Introductions to aberrations OPTI 517

Lecture 11

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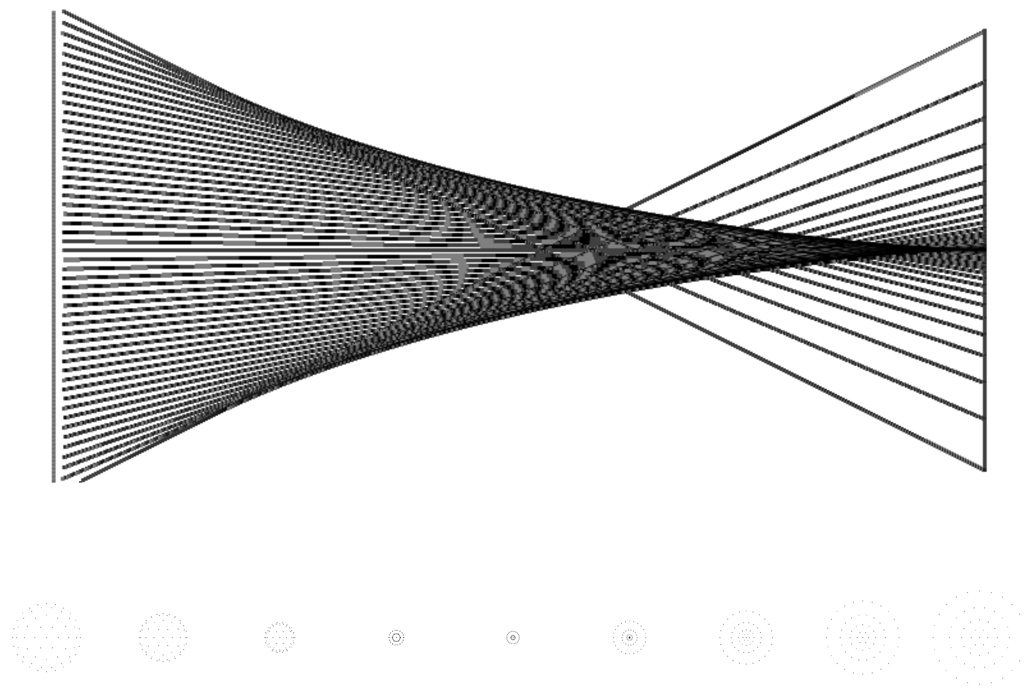
Spherical aberration



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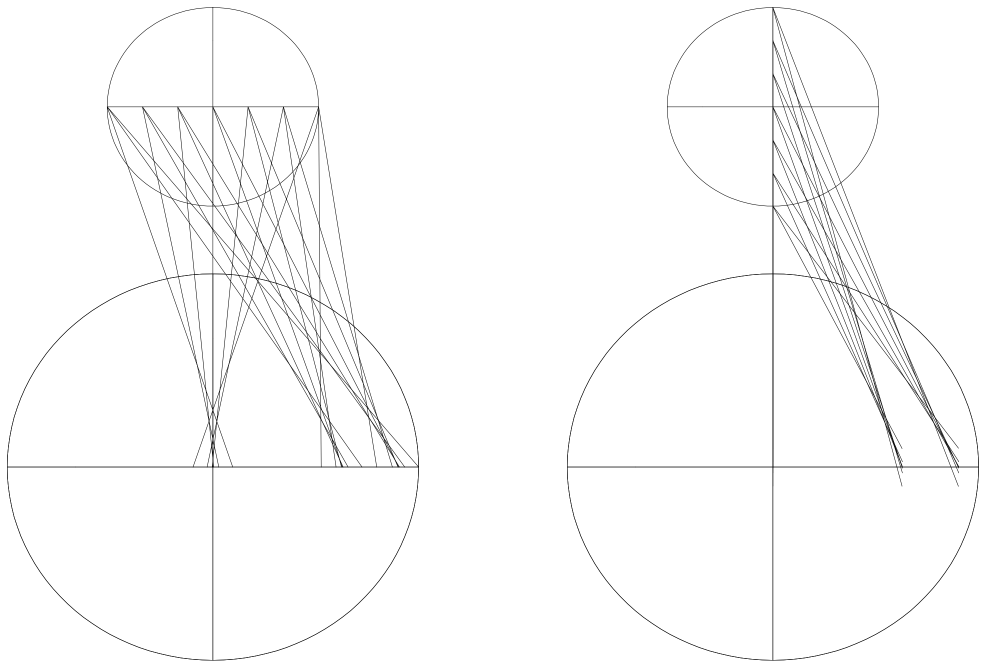
Meridional and sagittal ray fans



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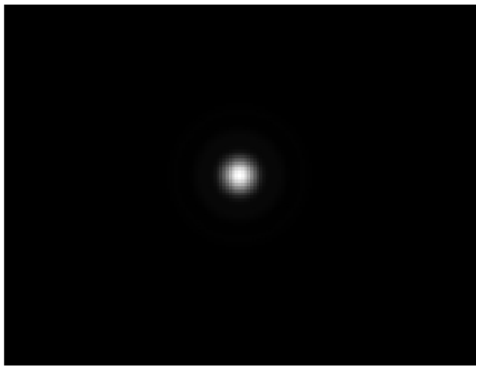
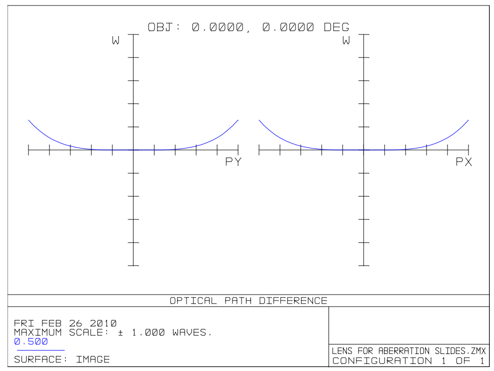
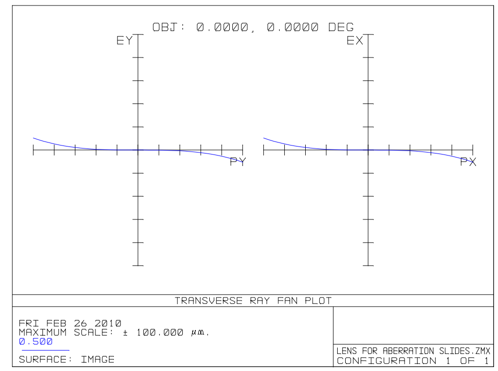
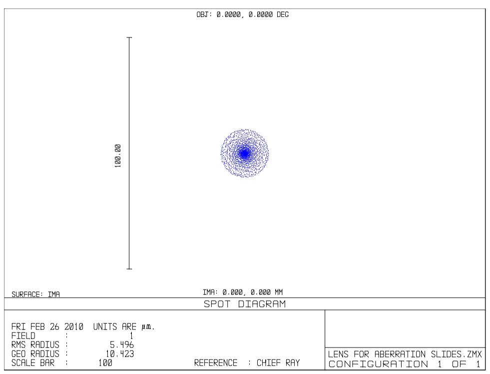
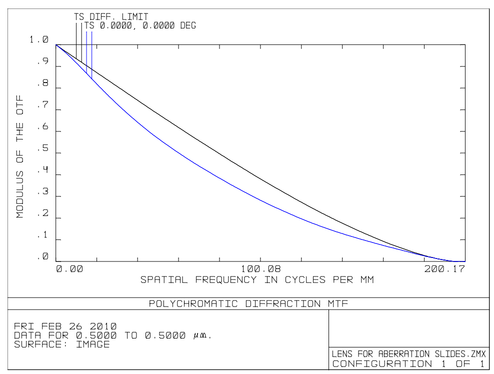
Spherical aberration 0.25 wave
f/10; f=100 mm; wave=0.0005 mm

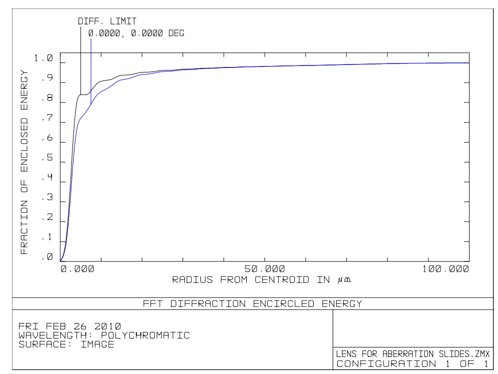




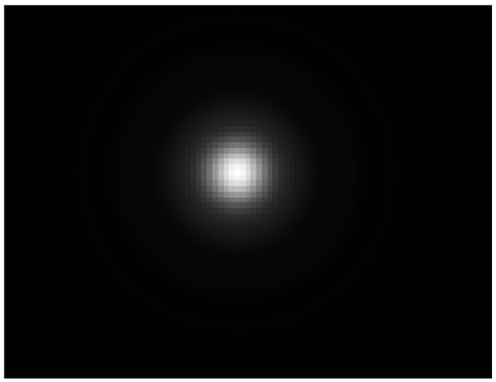


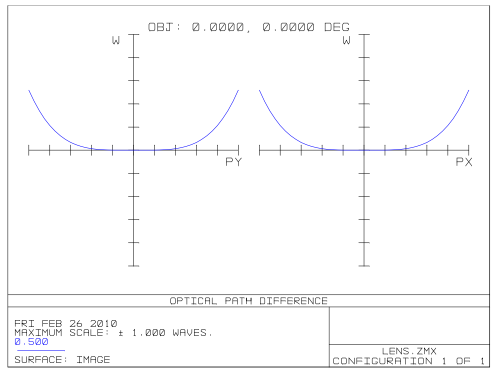


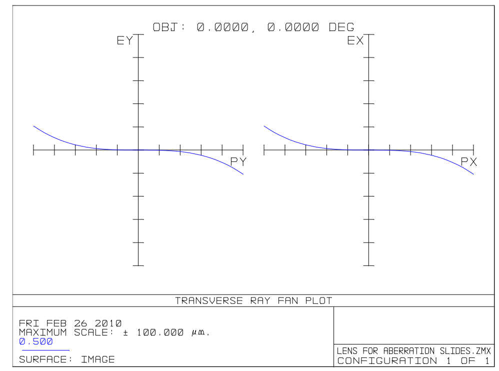


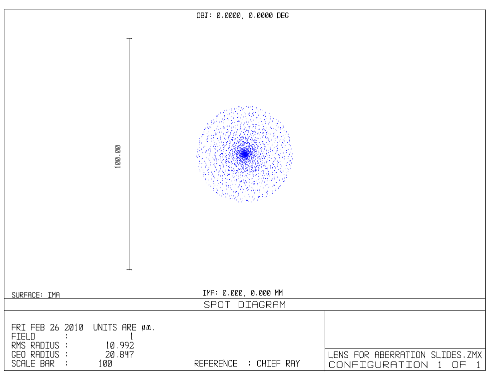


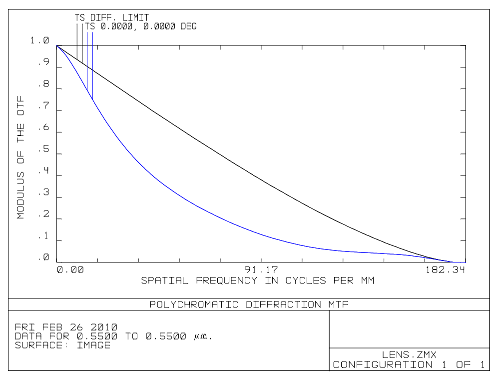
Spherical aberration 0.5 wave
f/10; f=100 mm; wave=0.0005 mm

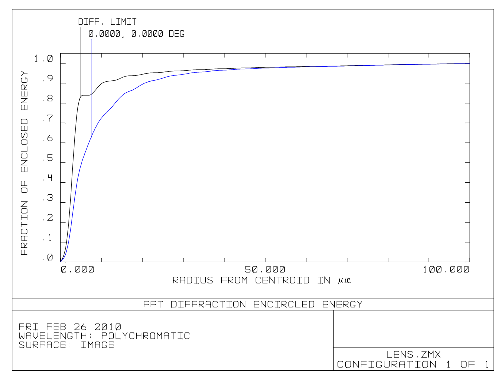












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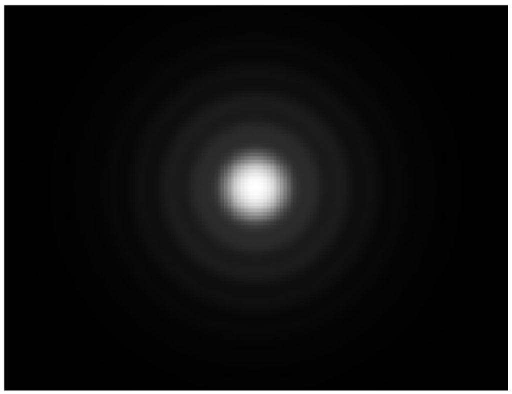
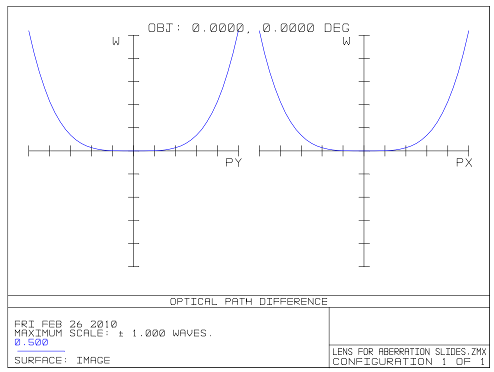
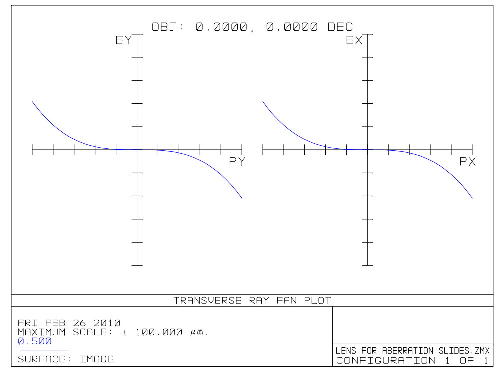
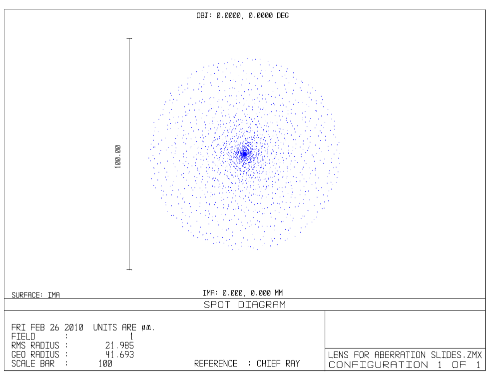
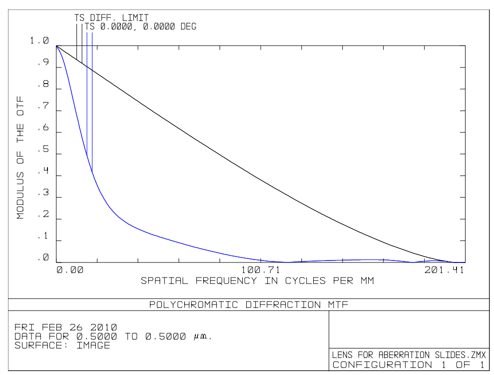
Spherical aberration 1 wave
f/10; f=100 mm; wave=0.0005 mm

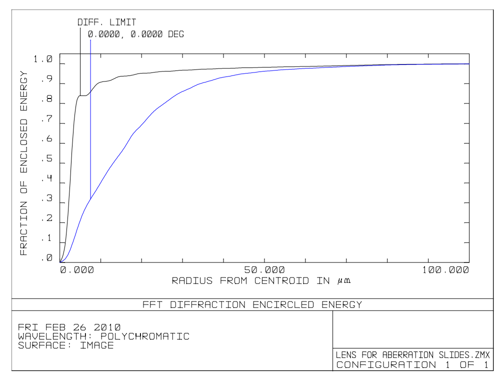






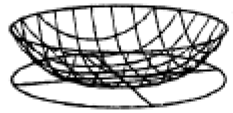


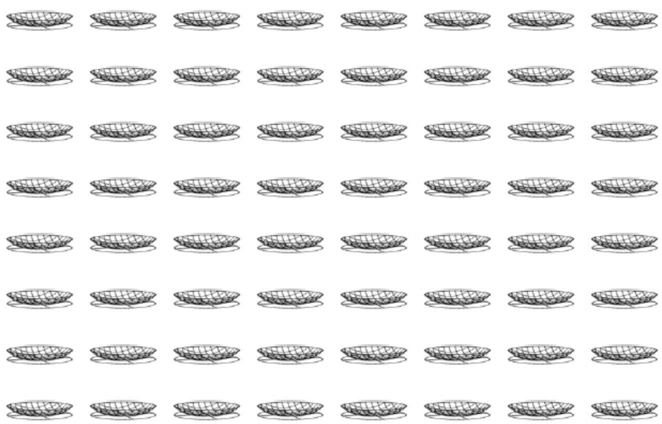




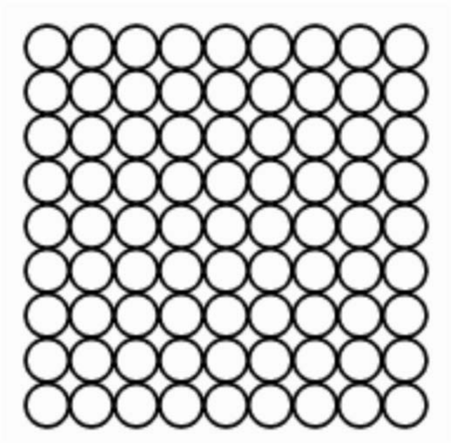
Spherical aberration is uniform over the field of view

$$W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^{2}$$



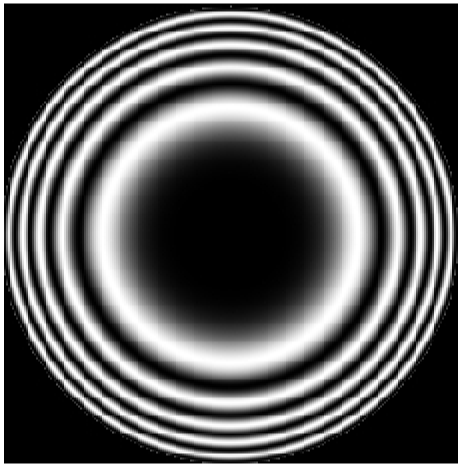


Wavefront



Spots

Interferometric representation



5 waves

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Cases of zero spherical aberration from a spherical surface

$$W_{040}\left(\vec{\rho}\cdot\vec{\rho}\right)^2 \qquad W_{040}=\frac{1}{8}S_I \qquad S_I=-\sum A^2 y\Delta\left(\frac{u}{n}\right)$$

$$y=0$$

$$A=0$$

$$\Delta\left(u/n\right)=u'/n'-u/n=0$$

y=0 the aperture is zero or the surface is at an image
A=0 the surface is concentric with the Gaussian image point on axis
u'/n-u/n=0 the conjugates are at the aplanatic points

Aplanatic means free from error;
freedom from spherical aberration and coma

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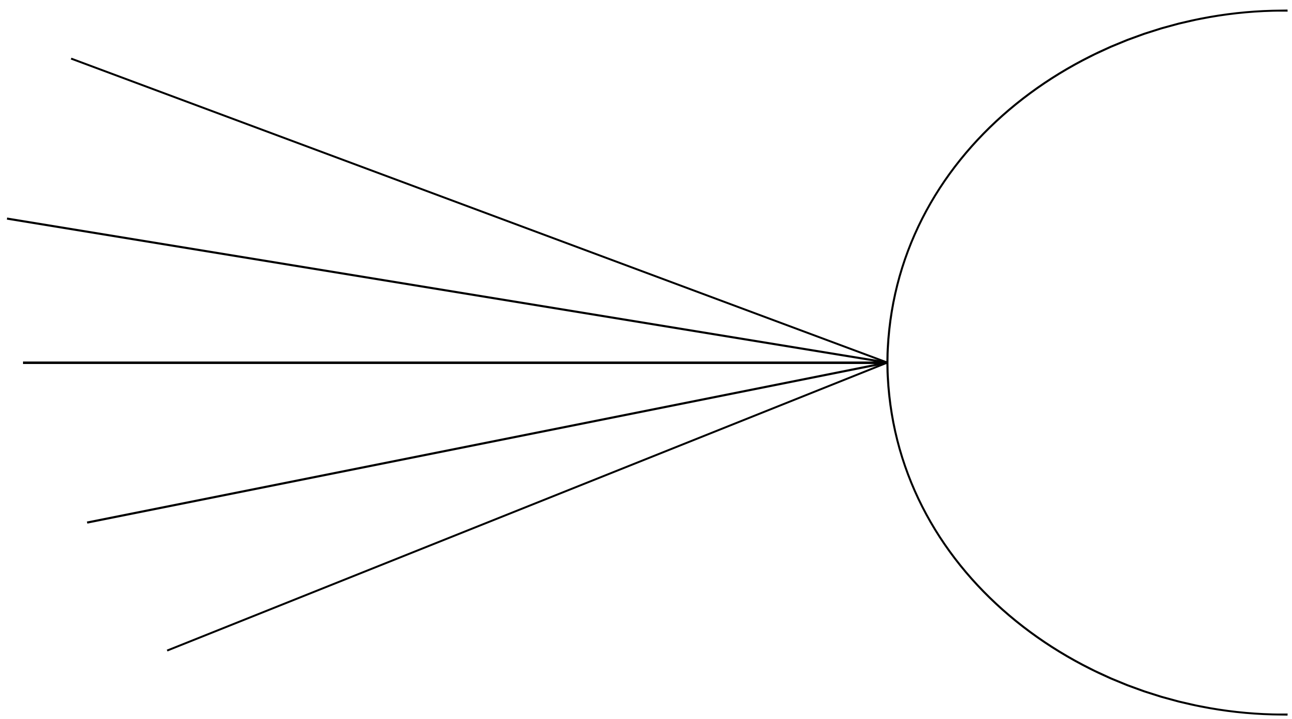
Surface at image

$$y = 0$$



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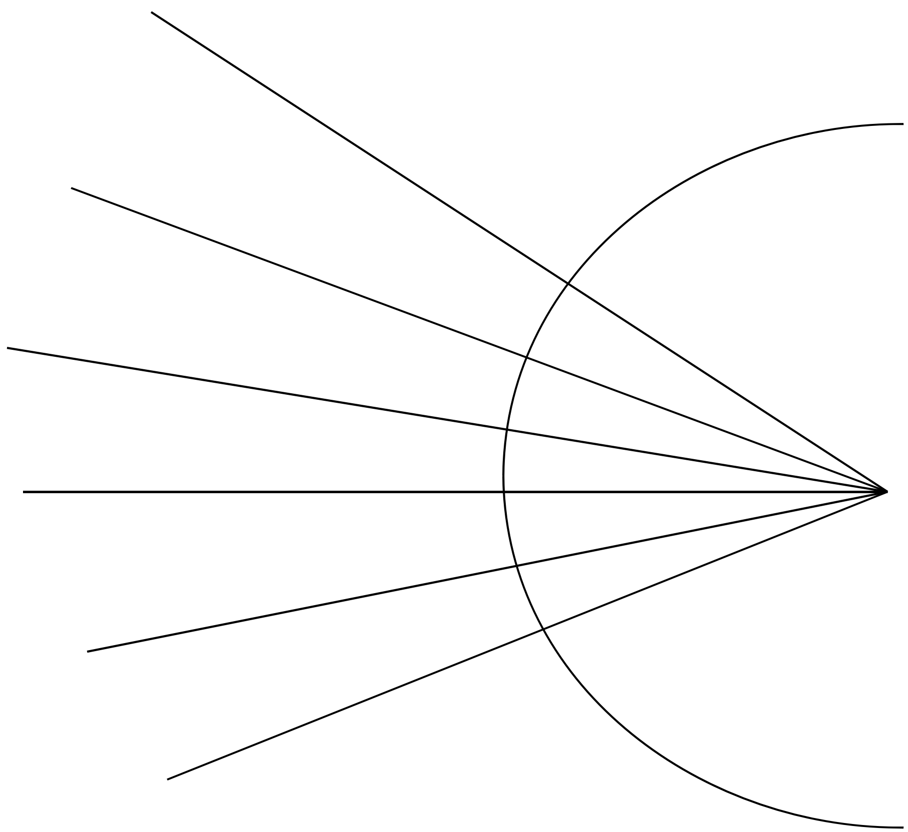
Concentric surface

$$A = 0$$



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Aplanatic points of a spherical surface

$$-\frac{1}{n's'}+\frac{1}{ns}=0$$

$$\frac{n'}{s'}-\frac{n}{s}=\frac{n'-n}{r}$$

$$S=r\frac{n'+n}{n}$$

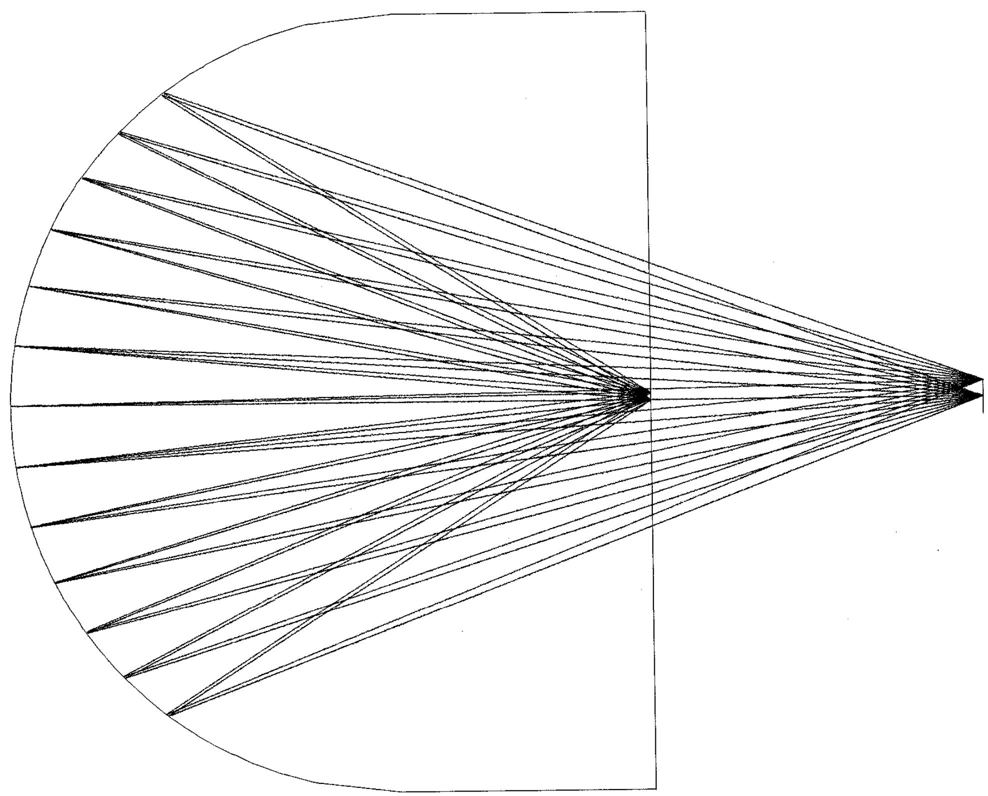
$$S'=r\frac{n'+n}{n'}$$

$$S=2.5r$$

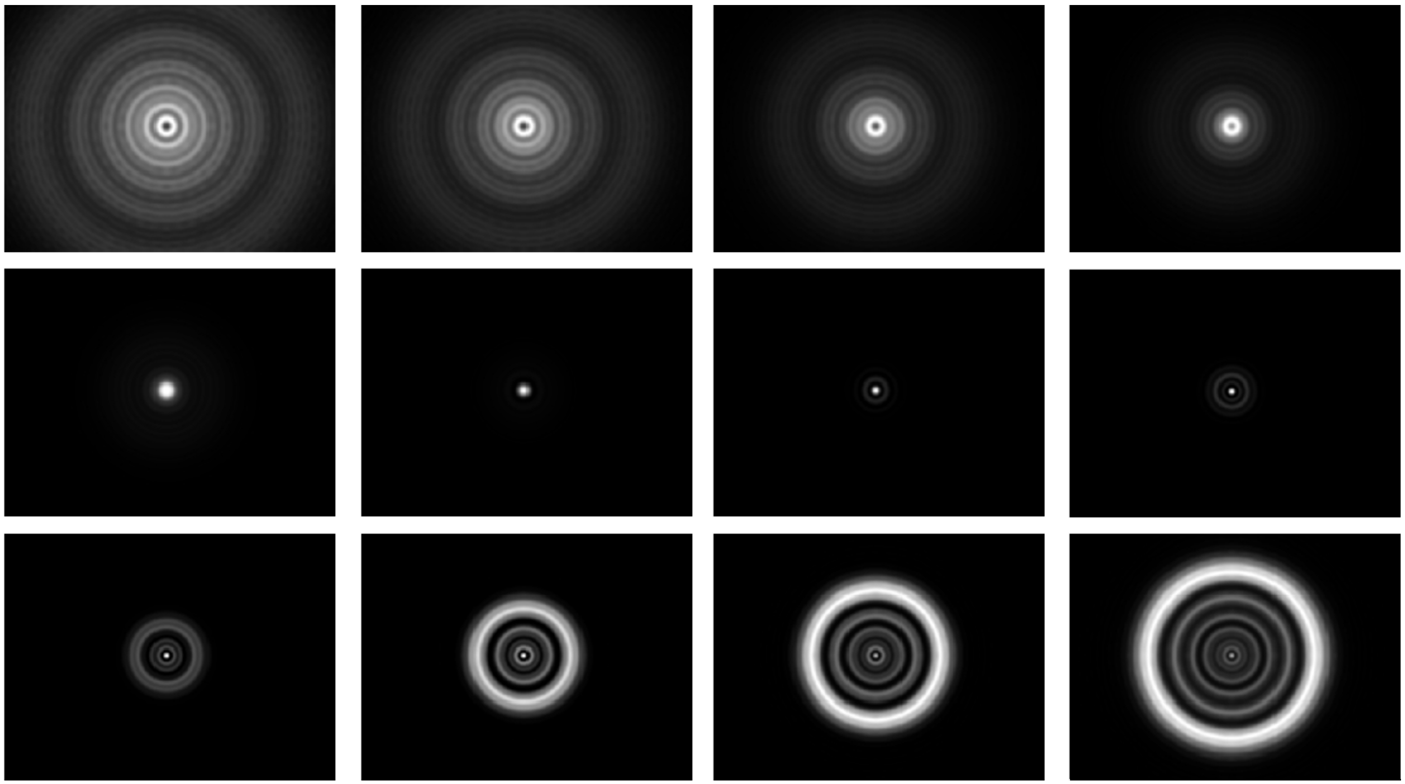
$$S'=(5/3)r$$

$$n=1.5$$

$$\Delta(u/n)=u'/n-u/n=0$$

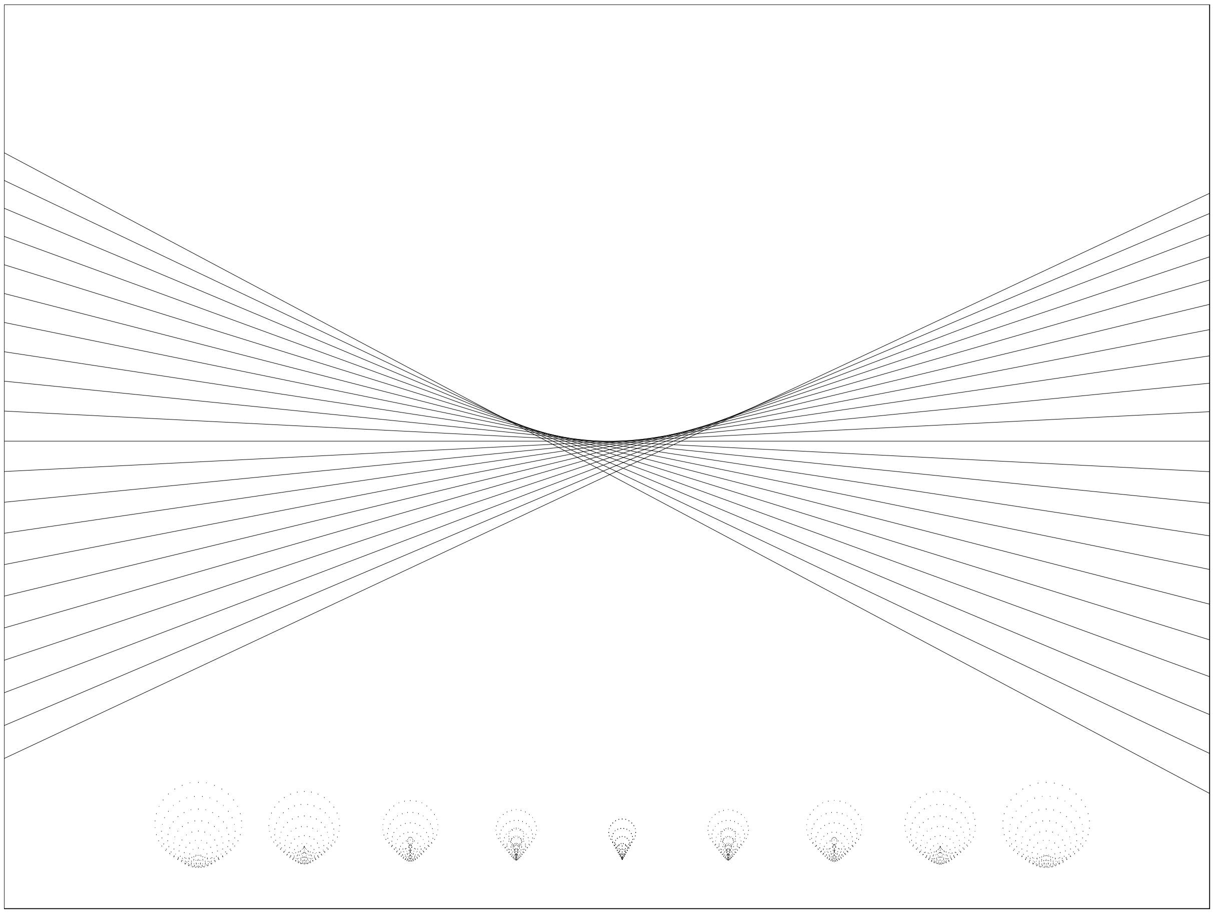


Diffraction images



Two waves of spherical aberration

Coma aberration



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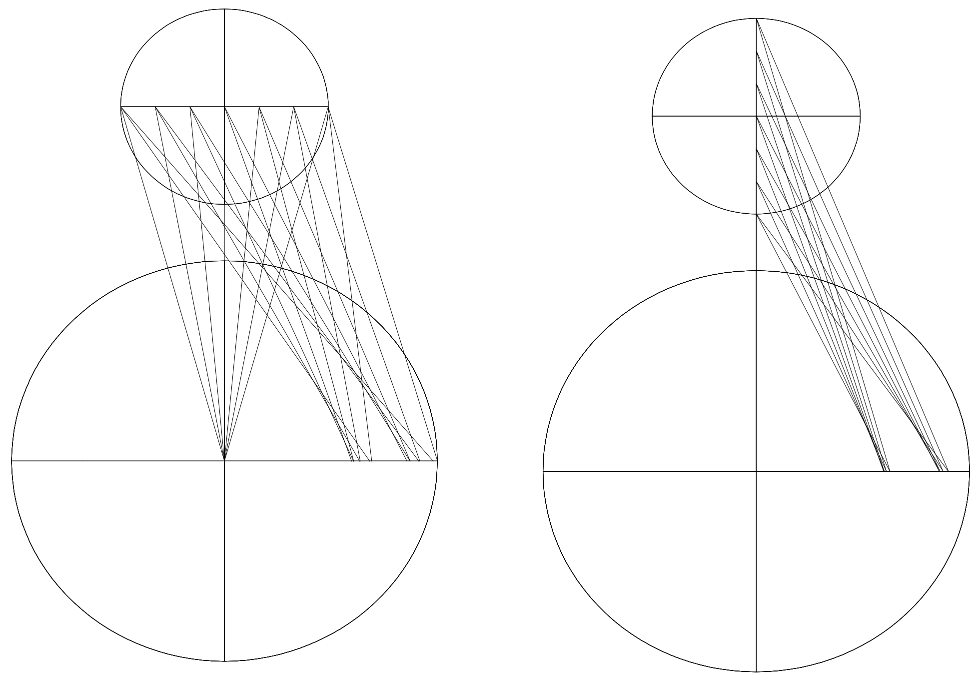
Meridional and sagittal ray fans



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WAVE ABERRATION

$$W = W_{131} H \rho^3 \cos\phi + \Delta W_{20} \rho^2$$

TRANSVERSE RAY ABERRATION

$$w'\vec{\epsilon} = [W_{131} H \rho^2] \vec{h} + 2[\Delta W_{20} \rho + (W_{131} H \rho^2) \cos\phi] \vec{g}$$

FOR $\rho = \text{CONST.}$ (ZONAL DIAGRAM), $a = W_{131} H \rho^2$, $b = \Delta W_{20} \rho$

$$w'\vec{\epsilon} = a\vec{h} + 2(b + a\cos\phi)\vec{g}$$

$$= a\vec{h} + r\vec{g}$$

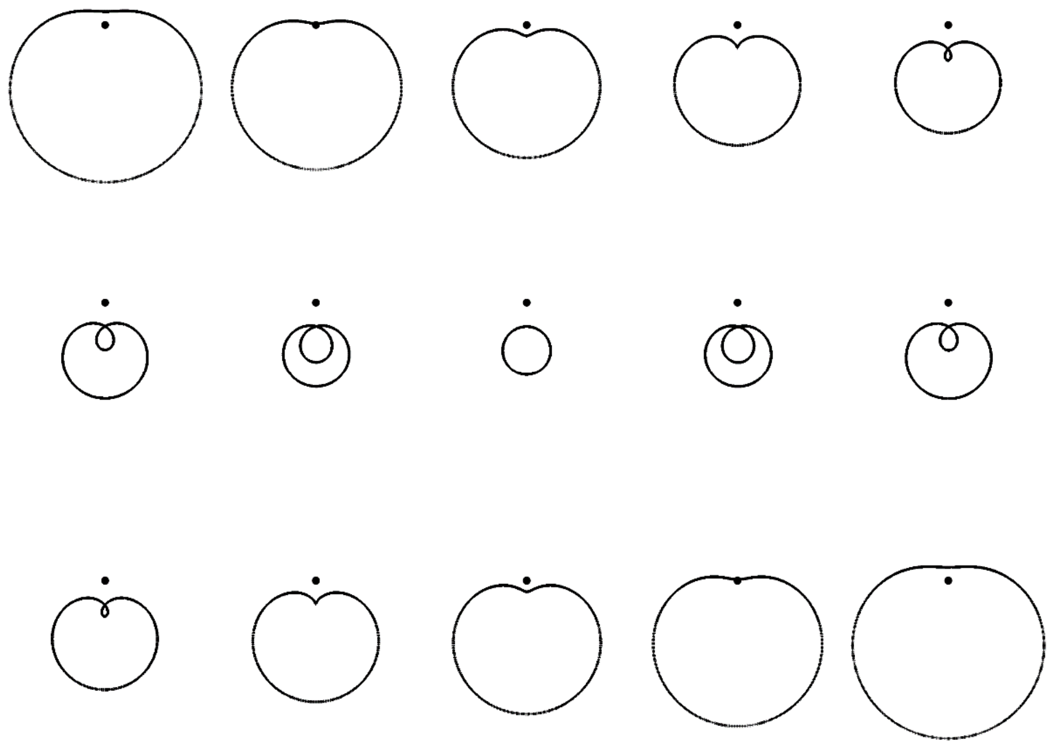
$r = 2(b + a\cos\phi)$ LIMAÇON OF PASCAL

FOR $b = 0$ $(\Delta W_{20} = 0)$, $r = 2a\cos\phi$ DOUBLE CIRCLE

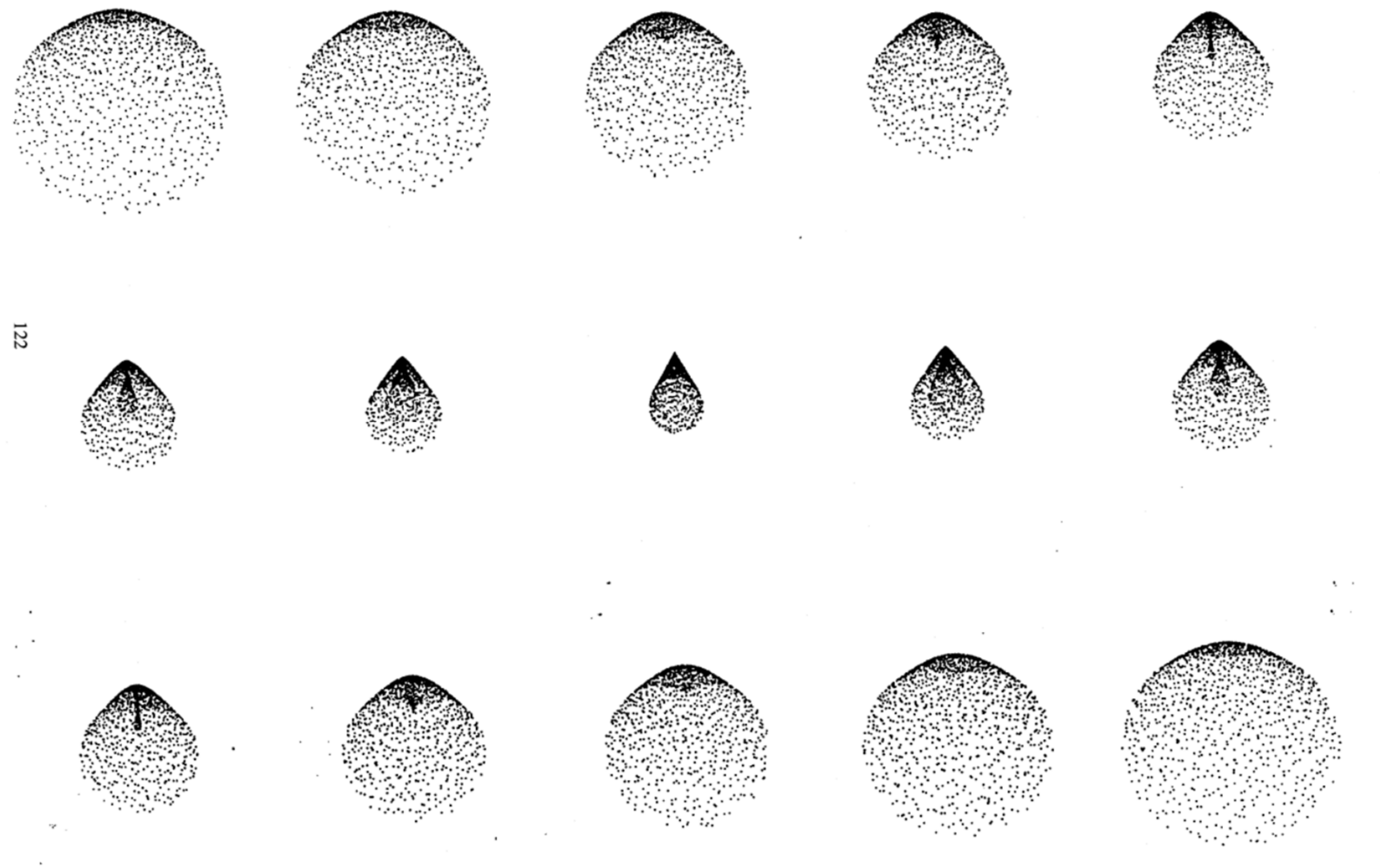
FOR $b = \pm a$ $(\Delta W_{20} = \pm W_{131} H \rho)$, $r = 2a(\pm 1 + \cos\phi)$ CARDIOID

Roland Shack's notes

Coma zonal diagrams



Spot diagrams through focus



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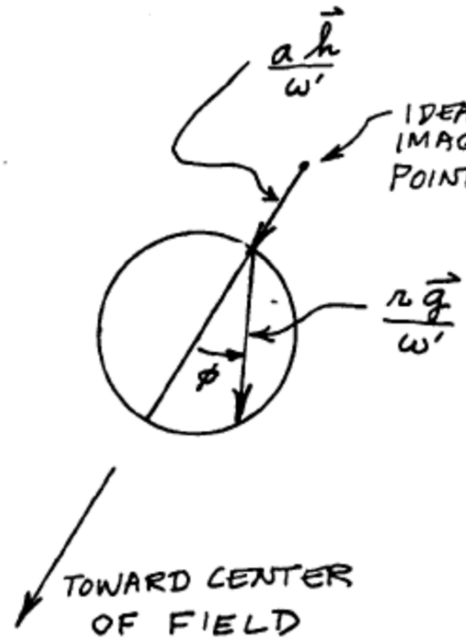
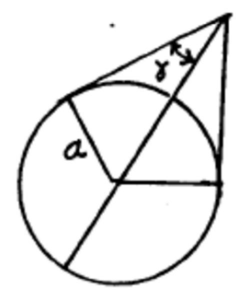
Coma zonal diagrams

$\rho = 1$, $\Delta W_{20} = 0$ (ZONAL DIAGRAM)



$\sin\gamma = \frac{a}{2a} = \frac{1}{2}$, $\gamma = 30°$



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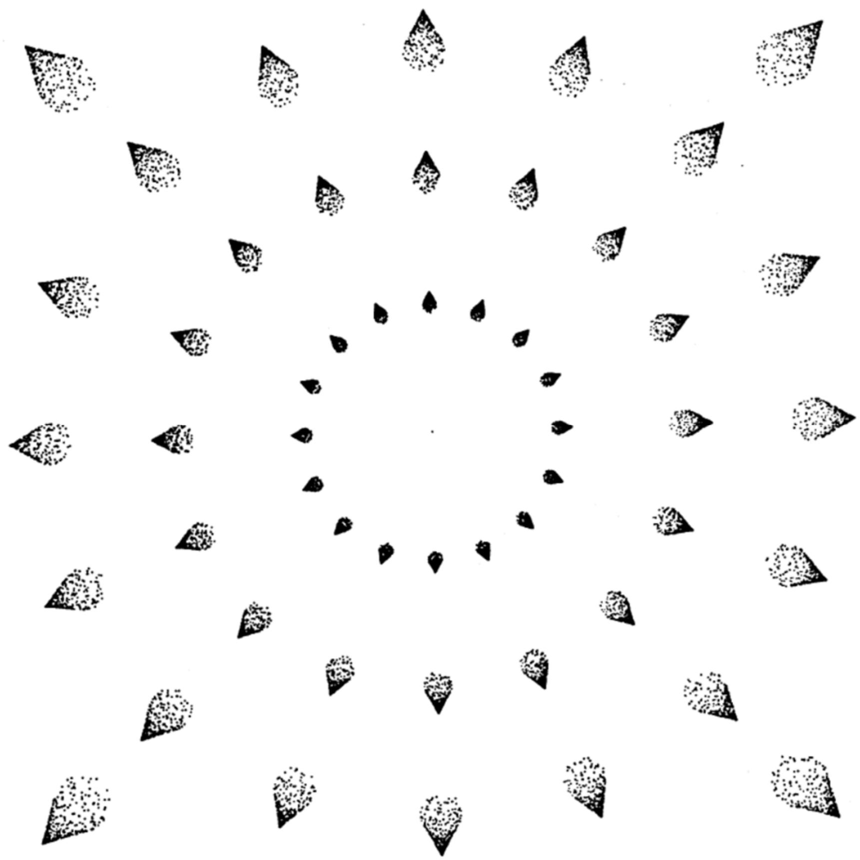
Positive coma over the field of view



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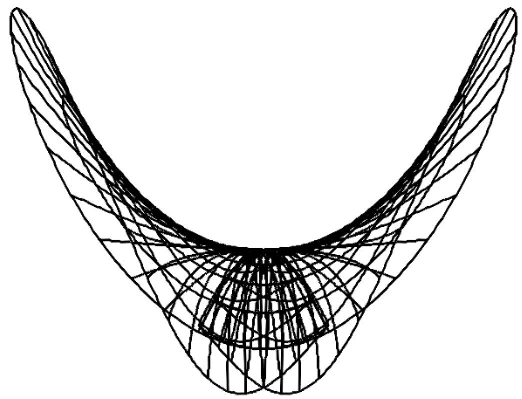
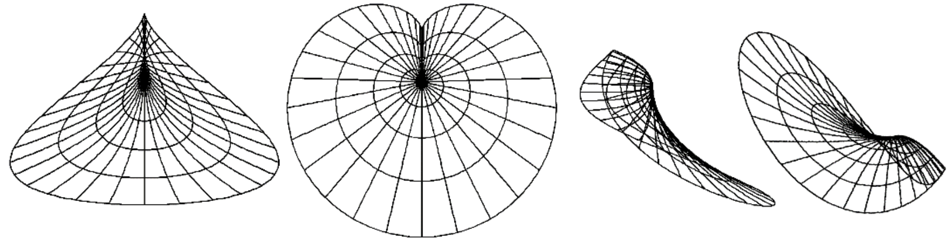
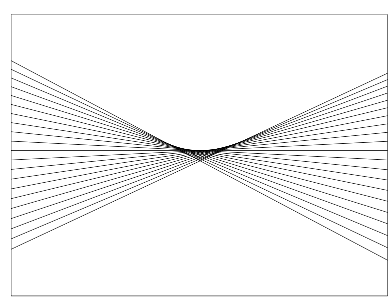
Caustic sheets







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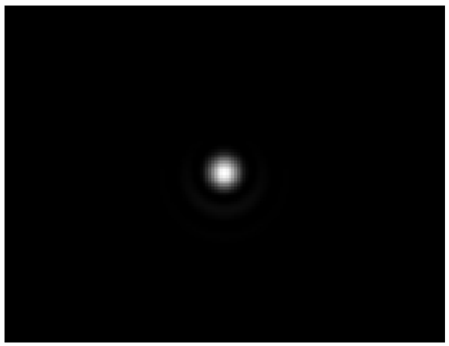
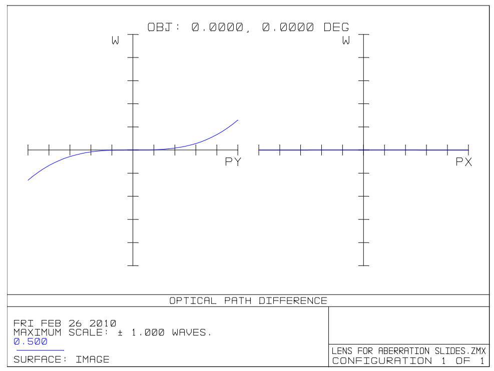
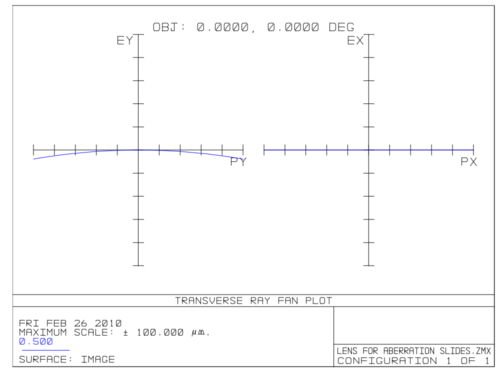
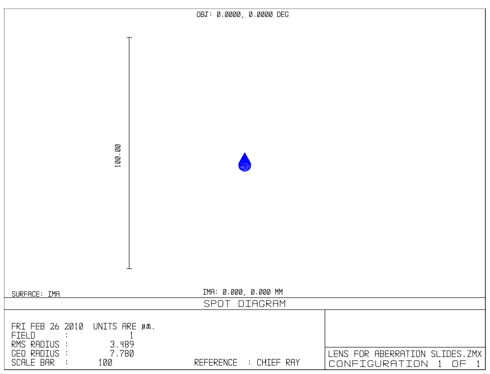
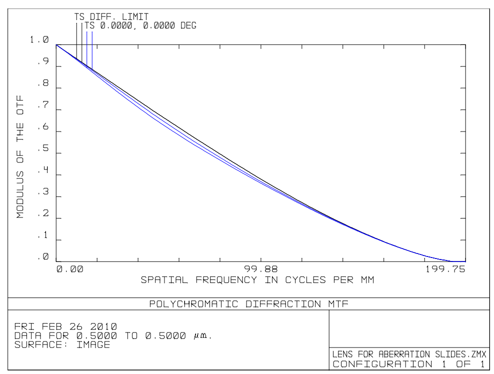
Coma aberration 0.25 wave
f/10; f=100 mm; wave=0.0005 mm

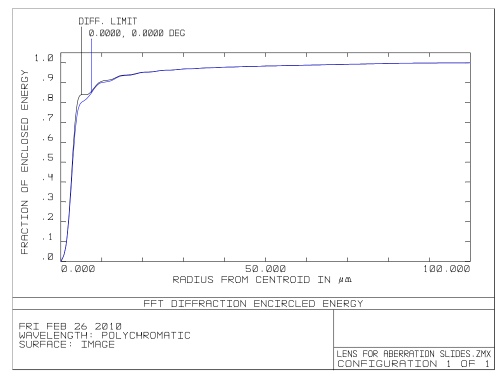












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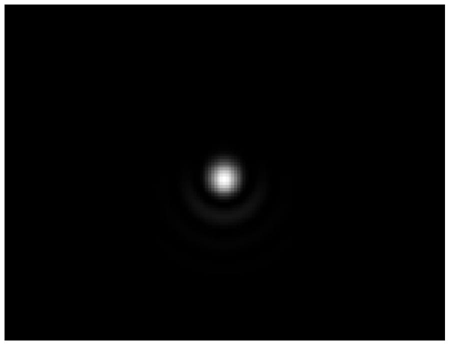
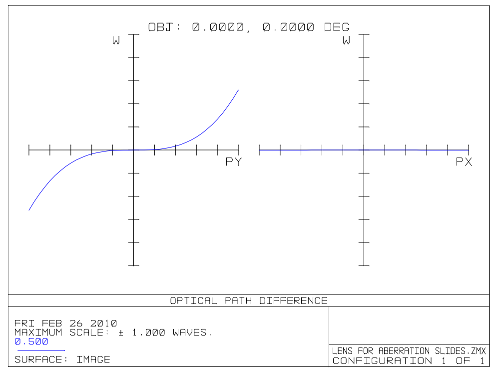
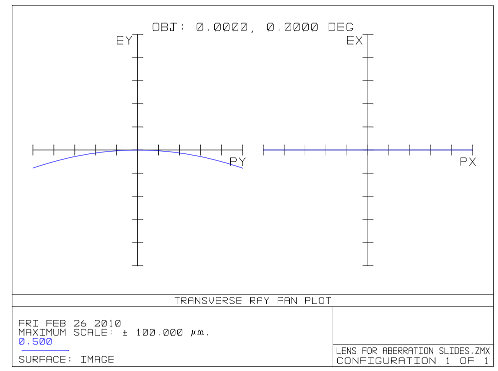
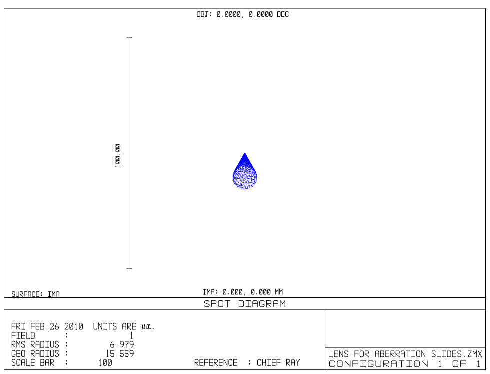
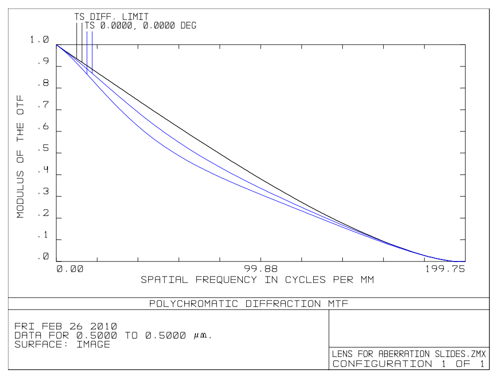
Coma aberration 0.5 wave
f/10; f=100 mm; wave=0.0005 mm

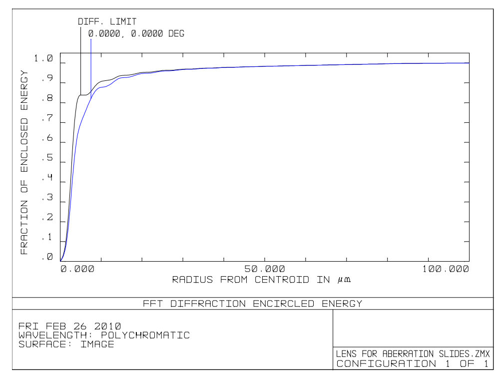












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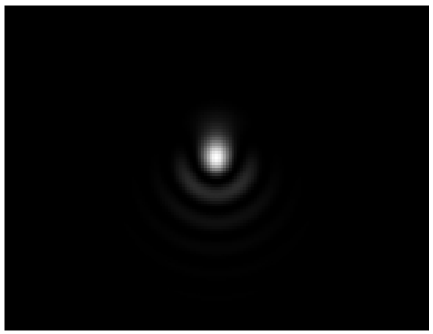
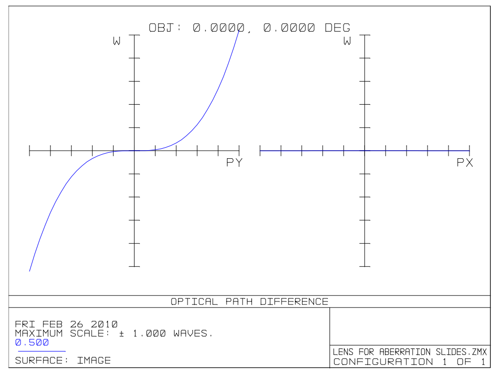
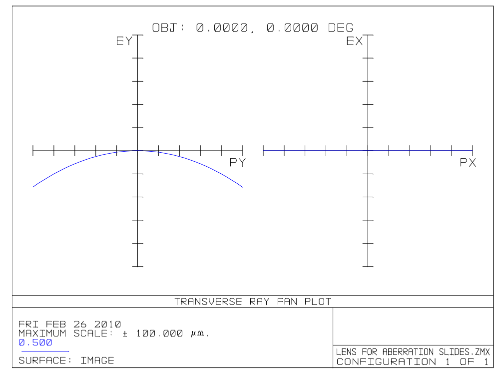
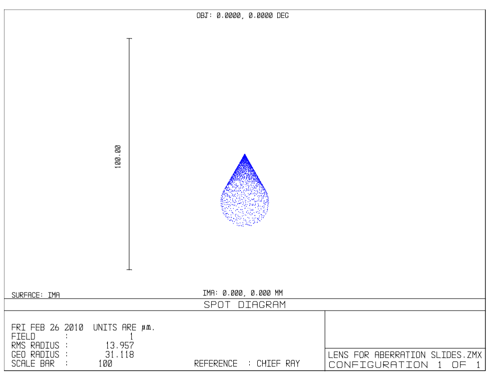
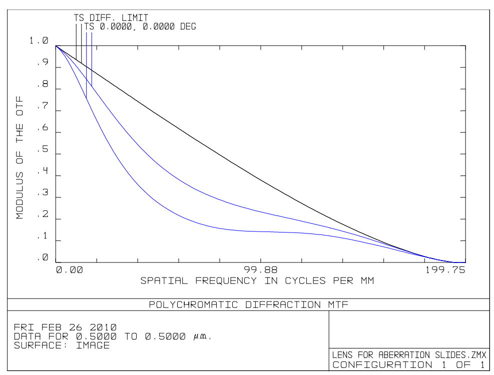
Coma aberration 1.0 wave
f/10; f=100 mm; wave=0.0005 mm

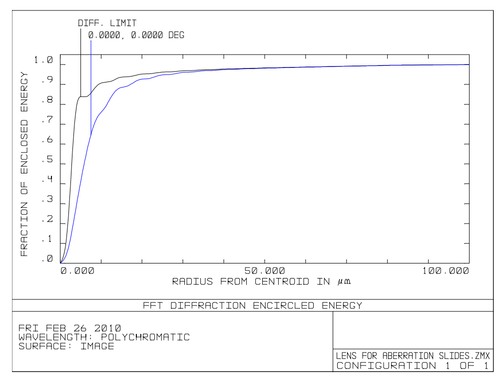












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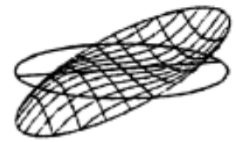
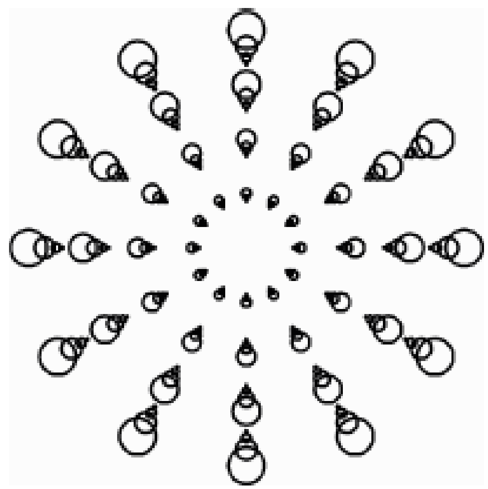
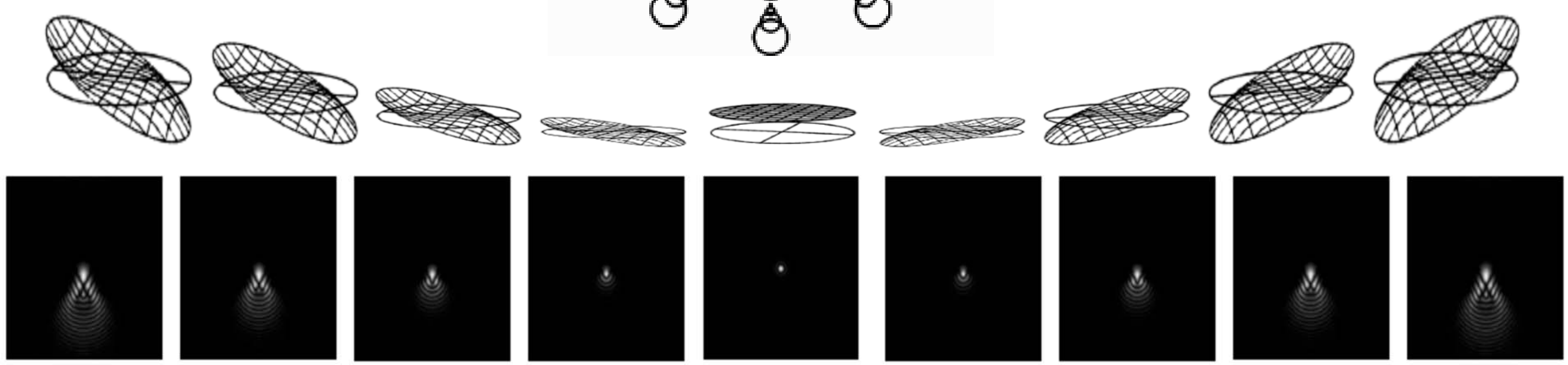
Coma varies linearly over the field of view

$$W_{131}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$$







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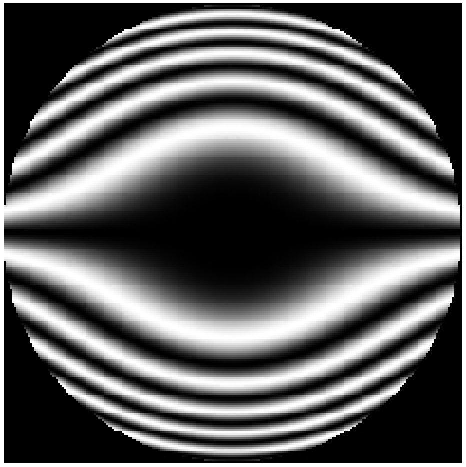
Interferometric representation



5 waves

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Cases of zero coma aberration from a spherical surface

$$W_{131}\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right) \qquad W_{131}=\frac{1}{2}S_{II} \qquad S_{II}=-\sum A\bar{A}y\Delta\left(\frac{u}{n}\right)$$

$$y=0$$

$$A=0$$

$$\bar{A}=0$$

$$\Delta\left(u/n\right)=u'/n'-u/n=0$$

y=0 the aperture is zero or the surface is at an image
A=0 the surface is concentric with the Gaussian image point on axis
Abar=0 surface is concentric with stop or pupils
u'/n-u/n=0 the conjugates are at the aplanatic points

Aplanatic means free from error;
freedom from spherical aberration and coma

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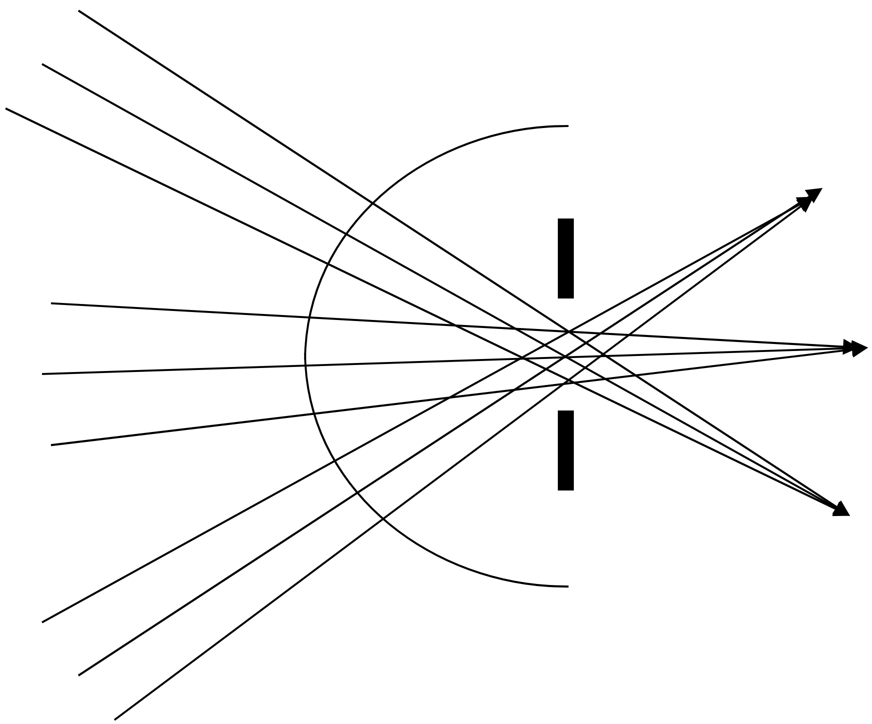
Concentric surface with stop or pupils

$$\bar{A} = 0$$



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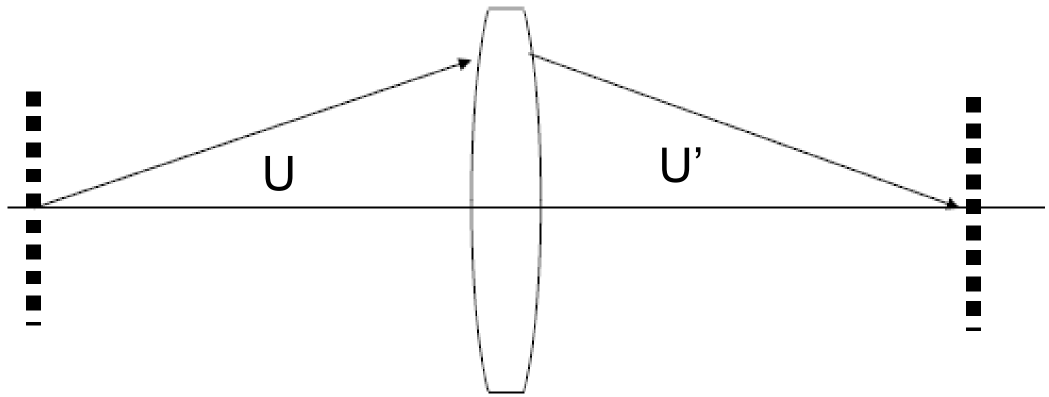
Sine condition

- In the absence of spherical aberration there are no linear phase errors that depend on the field of view if the sine condition is met:

$$\frac{\sin(U)}{\sin(U')} = \frac{u}{u'}$$

The first-order magnification is equal to the real marginal ray magnification

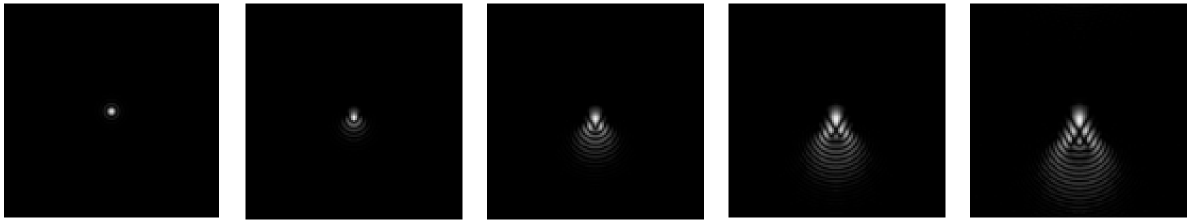
Imaging a grating



$$\sin(U) = \frac{m \cdot \lambda}{d}$$

$$d \cdot \sin(U) = m \cdot \lambda = d' \sin(U')$$

Diffraction images



2 waves

4 waves

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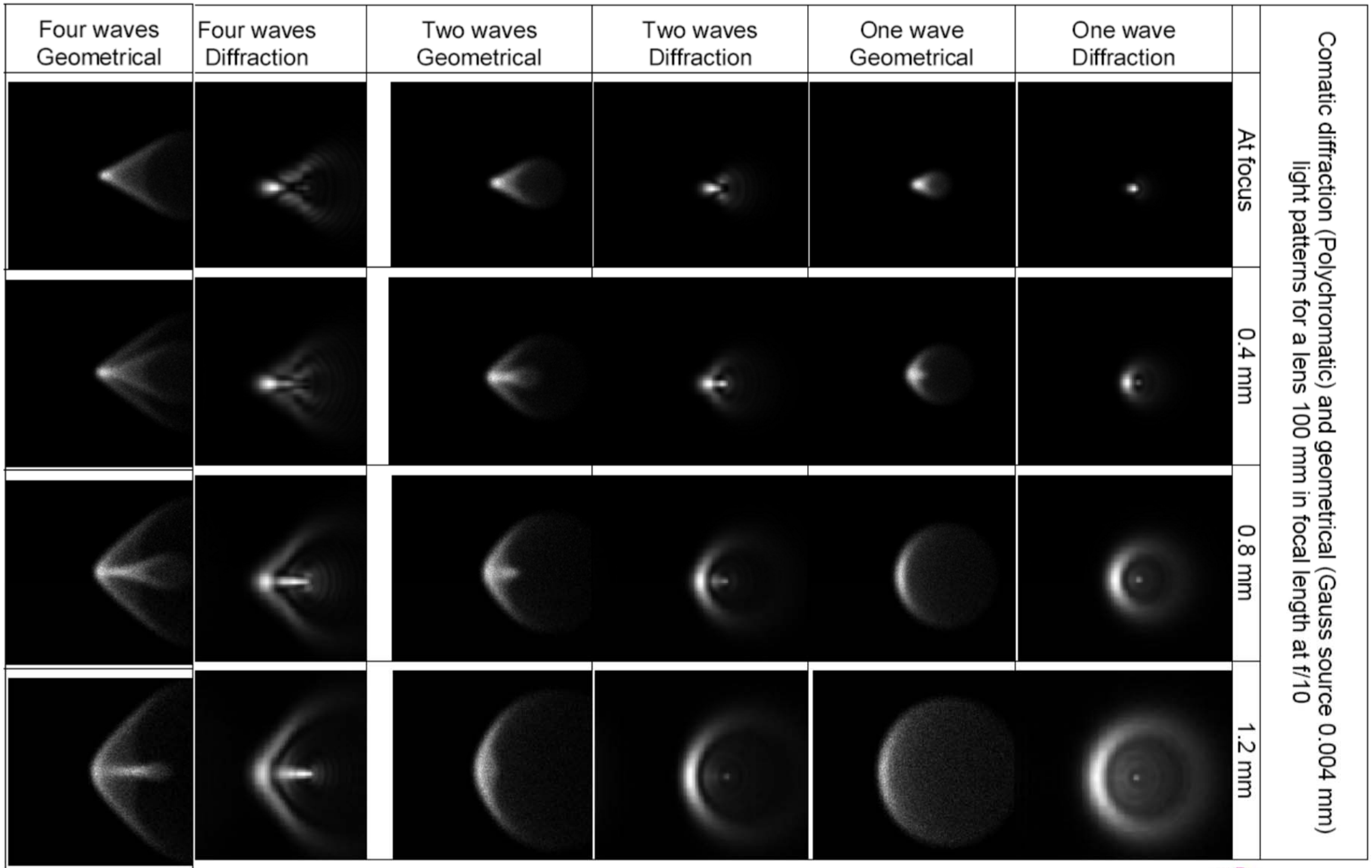
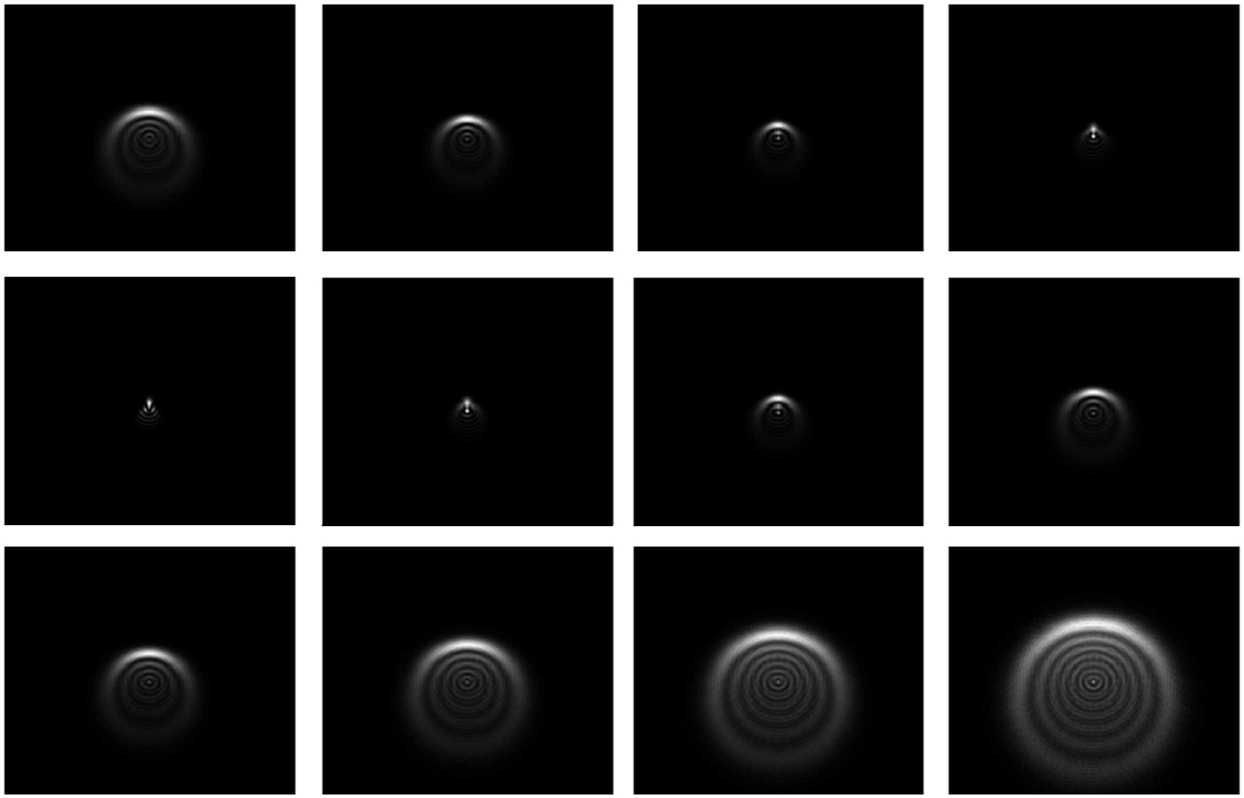
Geometrical and diffraction



Coma



Two waves of coma through focus

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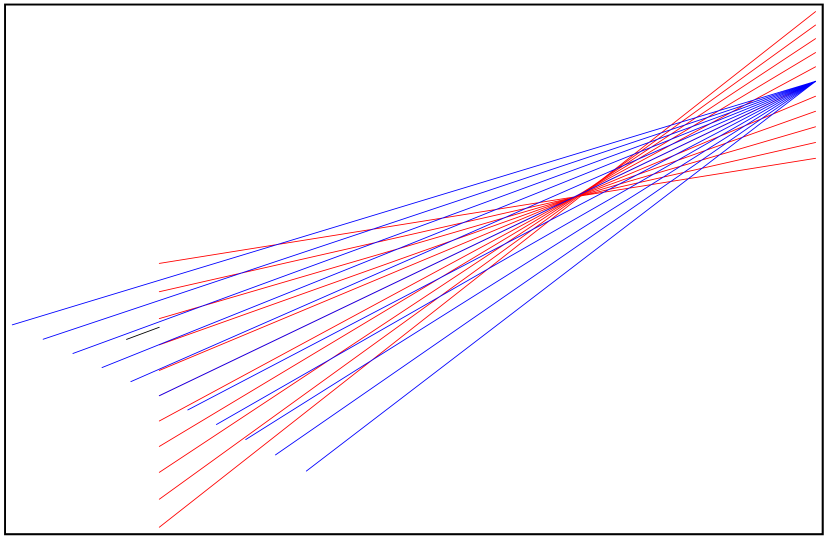
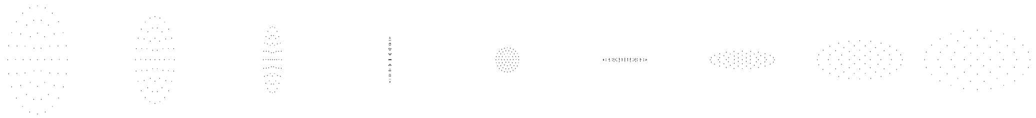
Astigmatism aberration





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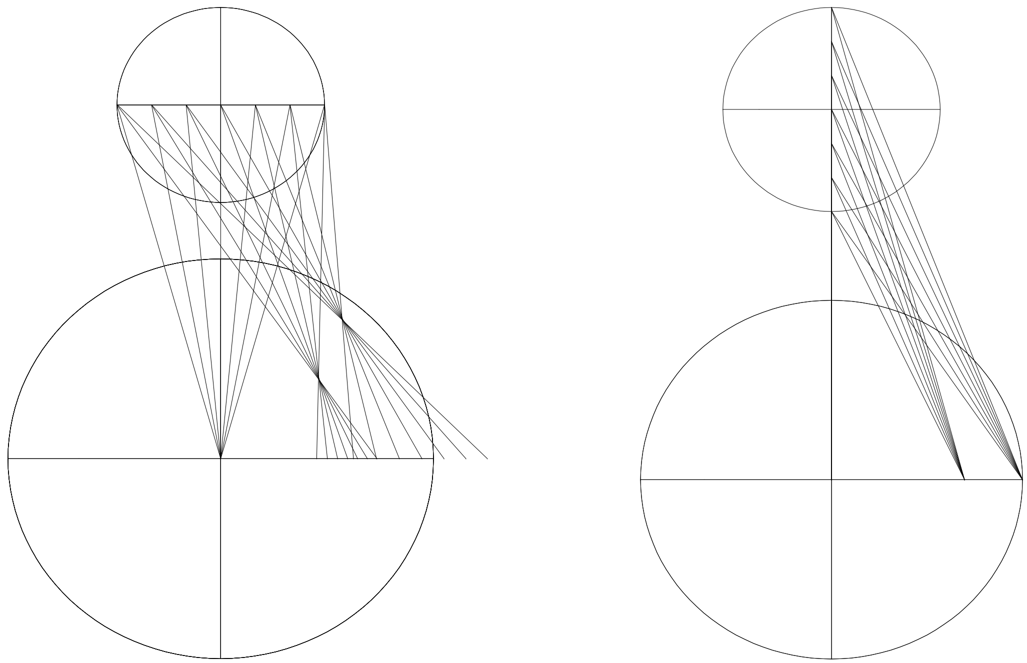
Meridional and sagittal ray fans



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Astigmatism

WAVE ABERRATION

$$W = W_{222} H^2 \rho^2 \cos^2\phi + W_{220} H^2 \rho^2 + \Delta W_{20} \rho^2$$

TRANSVERSE RAY ABERRATION

$$\omega' \vec{\epsilon} = 2\left[(W_{222} + W_{220}) H^2 + \Delta W_{20}\right] \rho \cos\phi \, \vec{h} + 2\left[W_{220} H^2 + \Delta W_{20}\right] \rho \sin\phi \, \vec{i}$$

$$\omega' \vec{\epsilon} = a \cos\phi \, \vec{h} + b \sin\phi \, \vec{i}$$ ELLIPSE

SAGITTAL FOCUS $(b = 0)$

$$\Delta W_{20} = -W_{220} H^2$$

$$\omega' \vec{\epsilon} = 2 W_{222} H^2 \rho \cos\phi \, \vec{h}$$ MERIDIONAL LINE SEGMENT

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Astigmatism

TANGENTIAL FOCUS $(a=0)$

$$\Delta W_{20} = -(W_{220} + W_{222})H^2$$

$$\omega' \vec{\epsilon} = -2W_{222} H^2 \rho \sin\phi \, \vec{i}$$ TRANSVERSE LINE SEGMENT

MEDIAL FOCUS $(b=-a)$

$$\Delta W_{20} = -(W_{220} + \tfrac{1}{2} W_{222})H^2$$

$$\omega' \vec{\epsilon} = W_{222} H^2 \rho \left[\cos\phi \, \vec{h} - \sin\phi \, \vec{i}\right]$$ CIRCLE (COUNTER-ROTATING)

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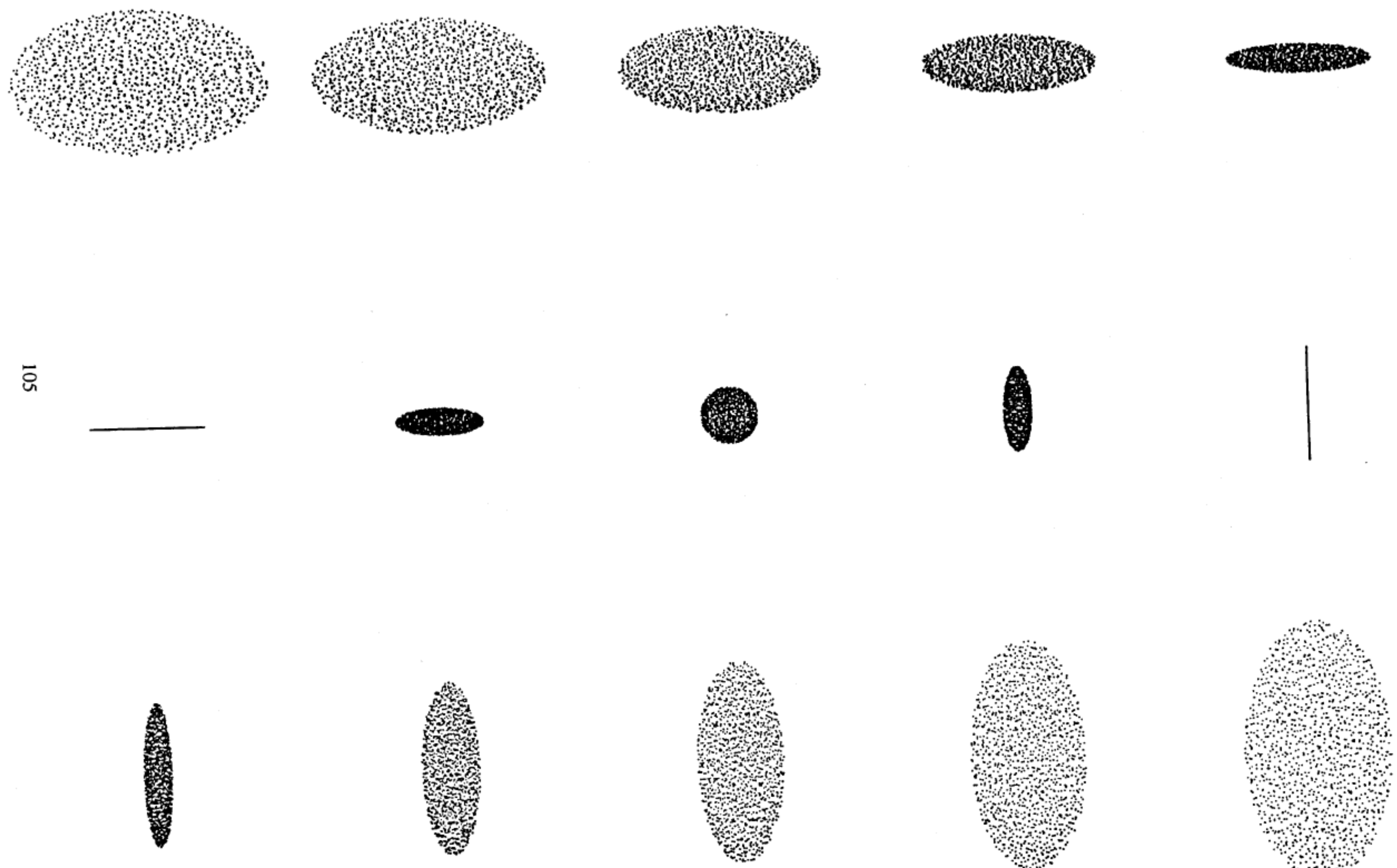
Spots through focus



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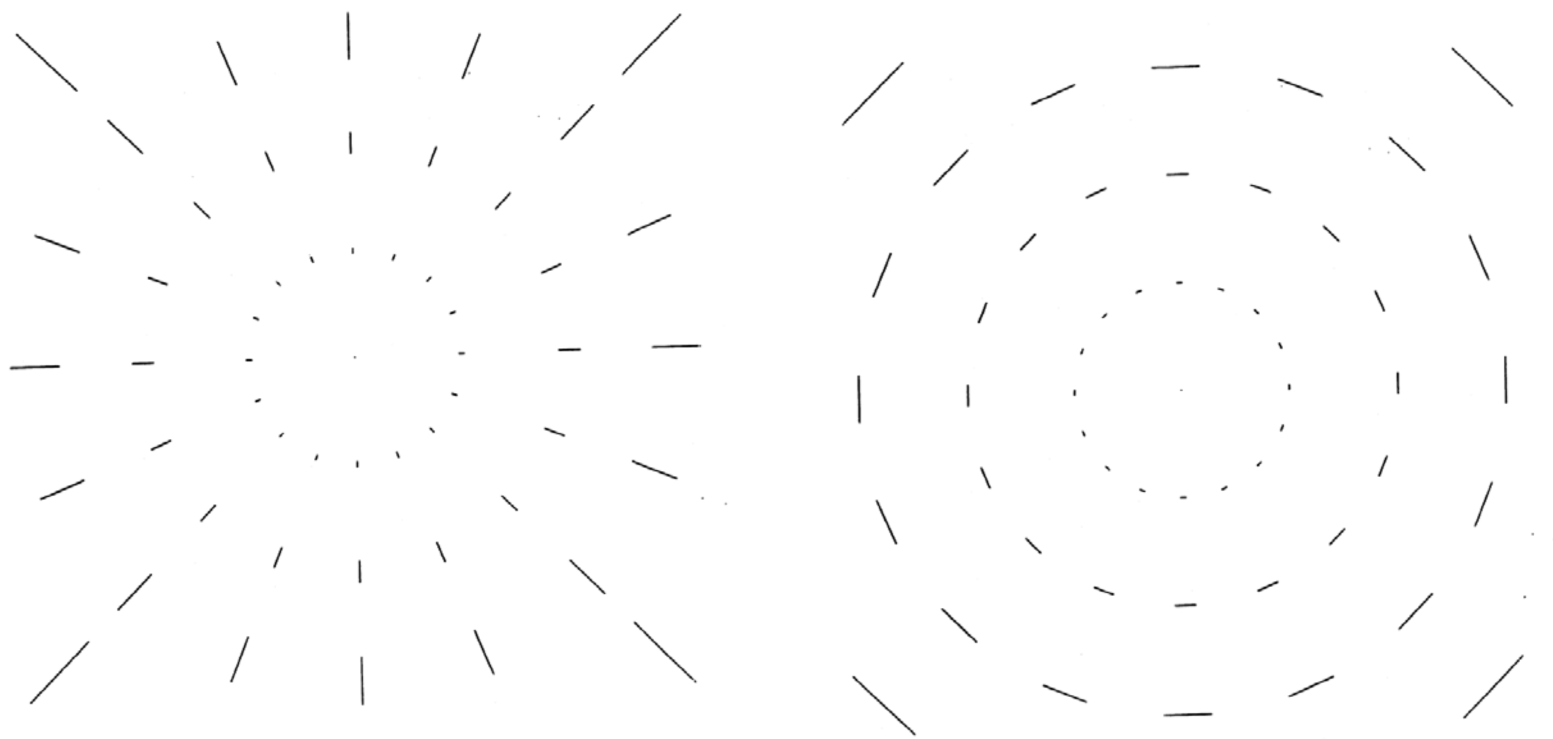
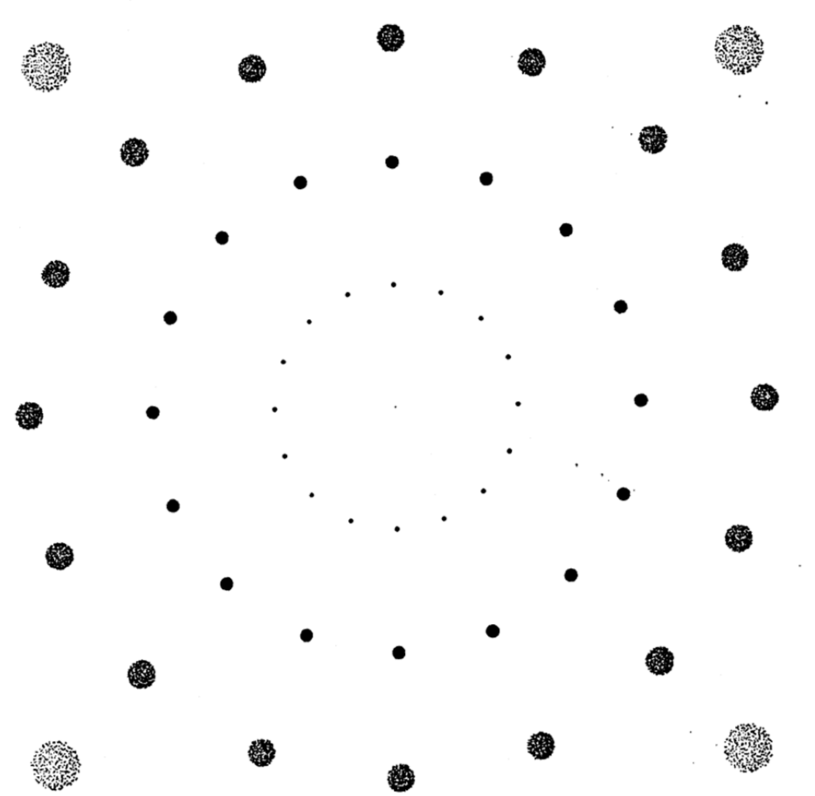
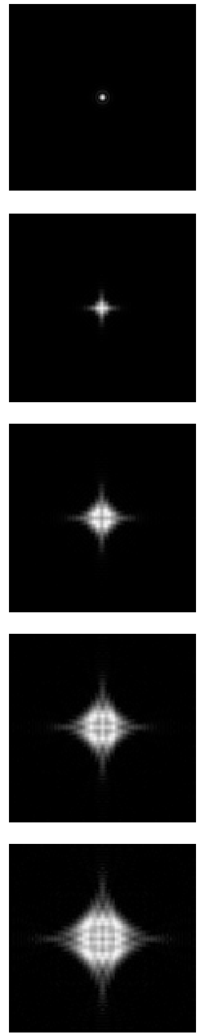
Quadratic (radial) Astigmatism



Theoretical behavior

Medial focus





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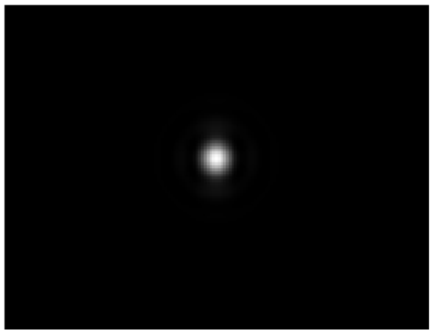
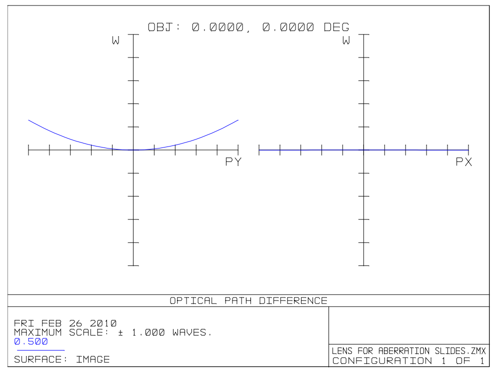
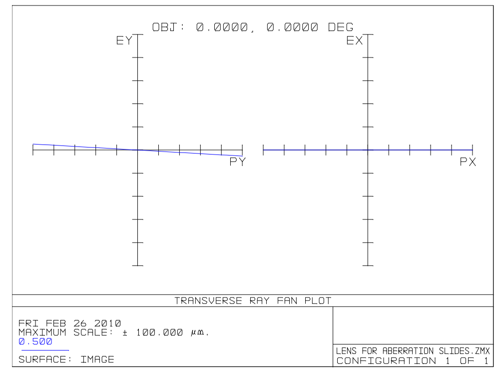
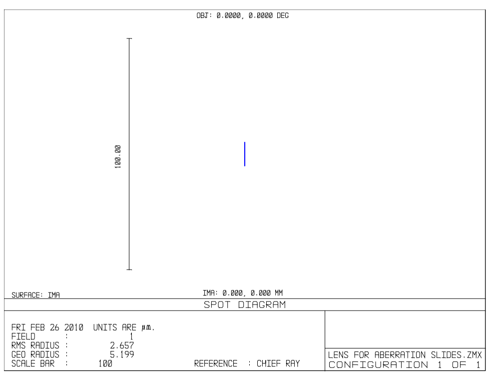
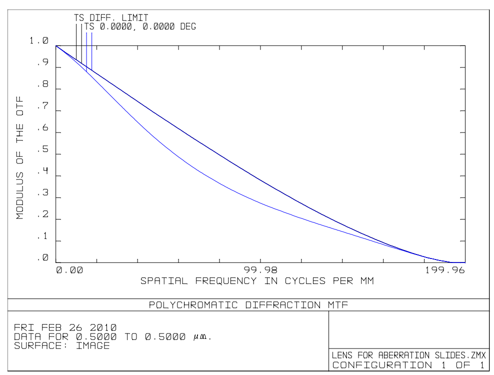
Astigmatism aberration 0. 25 wave
f/10; f=100 mm; wave=0.0005 mm

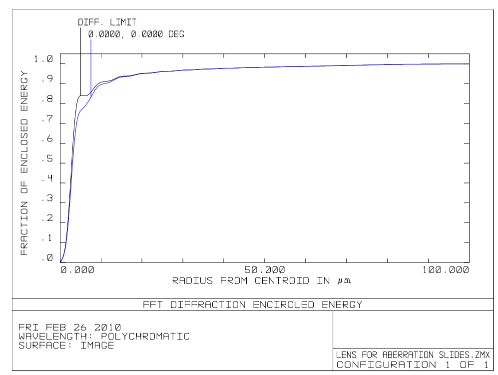




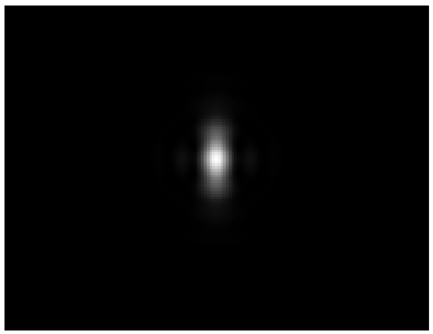


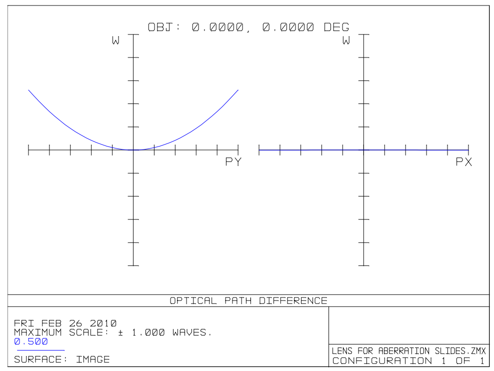


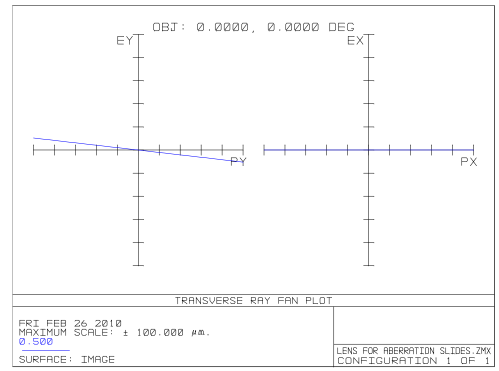


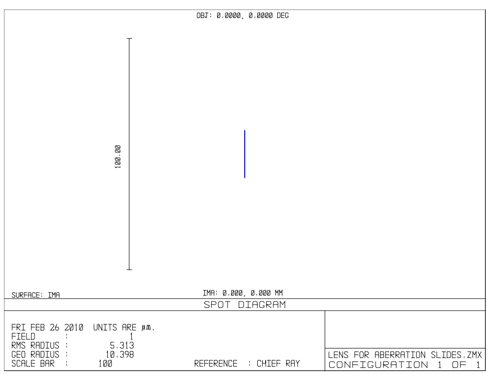


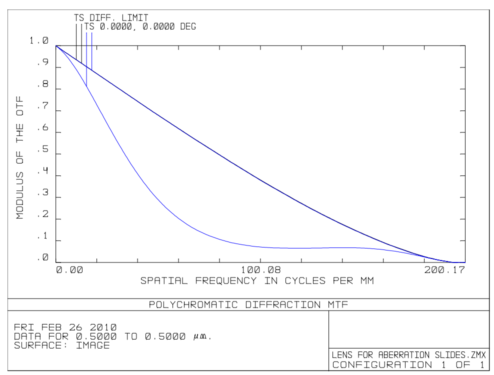
Astigmatism aberration 0. 5 wave f/10; f=100 mm; wave=0.0005 mm

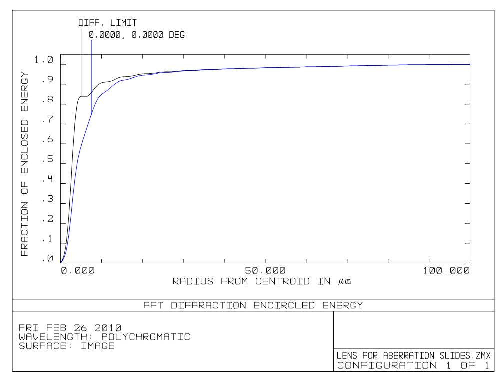




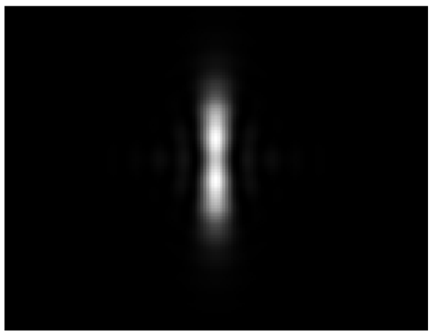


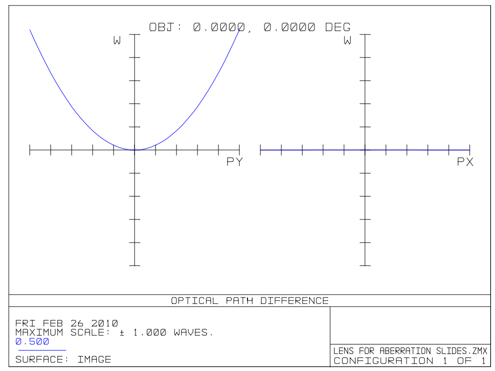


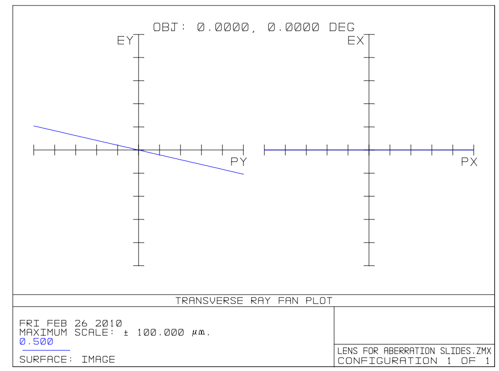


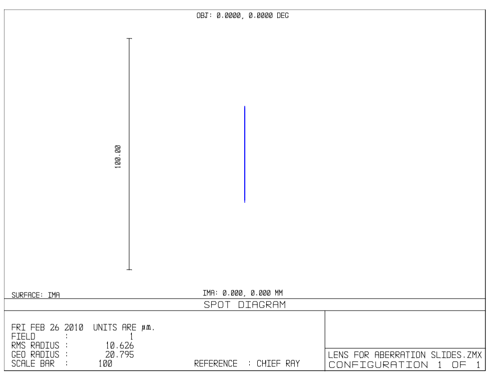


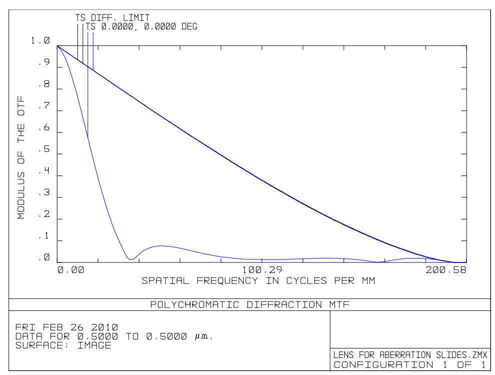
Astigmatism aberration 1.0 wave
f/10; f=100 mm; wave=0.0005 mm

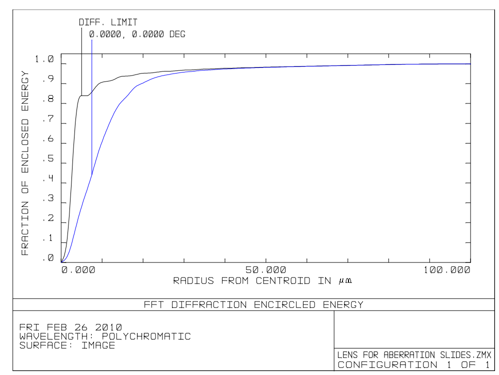












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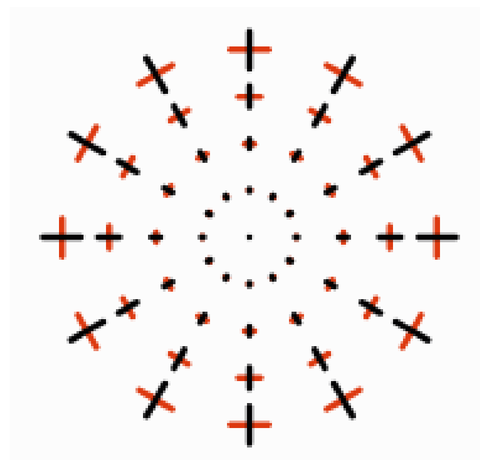
Astigmatism varies as the square of the field of view

$$W_{222}\left(\vec{H}\cdot\vec{\rho}\right)^{2}$$



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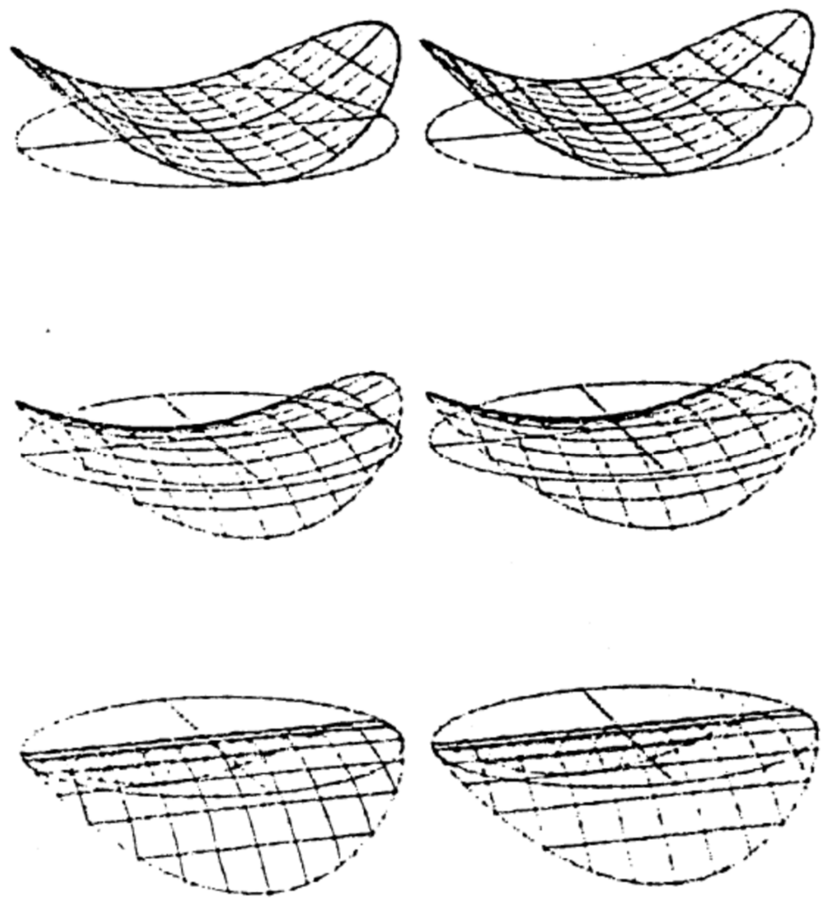
Astigmatism



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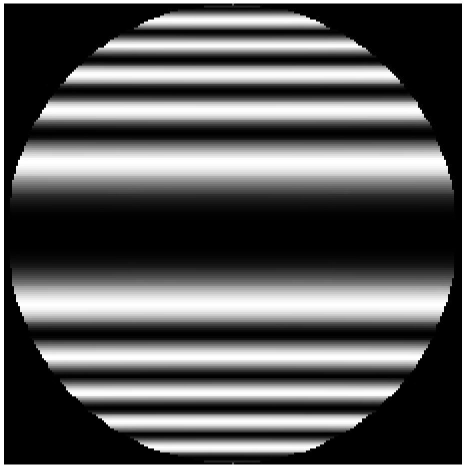
Interferometric representation



5 waves

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Cases of zero astigmatism aberration from a spherical surface

$$W_{222}\left(\vec{H}\cdot\vec{\rho}\right)^2 \qquad W_{222}=\frac{1}{2}S_{III} \qquad S_{III}=-\sum \bar{A}^2 y\Delta\left(\frac{u}{n}\right)$$

$$y=0$$

$$\bar{A}=0$$

$$\Delta\left(u/n\right)=u'/n'-u/n=0$$

y=0 the aperture is zero or the surface is at an image
Abar=0 surface is concentric with stop or pupils
u'/n-u/n=0 the conjugates are at the aplanatic points
Aplanatic means free from error;
freedom from spherical aberration and coma.

Aplanatic spherical surface is also anastigmatic
Anastigmatic: free from spherical, coma and astigmatism

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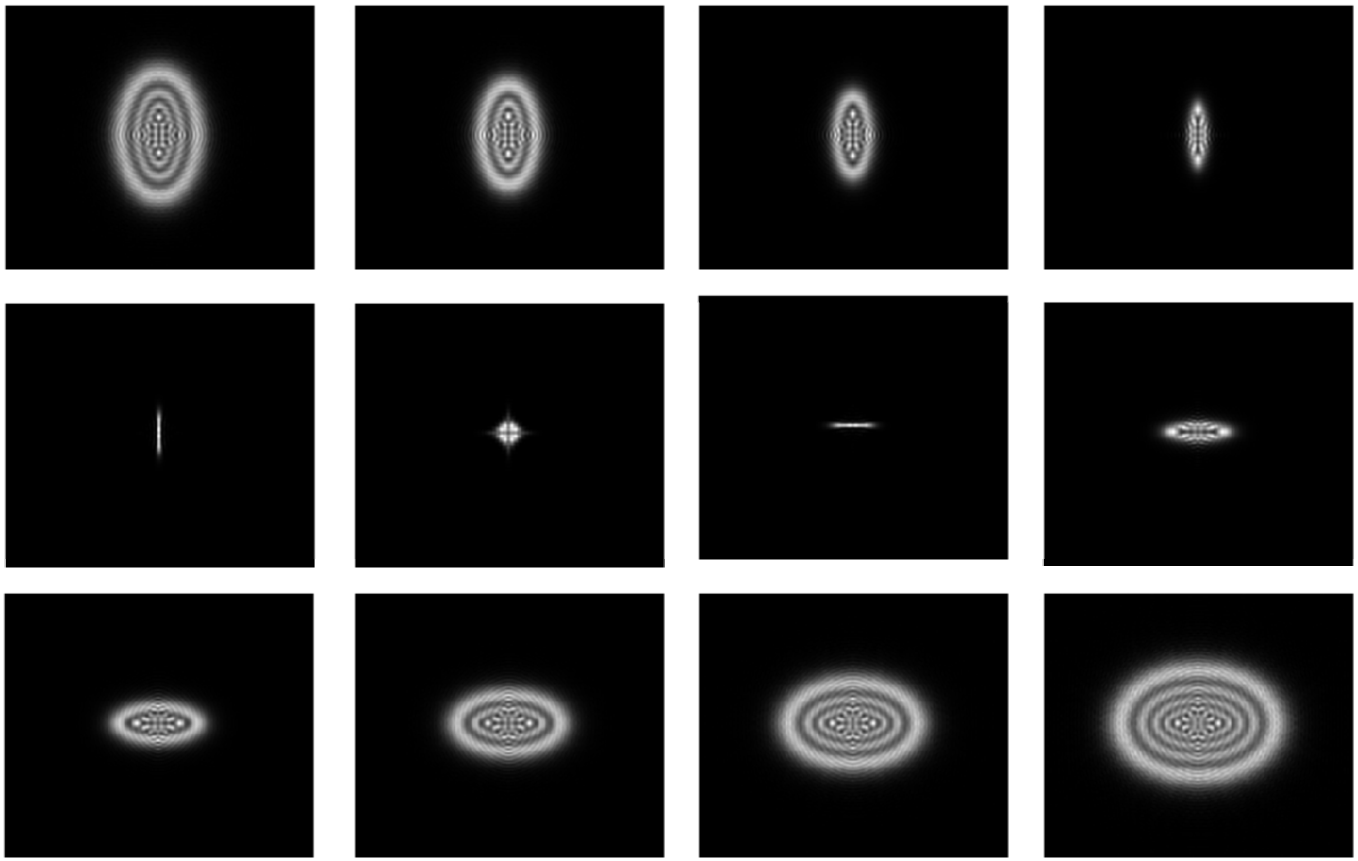
Two waves of astigmatism



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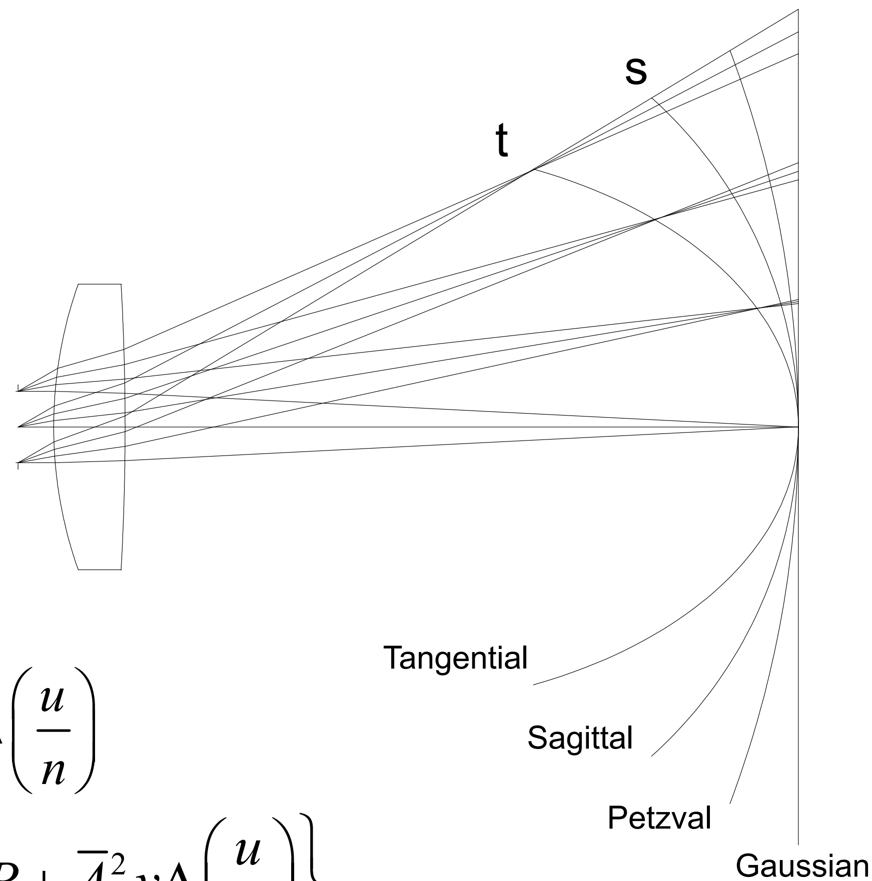
The field curves



$$W_{222} = -\frac{1}{2}\sum \bar{A}^2 y\Delta\left(\frac{u}{n}\right)$$

$$W_{220} = -\frac{1}{4}\sum \left\{ Ж^2 P + \bar{A}^2 y\Delta\left(\frac{u}{n}\right)\right\}$$

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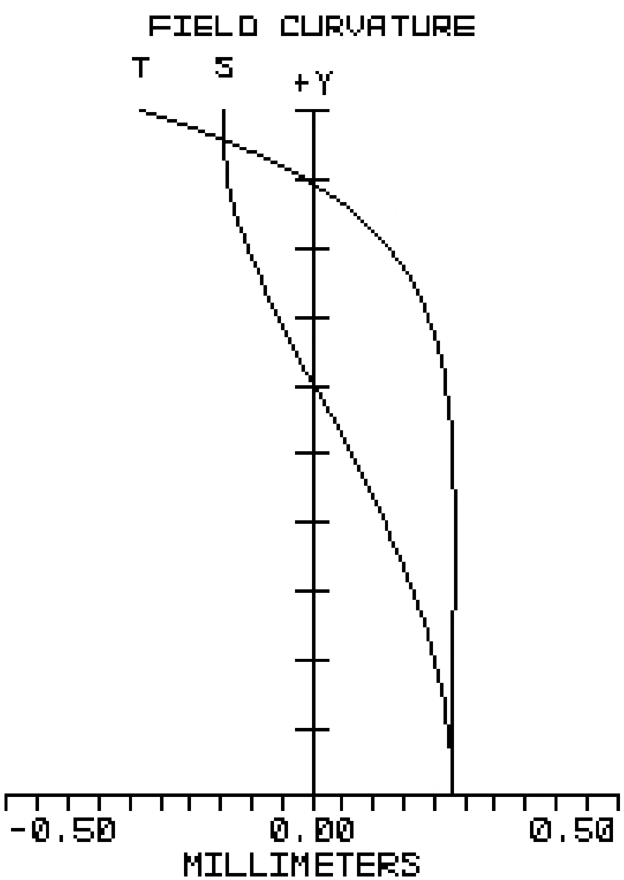
Field curves by an optical design program



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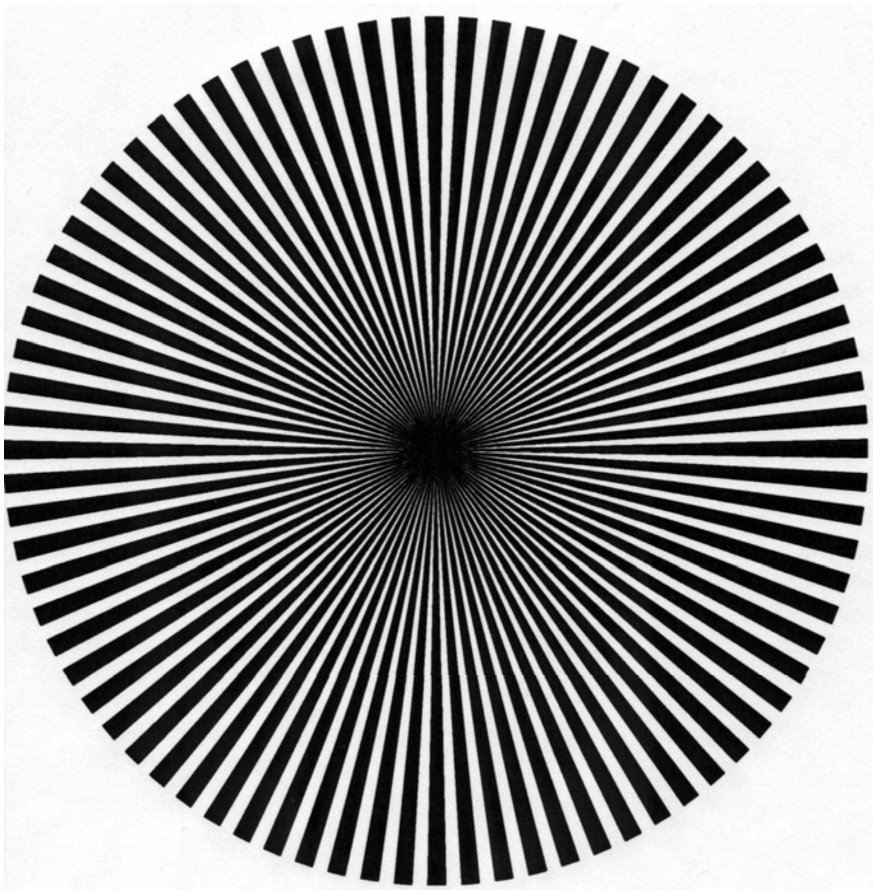
Eye astigmatism



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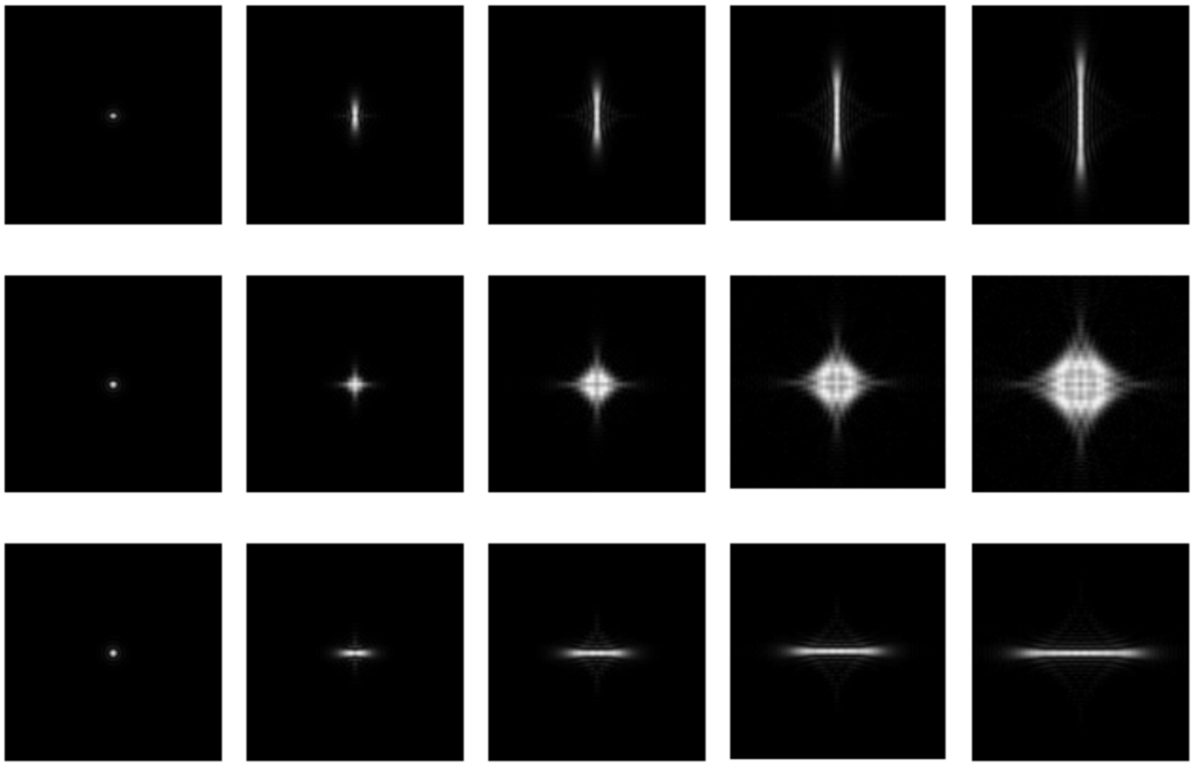
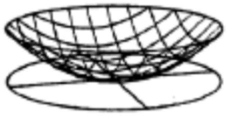
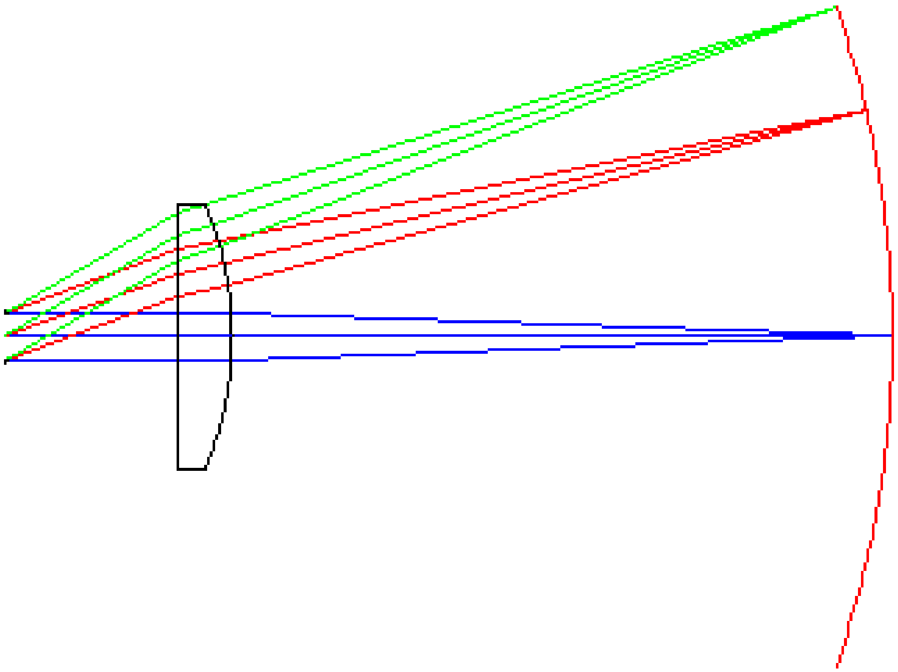
Diffraction images



Field curvature

$$W_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$$





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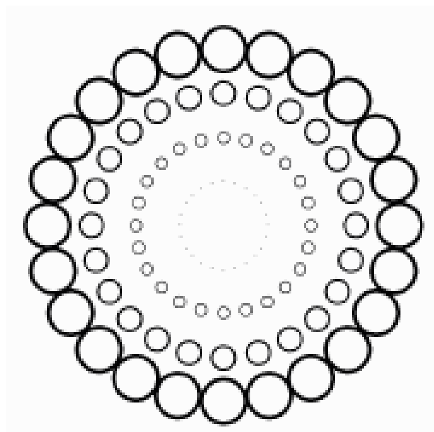
Field curvature varies as the square of the field of view

$$W_{220}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$$



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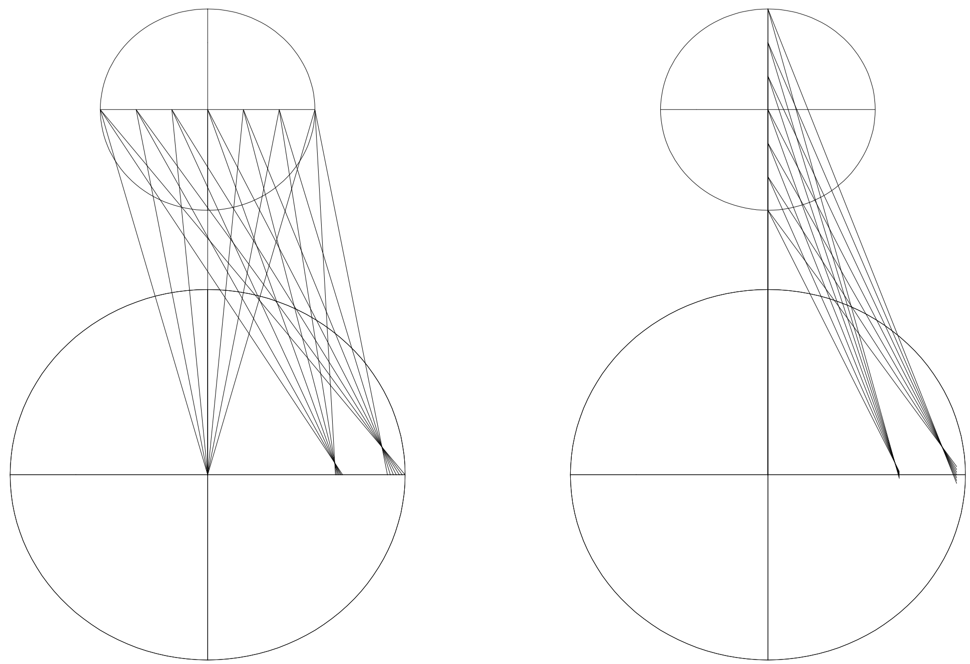
Meridional and sagittal ray fans



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Field curvature

$$W_{220} = -\frac{1}{4}\sum\left(Ж^2 P + \bar{A}^2 \Delta\left\{\frac{u}{n}\right\} y \right) \qquad P = C \cdot \Delta\left(\frac{1}{n}\right)$$

Petzval sum:

$$\frac{1}{n'_k \rho'_k} - \frac{1}{n_1 \rho_1} = -\sum_{i=1}^{k} \frac{n' - n}{n' n r}$$

For a system of thin lenses
In air:

$$\frac{1}{\rho'_k} = -\sum_{i=1}^{k} \frac{\phi_i}{n_i}$$

Petzval field curvature interpretation

For a single lens
$$-\frac{1}{\rho_{Petzval}}=\sum_{i=1}^{2}\frac{n'-n}{n'nr}=\frac{n'-1}{n'r_1}-\frac{n'-1}{n'r_2}=\frac{n'-1}{n'}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Multiply by $\frac{h^2}{2}$ or
$$-\frac{h^2}{2\rho_{Petzval}}=\frac{n'-1}{n'}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)\frac{h^2}{2}=\frac{n'-1}{n'}t$$

h is a given height; t is the lens sag or thickness at height h

Then $\frac{h^2}{2\rho_{Petzval}}$ Is the sag of the Petzval field curvature and $\frac{n'-1}{n'}t$

is the image displacement by a parallel plate of thickness t.

Thus Petzval field curvature can be interpreted as the image shift due to a “parallel plate” of varying thickness t.

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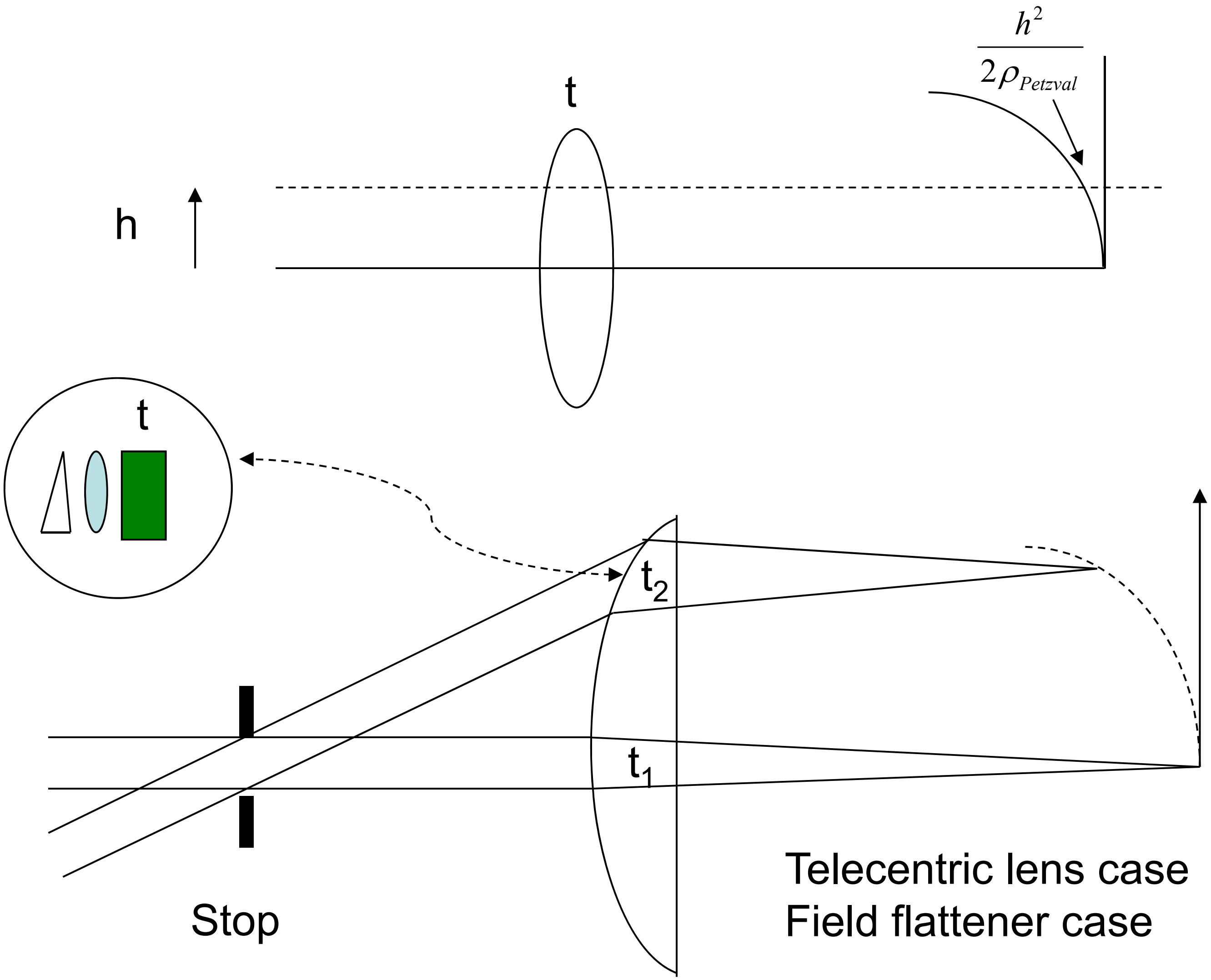
Petzval field curvature interpretation



Telecentric lens case
Field flattener case

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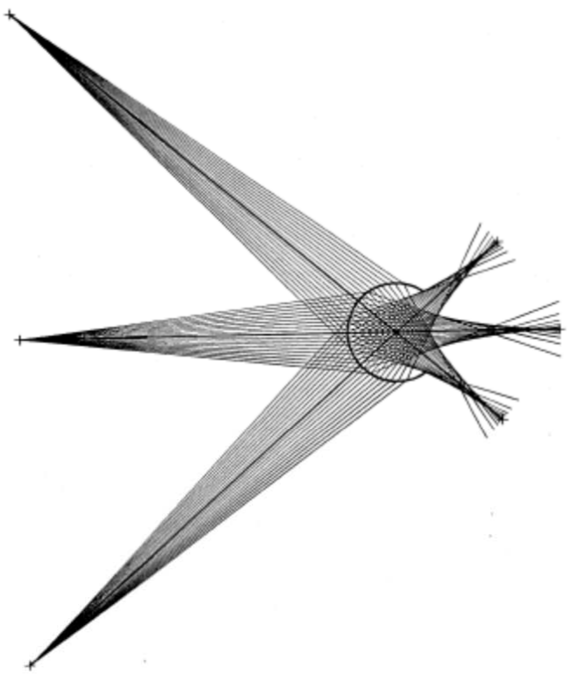
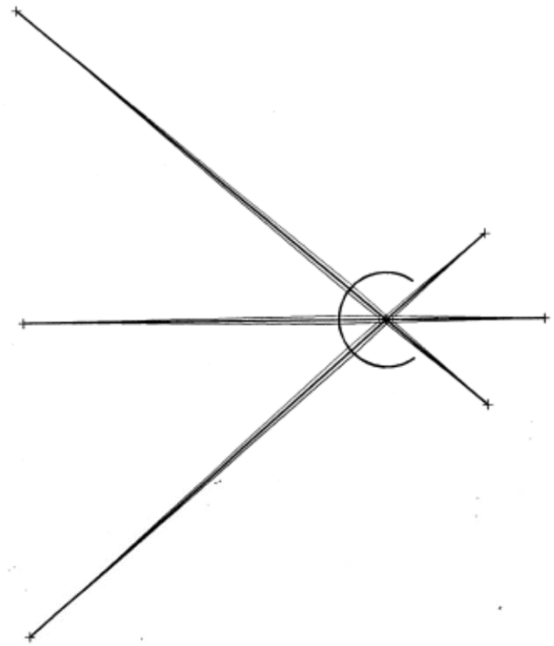
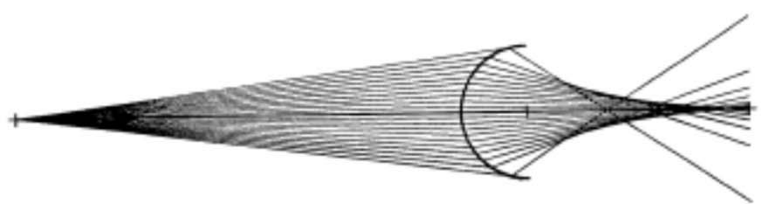
Petzval field curvature: Stop at center of curvature







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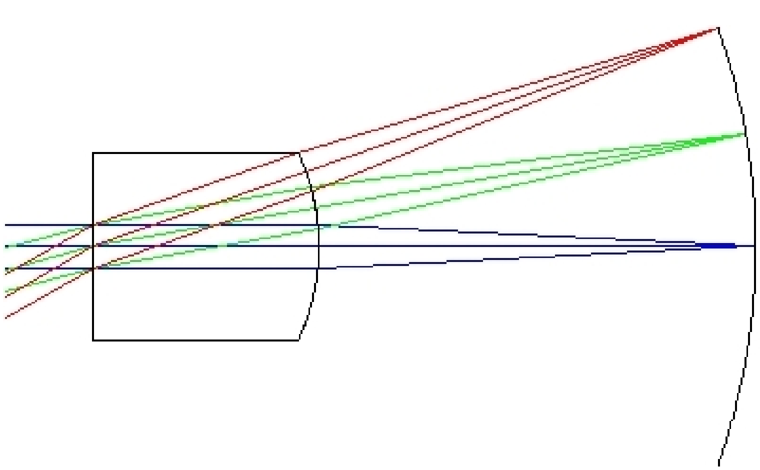
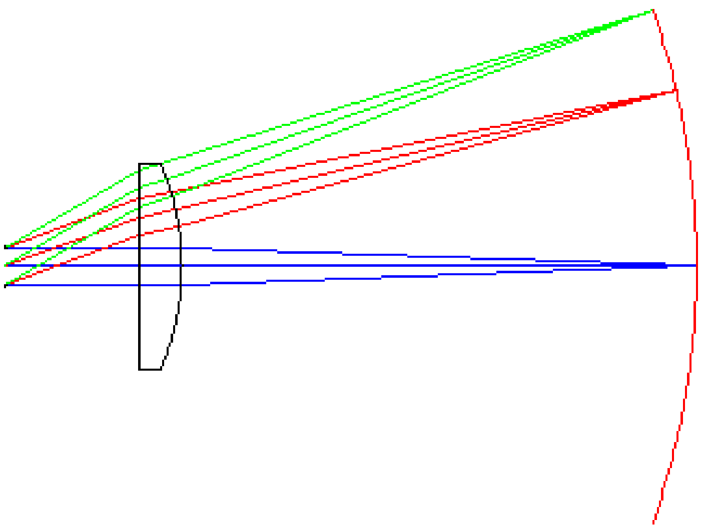
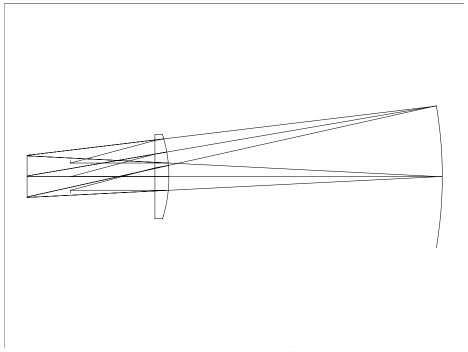
Petzval radius for a lens





$$\frac{1}{\rho'_k} = -\sum \frac{\phi}{n}$$

R=t=51.852239
F=100 mm
Petzval radius=151.852239=nf
Bk7 glass



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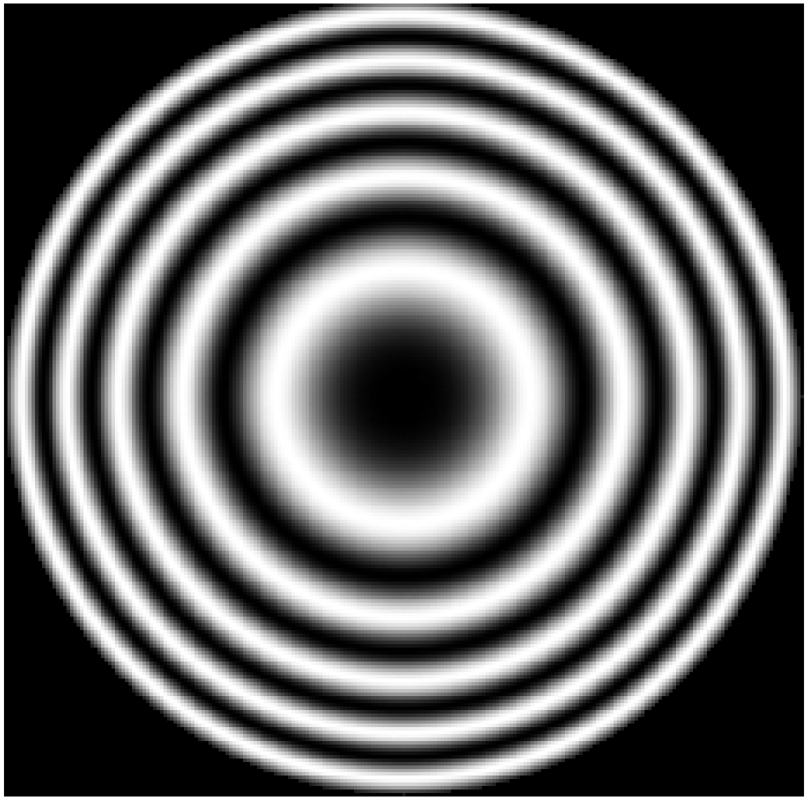
Interferometric representation



5 waves

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Field curves vertex curvature

$$\Delta z = -2\frac{W_{020}}{n'u'^2} = -2n'\frac{\bar{y}_I^2}{Ж^2}W_{020}$$

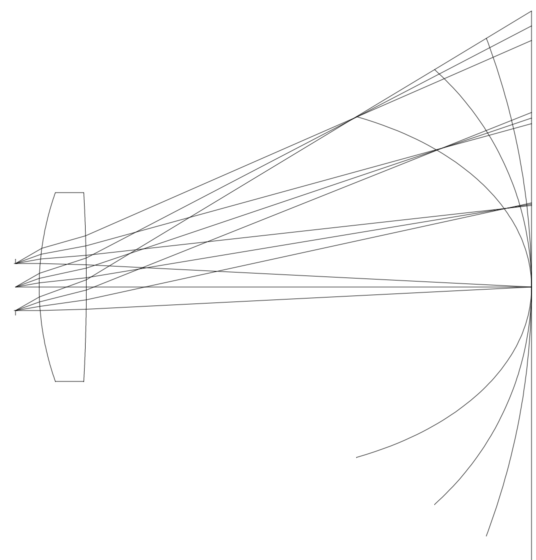
$$\frac{2\Delta z}{\bar{y}_I^2} = -4n'\frac{1}{Ж^2}W_{020}$$

$$C_t = -4\frac{n'}{Ж^2}\left(W_{220P} + \frac{3}{2}W_{222}\right)$$

$$C_m = -4\frac{n'}{Ж^2}\left(W_{220P} + W_{222}\right)$$

$$C_s = -4\frac{n'}{Ж^2}\left(W_{220P} + \frac{1}{2}W_{222}\right)$$

$$C_P = -4\frac{n'}{Ж^2}W_{220P}$$



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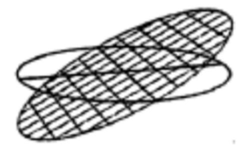
Distortion

$$W_{311}\left(\vec{H}\cdot\vec{H}\right)\left(\vec{H}\cdot\vec{\rho}\right)$$

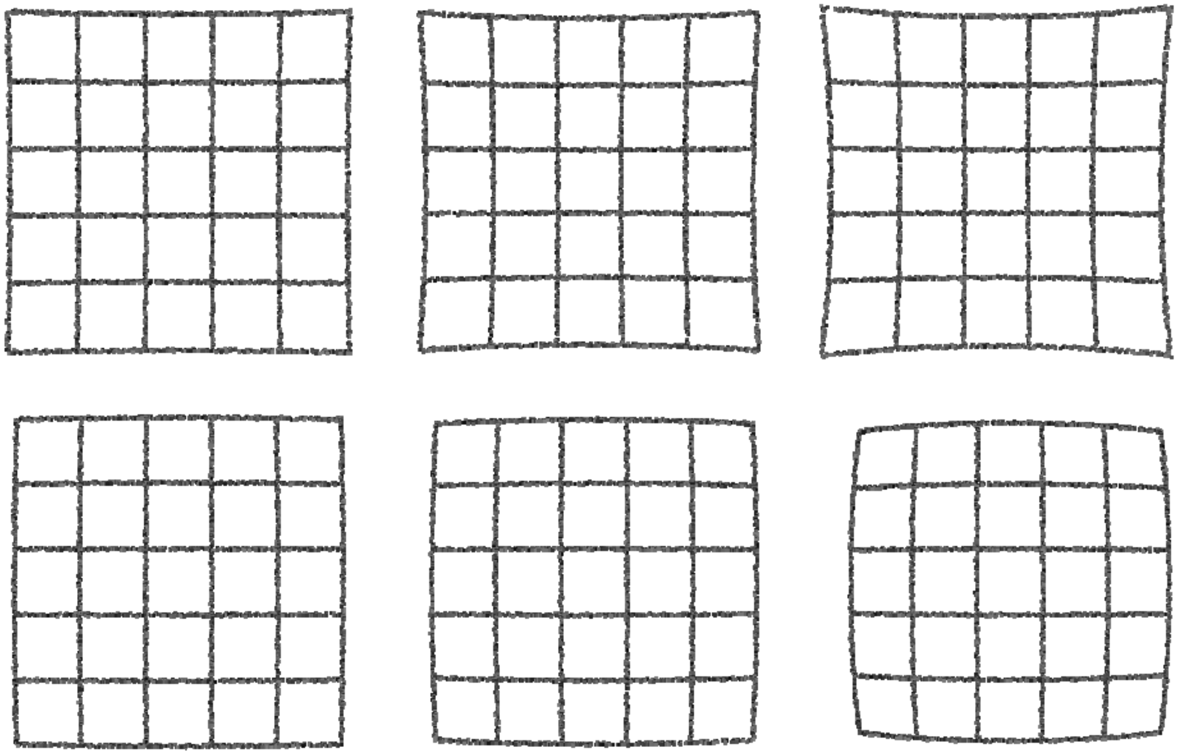


$$W_{311} = \frac{1}{2} S_V$$

$$S_V = -\sum \frac{\bar{A}}{A}\left[\text{Ж}^2 P + \bar{A}^2 y \Delta\left(\frac{u}{n}\right)\right]$$

$$S_V = -\sum \bar{A}\left[\bar{A}^2 \Delta\left(\frac{1}{n^2}\right) y - \left(\text{Ж} + \bar{A} y\right) \bar{y} P\right]$$

Distortion



2.5%, 5% and 10%

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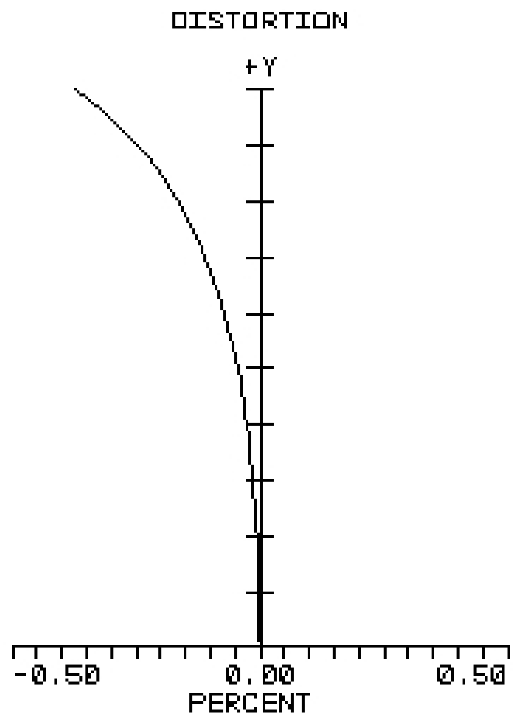
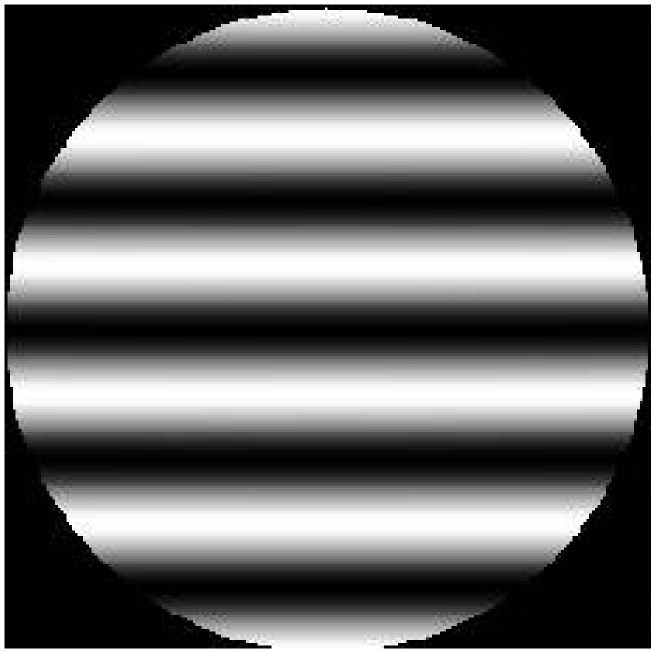
Distortion



$$Distortion = \frac{Y - \bar{y}}{\bar{y}} \cdot 100$$

Interferometric representation



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OPTI 518

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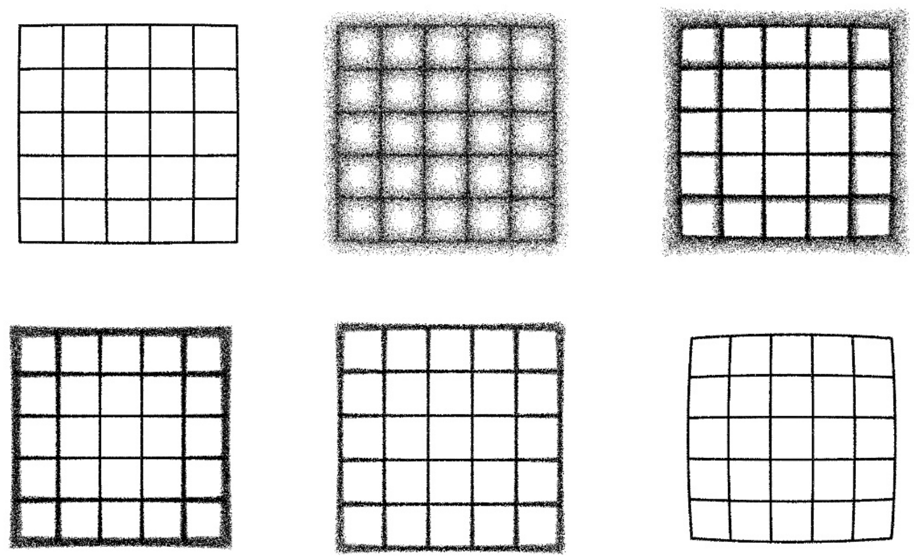
Geometrical imaging with aberrations



Spot diagram concept applied to an extended object

Prof. Jose Sasian
OPTI 518

College of Optical Sciences
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Parity of the aberrations

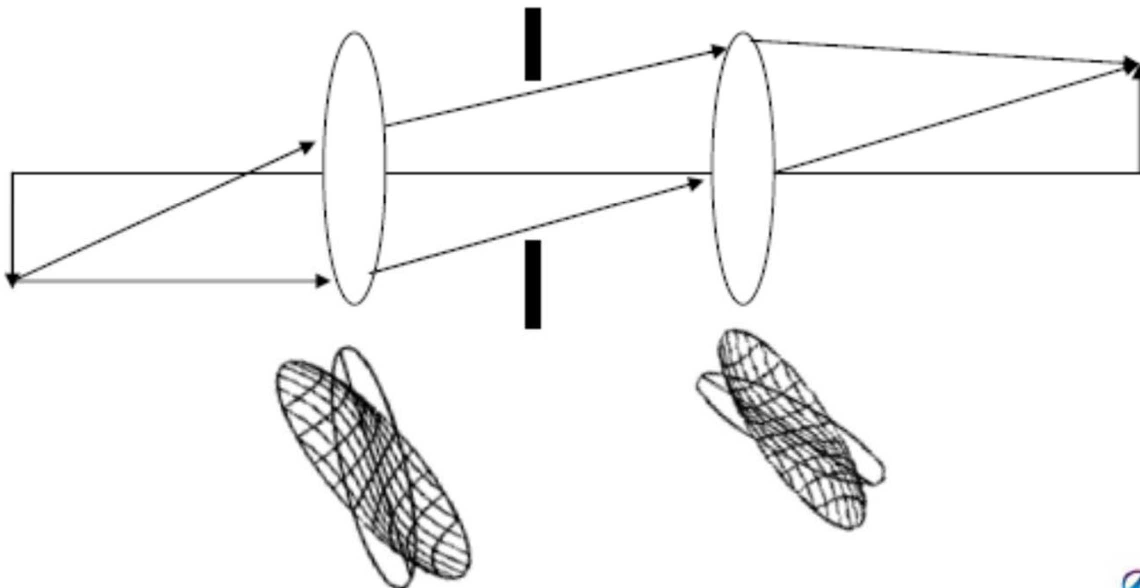
The aberrations can be classified as even and odd aberrations. Spherical aberration, astigmatism, field curvature, and longitudinal color are even aberrations. Coma, distortion, and transverse color are odd aberrations. The parity is found by observation of the algebraic power's parity of the field and aperture vectors in the aberration coefficients.

The odd aberrations have the important property that in a symmetrical system they Cancel (fourth-order), each half contributes the same amount of aberration but with opposite algebraic sign. The symmetry is about the aperture stop. In comparison in a symmetrical system the even aberrations from each half of the system add, rather than cancel

Aberrations and symmetry

$$W(H,\rho,\theta) = W_{200}H^2 + W_{020}\rho^2 + W_{111}H\rho\cos\theta +$$
$$+ W_{040}\rho^4 + W_{131}H\rho^3\cos\theta + W_{222}H^2\rho^2\cos^2\theta +$$
$$+ W_{220}H^2\rho^2 + W_{311}H^3\rho\cos\theta + W_{400}H^4 +$$
$$+ \ldots$$

- Coma is an odd aberration with respect to the stop
- Natural stop position to cancel coma by symmetry



Summary

- Discussion of the primary aberrations
- Use of several types of plots
- Main point is to gain understanding and familiarity
- Must be able to recognize the aberrations under a variety of representations.
- Must be able to appreciate how the ideal image degrades in the presence of aberration