Coma aberration

Lens Design OPTI 517

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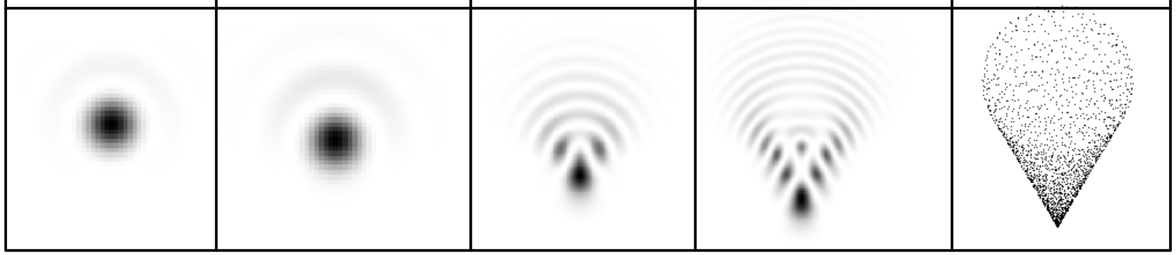
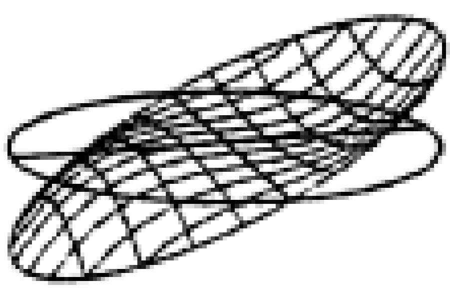
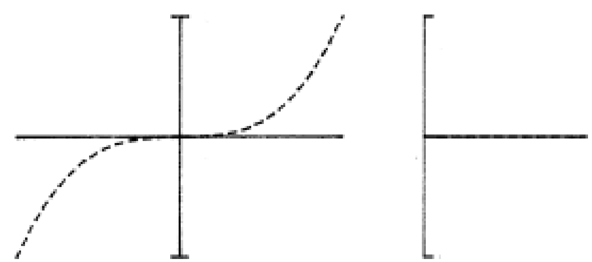
Coma

0.25 wave	1.0 wave	2.0 waves	4.0 waves	Spot diagram

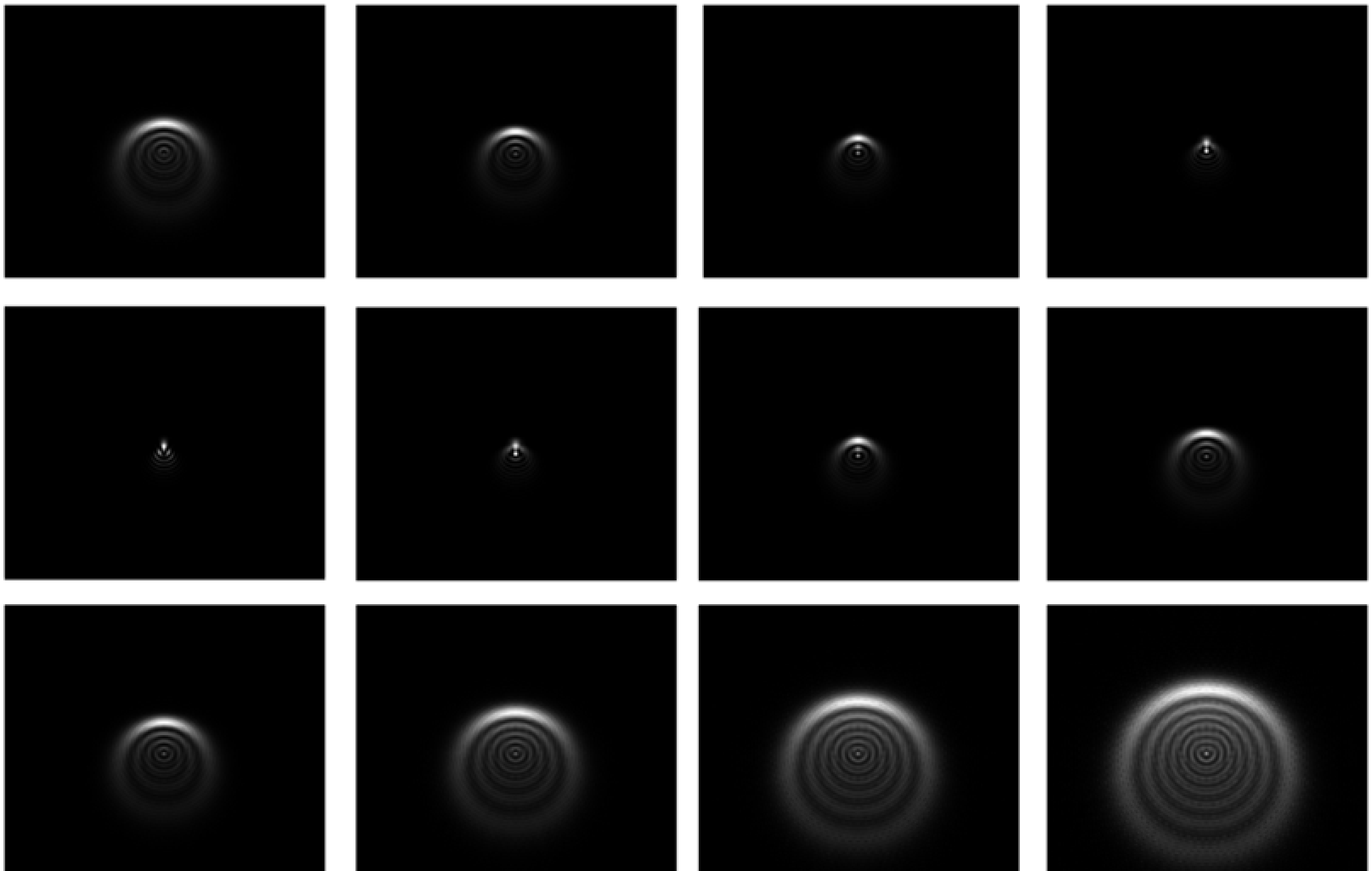


$$W(H,\rho,\theta)=W_{200}H^2+W_{020}\rho^2+W_{111}H\rho\cos\theta+$$
$$+W_{040}\rho^4+W_{131}H\rho^3\cos\theta+W_{222}H^2\rho^2\cos^2\theta+$$
$$+W_{220}H^2\rho^2+W_{311}H^3\rho\cos\theta+W_{400}H^4+$$
$$+\ldots$$





Coma though focus



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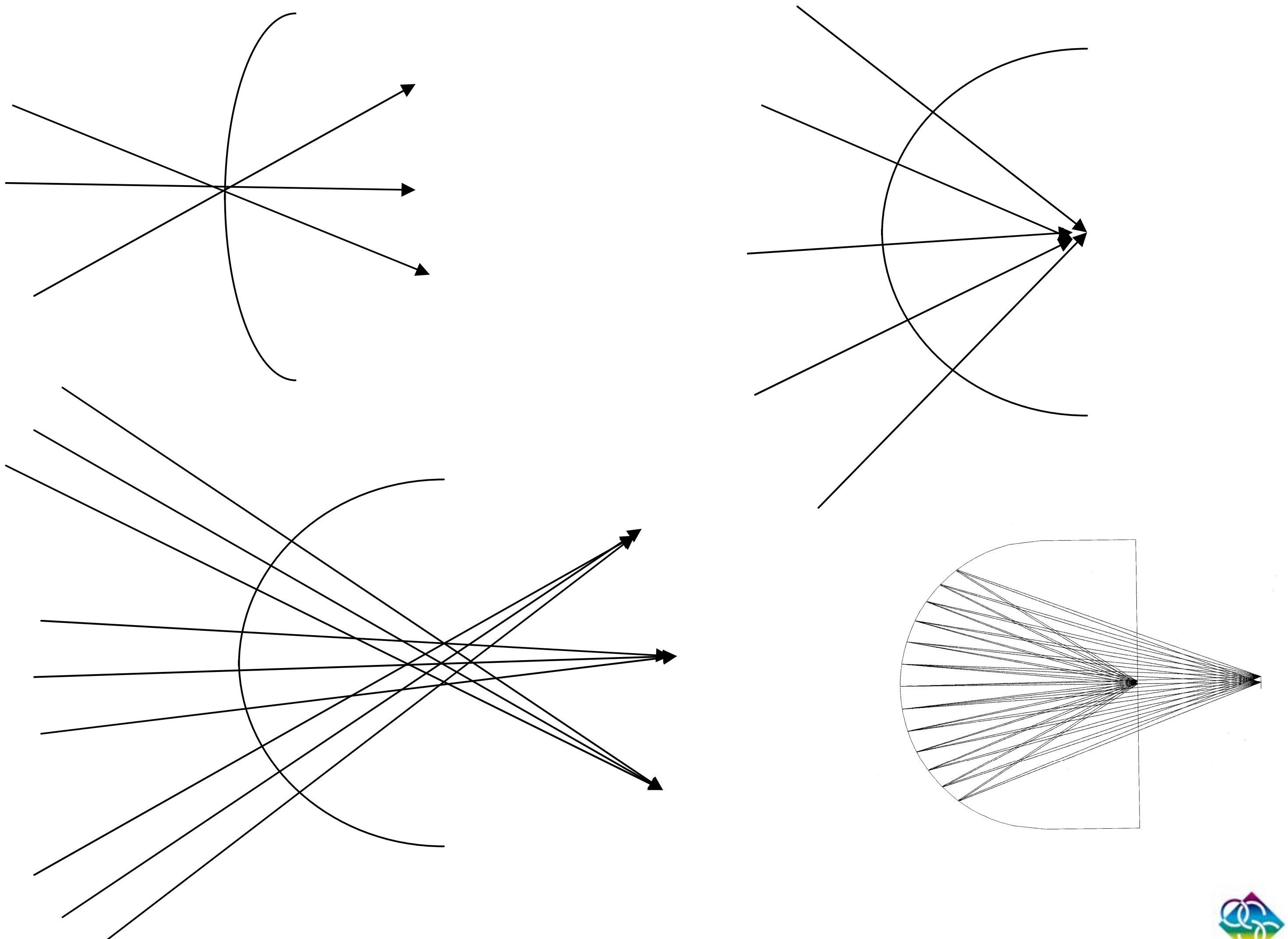
Cases of zero coma

$$W_{131} = -\frac{1}{2} A \bar{A} \Delta \left\{ \frac{u}{n} \right\} y$$

- At y=0, surface is at an image
- A=0, On axis beam concentric with center of curvature
- A-bar=0, Off-axis beam concentric, chief ray goes through the center of curvature
- Aplanatic points

Cases of zero coma



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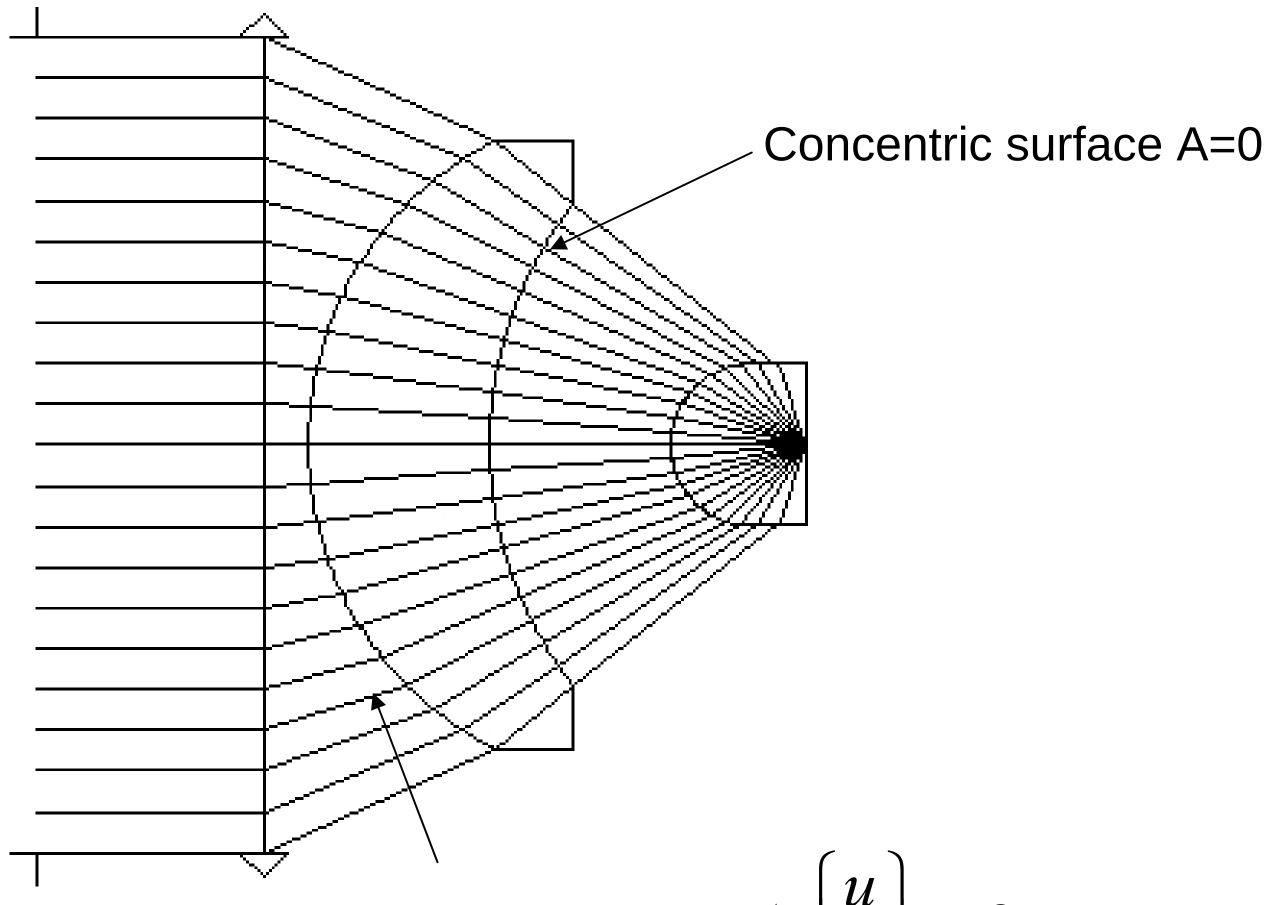
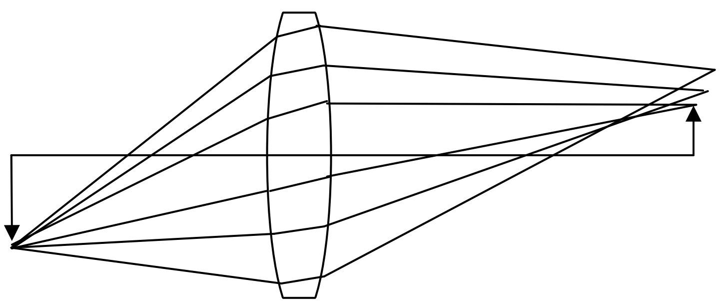
Aplanatic-concentric

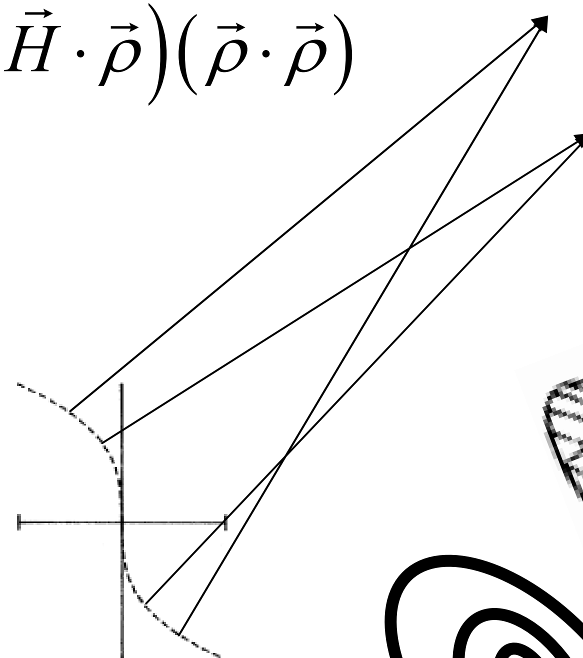


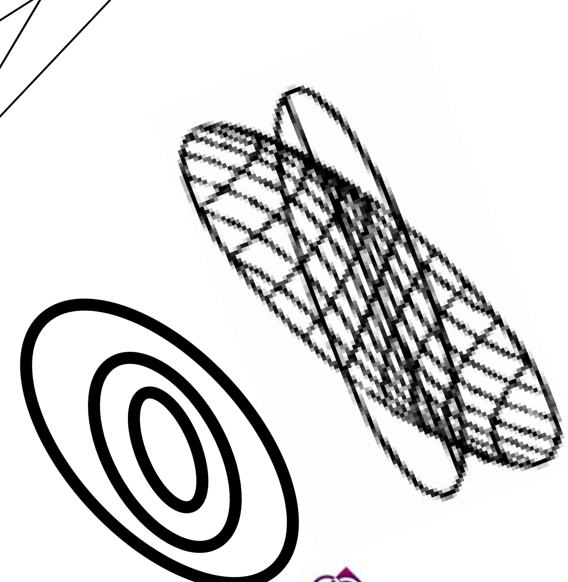
$$\Delta\left\{\frac{u}{n}\right\}=0$$

Coma as a variation of magnification with aperture I

$$W_{131}\left(\vec{H},\vec{\rho}\right)=-\frac{1}{2}A\bar{A}\Delta\left\{\frac{u}{n}\right\}y\left(\vec{H}\cdot\vec{\rho}\right)\left(\vec{\rho}\cdot\vec{\rho}\right)$$







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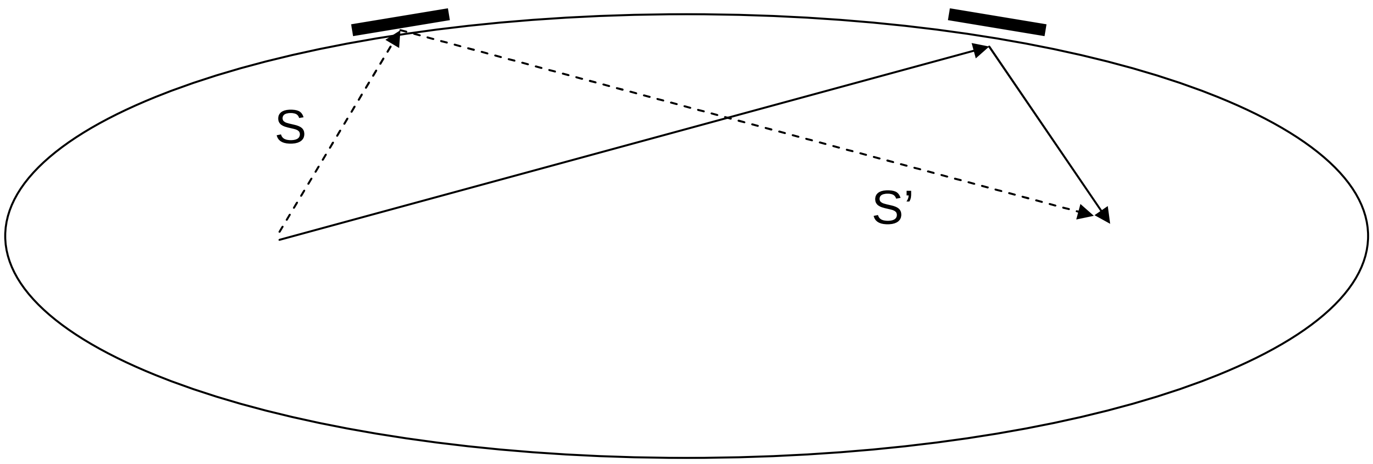
Coma as a variation of magnification with aperture II

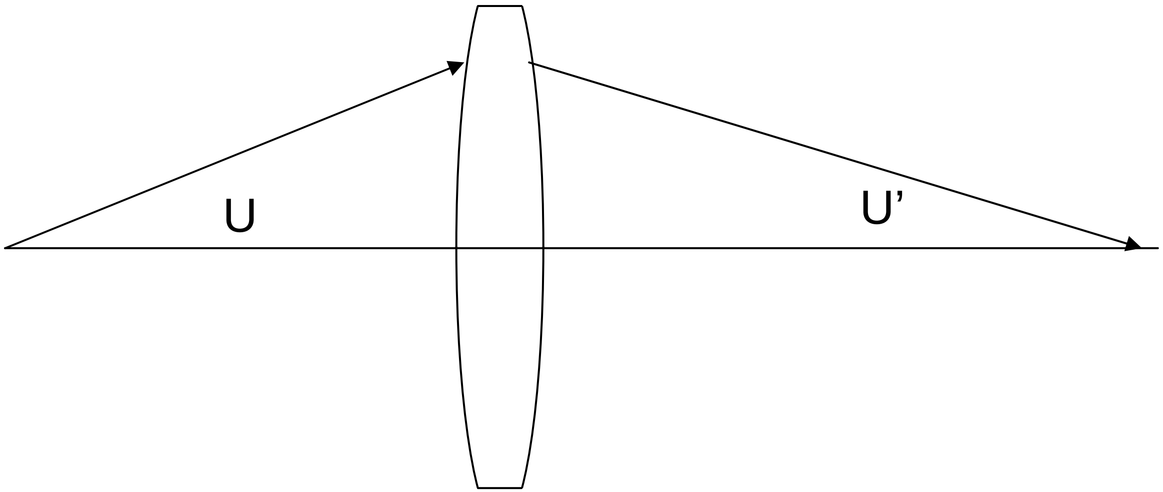


$$m = s'/s$$

Sine condition

Coma aberration can be considered as a variation of magnification with respect to the aperture. If the paraxial magnification is equal to the real ray marginal magnification, then an optical system would be free of coma.

Spherical aberration can be considered as a variation of the focal length with the aperture.



$$\frac{u}{u'} = \frac{\sin(U)}{\sin(U')}$$

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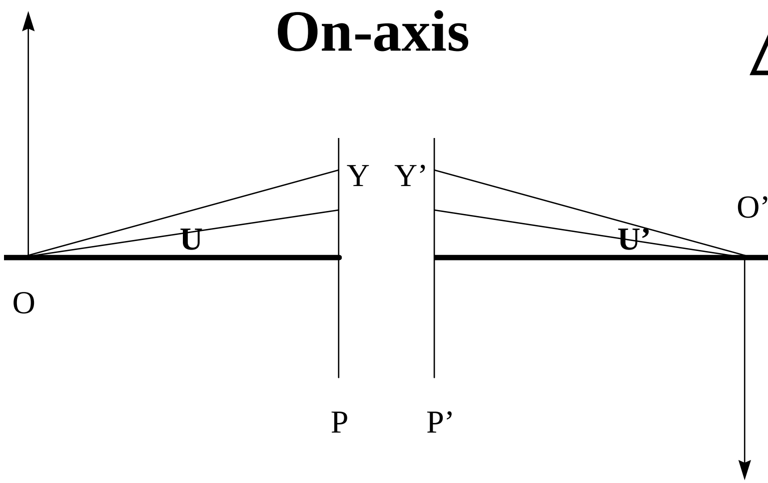
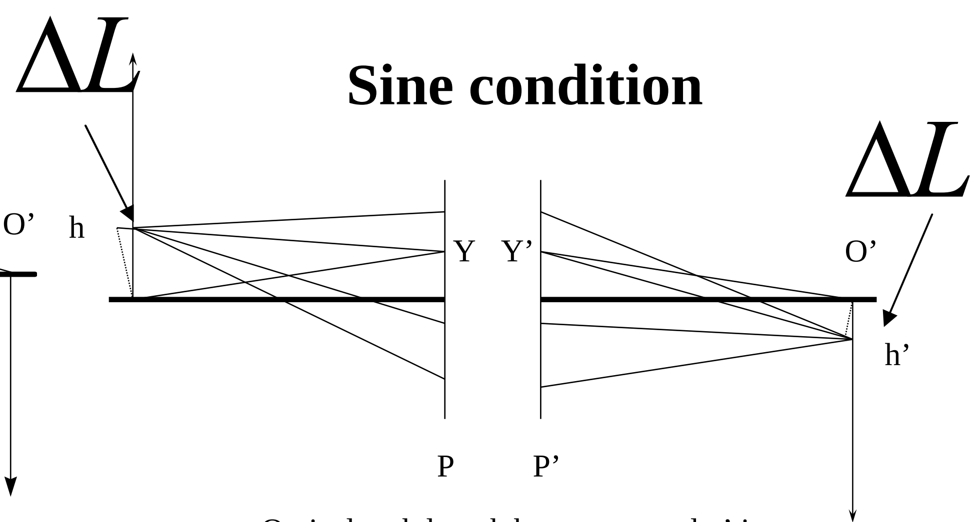
Sine condition





Optical path length between O and O’ is L_{axis} and does not depend on Y or Y’

Optical path length between y and y’ is

$$L_{off\text{-}axis} = L_{axis} + \Delta L' - \Delta L$$

$$L_{off\text{-}axis} = L_{axis} + h' n' \sin(\mathbf{U}') - h n \sin(\mathbf{U})$$

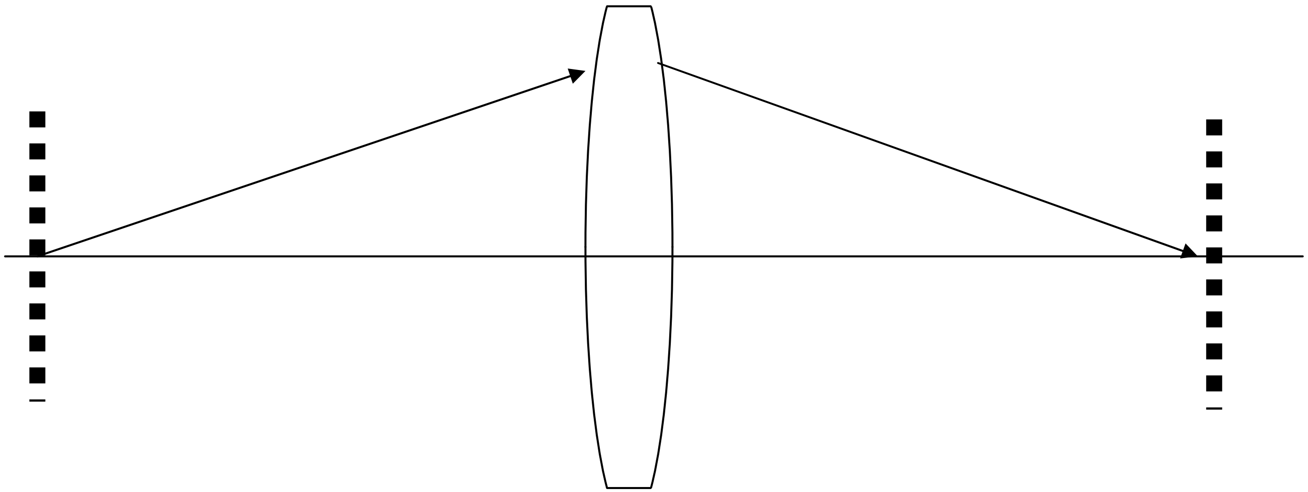
$$\Delta L = h\sin(U) \qquad \Delta L' = h'\sin(U')$$

$$h'n'\sin(U') = hn\sin(U)$$

$$\frac{u}{u'} = \frac{\sin(U)}{\sin(U')}$$

That is: OPD has no linear phase errors as a function of field of view!

Imaging a grating



$$\sin(U) = \frac{m \cdot \lambda}{d}$$

$$d \cdot \sin(U) = m \cdot \lambda = d' \sin(U')$$

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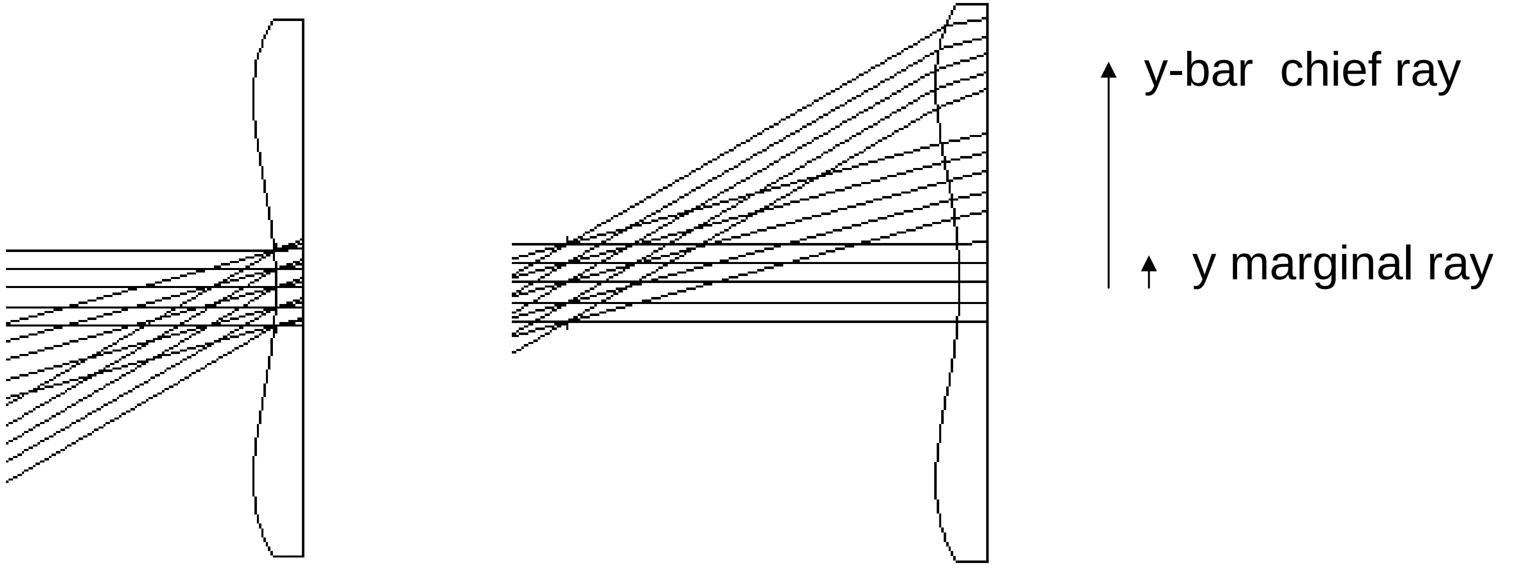
Contribution from an aspheric surface

$$\Delta W_{131} = -\frac{1}{2}\frac{\bar{y}}{y}\left\{8A_4 y^4 \Delta(n)\right\}$$



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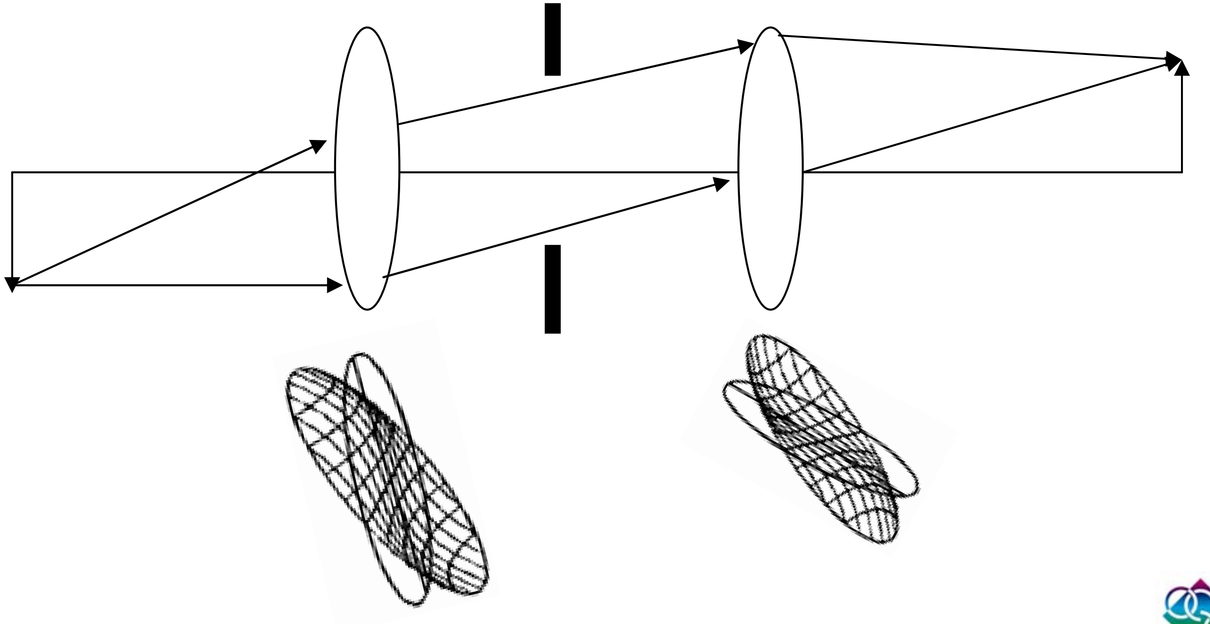
Aberrations and symmetry

$$W(H,\rho,\theta) = W_{200}H^2 + W_{020}\rho^2 + W_{111}H\rho\cos\theta +$$
$$+ W_{040}\rho^4 + W_{131}H\rho^3\cos\theta + W_{222}H^2\rho^2\cos^2\theta +$$
$$+ W_{220}H^2\rho^2 + W_{311}H^3\rho\cos\theta + W_{400}H^4 +$$
$$+ \ldots$$

- Coma is an odd aberration with respect to the stop
- Natural stop position to cancel coma by symmetry



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Structural coefficients: Thin lens (stop at lens)

$$S_I = \frac{1}{4} y^4 \phi^3 \left[AX^2 - BXY + CY^2 + D \right]$$

$$S_{II} = \frac{1}{2} Ж \; y^2 \varphi^2 \left[EX - FY \right]$$

$$S_{III} = Ж^2 \varphi$$

$$S_{IV} = Ж^2 \varphi \frac{1}{n}$$

$$S_V = 0$$

$$C_L = y^2 \phi \frac{1}{\nu}$$

$$C_T = 0$$

$$X = \frac{c_1 + c_2}{c_1 - c_2} = \frac{r_2 + r_1}{r_2 - r_1}$$

$$Y = \frac{1+m}{1-m} = \frac{u' + u}{u' - u}$$

$$\phi = \Delta n \Delta c = (n-1)(c_1 - c_x)$$

$$A = \frac{n+2}{n(n-1)^2}$$

$$B = \frac{4(n+1)}{n(n-1)}$$

$$C = \frac{3n+2}{n}$$

$$D = \frac{n^2}{(n-1)^2}$$

$$E = \frac{n+1}{n(n-1)}$$

$$F = \frac{2n+1}{n}$$

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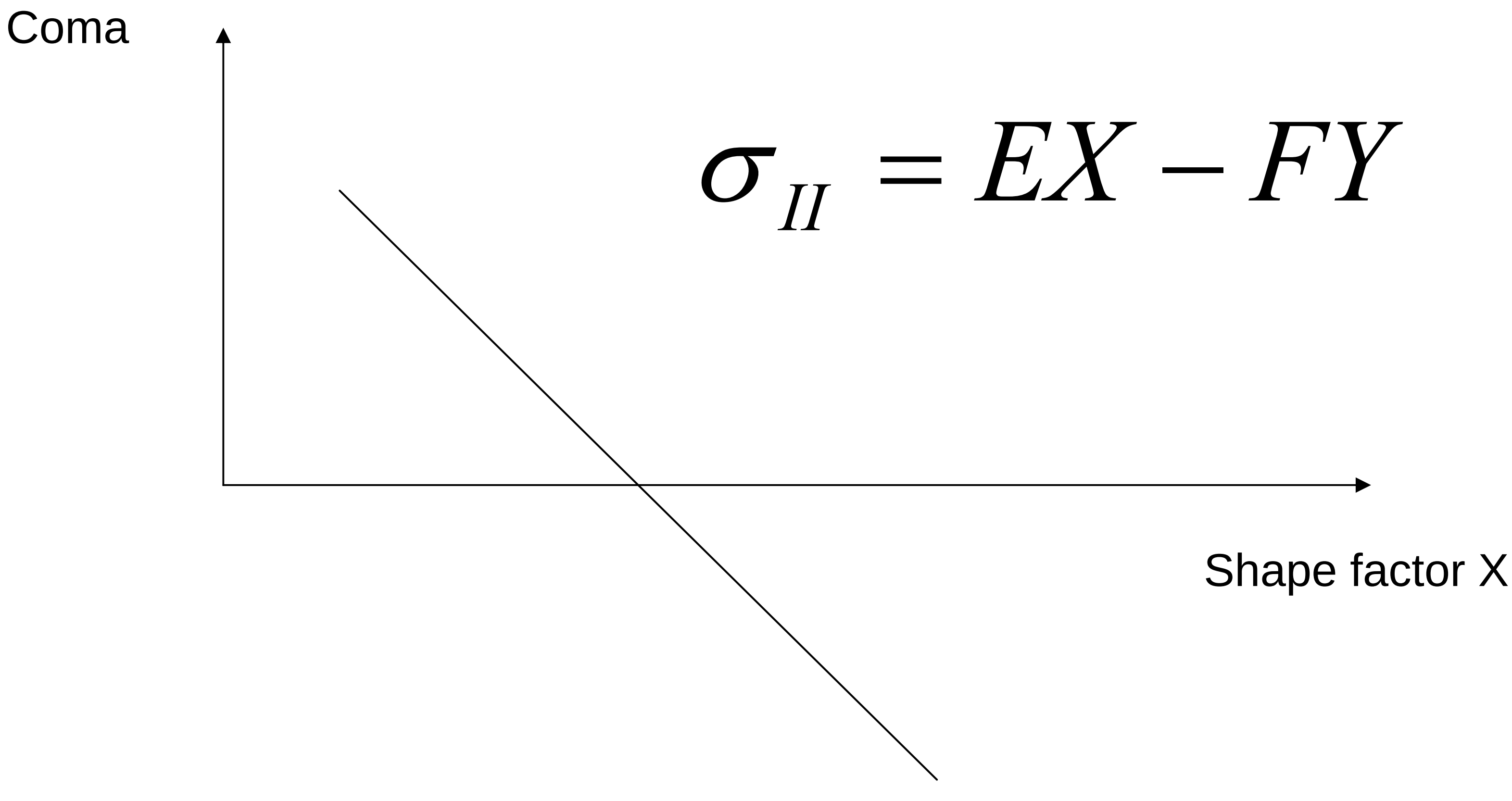
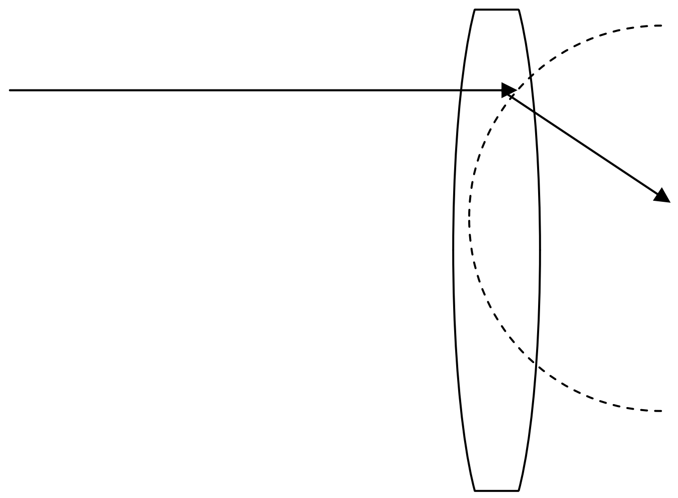
Coma vs Bending

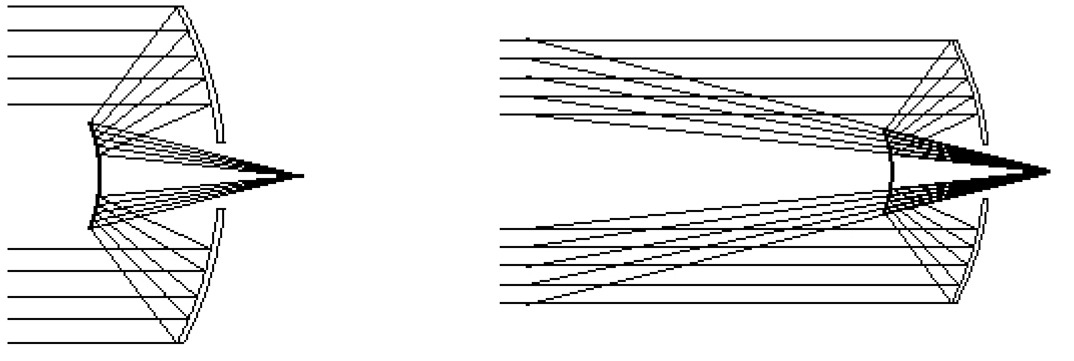


$$\sigma_{II} = EX - FY$$

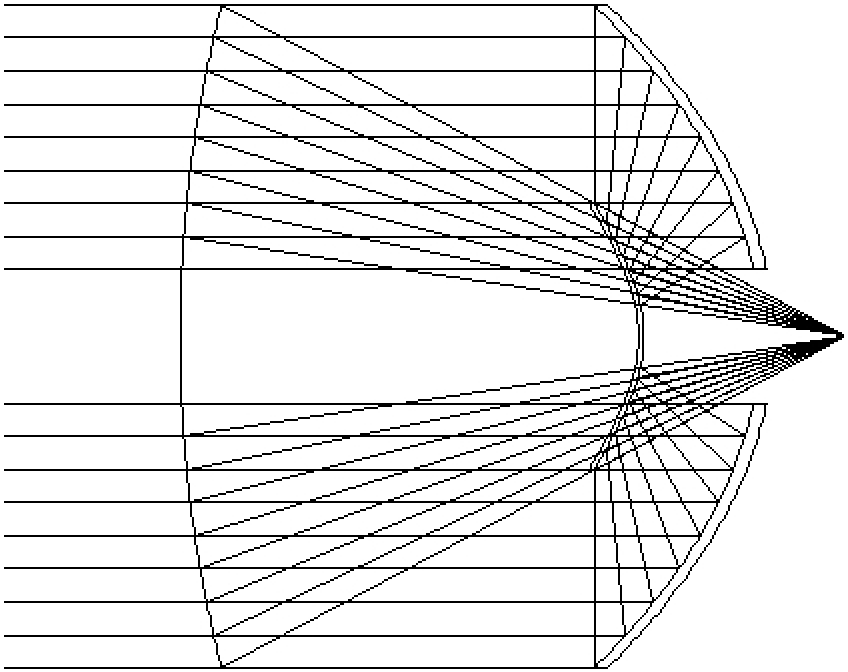
Principal surface



In an aplanat working at m=0
the equivalent
refracting surface
is a hemisphere



Cassegrain's principal surface



Since the equivalent refracting surface in a Cassegrain telescope is a paraboloid then the coma of that Cassegrain is the same of a paraboloid mirror with the same focal length.

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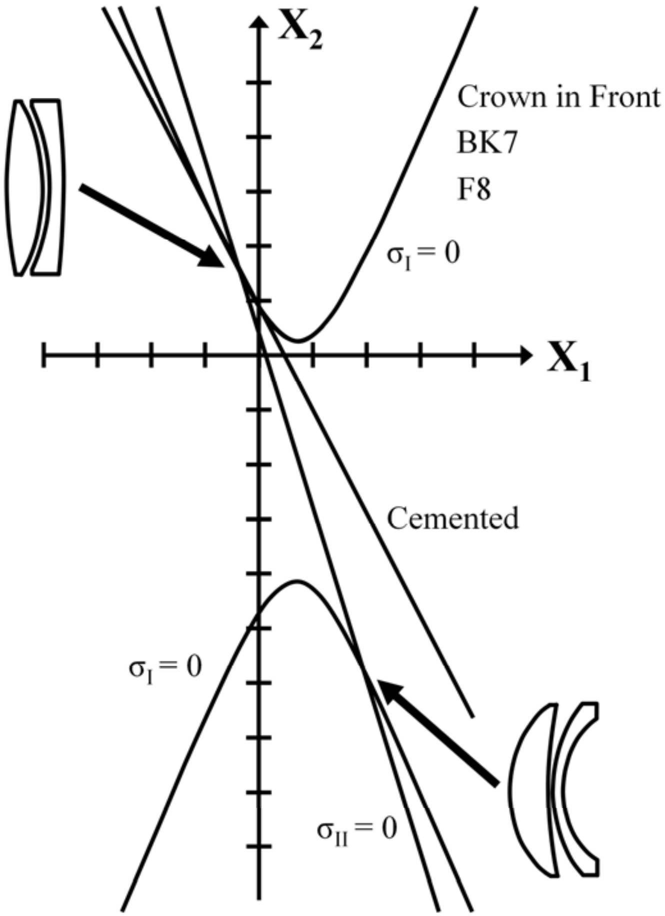
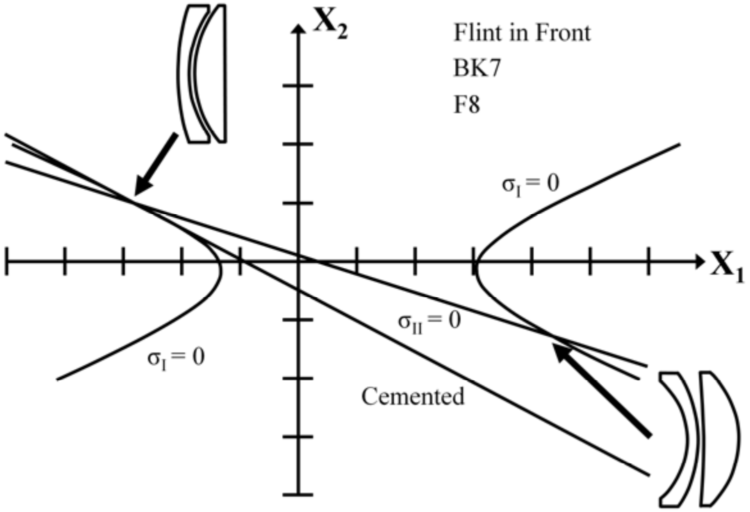
Aplanat doublets



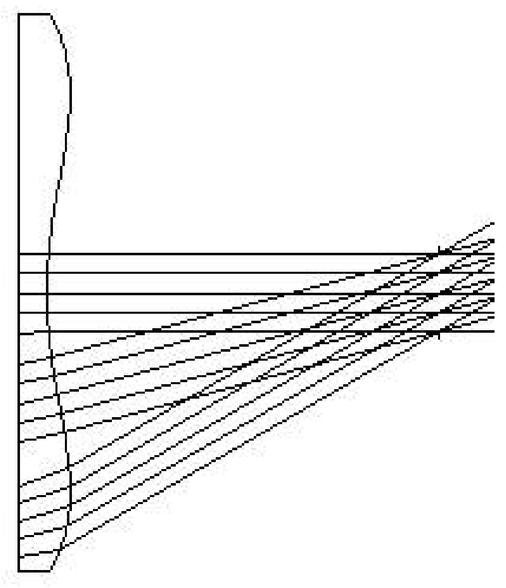


Kingslake's cemented aplanat

Chromatic correction
Spherical aberration correction
Coma correction
Still cemented

C=1/r	d	Glass	n_D	V_D
0.1509				
	0.32	SK-11	1.56376	60.75
-0.2246				
	0.15	SF-19	1.66662	33.08
-.052351				

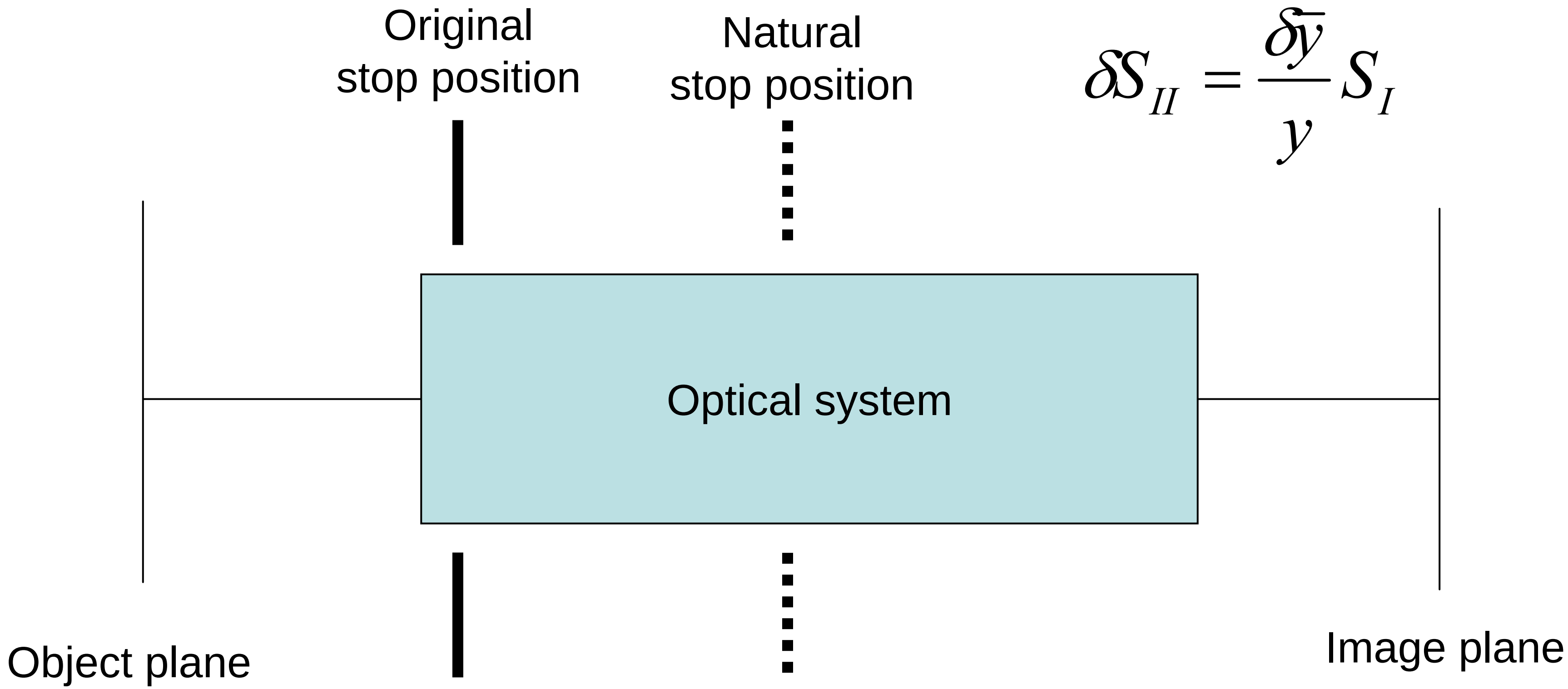
Control of coma in the presence of an aspheric mirror near a pupil



In the presence of a strong aspheric surface near the stop or pupil, coma aberration can be corrected by moving the surface

$$\Delta W_{131} = -\frac{1}{2}\frac{\bar{y}}{y}\left\{8A_4 y^4 \Delta(n)\right\}$$

Coma correction by phantom stop position



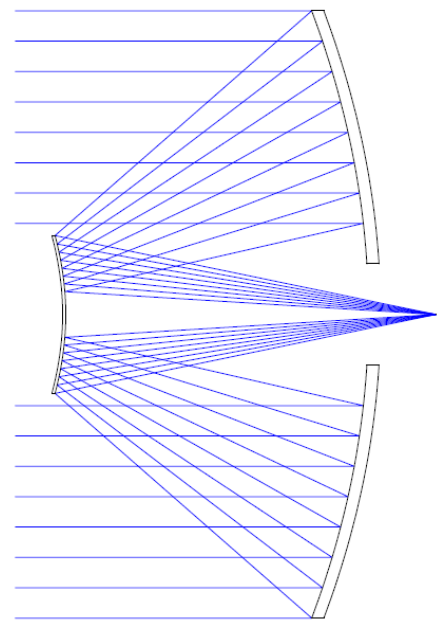
$$\delta S_{II} = \frac{\delta \bar{y}}{y} S_I$$

In the presence of spherical aberration there is a stop position for which coma is zero. At that stop position spherical aberration might be corrected. Then the system becomes aplanatic and the stop can be shifted back to its original position.

The aplanatic member(s) in a family of solutions

Doublet

Ritchey-Chretien



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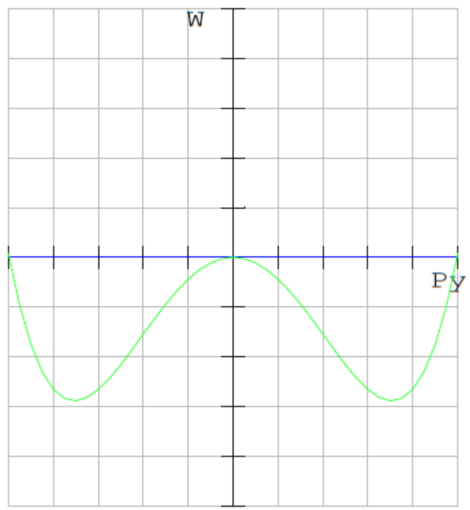
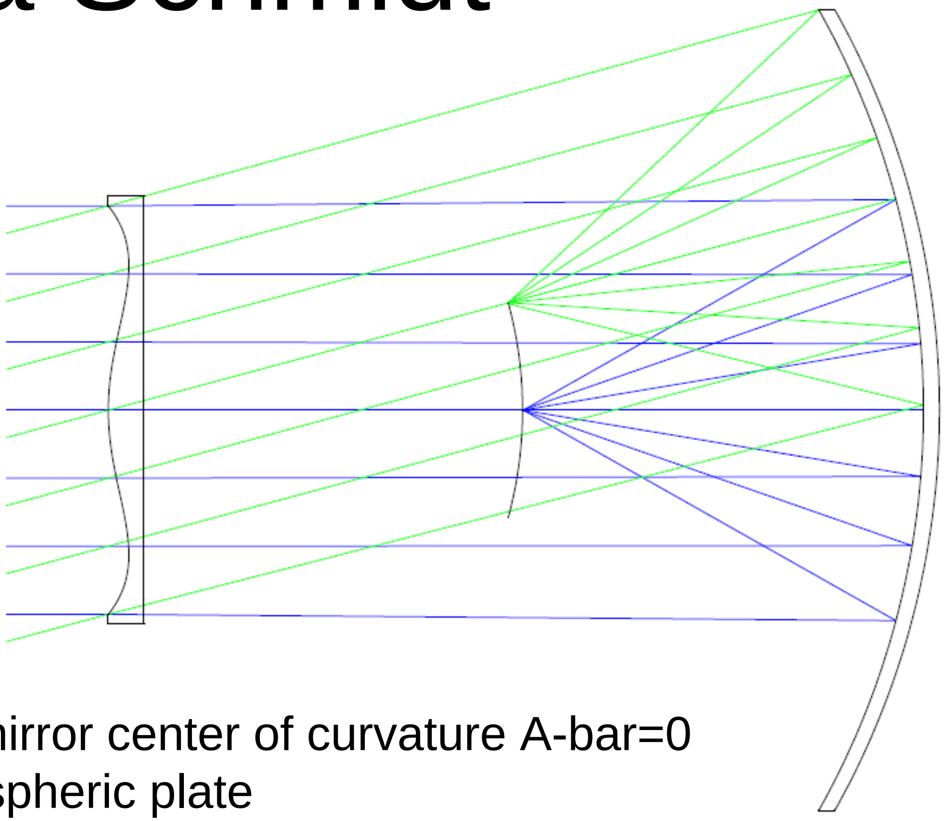
Camera Schmidt





Aspheric plate at mirror center of curvature A-bar=0
Stop aperture at aspheric plate
Note symmetry about mirror CC
No spherical aberration
No coma
No astigmatism.
Anastigmatic over a wide field of view!
Satisfies Conrady's D-d sum

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Notes

- Need to make aplanatic zero-field systems (that are fast). The alignment becomes easier.
- Lenses for lasers diodes/optical fibers
- Microscope objectives

Sine condition from optical flux conservation and radiance theorem

Etendu considerations

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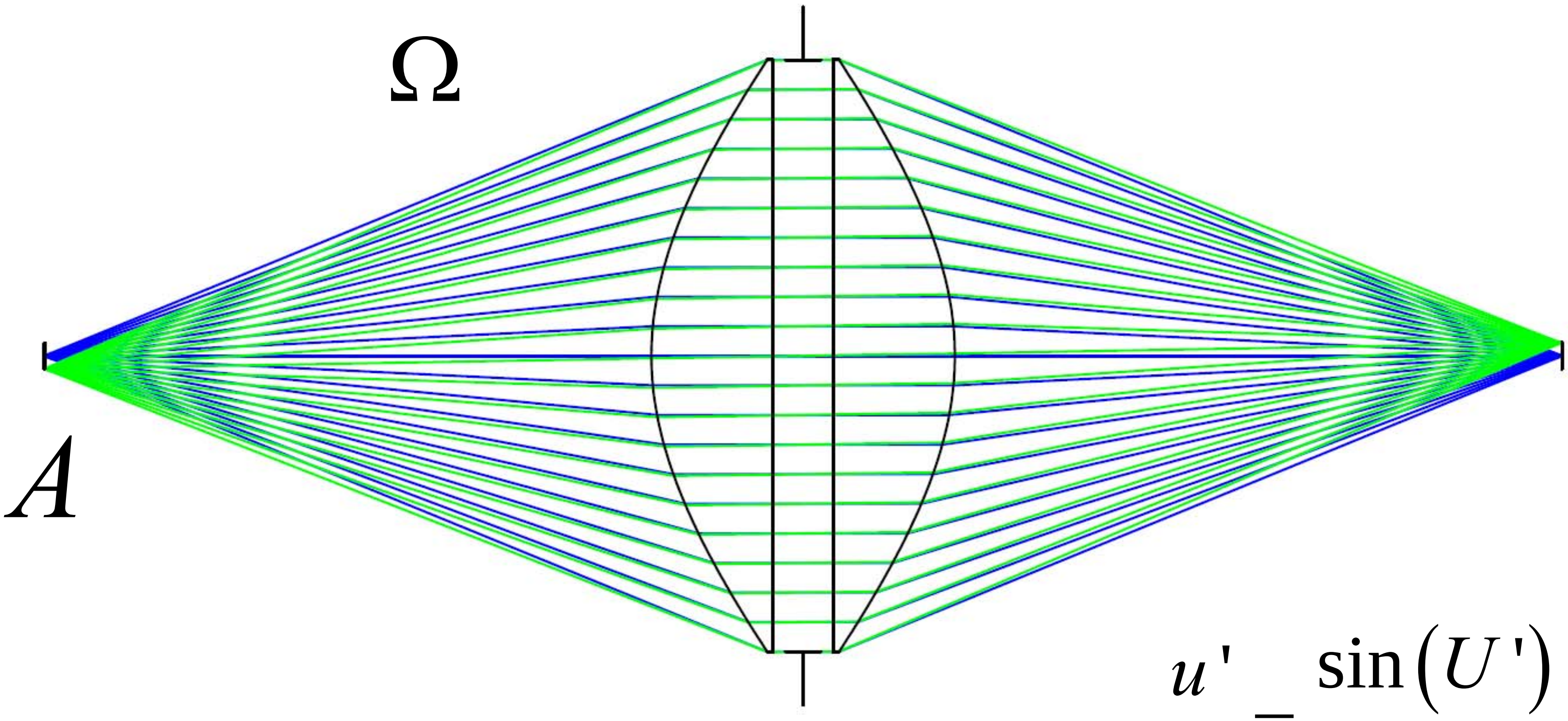
Sine condition



$$\frac{u'}{u} = \frac{\sin(U')}{\sin(U)}$$

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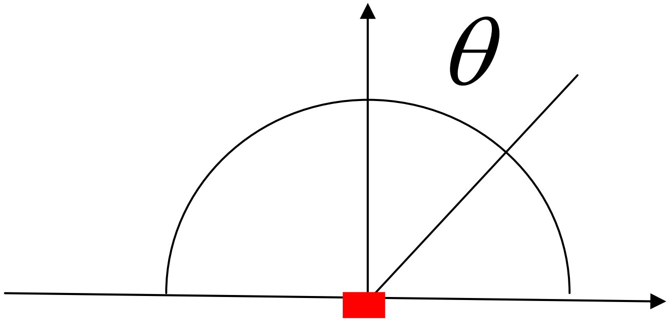
Optical flux from a Lambertian source

$$\Phi(\theta) = 2\pi A L_0 \int_0^{\theta} \cos(\theta)\sin(\theta)\,d\theta = \pi A L_0 \sin^2(\theta)$$



Compare with homogeneous source

$$\Phi(\theta) = 2\pi A L_0 \int_0^{\theta} \sin(\theta)\,d\theta = 2\pi A L_0 \left(1 - \cos(\theta)\right) = A L_0 \Omega$$

Optical flux=radiance x throughput

$$\Phi(\theta) = \frac{L_0}{n^2} T \qquad \Phi(\theta') = \frac{L_0'}{n'^2} T$$

$$\frac{n^2}{L_0} \Phi(\theta) = \frac{n'^2}{L_0'} \Phi(\theta') = T$$

$$\frac{n^2}{L_0} = \frac{n'^2}{L_0'}$$ Radiance theorem

$$U = \theta$$

$$\frac{n^2}{L_0} \Phi(U) = \frac{n'^2}{L_0'} \Phi(U')$$

Sine condition from optical flux conservation

$$\frac{n^2}{L_0}\Phi(U)=\frac{n'^2}{L_0'}\Phi(U')$$

$$n^2 A\sin^2(U)=n'^2 A'\sin^2(U')$$

$$\pi n^2 h^2\sin^2(U)=\pi h'^2 n'^2\sin^2(U')$$

$$n^2 h^2\sin^2(U)=h'^2 n'^2\sin^2(U')$$

$$nh\sin(U)=n'h'\sin(U')$$

$$\frac{u'}{u}=\frac{\sin(U')}{\sin(U)}$$

Sine condition

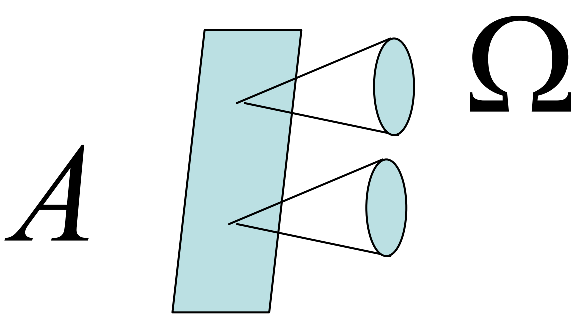
Throughput
Etendu

(area-omega product)

$$\varepsilon = n^2 A\Omega = n'^2 A'\Omega'$$

$$T = \pi \text{Ж}^2$$

"Capacity to transfer optical flux"



$$\Omega = 2\pi\left(1 - \cos\left(\theta\right)\right)$$

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Even better

$$\varepsilon = n^2 A\Omega = n'^2 A'\Omega'$$

Homogeneous source

$$\varepsilon = A(NA)^2 = n^2 A \sin^2(\theta)$$

Lambertian source

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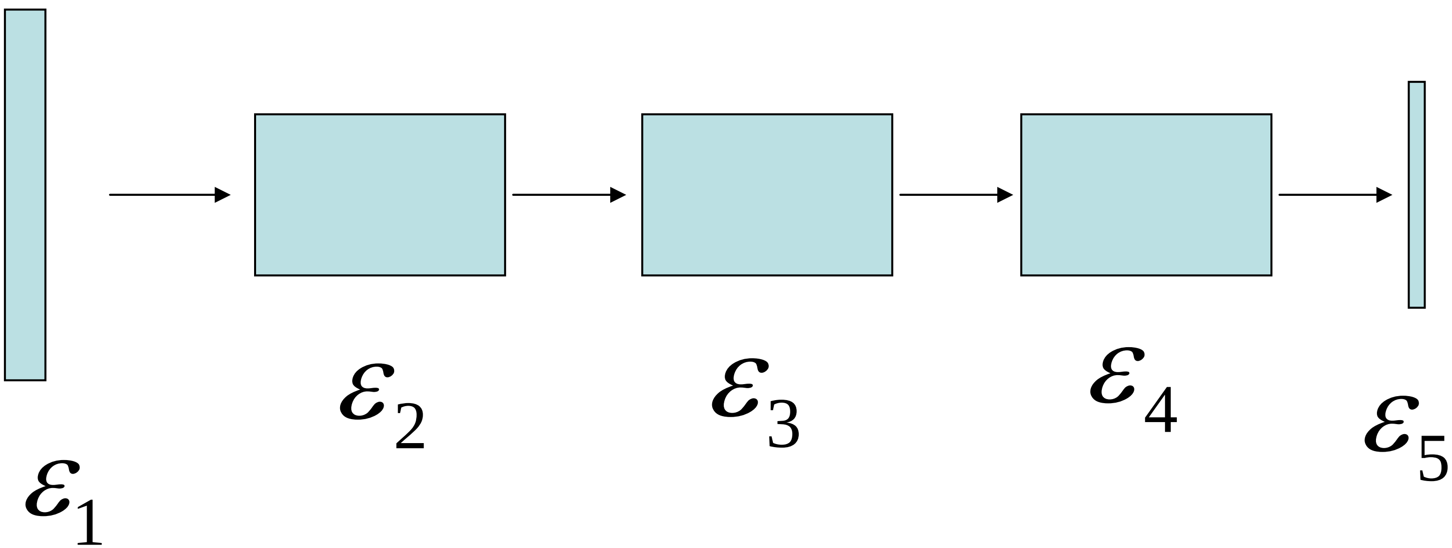
Etendu considerations are key to design an optical system



$$\varepsilon_5 \geq \varepsilon_4 \geq \varepsilon_3 \geq \varepsilon_2 \geq \varepsilon_1$$

Start with the sensor at the end $\varepsilon_5 = A_5 \left(NA \right)^2$

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Summary

- Coma aberration
- Coma as an odd aberration
- Sine condition
- Natural stop position
- Aplanatic doublets
- Zero-field systems