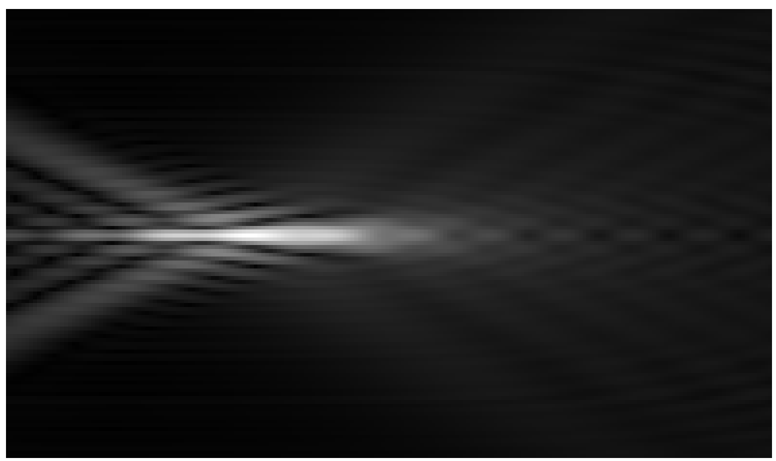
Diffraction images along the axis





Two waves:
Spherical
Coma
Astigmatism



Which one is which?

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Summary

- The concept of caustic
- Principal curvatures
- Significant location along the axis
- Spherical aberration caustic
- Coma caustic
- Astigmatism caustic

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