Sine condition from optical flux conservation and radiance theorem

Etendu considerations

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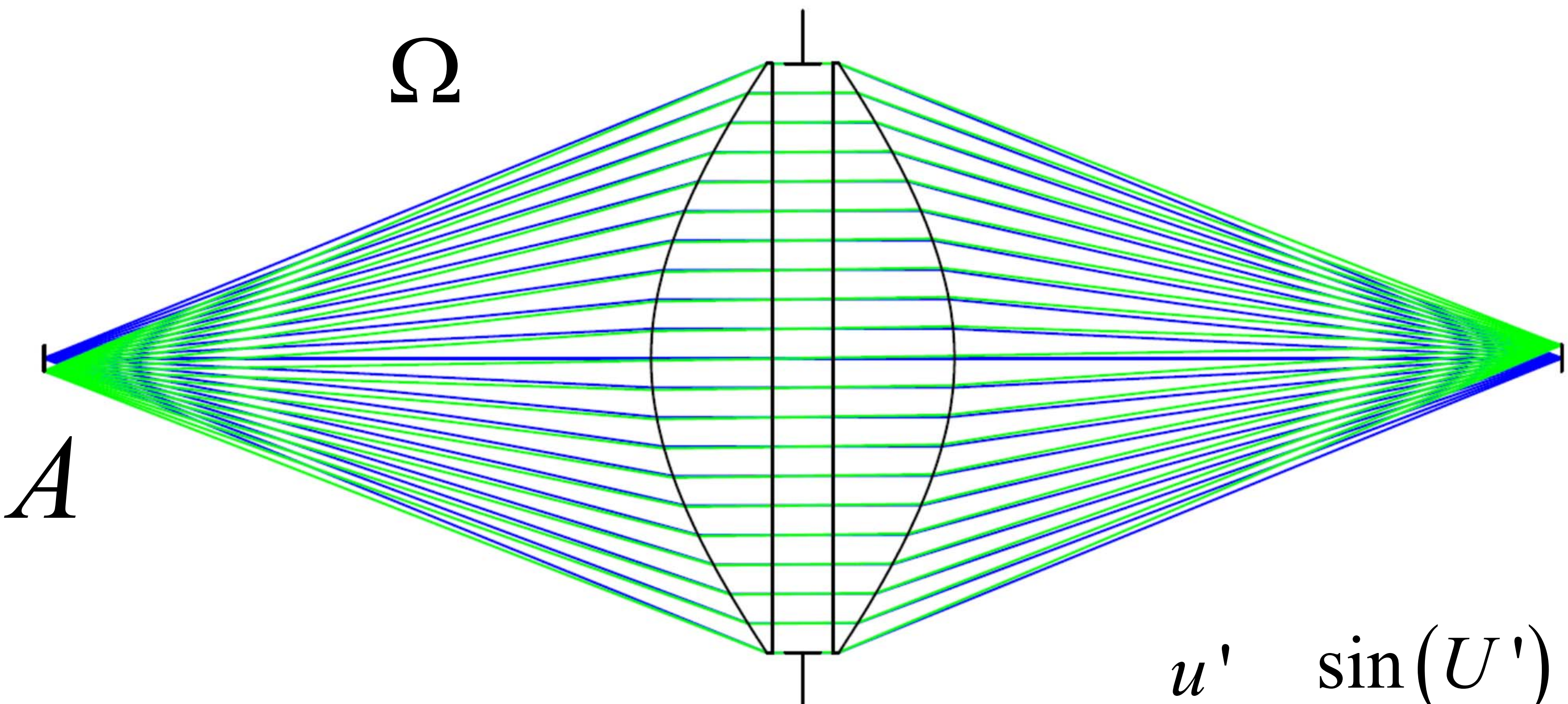
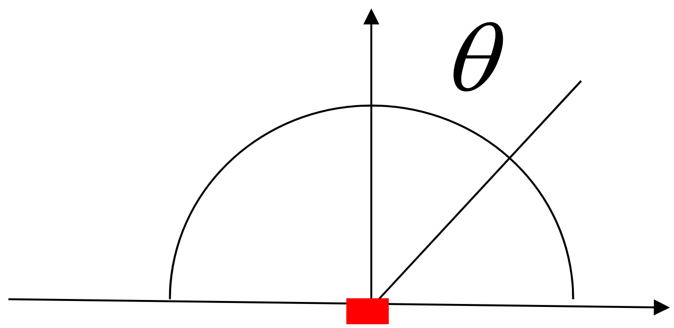
Sine condition



$$\frac{u'}{u} = \frac{\sin(U')}{\sin(U)}$$

Optical flux from a Lambertian source

$$\Phi(\theta) = 2\pi A L_0 \int_0^{\theta} \cos(\theta)\sin(\theta)\, d\theta = \pi A L_0 \sin^2(\theta)$$



Compare with homogeneous source

$$\Phi(\theta) = 2\pi A L_0 \int_0^{\theta} \sin(\theta)\, d\theta = 2\pi A L_0 \left(1 - \cos(\theta)\right) = A L_0 \Omega$$

Optical flux=radiance x throughput

$$\Phi(\theta) = \frac{L_0}{n^2} T \qquad \Phi(\theta') = \frac{L_0'}{n'^2} T$$

$$\frac{n^2}{L_0}\Phi(\theta) = \frac{n'^2}{L_0'}\Phi(\theta') = T$$

$$\frac{n^2}{L_0} = \frac{n'^2}{L_0'}$$ Radiance theorem

$$U = \theta$$

$$\frac{n^2}{L_0}\Phi(U) = \frac{n'^2}{L_0'}\Phi(U')$$

Sine condition from optical flux conservation

$$\frac{n^2}{L_0}\Phi(U) = \frac{n'^2}{L_0'}\Phi(U')$$

$$n^2 A \sin^2(U) = n'^2 A' \sin^2(U')$$

$$\pi n^2 h^2 \sin^2(U) = \pi h'^2 n'^2 \sin^2(U')$$

$$n^2 h^2 \sin^2(U) = h'^2 n'^2 \sin^2(U')$$

$$nh\sin(U) = n'h'\sin(U')$$

$$\frac{u'}{u} = \frac{\sin(U')}{\sin(U)}$$

Sine condition

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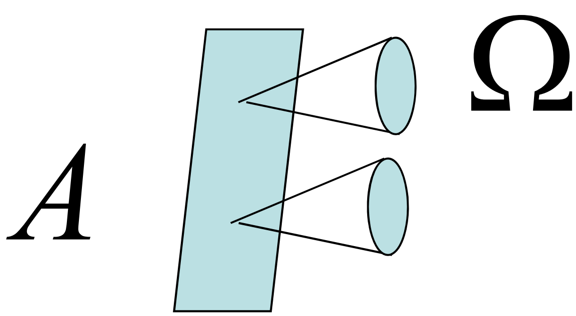
Throughput
Etendu
(area-omega product)

$$\varepsilon = n^2 A\Omega = n'^2 A'\Omega'$$

$$T = \pi Ж^2$$

"Capacity to transfer optical flux"



$$\Omega = 2\pi\left(1-\cos\left(\theta\right)\right)$$

Even better

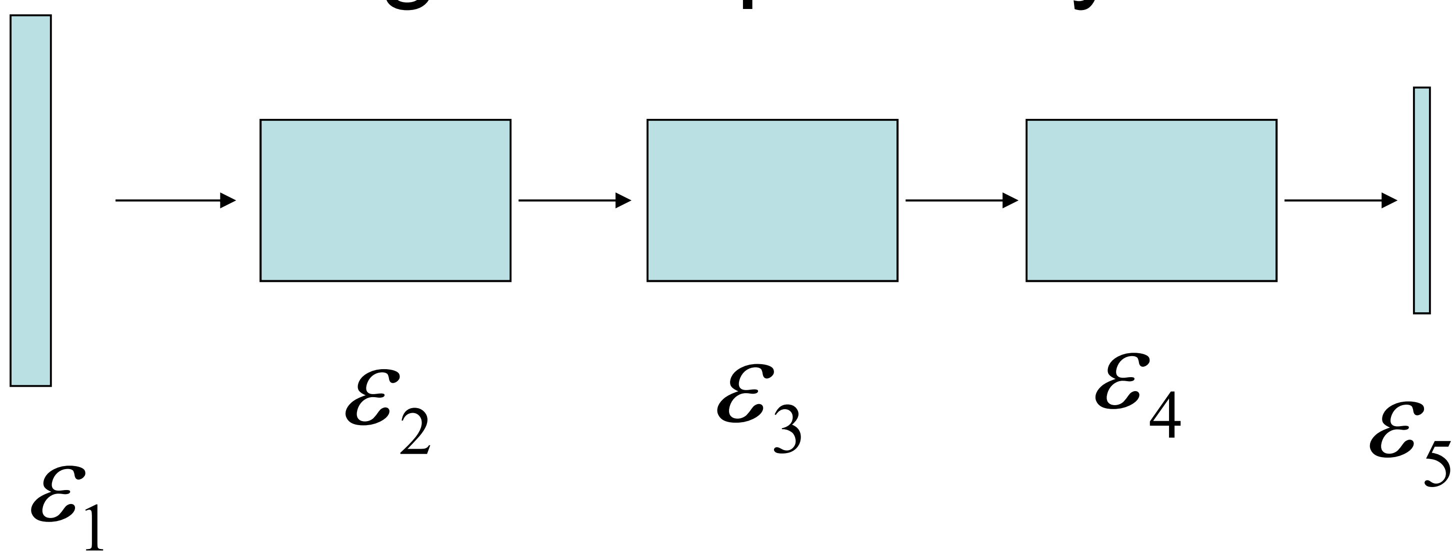
$$\varepsilon = n^2 A\Omega = n'^2 A'\Omega'$$

Homogeneous source

$$\varepsilon = A\left(NA\right)^2 = n^2 A \sin^2\left(\theta\right)$$

Lambertian source

Etendu considerations are key to design an optical system



$$\varepsilon_5 \geq \varepsilon_4 \geq \varepsilon_3 \geq \varepsilon_2 \geq \varepsilon_1$$

Start with the sensor at the end $\quad \varepsilon_5 = A_5 \left(NA \right)^2$

Every optics component has associated an etendue. The component with the smallest etendue limits the amount of optical flux transferred by the system.