OPTI 517
Image Quality

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Why is Image Quality Important?

- Resolution of detail
  - Smaller blur sizes allow better reproduction of image details
  - Addition of noise can mask important image detail

Original  Blur added  Noise added  Pixelated
Step One - What is Your Image Quality (IQ) Spec?

There are many metrics of image quality:

- Geometrical based (e.g., spot diagrams, RMS wavefront error)
- Diffraction based (e.g., PSF, MTF)
- Other (F-theta linearity, uniformity of illumination, etc.)

It is imperative that you have a specification for image quality when you are designing an optical system:

- Without it, you don't know when you are done designing!
You vs. the Customer

- Different kinds of image quality metrics are useful to different people
  - Customers usually work with performance-based specifications
    - MTF, ensquared energy, distortion, etc.
  - Designers often use IQ metrics that mean little to the customer
    - E.g., ray aberration plots and field plots
    - These are useful in the design process, but they are not end-product specs
- In general, you will be working to an end-product specification, but will probably use other IQ metrics during the design process
  - Often the end-product specification is difficult to optimize to or may be time consuming to compute
- Some customers do not express their image quality requirements in terms such as MTF or ensquared energy
  - They know what they want the optical system to do
- It is up to the optical engineer (in conjunction with the system engineer) to translate the customer's needs into a numerical specification suitable for optimization and image quality analysis
When to Use Which IQ Metric

• The choice of appropriate IQ metric usually depends on the application of the optical system
  • Long-range targets where the object is essentially a point source
    • Example might be an astronomical telescope
    • Ensquared energy or RMS wavefront error might be appropriate
  • Ground-based targets where the details of the object are needed to determine image features
    • Example is any kind of image in which you need to see detail
    • MTF would be a more appropriate metric
  • Laser scanning systems
    • A different type of IQ metric such as the variation from F-theta distortion

• The type of IQ metric may be part of the lens specification or may be a derived requirement flowed down to the optical engineer from systems engineering
  • Do not be afraid to question these requirements
  • Often the systems engineering group doesn't really understand the relationship between system performance and optical metrics
Image Quality Metrics

- The most commonly used geometrical-based image quality metrics are:
  - Ray aberration curves
  - Spot diagrams
  - Seidel aberrations
  - Encircled (or ensquared) energy
  - RMS wavefront error
  - Modulation transfer function (MTF)

- The most commonly used diffraction-based image quality metrics are:
  - Point spread function (PSF)
  - Encircled (or ensquared) energy
  - MTF
  - Strehl Ratio
Ray Aberration Curves

These are by far the image quality metric most commonly used by optical designers during the design process.

Ray aberration curves trace fans of rays in two orthogonal directions:

- They then map the image positions of the rays in each fan relative to the chief ray vs. the entrance pupil position of the rays.
Graphical Description of Ray Aberration Curves

Ray aberration curves map the image positions of the rays in a fan
- The plot is image plane differences from the chief ray vs. position in the fan

Ray aberration curves are generally computed for a fan in the YZ plane and a fan in the XZ plane
- This omits skew rays in the pupil, which is a failing of this IQ metric
Transverse vs. Wavefront Ray Aberration Curves

Ray aberration curves can be transverse (linear) aberrations in the image vs. pupil position or can be OPD across the exit pupil vs. pupil position.

- The transverse ray errors are related to the slope of the wavefront curve:
  \[
  \varepsilon_y(x_p, y_p) = -(R/r_p) \frac{\partial W(x_p, y_p)}{\partial y_p}
  \]
  \[
  \varepsilon_x(x_p, y_p) = -(R/r_p) \frac{\partial W(x_p, y_p)}{\partial x_p}
  \]
  \[
  R/r_p = -1/(n'u') \approx 2 f/#
  \]

Example curves for pure defocus:

- Transverse:
  - 0.001 inch

- Wavefront error:
  - 1.0 wave
More on Ray Aberration Curves

The shape of the ray aberration curve can tell what type of aberration is present in the lens for that field point (transverse curves shown)

- Defocus
- Coma
- Third-order spherical
- Astigmatism
The Spot Diagram

- The spot diagram is readily understood by most engineers (and customers)
- It is a diagram of how spread out the rays are in the image
  - The smaller the spot diagram, the better the image
  - This is **geometrical only**; diffraction is ignored
- It is useful to show the detector size (and/or the Airy disk diameter) superimposed on the spot diagram

- The shape of the spot diagram can often tell what type of aberrations are present in the image

Detector outline

Different colors represent different wavelengths
Main Problem With Spot Diagrams

Å The main problem is that spots in the spot diagram don't convey intensity
  ï A ray intersection point in the diagram does not tell the intensity at that point

The on-axis image appears spread out in the spot diagram, but in reality, it has a tight core with some surrounding low-intensity flare.
Diffraction

Some optical systems give point images (or near point images) of a point object when ray traced geometrically (e.g., a parabola on-axis)

However, there is in reality a lower limit to the size of a point image

This lower limit is caused by diffraction

The diffraction pattern is usually referred to as the Airy disk
Size of the Diffraction Image

Â The diffraction pattern of a perfect image has several rings
   ï The center ring contains ~84% of the energy, and is usually considered to be the "size" of the diffraction image

![Diagram of diffraction pattern with labeled dimensions d, showing the relationship between the diameter of the first ring and the focal length and f/number]

Â The diameter of the first ring is given by $d \approx 2.44 \lambda f/#$
   ï This is independent of the focal length; it is only a function of the wavelength and the f/number
   ï The angular size of the first ring $\beta = d/F \approx 2.44 \lambda/D$

Â When there are no aberrations and the image of a point object is given by the diffraction spread, the image is said to be **diffraction-limited**

Very important !!!!
For both systems, the Airy disk diameter is the same size
\[ d = 2.44 \lambda \frac{l}{f/\#} \]

For both systems, the irradiance of the background at the image is the same
\[ E_B = L_B \left(\frac{\pi}{4f^2}\right) \]

The flux forming the image from the larger system is larger by \((D_2/D_1)^2\)
- We get more energy in the image, so the signal-to-noise ratio (SNR) is increased by \((D_2/D_1)^2\)
- This is important for astronomy and other forms of point imagery
Spot Size vs. the Airy Disk

- **Regime 1** — Diffraction-limited
  - Airy disk diameter
  - Point image (geometrically)

- **Regime 2** — Near diffraction-limited
  - Non-zero geometric blur, but smaller than the Airy disk

- **Regime 3** — Far from diffraction-limited
  - Airy disk diameter
  - Geometric blur significantly larger than the Airy disk

Image intensity

Strehl = 1.0

Strehl ≥ 0.8

Strehl ~ 0
Point Spread Function (PSF)

This is the image of a point object including the effects of diffraction and all aberrations.

- Intensity peak of the PSF relative to that of a perfect lens (no wavefront error) is the Strehl Ratio.
- Airy disk (diameter of the first zero).
Diffraction Pattern of Aberrated Images

When there is aberration present in the image, two effects occur:

- Depending on the aberration, the shape of the diffraction pattern may become skewed.
- There is less energy in the central ring and more in the outer ring.

\[
\text{Strehl Ratio} = \exp(-2\pi\Phi^2) \text{ for small amounts of RMS wavefront error } \Phi
\]

- Strehl Ratio(\(\Phi=0.07\)) \(\approx 0.80\) often considered to be \(\approx\) diffraction-limited.
Figure 11.24  Point spread functions for different amounts of defocus. (a) 0.125 wave (P-V); 0.037 wave rms; 0.95 Strehl. (b) 0.25 wave (P-V); 0.074 wave rms; 0.80 Strehl. (c) 0.50 wave (P-V); 0.148 wave rms; 0.39 Strehl. (d) 1.00 wave (P-V); 0.297 wave rms; 0.00 Strehl.
PSF vs. Third-order Spherical Aberration

Figure 11.25  Point spread functions for different amounts of third-order spherical aberration. (a) 0.125 wave (P-V); 0.040 wave rms; 0.94 Strehl. (b) 0.25 wave (P-V); 0.080 wave rms; 0.78 Strehl. (c) 0.50 wave (P-V); 0.159 wave rms; 0.37 Strehl. (d) 1.00 wave (P-V); 0.318 wave rms; 0.08 Strehl. Note: Reference sphere centered at 0.5LA₀ (midway between marginal and paraxial foci).
PSF vs. Third-order Coma

Figure 11.26  Point spread functions for different amounts of third-order coma. (a) 0.125 wave (P-V); 0.031 wave rms; 0.96 Strehl. (b) 0.25 wave (P-V); 0.061 wave rms; 0.86 Strehl. (c) 0.50 wave (P-V); 0.123 wave rms; 0.65 Strehl. (d) 1.00 wave (P-V); 0.25 wave rms; 0.18 Strehl. Note: P-V OPD reference sphere centered at 0.25Coma_T from chief ray intersection point. rms OPD reference sphere centered at 0.226Coma_T from chief ray intersection point.
Figure 11.27  Point spread functions for different amounts of astigmatism. (a) 0.125 wave (P-V); 0.026 wave rms; 0.97 Strehl. (b) 0.25 wave (P-V); 0.052 wave rms; 0.90 Strehl. (c) 0.50 wave (P-V); 0.104 wave rms; 0.65 Strehl. (d) 1.00 wave (P-V); 0.207 wave rms; 0.18 Strehl. Note: Reference sphere centered midway between sagittal and tangential foci.
PSF for Strehl = 0.80

Defocus  3rd-order SA  Balanced 3rd and 5th-order SA

Astigmatism  Coma

Figure 11.29  Point spread functions for five different aberrations, each with a Strehl ratio of 0.80 (the Marechal criterion). In each case the center of the reference sphere is located to minimize the rms OPD, which is 0.075 wave for all five aberrations. (a) Defocus: 0.25 wave (P-V). (b) Third-order spherical: 0.235 wave (P-V). (c) Balanced third- and fifth-order spherical: 0.221 wave (P-V). (d) Astigmatism: 0.359 wave (P-V). (e) Coma: 0.305 wave (P-V).
Encircled or Ensquared Energy

Encircled or ensquared energy is the ratio of the energy in the PSF that is collected by a single circular or square detector to the total amount of energy that reaches the image plane from that object point.

- This is a popular metric for system engineers who, reasonably enough, want a certain amount of collected energy to fall on a single pixel.
- It is commonly used for systems with point images, especially systems which need high signal-to-noise ratios.

For %EE specifications of 50-60% this is a reasonably linear criterion:

- However, the specification is more often 80%, or even worse 90%, energy within a near diffraction-limited diameter.
- At the 80% and 90% levels, this criterion is highly non-linear and highly dependent on the aberration content of the image, which makes it a poor criterion, especially for tolerancing.
Ensquared energy on a detector of same order of size as the Airy disk
Perfect lens, f/2, 10 micron wavelength, 50 micron detector

Approximately 85% of the energy is collected by the detector
Modulation Transfer Function (MTF)

- MTF is the Modulation Transfer Function
- Measures how well the optical system images objects of different sizes
  - Size is usually expressed as spatial frequency (1/size)
- Consider a bar target imaged by a system with an optical blur
  - The image of the bar pattern is the geometrical image of the bar pattern convolved with the optical blur

- MTF is normally computed for sine wave input, and not square bars to get the response for a pure spatial frequency
- Note that MTF can be geometrical or diffraction-based
Computing MTF

The MTF is the amount of modulation in the image of a sine wave target.

- At the spatial frequency where the modulation goes to zero, you can no longer see details in the object of the size corresponding to that frequency.

The MTF is plotted as a function of spatial frequency (1/sine wave period).

\[
MTF = \frac{\text{Max} - \text{Min}}{\text{Max} + \text{Min}}
\]
MTF of a Perfect Image

For an aberration-free image and a round pupil, the MTF is given by

\[
\text{MTF}(f) = \frac{2}{\pi} \left[ \phi - \cos \phi \sin \phi \right]
\]

\[
\phi = \cos^{-1} \left( \frac{f}{f_{co}} \right) = \cos^{-1} \left( \frac{\lambda f}{2NA} \right)
\]

Cutoff frequency
\[
f_{co} = \frac{1}{\lambda.f/\#}
\]
Abbe’s Construct for Image Formation

Abbe developed a useful framework from which to understand the diffraction-limiting spatial frequency and to explain image formation in microscopes.

If the first-order diffraction angle from the grating exceeds the numerical aperture (NA = 1/(2f/#)), no light will enter the optical system for object features with that characteristic spatial period.
Example MTF Curve

MTF depends on target orientation

S = Sagittal
R = Radial
T = Tangential
MTF as an Autocorrelation of the Pupil

- The MTF is usually computed by lens design programs as the autocorrelation of the OPD map across the exit pupil

Relative spatial frequency = spacing between shifted pupils (cutoff frequency = pupil diameter)

Perfect MTF = overlap area / pupil area

MTF is computed as the normalized integral over the overlap region of the difference between the OPD map and its shifted complex conjugate

Complex OPD computed for many points across the pupil

Overlap area
Typical MTF Curves

MTF curves are different for different points across the FOV.

MTF is a function of the focus.

MTF is a function of the spectral weighting.

Diffraction-limited MTF (as good as it can get)
Phase Shift of the OTF

Å Since OPD relates to the phase of the ray relative to the reference sphere, the pupil autocorrelation actually gives the OTF (optical transfer function), which is a complex quantity
  ï MTF is the real part (modulus) of the OTF

OTF = Optical Transfer Function
MTF = Modulus of the OTF
PTF = Phase of the OTF

When the OTF goes negative, the phase is \( \pi \) radians
What Does OTF $< 0$ Mean?

- When the OTF goes negative, it is an example of contrast reversal.
Example of Contrast Reversal

At best focus

Defocused
More on Contrast Reversal

![Diagram showing contrast reversal](image-url)
Effect of Strehl = 0.80

When the Strehl Ratio = 0.80 or higher, the image is often considered to be equivalent in image quality to a diffraction-limited image (Maréchal Criterion).

The MTF in the mid-range spatial frequencies is reduced by the Strehl ratio.

![Graph](image-url)
Shannon has shown that the MTF can be approximated as a product of the diffraction-limited MTF (DTF) and an aberration transfer function (ATF)

\[ DTF(\nu) = \frac{2}{\pi} \left[ \cos^{-1} \nu - \nu \sqrt{1 - \nu^2} \right] \]

\[ \nu = \frac{f}{f_{co}} \]

\[ ATF(\nu) = 1 - \left( \frac{W_{\text{rms}}}{0.18} \right)^2 \left( 1 - 4(\nu - 0.5)^2 \right) \]
Demand Contrast Function

The eye requires more modulation for smaller objects to be able to resolve them.

- The amount of modulation required to resolve an object is called the demand contrast function.
- This and the MTF limits the highest spatial frequency that can be resolved.

The limiting resolution is where the Demand Contrast Function intersects the MTF.

System A will produce a superior image although it has the same limiting resolution as System B.

System A has a lower limiting resolution than System B even though it has higher MTF at lower frequencies.
Example of Different MTFs on RIT Target

MTFs for RIT Targets

Frequency (cycles/mm)

MTF for right column image

MTF for middle column image

MTF for left column image

Images of RIT targets with different MTFs.
Central Obscurations

- In on-axis telescope designs, the obscuration caused by the secondary mirror is typically 30-50% of the diameter
  - Any obscuration above 30% will have a noticeable effect on the Airy disk, both in terms of dark ring location and in percent energy in a given ring (energy shifts out of the central disk and into the rings)

- Contrary perhaps to expectations, as the obscuration increases the diameter of the first Airy ring decreases (the peak is the same, and the loss of energy to the outer rings has to come from somewhere)
Central Obscurations

Central obscurations, such as in a Cassegrain telescope, have two deleterious effects on an optical system:

- The obscuration causes a loss in energy collected (loss of area)
- The obscuration causes a loss of MTF

\[
\begin{align*} 
A & & S_o/S_m = 0.00 \\
B & & S_o/S_m = 0.25 \\
C & & S_o/S_m = 0.50 \\
D & & S_o/S_m = 0.75 
\end{align*}
\]
Coherent Illumination

Â Incoherent illumination fills the whole entrance pupil
Â Partially coherent illumination fills only part of the entrance pupil
    i Coherent illumination essentially only fills a point in the entrance pupil

Figure 11.20 (a–c) The MTF with coherent illumination. (d–f) The MTF with semicoherent illumination (which partially fills the pupil).
MTF of Partially Coherent Illumination

Figure 11.21  MTF vs. frequency for a partially filled pupil (semitransparent illumination). Numbers are the ratio of illuminating system NA to optical system NA.
Partial Coherent Image of a 3-Bar Target

DIFFRACTION INTENSITY PROFILE
PARTIALLY COHERENT ILLUMINATION

WAVELENGTH WEIGHT
500.0 NM 1

GEOMETRICAL SHADOW
INC (0.00, 0.00) R
1.50 (0.00, 0.00) R
1.00 (0.00, 0.00) R
0.50 (0.00, 0.00) R
0.00 (0.00, 0.00) R

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RELATIVE INTENSITY

DISPLACEMENT ON IMAGE SURFACE (MICRONS)

DEFOCUSING 0.00000
Example of Elbows Imaged in Partially Coherent Light

With 1 wave of spherical aberration
The Main Aberrations in an Optical System

- **Defocus**: the focal plane is not located exactly at the best focus position
- **Chromatic aberration**: the axial and lateral shift of focus with wavelength
- **The Seidel aberrations**
  - Spherical Aberration
  - Coma
  - Astigmatism
  - Distortion
  - Curvature of field
Defocus

Technically, defocus is not an aberration in that it can be corrected by simply refocusing the lens.

However, defocus is an important effect in many optical systems.

When maximum OPD = $\lambda/4$, you are at the Rayleigh depth of focus $= 2\lambda (f/#)^2$.
Defocus Ray Aberration Curves

Wavefront map

Spot diagram

Wavefront error

Transverse ray aberration
MTF of a Defocused Image

As the amount of defocus increases, the MTF drops accordingly.

A  OPD = 0
B  OPD = \lambda/4
C  OPD = \lambda/2
D  OPD = 3\lambda/4
E  OPD = \lambda
Sources of Defocus

- One obvious source of defocus is the location of the object
  - For lenses focused at infinity, objects closer than infinity have defocused images
  - There's nothing we can do about this (unless we have a focus knob)

- Changes in temperature
  - As the temperature changes, the elements and mounts change dimensions and the refractive indices change
  - This can cause the lens to go out of focus
  - This can be reduced by design (material selection)

- Another source is the focus procedure
  - There are two possible sources of error here
    - Inaccuracy in the measurement of the desired focus position
    - Resolution in the positioning of the focus (e.g., shims in 0.001 inch increments)
  - The focus measurement procedure and focus position resolution must be designed to not cause focus errors which can degrade the image quality beyond the IQ specification
Chromatic Aberration

Chromatic aberration is caused by the lens's refractive index changing with wavelength. The shorter wavelengths focus closer to the lens because the refractive index is higher for the shorter wavelengths.

The shorter wavelengths focus closer to the lens because the refractive index is higher for the shorter wavelengths.
Computing Chromatic Aberration

The chromatic aberration of a lens is a function of the dispersion of the glass.

Dispersion is a measure of the change in index with wavelength.

It is commonly designated by the Abbe V-number for three wavelengths.

For visible glasses, these are F (486.13), d (587.56), C (656.27).

For infrared glasses, they are typically 3, 4, 5 or 8, 10, 12 microns.

\[ V = \frac{(n\text{middle}-1)}{(n\text{short} - n\text{long})} \]

For optical glasses, V is typically in the range 35-80.

For infrared glasses, they vary from 50 to 1000.

The axial (longitudinal) spread of the short wavelength focus to the long wavelength focus is \( F/V \).

Example 1: N-BK7 glass has a V-value of 64.4. What is the axial chromatic spread of an N-BK7 lens of 100 mm focal length?

Answer: \( \frac{100}{64.4} = 1.56 \text{ mm} \)

Note that if the lens were f/2, the diffraction DOF = \( \pm 2\lambda f^2 = \pm 0.004 \text{ mm} \).

Example 2: Germanium has a V-value of 942 (for 8-12 \( \mu \)). What is the axial chromatic spread of a germanium lens of 100 mm focal length?

Answer: \( \frac{100}{942} = 0.11 \text{ mm} \)

Note: DOF(f/2) = \( \pm 2\lambda f^2 = \pm 0.08 \text{ mm} \).
Chromatic Aberration Example - Germanium Singlet

We want to use an f/2 germanium singlet over the 8 to 12 micron band.

Question - What is the longest focal length we can have and not need to color correct? (assume an asphere to correct any spherical aberration)

Answer

- Over the 8-12 micron band, for germanium $V = 942$
- The longitudinal defocus = $F / V = F / 942$
- The 1/4 wave depth of focus is $\pm 2\lambda f^2$
- Equating these and solving gives $F = 4*942*\lambda f^2 = 150$ mm

Strehl = 0.86
Focus Shift vs. Wavelength (Germanium singlet)

![Graph showing the relationship between focus shift and wavelength. The x-axis represents wavelength in nm, ranging from 800 to 12000 nm. The y-axis represents focus shift in nm, ranging from -0.1000 to 0.1000 nm. The graph includes a red and a blue dashed line, indicating diffraction-limited depth of focus.](image-url)
Correcting Chromatic Aberration

Chromatic aberration is corrected by a combination of two glasses:
- The positive lens has low dispersion (high V number) and the negative lens has high dispersion (low V number).
- The red and blue wavelengths focus together.
- The green (or middle) wavelength still has a focus error.

This will correct primary chromatic aberration:
- The red and blue wavelengths focus together.
- The green (or middle) wavelength still has a focus error.
- This residual chromatic spread is called secondary color.
Secondary Color

Secondary color is the residual chromatic aberration left when the primary chromatic aberration is corrected.

Secondary color can be reduced by selecting special glasses:

- These glasses cost more (naturally)
Lateral Color

Lateral color is a change in focal length (or magnification) with wavelength
- This results in a different image size with wavelength
- The effect is often seen as color fringes at the edge of the FOV
- This reduces the MTF for off-axis images
Higher-order Chromatic Aberrations

Å For broadband systems, the chromatic variation in the third-order aberrations are often the most challenging aberrations to correct (e.g., spherochromatism, chromatic variation of coma, chromatic variation of astigmatism)

ï These are best studied with ray aberration curves and field plots
The Seidel Aberrations

These are the classical aberrations in optical design:

- Spherical aberration
- Coma
- Astigmatism
- Distortion
- Curvature of field

These aberrations, along with defocus and chromatic aberrations, are the main aberrations in an optical system.
The Importance of Third-order Aberrations

- The ultimate performance of any unconstrained optical design is almost always limited by a specific aberration that is an intrinsic characteristic of the design form.

- A familiarity with aberrations and lens forms is an important ingredient in a successful optimization that makes optimal use of the time available to accomplish the design.

- A knowledge of the aberrations
  - Allows "spotting" lenses that are at the end of the road with respect to optimization.
  - Gives guidance in what direction to "kick" a lens that has strayed from the optimal solution.
Orders of Aberrations

Å The various Seidel aberrations have different dependencies on the aperture (EPD) radius $y$ and the field angle $\theta = \text{field height/focal length}$

Å For the third-order aberrations, the variation with $y$ and $\theta$ are as follows:

<table>
<thead>
<tr>
<th>Aberration</th>
<th>Aperture</th>
<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal spherical aberration</td>
<td>$y^2$</td>
<td>-</td>
</tr>
<tr>
<td>Transverse spherical aberration</td>
<td>$y^3$</td>
<td>-</td>
</tr>
<tr>
<td>Coma</td>
<td>$y^2$</td>
<td>$\theta$</td>
</tr>
<tr>
<td>Astigmatism</td>
<td>-</td>
<td>$\theta^2$</td>
</tr>
<tr>
<td>Field curvature</td>
<td>-</td>
<td>$\theta^2$</td>
</tr>
<tr>
<td>Linear distortion</td>
<td>-</td>
<td>$\theta^3$</td>
</tr>
<tr>
<td>Percent distortion</td>
<td>-</td>
<td>$\theta^2$</td>
</tr>
<tr>
<td>Axial chromatic aberration</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Lateral chromatic aberration</td>
<td>-</td>
<td>$\theta$</td>
</tr>
</tbody>
</table>

Å Knowing the functional dependence of an aberration will allow you to estimate the change in a given aberration for a change in $f$/number or field angle
Spherical Aberration

Â Spherical aberration is an on-axis aberration

Â Rays at the outer parts of the pupil focus closer to or further from the lens than the paraxial focus

This is referred to as undercorrected spherical aberration (marginal rays focus closer to the lens than the paraxial focus)

Â The magnitude of the (third-order) spherical aberration goes as the cube of the aperture (going from f/2 to f/1 increases the SA by a factor of 8)
Third-order SA Ray Aberration Curves

Wavefront map

Spot diagram

Wavefront error

Transverse ray aberration curve
Spherical Aberration

- Marginal focus
- Minimum spot size
- Minimum RMS WFE
- Paraxial focus

L

½ L

¾ L

Marginal focus
Minimum spot size
Minimum RMS WFE
Paraxial focus

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Scaling Laws for Spherical Aberration

Spot size goes as the cube of the EPD (or inverse cube of the f/#)

Spot size not dependent on field position
Spherical Aberration vs. Lens Shape

The spherical aberration is a function of the lens bending, or shape of the lens.
Spherical Aberration vs. Refractive Index

Å Spherical aberration is reduced with higher index materials
   Ŷ Higher indices allows shallower radii, allowing less variation in incidence angle across the lens

\[ n = 1.50 \]

Notice the bending for minimum SA is a function of the index

\[ n = 1.95 \]
Spherical Aberration vs. Index and Bending

\[ \beta \text{ at } K_{\min} = r^3 \varphi^3 \frac{4n^2 - n}{16(n - 1)^2(n + 2)} \]

\[ n = 1.5 \]

\[ n = 2.0 \]

\[ n = 3.0 \]

\[ n = 4.0 \]
Example - Germanium Singlet

Â We want an f/2 germanium singlet to be used at 10 microns (0.01 mm)

Â Question - What is the longest focal length we can have and not need aspherics (or additional lenses) to correct the spherical aberration?

Â Answer

ï Diffraction Airy disk angular size is $\beta_{\text{diff}} = 2.44 \frac{\lambda}{D}$
ï Spherical aberration angular blur is $\beta_{\text{sa}} = 0.00867 / f^3$
ï Equating these gives $D = 2.44 \lambda f^3 / 0.00867 = 22.5$ mm
ï For f/2, this gives $F = 45$ mm

Strehl = 0.91
Spherical Aberration vs. Number of Lenses

Spherical aberration can be reduced by splitting the lens into more than one lens.

- \( \text{SA} = 1 \) (arbitrary units)
- \( \text{SA} = 1/4 \) (arbitrary units)
- \( \text{SA} = 1/9 \) (arbitrary units)
Spherical Aberration vs. Number of Lenses

\[ \Sigma_{sc} = \sum_{i=1}^{i=N} \frac{y^2 \Phi_i}{8} \frac{4N-1-4i(i-1)(N-1)^2}{(N-1)^2(N+2)} \]

WHERE \( i = \) THE NUMBER OF ELEMENTS
\( \Phi = \) TOTAL POWER = \( i \times \phi_i \)
\( j = \) ELEMENT NUMBER

**Figure 3.2** The spherical aberration of one, two, three, and four thin positive elements, each bent for minimum spherical aberration, plotted as a function of the index of refraction, and showing the reduction in the amount of aberration produced by splitting a single element into two or more elements (of the same total power). Each plot is labeled with \( i \), the number of elements in the set. (The object is at infinity.)
Spherical Aberration and Aspherics

The spherical aberration can be reduced, or even effectively eliminated, by making one of the surfaces aspheric.
Aspheric Surfaces

Aspheric surfaces technically are any surfaces which are not spherical, but usually refer to a polynomial deformation to a conic

\[ z(r) = \frac{r^2 / R}{1 + \sqrt{1 - (k + 1)(r / R)^2}} + A r^4 + B r^6 + C r^8 + D r^{10} + ... \]

The aspheric coefficients (A, B, C, D, é) can correct 3rd, 5th, 7th, 9th, é order spherical aberration

When used near a pupil, aspherics are used primarily to correct spherical aberration

When used far away (optically) from a pupil, they are primarily used to correct astigmatism by flattening the field

Before using aspherics, be sure that they are necessary and the increased performance justifies the increased cost

Never use a higher-order asphere than justified by the ray aberration curves
For an asphere at (or near) a pupil, there need to be enough rays to sample the pupil sufficiently. This asphere primarily corrects spherical aberration.

For an asphere far away (optically) from a pupil, the ray density need not be high, but there must be a sufficient number of overlapping fields to sample the surface accurately. This asphere primarily corrects field aberrations (e.g., astigmatism).
Asphere Example

A 2 inch diameter, f/2 plano-convex lens (glass is N-BK7)

Note: Airy disk diameter is ~ 0.0001 inch
Aspheric Orders

Corresponds to ~114 waves of asphericity
MTF vs. Aspheric Order

sphere

asphere A term only

asphere A,B terms

asphere A,B,C terms
Normalized Aspheric Coefficients

\[ z(r) = z_{\text{conic}}(r) + \sum C_i r^i \quad i = 4, 6, 8 \ldots \]

\[ z(r) = z_{\text{conic}}(r) + \sum C_i r^i \left( \frac{r_{\text{max}}}{r_{\text{max}}} \right)^i \]

\[ z(r) = z_{\text{conic}}(r) + \sum (C_i r_{\text{max}}^i) \left( \frac{r}{r_{\text{max}}} \right)^i \]

\[ z(r) = z_{\text{conic}}(r) + \sum A_i x^i \quad A_i = C_i r_{\text{max}}^i \quad x = \frac{r}{r_{\text{max}}} \]
Another Asphere Example

<table>
<thead>
<tr>
<th>Coef</th>
<th>Coef value</th>
<th>Norm. Coef</th>
<th>Value</th>
<th>Fringes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2.361813e-005</td>
<td>A4</td>
<td>0.5766</td>
<td>1478.499</td>
</tr>
<tr>
<td>B</td>
<td>-1.130308e-008</td>
<td>A6</td>
<td>-0.0431</td>
<td>-110.559</td>
</tr>
<tr>
<td>C</td>
<td>-1.111391e-011</td>
<td>A8</td>
<td>-0.0066</td>
<td>-16.986</td>
</tr>
<tr>
<td>D</td>
<td>-2.398171e-014</td>
<td>A10</td>
<td>-0.0022</td>
<td>-5.727</td>
</tr>
<tr>
<td>E</td>
<td>3.035791e-017</td>
<td>A12</td>
<td>0.0004</td>
<td>1.133</td>
</tr>
<tr>
<td>F</td>
<td>1.366082e-019</td>
<td>A14</td>
<td>0.0003</td>
<td>0.796</td>
</tr>
<tr>
<td>G</td>
<td>-1.888159e-022</td>
<td>A16</td>
<td>-0.0001</td>
<td>-0.172</td>
</tr>
</tbody>
</table>

Lens also has a conic value k = -1.35

Maximum departure from base conic: 0.5253 mm (1346.985 Fringes)
Maximum departure from best fit sphere: -0.1754 mm (-449.660 Fringes)

RMS WFE = 0.0025
Reoptimize to No Conic and 10th Order Only

<table>
<thead>
<tr>
<th>Coef</th>
<th>Value</th>
<th>Fringes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A4</td>
<td>-0.5131</td>
<td>-1315.542</td>
</tr>
<tr>
<td>A6</td>
<td>-0.3338</td>
<td>-855.934</td>
</tr>
<tr>
<td>A8</td>
<td>0.0334</td>
<td>85.726</td>
</tr>
<tr>
<td>A10</td>
<td>-0.1923</td>
<td>-492.966</td>
</tr>
<tr>
<td>A12</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
<tr>
<td>A14</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
<tr>
<td>A16</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
<tr>
<td>A18</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
<tr>
<td>A20</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Maximum departure from base radius: -1.0057 mm (-2578.715 Fringes)
Maximum departure from best fit sphere: -0.1731 mm (-443.933 Fringes)
Add a Field Height = 0.1 mm
Reoptimize to Reduce Coma

RMS WFE = 0.022

RMS WFE = 0.038

<table>
<thead>
<tr>
<th>Coef</th>
<th>Value</th>
<th>Fringes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A4</td>
<td>-0.2553</td>
<td>-654.735</td>
</tr>
<tr>
<td>A6</td>
<td>-0.2176</td>
<td>-557.884</td>
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<tr>
<td>A8</td>
<td>0.0464</td>
<td>118.858</td>
</tr>
<tr>
<td>A10</td>
<td>-0.1526</td>
<td>-391.219</td>
</tr>
<tr>
<td>A12</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
<tr>
<td>A14</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
<tr>
<td>A16</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
<tr>
<td>A18</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
<tr>
<td>A20</td>
<td>0.0000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Maximum departure from base conic: -0.5791 mm (-1484.979 Fringes)
Maximum departure from best fit sphere: -0.2066 mm (-529.688 Fringes)
K = -0.2378
Coma

- Coma is an off-axis aberration
- It gets its name from the spot diagram which looks like a comet (coma is Latin for comet)
- A comatic image results when the periphery of the lens has a higher or lower magnification than the portion of the lens containing the chief ray

- The magnitude of the (third-order) coma is proportional to the square of the aperture and the first power of the field angle
Transverse vs. Wavefront 3rd-order Coma

Wavefront map

Spot diagram

Wavefront error

Transverse ray aberration
Scaling Laws for Coma

\[ \propto (f/#)^{-2} \]

\[ \propto \theta^{1} \]

**Full Field**

Spot size goes as the square of the EPD (or inverse square of the f/#)

**0.5 Field**

**On-axis**

Spot size is linearly dependent on field height
Both spherical aberration and coma are a function of the lens bending.
Coma vs. Stop Position

Å The size of the coma is also a function of the stop location relative to the lens

Coma is reduced due to increased lens symmetry around the stop
Coma is an Odd Aberration

Any completely symmetric optical system (including the stop location) is free of all orders of odd field symmetry aberrations (coma and distortion)
Astigmatism

- Astigmatism is caused when the wavefront has a cylindrical component
  - The wavefront has different spherical power in one plane (e.g., tangential) vs. the other plane (e.g., sagittal)
- The result is different focal positions for tangential and sagittal rays

- The magnitude of the (third-order) astigmatism goes as the first power of the aperture and the square of the field angle
Cause of Astigmatism

Non-rotationally symmetric through an off-center part of the surface

Rotationally symmetric through a centered part of the surface

Radius = $R_{\text{Sphere}}$

$R_{\text{cut}} < R_{\text{Sphere}}$

No astigmatism

Astigmatism

Radius = $R_{\text{Cut}}$
Image of a Wagon Wheel With Astigmatism

Wagon Wheel

Tangential Focus

Sagittal or Radial Focus

Tangential lines In Focus

Radial lines In Focus

Radial lines

Tangential lines
Astigmatism vs. Field

Field Position

0.00, 1.00
0.000, 5.000 DG

0.00, 0.75
0.000, 3.750 DG

0.00, 0.50
0.000, 1.250 DG

0.00, 0.25
0.000, 0.500 DG

0.00, 0.00
0.000, 0.000 DG

Focus (Millimeters)

-0.100 -0.090 -0.080 -0.070 -0.060 -0.050 -0.040 -0.030 -0.020 -0.010 -0.000
Scaling Laws for Astigmatism

\[ \propto (f/\#)^{-1} \theta^2 \]

- **Tangential focus**
- **Sagittal focus**
Astigmatism Ray Aberration Plots

Tangential focus

Medial focus
(best diffraction focus)

Occurs halfway between sagittal and tangential foci

Sagittal focus

Note: the sagittal focus does not always occur at the paraxial focus

OPTI 517

95
Transverse vs. Wavefront Astigmatism

At medial focus

Spot diagram

Wavefront map

Wavefront error

Transverse ray aberration

OPTI 517
PSF of Astigmatism vs. Focus Position

- Tangential focus
- Medial focus (best diffraction focus)
- Sagittal focus
Astigmatism of a Tilted Flat Plate

Placing a tilted plane parallel plate into a diverging or converging beam will introduce astigmatism.

The amount of the longitudinal astigmatism (focus shift between the tangential and sagittal foci) is given by

\[
\text{Ast} = \frac{t}{\sqrt{n^2 - \sin^2 \theta}} \left[ \frac{n^2 \cos^2 \theta}{n^2 - \sin^2 \theta} - 1 \right]
\]

Exact

\[
\text{Ast} = -\frac{t \theta^2 (n^2 - 1)}{n^3}
\]

Third-order
Correcting the Astigmatism of a Tilted Flat Plate

The astigmatism introduced by a tilted flat plate can be corrected by:

- Adding cylindrical lenses
- Adding another plate tilted in the orthogonal plane
- Adding tilted spherical lenses

To correct for this

Do not do this (it will double the astigmatism)

Do this
Reducing the Astigmatism of a Tilted Flat Plate

Astigmatism of a flat plate can be reduced by adding a slight wedge to the plate.
Correcting Astigmatism with Tilted Spherical Lenses
Most optical systems want to image rectilinear objects into rectilinear images.

This requires that \( m = -s'/s = -h'/h = \text{constant} \) for the entire FOV.

For infinite conjugate lenses, this requires that \( h' = F \tan \theta \) for all field angles.
Distortion

Â If rectilinear imaging is not met, then there is distortion in the lens

Â Effectively, distortion is a change in magnification or focal length over the field of view

Â Negative distortion (shown) is often called barrel distortion

Â Positive distortion (not shown) is often called pincushion distortion
Cause of Distortion

1. Thin Lens stop at the lens
   - Object (Rect Grid) → Thin Lens stop at the lens

2. Stop in front of the lens
   - Barrel Distortion

3. Stop behind the lens
   - Pincushion Distortion
Correcting Distortion

Object (Rect Grid)  Image

FIGURE 9U
(a) A stop in front of a lens giving rise to barrel distortion. (b) A stop behind a lens giving rise to pincushion distortion. (c) A symmetrical doublet with a stop between is relatively free of distortion.
More on Distortion

- Distortion does not result in a blurred image and does not cause a reduction in any measure of image quality such as MTF.
- Distortion is a measure of the displacement of the image from its corresponding paraxial reference point.
- Distortion is independent of f/number.
- Linear distortion is proportional to the cube of the field angle.
- Percent distortion is proportional to the square of the field angle.

![Graph showing the relationship between angle and distortion](graph.png)
Implications of Distortion

Å Consider negative distortion

ï A rectilinear object is imaged inside the detector

Å This means a rectilinear detector sees a larger-than-rectilinear area in object space
Curvature of Field

In the absence of astigmatism, the focal surface is a curved surface called the Petzval surface.
Third-order Field Curvature

Aberrations relative to a flat image surface
The Petzval Surface

The radius of the Petzval surface is given by

\[
\frac{1}{R_{\text{Petzval}}} = \sum_i \left( \frac{1}{n_i F_i} \right)
\]

- For a singlet lens, the Petzval radius = n F

Obviously, if we have only positive lenses in an optical system, the Petzval radius will become very short
  - We need some negative lenses in the system to help make the Petzval radius longer (i.e., flatten the field)

This, and chromatic aberration correction, is why optical systems need some negative lenses in addition to all the positive lenses
Field Curvature and Astigmatism

• As an aberration, field curvature is not very interesting
• As a design obstacle, it is the basic reason that optical design is still a challenge
• The astigmatic contribution starts from the Petzval surface
  - If the axial distance from the Petzval surface to the sagittal surface is 1 (arbitrary units), then the distance from the Petzval surface to the tangential surface is 3

Field curvature and astigmatism can be used together to help flatten the image plane and improve the image quality
Flattening the Field

Å \( \Phi_{sys} = \sum h_i \Phi_i \) where \( h_1 = 1 \), want \( \Phi_{sys} > 0 \)

Å To flatten the field, want \( \Sigma \Phi_{positive} \approx -\Sigma \Phi_{negative} \)

Å The contribution of a lens to the Petzval sum is proportional to \( \Phi/n \)

Å Thus, if we include negative lenses in the system where \( h \) is small we can reduce the Petzval sum and flatten the field while holding the focal length

Å Yet another reason why optical systems have so darn many lenses

Cooke Triplet

Lens With Field Flattener (Petzval Lens)

Flat-field lithographic lens
Negative lenses in RED
Original Object
Spherical Aberration

Image blur is constant over the field
Coma

Image blur grows linearly over the field
Astigmatism

Image blurs more in one direction over the field
Distortion

No image degradation but image locations are shifted
Curvature of Field

Image blur grows quadratically over the field
Combined Aberrations – Spot Diagrams

Defocus has been added to each to produce the minimum RMS spot size.
Balancing of Aberrations

- Different aberrations can be combined to improve the overall image quality
  - Spherical aberration and defocus
  - Astigmatism and field curvature
  - Third-order and fifth-order spherical aberration
  - Longitudinal color and spherochromatism
  - Etc.

- Lens design is the art (or science) of putting together a system so that the resulting image quality is acceptable over the field of view and range of wavelengths
Resolution

Å Resolution is an important aspect of image quality
Å Every image has some resolution associated with it, even if it is the Airy disk
   ii In this case, the resolution is dependent on the aberrations of the system
Å Resolution is the smallest detail you can resolve in the image

- Well-resolved
- Rayleigh spacing
  Peak of the second Airy disk is at the first zero of the first Airy disk
  (26% intensity dip between the peaks)
- Sparrow criterion
  No discernible intensity dip between the peaks
Resolution vs. P-V Wavefront Error

The 1/4 wave rule was empirically developed by astronomers as the greatest amount of P-V wavefront error that a telescope could have and still resolve two stars separated by the Rayleigh spacing (peak of one at 1st zero of the other).
Resolution Examples

- Angular resolution is given by $\beta \approx 2.44 \frac{\lambda}{D}$
  - Limited only by the diameter, not by the focal length or f/number

- U of A is building 8.4 meter diameter primary mirrors for astronomical telescopes

- For visible light (~0.5 μm), the Rayleigh spacing corresponds to an angular separation of $(2.44 \times 0.5 \times 10^{-6} / 8.4)/2 = 0.073 \times 10^{-6}$ radian (~0.015 arc second)

- Assume a binary star at a distance of 200 light years (~1.2 x 10$^{15}$ miles)
  - This would have a resolution of 90 million miles
  - Perhaps enough resolution to "split the binary"

- A typical cell phone camera has an aperture of about 0.070 inch
  - This gives a Rayleigh spacing of about 0.34 mrad (for reference, the human eye has an angular resolution of about 0.3 mrad)
  - For an object 10 feet away, this is an object resolution of about 1 mm
Film Resolution

Â Due to the grain size of film, there is an MTF associated with films

Â A reasonable guide for MTF of a camera lens is the 30-50 rule: 50% at 30 lp/mm and 30% at 50 lp/mm

Â For excellent performance of a camera lens, use 50% MTF at 50 lp/mm

Â Another criterion for 35 mm camera lenses is 20% MTF at 30 lp/mm over 90% of the field (at full aperture)

Â As a rough guide for the resolution required in a negative, use 200 lines divided by the square root of the long dimension in mm
Detectors

All optical systems have some sort of detector
- The most common is the human eye
- Many optical systems use a 2D detector array (e.g., CCD)

No matter what the detector is, there is always some small element of the detector which defines the detector resolution
- This is referred to as a picture element (pixel)

The size of the pixel divided by the focal length is called the Instantaneous FOV (IFOV)
- The IFOV defines the angular limit of resolution in object space
- IFOV is always expressed as a full angle
Implications of IFOV

- If the target angular size is smaller than an IFOV, it is **not resolved**
  - It is essentially a point target
  - Example is a star

- If the target annular size is larger than an IFOV it may be resolved
  - This does not mean that you can always tell what the object is
Practical Resolution Considerations

- Resolution required to photograph written or printed copy:
  - Excellent reproduction (serifs, etc.) requires 8 line pairs per lower case e
  - Legible reproduction requires 5 line pairs per letter height
  - Decipherable (e, c, o partially closed) requires 3 line pairs per height

- The correlation between resolution in cycles/minimum dimension and certain functions (often referred to as the Johnson Criteria) is:
  - Detect: 1.0 line pairs per dimension
  - Orient: 1.4 line pairs per dimension
  - Aim: 2.5 line pairs per dimension
  - Recognize: 4.0 line pairs per dimension
  - Identify: 6-8 line pairs per dimension
  - Recognize with 50% accuracy: 7.5 line pairs per height
  - Recognize with 90% accuracy: 12 line pairs per height
Johnson Resolution Criteria

PROBABILITY OF PERFORMING TASK (%)

REQUIRED NO. OF CYCLES ACROSS TARGET CRITICAL DIMENSION
(FOR NO. PIXELS ACROSS TARGET MULTIPLY BY 2)

Mission:
- detection
- classification
- recognition
- identification
Examples of the Johnson Criteria

Detect
1 bar pair

Maybe something of military interest

Recognize
4 bar pairs

Tank

Identify
7 bar pairs

Abrams Tank
MTF of a Pixel

Consider a fixed size pixel scanning across different sized bar targets.

When the pixel size equals the width of a bar pair (light and dark) there is no more modulation.
MTF of a Pixel

If the pixel is of linear width $\Delta$, the MTF of the pixel is given by

$$MTF(f) = \frac{\sin(\pi f \Delta)}{\pi f \Delta}$$

The cutoff frequency (where the MTF goes to zero) is at a spatial frequency $1/\Delta$.
Optical MTF and Pixel MTF

The total MTF is the product of the optical MTF and the pixel MTF.

Case 1 - Optics limited
Best for high resolution over-sampling

Case 2 - Optics and detector are matched
Best for most FLIR-like mapping systems

Case 3 - Detector limited
Best for detecting dim point targets

Of course, there are other MTF contributors to total system MTF:
- Electronics, display, jitter, smear, eye, turbulence, etc.
Aliasing

- Aliasing is a very common effect but is not well understood by most people.
- Aliasing is an image artifact that occurs when we insufficiently sample a waveform.
  - It is evidenced as the imaging of high frequency objects as low frequency objects.

Array of detectors

Signal from the detectors
MTF Fold Over

- The effect of sampling is to replicate the MTF back from the sampling frequency
  - This will cause higher frequencies to appear as lower frequencies

- The solution to this is to prefilter the MTF so it goes to zero at the Nyquist frequency
  - This is often done by blurring
Effects of Signal/CCD Alignment on MTF

A sampled imaging system is not shift-invariant
MTF of Alignment

When performing MTF testing, the user can align the image with respect to the imager to produce the best image

- In this case, a sampling MTF might not apply

A natural scene, however, has no net alignment with respect to the sampling sites

To account for the average alignment of unaligned objects a sampling MTF must be added

- \( MTF_{\text{sampling}} = \frac{\sin(\pi f \Delta x)}{\pi f \Delta x} \) where \( \Delta x \) is the sampling interval

- This MTF an ensemble average of individual alignments and hence is statistical in nature
Final Comments on Image Quality

- Image quality is essentially a measure of how well an optical system is suited for the expected application of the system.
- Different image quality metrics are needed for different systems.
- The better the needed image quality, the more complex the optical system will be (and the harder it will be to design and the higher the cost will be to make it).
- The measures of image quality used by the optical designer during the design process are not necessarily the same as the final performance metrics.
  - It's up to the optical designer to convert the needed system performance into appropriate image quality metrics for optimization and analysis.