EG&G PRINCETON APPLIED RESEARCH

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Technical Note 243

Noise in Amplifiers

A number of factors contribute to the noise problem, which can drastically limit the effectiveness of amplifiers and other instrumentation, especially at low signal levels.

In practically every type of research program in the physical sciences as well as in sophisticated engineering analyses, very small electrical signals must be measured and, in general, the limit of attainable precision and detectability is set by noise. This is true for the physicist and chemist performing nuclear magnetic resonance or spectroscopy experiments, for medical and biological researchers interested in evoked potentials, for geologists measuring small remanent magnetic fields in rock samples, for the metallurgist making Fermi surface measurements, and for the engineer performing vibration analysis and sensitive bridge measurements. These are only a few examples of applications in which noise plays a critical role in limiting measurement precision and signal detectability.This article discusses some of the inherent problems and describes techniques for improving signal-to-noise ratio.

Generally speaking, noise includes all those voltages and currents that accompany a signal of interest and obscure it. There are many different types of noise and they arise in many different sources. Many are directly electrical in nature, such as the noise produced by amplifiers and other instrumentation used to process signals. Others are not inherently electrical, but manifest themselves as electrical fluctuations when some element of the experimental system acts as a transducer (frequently without the knowledge of the experimenter). For example, structure vibration transmitted to coaxial cables can cause signals to be induced in the cables as a result of dimensional, and hence capacitive, changes in the cables. Some of the noises with which the experimenter must contend include:

(1) Thermal noise arising in the signal source impedance.

(2) Noise produced in the instruments used to process the signal. In most instances, front-end preamplifier noise will dominate.

(3) Environmental noise, which opens up a host of possibilities, including:
   - (a) Interference at the power frequency or its harmonics.
   - (b) Automotive ignition noise.
   - (c) Radio stations.
   - (d) Lightning (and this can be remarkably distant).
   - (e) Changes in barometric pressure.
   - (f) Structure vibration.
   - (g) Temperature fluctuations.

(4) Statistical fluctuations resulting from the ultimately quantized nature of all measured quantities.

All of these, and many others, can limit the accuracy, precision, and useful sensitivity of measurements. Fortunately, only a few will have a significant effect in any one experiment, and the problem of the experimenter in minimizing noise effects will be less difficult than the foregoing list might lead one to believe.

The environmental interference can frequently be reduced to a negligible level by following "sound experimental practice,"[1] which includes proper grounding, shielding, guarding, and other procedures. With environmental factors under control, the experimenter still faces "quantization fluctuations," along with source-resistance noise and amplifier noise. The former only rarely will prove to be a problem, and where it is a problem there is little the experimenter can do other than to optimize his experiment so that the maximum number of "events" per unit time are measured. This leaves the final two types of noise, source thermal noise and amplifier noise, from which we arrive at the purpose of this article — namely, to provide some insight into the characterization of these noises and their effects, and to indicate how these noise effects can be minimized for a given experimental situation.

SNR at amplifier output terminals (voltage-source driven)

The SNR is a universally accepted quantitative expression for the degree of noise contamination of a signal. Figure 1 shows an amplifier fed from a voltage source E_s with a finite source resistance R_s. Before going on to calculate the SNR for this amplifier, we must first quantitatively characterize the source-resistance noise and the amplifier noise.

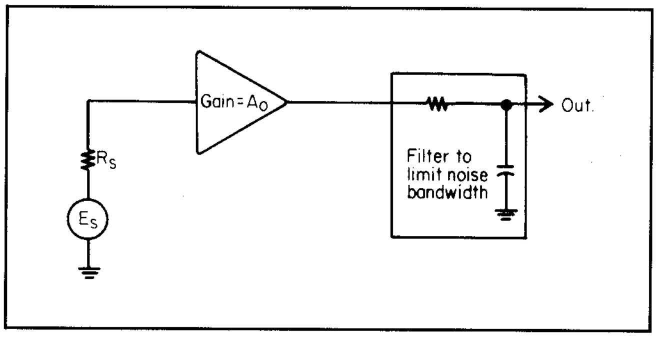

FIGURE 1. Amplifier fed from voltage source through source resistance.

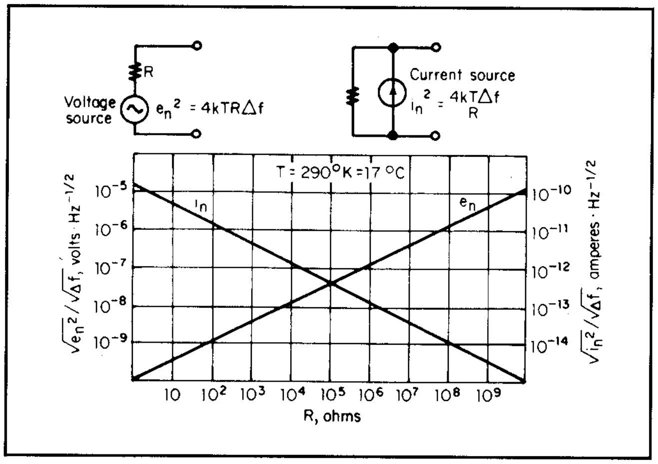

FIGURE 2. Equivalent-circuit representations of thermal noise in resistor R, together with magnitude of rms voltage and current per cycle as a function of R.

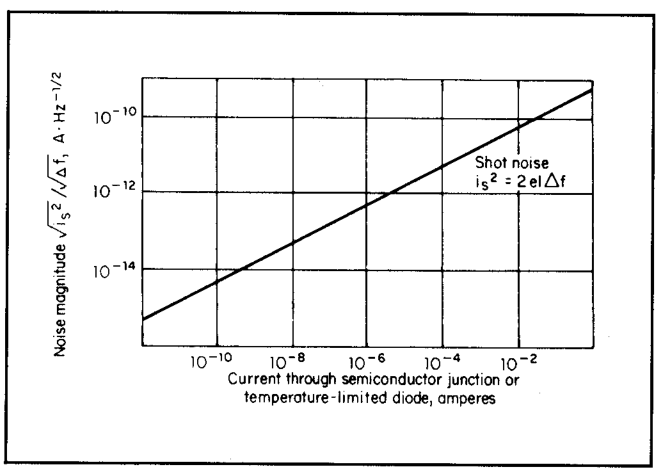

FIGURE 3. Shot noise as a function of current.

Johnson noise. Johnson noise is caused by random motion of thermally agitated electrons in resistive materials. Its instantaneous amplitude is unpredictable, but the probability that it will have an amplitude in an interval dV volts is given by p(V) dV, where p(V) is the familiar Gaussian probability density function:

$$p(V) = \frac{1}{(2\sigma^2)^{1/2}} e^{-V^2/2\sigma^2} \qquad (1)$$

where the parameter σ is the rms value of the fluctuations and the quantity universally accepted to describe the noise output from a resistor. It can be shown from thermodynamic considerations that the rms value is bandwidth-dependent as follows:

$$\sigma = E_{JN} = (4kTR_sB)^{1/2} \qquad (2)$$

where k = Boltzmann's constant = 1.38×10^{-23} J/°K; T = resistor temperature, °K; R_s = resistance, ohms; and B = noise bandwidth, hertz. Johnson noise is "white noise"; that is, its rms value per unit bandwidth (rms density) is constant from dc to frequencies extending into the infrared region. For analytical purposes, the noisy resistor is represented by a noiseless resistor and a noise voltage or current generator as shown in Figure 2.

The source-resistance Johnson noise is the minimum possible noise that can accompany the signal. Other types of noise from other sources may obscure the signal as well, but the Johnson noise will always be present. Besides the Johnson noise that arises in the source resistance, there is additional Johnson noise produced by resistors in the amplifier. This noise also degrades the SNR. However, other sources of noises are also present in the amplifier, and contribute to the total amplifier noise, which is a conglomeration of thermal, shot, and flicker noise of resistors, vacuum tubes, and semiconductor devices.

Shot noise. Shot noise manifests itself as random current fluctuations in vacuum tubes and semiconductor junctions. It is caused by the random arrival of discrete electron charges at anodes, collectors, and drains. Equation (3) depicts the rms value of the shot-noise current in an emission-limited vacuum diode.

$$I_{shot} = (2eI_{dc}B)^{1/2} \text{ amperes rms} \qquad (3)$$

where e = electronic charge = 1.59×10^{-19} coulomb; I_{dc} = average current through diode, amperes; and B = noise bandwidth, hertz.

Figure 3 shows the shot-noise-density variation of the direct current. Like thermal noise, shot noise exhibits a flat power spectrum, and thus can be referred to as "white" noise.

Flicker noise. Flicker noise is characterized by its spectral composition, and, for most electronic devices, it dominates thermal and shot noise from dc to about 100

Hz. Although flicker noise is detectable in virtually all conducting materials having power applied to them, it is most prominent where electron conduction occurs in granular or semiconductor devices. Composition, deposited carbon, and even metal-film resistors, in that descending order, all show voltage-dependent flicker noise. Flicker noise exhibits a $1/f^n$ power spectrum, with n typically (but not always) having a value in the range of 0.9 to 1.35.

The particularly disturbing thing about flicker noise is that the 1/f characteristic seems to hold down to as low a frequency as one cares to measure it. Even the long-term drift in dc amplifiers would seem to be a manifestation of very low frequency flicker noise. As a result, flicker noise imposes a unique barrier to measurement accuracy. Consider the case where the signal of interest is at dc or at a low frequency in the flicker-noise-dominated frequency region. Because of the $1/f^n$ power-spectrum characteristic, no improvement in signal-to-noise ratio can be derived by increasing the time constant of the measuring device whenever n is equal to or greater than unity, the usual case — in contrast to operation in the white noise dominated frequency region, where a significant improvement can be achieved in this manner. It becomes apparent that small signal measurements should be made at frequencies at which white-noise phenomena (Johnson and shot noise) dominate, with dc measurements particularly to be avoided.

Total noise and SNR

The aforementioned amplifier noise sources can be lumped together, allowing the amplifier to be characterized as a noiseless amplifier and two fictitious noise generators — voltage and current — connected to the input terminals, as shown in Figure 4. Noise-density generators e_n and i_n are expressed in units of rms volts/$Hz^{1/2}$ and rms amperes/$Hz^{1/2}$, respectively.

This form of characterization facilitates SNR and NF calculations, as we shall see later. We shall assume that e_n and i_n are white-noise sources and that their cross-correlation function is identically equal to zero. These assumptions, though by no means universally justifiable, do simplify the ensuing mathematics, and the resulting conclusions and statements are valid. Figure 5 illustrates the frequency dependence of these generators in an actual amplifier. Most manufacturers specify the noise content of the amplifier by the noise figure, whose definition is stated later in the article. In any case, whether given e_n and i_n, or NF, the experimenter can predict amplifier performance and optimize his experiment as shown in the following pages.

Because the noise sources are considered to be random and uncorrelated, noise *power* in a system is additive, and the total rms noise is the square root of the sum of the squares of each generator output. Note that the amplifier noise current is treated as a voltage by computing the voltage drop across the source resistance R_s; see Figure 5. It is not necessary to consider the noise contribution of the low-pass filter resistor when the amplifier gain is large.

The total noise output voltage is

$$E_{tno} = [4kTR_s + e_n^2 + (i_nR_s)^2]^{1/2} A_o f_n^{1/2} \text{ volts rms} \quad (4)$$

where A_o = midband gain and f_n = noise bandwidth = 1/4RC for Figure 4, in which R and C are values of output filter components.

The signal at the output terminal is E_sA_o. Consequently, the output signal-to-noise voltage ratio $(SNR)_o$ can be determined by dividing E_sA_o by Equation (4). The result is as follows:

$$(SNR)_o = \frac{E_s}{[4kTR_s + e_n^2 + (i_nR_s)^2]^{1/2} f_n^{1/2}} \quad (5)$$

Often it is convenient to know the total equivalent noise referred to the amplifier input terminals (E_{tni}). This is easily obtained by dividing Equation (4) by the midband gain A_o. The result is

$$E_{tni} = [4kTR_s + e_n^2 + (i_nR_s)^2]^{1/2} f_n^{1/2} \text{ volts rms} \quad (6)$$

At this point it should be clear that E_{tni} can be found by measuring the noise at the amplifier output terminal with a true rms voltmeter and dividing the reading by the gain A_o. It should be equally obvious that E_{tni} cannot be found by putting the voltmeter directly across the amplifier's input terminals.

It is evident from the third term of Equation (4) that the noise contribution of the amplifier is dependent on the magnitude of the source resistance R_s. This leads us to the first of three statements that express the essence of these relationships.

Statement 1. For a given amplifier driven from a voltage source, the SNR is maximum when $R_s = 0$.

Equation (5) also shows the advantage of restricting the bandwidth. By allowing the signal of interest to be transmitted with no more of the high-frequency components getting through than are necessary to carry the "information," the SNR will be improved — hence the output low-pass filter shown in Figures 1 and 4. Further improvement could be realized by also incorporating a high-pass filter to attenuate noise components below the signal frequency. Bandpass-selective amplifiers having a fast rolloff above and below the signal frequency can frequently be used to great advantage.

Noise figure

A popular figure of merit used to describe an amplifier's quality, insofar as noise is concerned, is the noise figure of the amplifier. Relevant to the circuit in Figure 4, noise figure, expressed in decibels, can be defined as follows:

$$NF = 20 \log_{10} \left[\frac{\text{Input voltage SNR (amplifier disconnected)}}{\text{Voltage SNR at amplifier output terminals}} \right] \quad (7a)$$

or, in terms of power,

$$NF = 10 \log_{10} \left[\frac{\text{Input power SNR (amplifier disconnected)}}{\text{Power SNR at amplifier output terminals}} \right] \quad (7b)$$

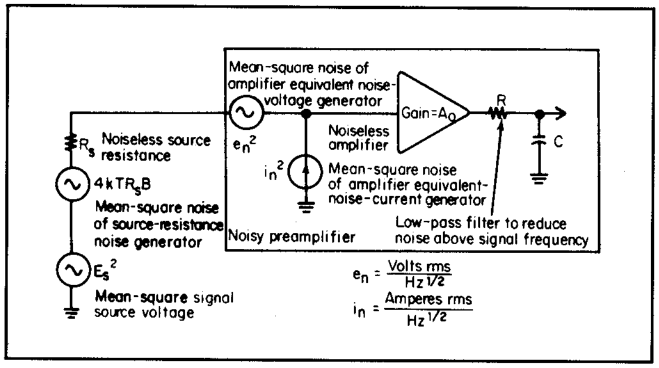

FIGURE 4. Equivalent circuit for noisy amplifier fed from signal E_s with source resistance R_s.

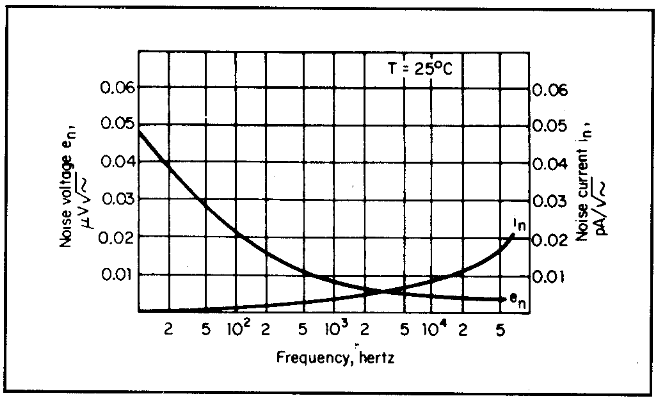

FIGURE 5. Noise current and voltage as a function of frequency for a typical low-noise preamplifier.

With the aid of Figure 4 and Equation (5), and by expressing all parameters as power, Equation (7b) can be written as

$$NF = 10 \log_{10} \left[\frac{E_s^2/4kTR_s f_n}{E_s^2/[4kTR_s + e_n^2 + (i_n R_s)^2]\, f_n} \right]$$

Further simplification yields

$$NF = 10 \log_{10} \left[1 + \frac{e_n^2 + (i_n R_s)^2}{4kTR_s} \right] \quad (8)$$

Akin to noise figure is noise factor, F, defined as

$$F_{power} = 10^{NF/10} = \left[1 + \frac{e_n^2 + (i_n R_s)^2}{4kTR_s} \right] \quad (9)$$

or, in terms of voltage,

$$F_{voltage} = 10^{NF/20}$$

Continuing, the total equivalent noise referred to the amplifier input terminals (E_{tni}) can be shown to be

$$E_{tni} = [4kTR_s f_n]^{1/2} 10^{NF/20} \text{ volts rms} \quad (10)$$

which brings us to our second statement.

Statement 2. For a *given source resistance*, the least noisy amplifier is the one with the smallest NF. A noiseless amplifier ($e_n = i_n = 0$) has an NF of 0 dB.

Let us now refer back to Equation (8) as the basis for the next statement.

Statement 3. The noise figure increases without limit as the source resistance approaches zero.

Compare statements 1, 2, and 3. Initially, they may seem to appear contradictory. This apparent paradox is easily resolved if one remembers that the noise figure is only a measure of comparison of amplifier noise with the thermal noise developed in an arbitrary source resistance and nothing more. In fact, many researchers over-emphasize the amplifier noise figure and neglect consideration of minimum experimental signal. The following example illustrates this point:

Assume that a given experiment provides a 1 μV rms signal and that it is necessary to choose which of two amplifiers will be used to process the signal. The first amplifier, a moderately priced unit, has a noise figure of 20 dB. The second, a very expensive instrument, has a noise figure of 3 dB. Assume the source resistance to be 100 ohms and the noise bandwidth to be 100 Hz. Equation (10) will be used to compute the total input noise for each of these amplifiers. Tabulating the required data, we obtain:

Amplifier 1	**Amplifier 2**
NF = 20 dB	NF = 3 dB
R_s = 100 ohms	R_s = 100 ohms
f_n = 100 Hz	f_n = 100 Hz

From Equation (10) the total equivalent input noise (E_{tni}) for amplifier 1 is 130 nV and for amplifier 2 it is 18 nV. Selecting amplifier 2 over amplifier 1 seems logical, but this decision may be uneconomical in light of the assumed 1 μV signal level.

Noise figure contours

Noise figure contours are essentially the loci of points of constant noise figure as a function of source resistance and operating frequency. They allow the user to determine suitable points of operation, such as source resistance and frequency. Equivalent input noise and signal-to-noise ratios can be determined by using the contours in conjunction with Equation (10). Figure 6 shows the noise figure contours for a PARC Model 113 preamplifier.

Figure 7 is a simplified sketch of the system used to measure noise figure. A noise generator provides a white noise source calibrated in microvolts/$Hz^{1/2}$ When the noise generator is shut off, the source resistor R_s and the amplifier produce a voltage reading on the true rms voltmeter of x volts rms. The noise generator is then turned on and its noise voltage is increased until the voltmeter reading is 1.414 times the previously taken x reading. In other words, a calibrated noise voltage equal to E_{tni} is added to E_{tni}, allowing the value of E_{tni} in $V/Hz^{1/2}$ to be read directly from the calibrated noise source. The following equation can then be used to compute the noise figure at one frequency and for one value of R_s:

$$NF = 10 \log_{10}\left[\frac{(E_{tni})^2}{4kTR_s}\right] \qquad (11)$$

By varying R_s while maintaining a fixed center frequency on the tuned amplifier, one can determine values of noise figure as a function of R_s. By varying the tuned-amplifier center frequency while holding R_s constant, one can determine values of NF as a function of frequency. All values of NF can then be plotted on a single graph to obtain the noise figure contours.

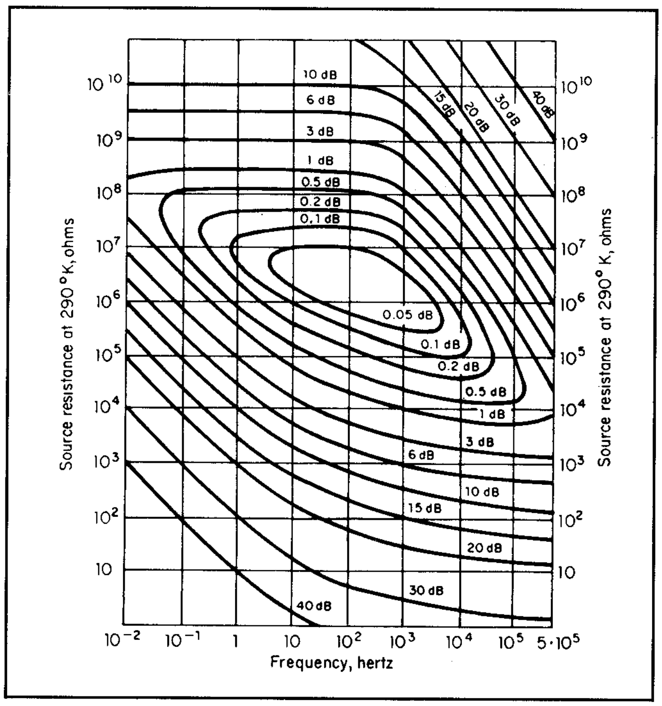

FIGURE 6. Noise-figure contours for Model 113 preamplifier.

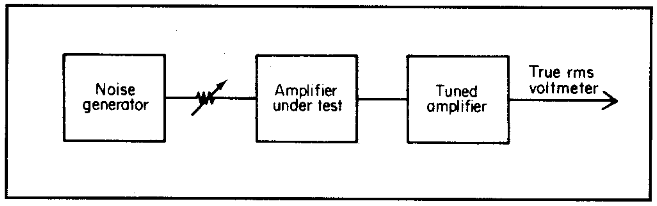

FIGURE 7. Equipment hookup for measuring noise figure.

Optimum source resistance and minimum noise factor

This section concerns finding the value of a source resistance (R_{opt}) that, when "inserted" across the amplifier's input terminals, will yield the minimum noise factor (F_{min}). This investigation is important; the next section will show that transforming an arbitrary source resistance to the optimum source resistance, via a transforming device, will maximize the SNR.

R_{opt} can be found by differentiating Equation (9) with respect to R_s, setting the derivative to zero, and solving for R_s, as follows:

$$\frac{\partial F}{\partial R_s} = 0 = 4kT \frac{[2(i_nR_s)^2 - e_n^2 - (i_nR_s)^2]}{\text{Denominator } (R_s)} \qquad (12)$$

$$R_{opt} = \frac{e_n}{i_n} \text{ ohms} \qquad (13)$$

Minimum noise factor (power) is found by substituting Equation (13) into Equation (9), resulting in

$$F_{min} = 1 + \frac{e_n i_n}{2kT} = 1 + \frac{2e_n^2}{4kTR_{opt}} \qquad (14)$$

Throughout this article it has been assumed that e_n and i_n are frequency-invariant; Figure 8 shows how R_{opt} and NF_{min} vary with frequency when e_n and i_n vary as shown in Figure 5.

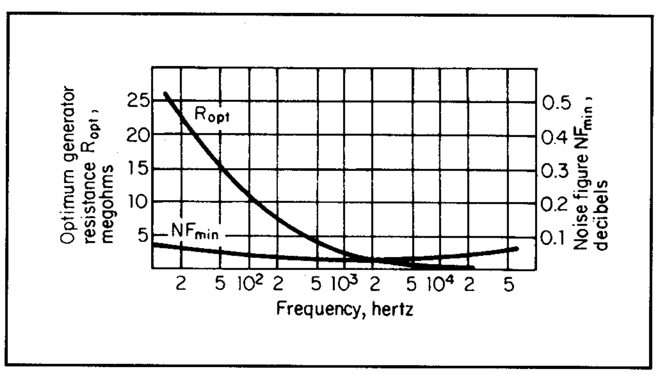

FIGURE 8. Variation of R_{opt} and NF_{min} with frequency for a low-noise preamplifier.

SNR improvement by properly matching R_s to preamplifier

This section will show that transforming the voltage source's intrinsic source resistance (R_s) to the optimum resistance (R_{opt}) via a matching transformer, as shown in Figure 9, achieves a significant improvement in SNR. To simplify the mathematics, it is assumed that the transformer is ideal; that it has infinite self-inductance, that it is lossless, and that it has infinite bandwidth. In addition, the amplifier input impedance is assumed to be much greater than the reflected source impedance. Now compute the signal-to-noise improvement (SNI) obtainable by using the transformer. To begin, let $a = N_s/N_p$, where N_s and N_p represent the number of secondary and primary turns, respectively. Then the optimum source resistance reflected across the amplifier input terminals is

$$a^2R_s = R_{opt} = \frac{e_n}{i_n}$$

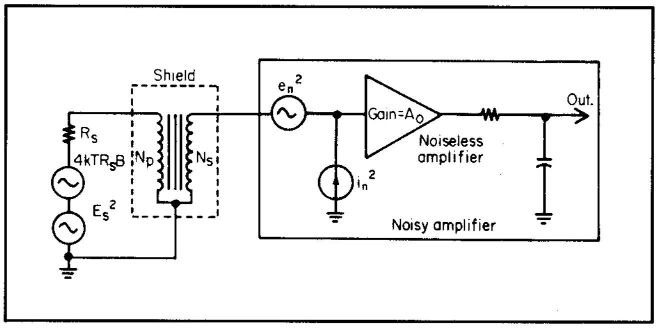

FIGURE 9. Use of transformer to match R_s to R_{opt} of preamplifier.

The signal-to-noise improvement factor (SNI) is defined as follows:

$$SNI = \frac{\text{SNR using transformer}}{\text{SNR without transformer}} \qquad (15)$$

If the following analysis shows SNI to be greater than one, the transformer improves the SNR. Therefore,

$$SNI = \frac{a^2E_s^2}{4kTR_{opt} + e_n^2 + (i_nR_s)^2} \div \frac{E_s^2}{4kTR_s + e_n^2 + (i_nR_s)^2}$$

$$SNI = \underbrace{\left[1 + \frac{e_n^2 + (i_nR_s)^2}{4kTR_s}\right]}_{F_{unmatched}} \div \underbrace{\left[1 + \frac{2e_n^2}{4kTR_{opt}}\right]}_{F_{min}}$$

The result is that SNI equals the unmatched noise factor divided by the minimum noise factor:

$$SNI = \frac{F_{unmatched}}{F_{min}} \qquad (16)$$

It is important to remain consistent with respect to voltage and power when using these equations. In other words, both values of F must be expressed as power to obtain SNI (power), and both values of F must be in voltage to obtain SNI (volts). Usually it is most convenient to determine SNI (voltage) by computing the square root of SNI (power).

EXAMPLE: Given a typical low-noise preamplifier to be driven from a low (10 ohm) source resistance. Data supplied by the manufacturer indicate e_n and i_n for the amplifier at the intended operating frequency: $e_n = 10^{-8}$ $V/Hz^{1/2}$ and $i_n = 10^{-13}$ $A/Hz^{1/2}$.

The problem is to pick a transformer that will yield maximum SNR improvement and to calculate that SNR improvement.

From the preceding discussion,

$$R_{opt} = \frac{e_n}{i_n} = \alpha^2R_s = \frac{10^{-8}}{10^{-13}} = 10^5 \text{ ohms}$$

A turns ratio of 1:100 (a = 100) is required to transform R_s to R_{opt}. The noise factor for the amplifier without the transformer can be computed by means of Equation (9), using the given data. Thus,

$$F_{unmatched} = 1 + \left[\frac{10^{-16} + (10^{-26} \times 10^2)}{1.66 \times 10^{-20} \times 10^1}\right] \cong 600$$

(NF $\cong$ 28 dB)

The noise factor for the amplifier when the transformer is used can be computed from Equation (14) as follows:

$$F_{min} = 1 + \left[\frac{10^{-8} \times 10^{-13}}{0.83 \times 10^{-20}}\right] \cong 1.21 \text{ (NF} \cong 0.5 \text{ dB)}$$

From Equation (16),

$$SNI_{power} = \frac{600}{1.12} = 536 \text{ (SNI voltage} = \sqrt{536} \cong 23)$$

It is not always possible to achieve F_{min}, but a compromise can be reached such that, in general, the SNR improvement can be shown to be

$$SNI = \frac{F_{unmatched}}{F_{matched}} \quad (17)$$

where $F_{unmatched} > F_{matched} \geq F_{min}$. Or, in terms of NF,

$$SNI_{power} = 10^{(NF_{unmatched} - NF_{matched})/10} \quad (18a)$$

and

$$SNI_{voltage} = 10^{(NF_{unmatched} - NF_{matched})/20} \quad (18b)$$

It must be pointed out that it is impossible to achieve the SNI indicated by Equation (16); the imperfections of all real transformers serve to reduce the expected SNI. Contributing sources of noise include winding-resistance thermal noise, Barkhausen noise (which results from the behavior of the magnetic domains in the core material when a signal is applied), mechanical stresses (including vibration), and sensitivity to pickup or interference from magnetic fields. This last source proves particularly bothersome at the power-line frequency and its lower-order harmonics. Heavy shielding is often required to reduce this pickup to an acceptable level. Frequently the physical orientation of the transformer also proves instrumental in achieving minimum pickup. Sensitivity to mechanical stresses and vibration does not often prove to be a serious problem, and where it does, supporting the transformer by suitable shock-absorbent material is usually an adequate solution. Most commonly, the winding resistance thermal noise proves to be the most significant source of transformer noise. Its magnitude is easily computed. The winding resistance of the secondary winding is divided by the square of the turns ratio, and that resistance is added to the primary winding resistance. For example, a 1:100 transformer having a primary winding resistance of 1 ohm and a secondary resistance of 10,000 ohms would have a thermal resistance of 2 ohms referred to the primary (20,000 ohms referred to the secondary). Despite the additional noise introduced by incorporating a transformer into the system, the SNI is increased sufficiently to make use of the transformer well worthwhile.

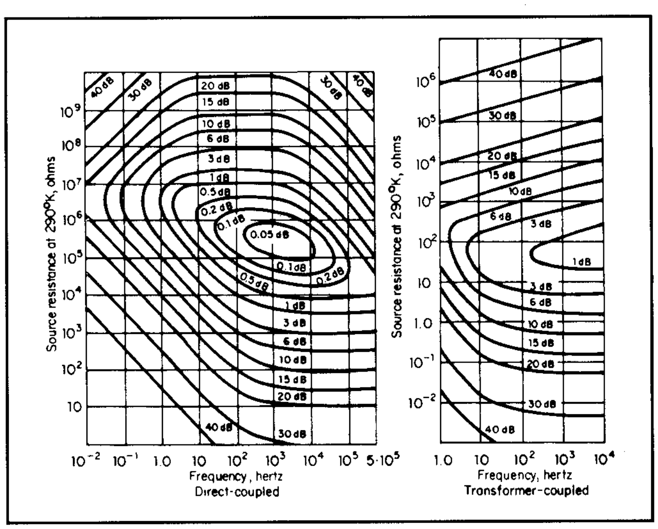

FIGURE 10. Noise-figure contours for Model 116 preamplifier.

Figure 10 shows the noise figure contours for a PARC Model 116 preamplifier. The contours on the left were measured under "direct input" conditions. Those on the right were measured with the amplifier's internal transformer incorporated into the input circuit. Front-panel switching allows the transformer to be introduced for improved SNR when working from low source resistances. The following example shows the degree of SNR improvement attainable by using the Model 116 preamplifier with the transformer in a low-source-resistance experiment:

Assuming a small signal at 100 Hz from a source resistance of 100 ohms, the problem is to choose between operating with or without the transformer, and to compute the signal-to-noise improvement.

From Figure 10, the preamplifier, directly coupled, has an NF of 15 dB under the given conditions, whereas the same preamplifier, transformer-coupled, has an NF of only 2 dB. Clearly, transformer coupling should be used. Employing Equation (18b) to compute SNI yields

$$SNI_{voltage} = 10^{(NF_{unmatched} - NF_{matched})/20}$$
$$= 10^{(15-2)/20} = 4.5$$

The computed SNI in this example will be significantly more accurate than that obtainable in the earlier example, in which it was necessary to work from e_n and i_n and to assume a perfect transformer. The results obtained were idealized and could be approached only under the best of conditions. In the latter example, however, the raw data were measured values of noise figure obtained with the transformer installed, and these values fully reflect the transformer imperfections.

A transformer's passband becomes narrower with increasing source resistance. Therefore, it is essential that the chosen matching transformer's frequency response be relatively flat within the frequency range of interest. Otherwise, the SNI expected may differ significantly from that obtained. It has been shown that, for a given voltage source and amplifier, the SNR is maximized when $R_s = 0$. In practice, $R_s \neq 0$ because all signal sources have some value of resistance associated with them. The SNR, however, can be enhanced by transforming a low value of R_s to a value as close to R_{opt} as practical considerations allow. This does not mean a physical resistor is to be inserted in series with R_s to obtain a cumulative resistance equal to R_{opt}. Increasing the source resistance in this manner causes a twofold degradation in the signal-to-noise ratio, as can be readily determined by referring to Equation (6). First, there is a considerable increase in thermal noise, which results when the thermal noise in the series resistor is added to that developed in the original source resistance. Second, there is an increased influence of the amplifier's noise current because of the voltage drop of this current across the series resistor. The current effect can be particularly damaging, since it varies directly with the total source resistance, in contrast to the thermal noise, which varies as the square root of the source resistance.

Equivalent noise resistance and equivalent noise temperature

In addition to the widely used equivalent noise generator and noise-figure methods of expressing the noise performance of amplifiers, other methods are also used. One of the most popular is to state an amplifier's series noise resistance R_e and parallel noise resistance R_i, where these quantities are defined by the following expressions[2]:

$$R_e = \frac{e_n^2}{4kT} \quad (19)$$

and

$$R_i = \frac{4kT}{i_n^2} \quad (20)$$

where e_n = output of amplifier's equivalent noise-voltage generator; k = Boltzmann's constant; T = absolute temperature, °K; and i_n = output of amplifier's equivalent noise-current generator.

By the use of R_e and R_i, Equation (8) takes the form:

$$NF = 10 \log 1 + \left[\frac{R_e}{R_s} + \frac{R_s}{R_i}\right] \quad (21)$$

Equation (21) makes it particularly easy to understand how NF varies as a function of R_s. Large noise figures are obtained with R_s either large or small. Minimum NF is obtained when the two terms containing R_s are equal. Differentiating with respect to R_s yields

$$R_{opt} = \sqrt{R_e R_i} \quad (22)$$

Equation (22) in turn can be substituted into (21) to obtain another expression for minimum noise figure:

$$NF_{min} = 10 \log [1 + 2\sqrt{R_e/R_i}] \quad (23)$$

For the sake of comparison, refer back to Equations (13) and (14) to see how R_{opt} and NF are computed from e_n and i_n.

It is interesting to note that a plot of constant noise figure contours "contains" R_e and R_i to a good approximation in the 3 dB contour. Over the range in which the upper and lower curves are widely separated, the upper portion of the 3 dB contour represents R_i and the lower portion represents R_e, with the actual resistance values being read directly from the source-resistance scale. At the ends, where the curves converge and close, the approximation is inaccurate because R_e and R_i extend beyond the convergence region and, in fact, cross over, with the result that, at very high and very low frequencies, R_e is larger than R_i. The significance of this relationship is that it gives a convenient way of quickly evaluating the relative noise performance of two amplifiers, where the noise performance of one is expressed as a plot of R_e and R_i, and that of the other is expressed as a plot of contours of constant noise figure.

It is implicit in the preceding discussion of R_e and R_i that attenuating a signal source, either by means of a passive network, or by loading by the input resistance of the amplifier, will degrade the SNR and not just the signal level. This is worth stressing because of the common misconception that such attenuation "reduces the signal and noise equally, leaving the SNR unchanged."

Equivalent noise temperature T_e is another way of expressing the noise characteristics of an amplifier. The effective noise temperature T_e of an amplifier whose input is fed from a particular source resistance is defined as the increase in source resistance temperature required to produce the observed available noise power at the output of the amplifier, the amplifier being noiseless.[3] Like noise figure, equivalent noise temperature is a function of source resistance and frequency, and thus one can plot contours of equivalent noise temperature that define the noise performance of an amplifier in much the same manner as do contours of constant noise figure. Unlike noise figure, equivalent noise temperature is *not* a function of the source-resistance temperature. Figure 11 is a plot of the contours of constant noise temperature for the amplifier whose noise-figure contours are plotted in Figure 6. The expression relating the equivalent noise temperature and noise figure is

$$T_e = 290(10^{NF/10} - 1) \quad (24)$$

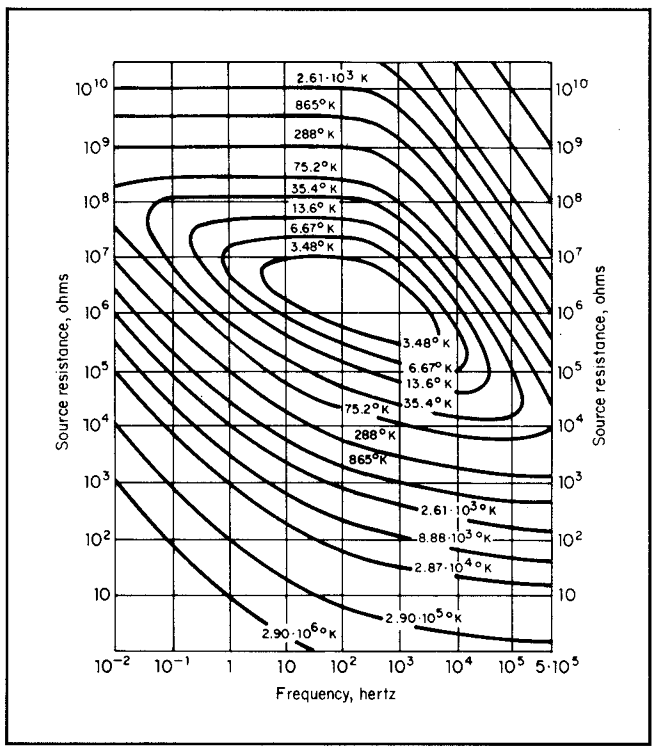

FIGURE 11. Contours of constant equivalent noise temperature for Model 113 preamplifier.

Note that the 290 value in Equation (24) is the absolute temperature of the NF measurement. If the NF had been measured at some other temperature, that temperature would be used in place of 290.

The utility of the noise-temperature method of expressing amplifier noise performance becomes clear by considering a case in which the source resistance is at a different temperature than the amplifier — for example, in cryogenic research. Suppose one wanted to estimate E_{tni} for an amplifier, first from the noise figure, and second from the equivalent noise temperature. The appropriate equations are

$$E_{tni} = [4kTR_s f_n]^{1/2}\, 10^{NF/20} \text{ volts rms} \qquad (10)$$

and

$$E_{tni} = [4k(T_s + T_e)R_s f_n]^{1/2} \text{ volts rms} \qquad (25)$$

where T_s is the source-resistance temperature and T_e is the amplifier equivalent noise temperature, both in degrees Kelvin.

If the source resistance and the amplifier were at the temperature of the NF measurements, either equation would quickly yield the desired result. However, if the amplifier were at room temperature, and the source resistance at liquid-helium temperature, only Equation (25) could be used to compute the total noise referred to the input directly. Equation (10) could be used only if NF were re-evaluated for the low source-resistance temperature. The required computation, though straightforward, is lengthy, and contains many more opportunities for error than does Equation (25). In general then, when the noise performance data are in terms of noise figure and it is necessary to operate at temperatures other than at which the noise-figure measurement was made, it is advisable to use Equations (24) and (25) to determine E_{tni}.

The following example further illustrates the advantages of T_e over NF to the cryogenic researcher. Consider the researcher who wishes to optimize an experiment to be conducted at 1 kHz at a source-resistance temperature of 4 °K. A cursory glance at Figure 6 could mistakenly lead him to believe that excellent noise performance is obtainable over a wide range of source resistance. At 1 kHz, the noise figure changes only 1 dB as R_s varies from 10^4 to 10^6 ohms. Figure 11, however, shows that the noise temperature changes from 75.2 °K to 3.48 °K, a marked difference, as R_s varies over the same range. It is obvious from Figure 11, if not from Figure 6, that at cryogenic temperatures it is particularly important to operate from the optimum source resistance, which, as explained earlier, may be obtained by following a low-resistance sensor by a suitable step-up transformer. Of course, if one were to "correct" Figure 6 by using Equation (24), substituting "4" for "290," this advantage would be equally obvious from the NF contours; at 4 °K, the NF at 1 kHz changes from 13 to 2.7 dB as R_s varies from 10^4 to 10^6 ohms. The point to be made is that T_e gives an *immediate* indication of the expected noise performance, valid for *all* temperatures, whereas NF must be recomputed to obtain a valid indication of expected noise performance if the source-resistance temperature differs significantly from that at which the NF contours apply.

Output SNR for current-source-driven amplifier

Figure 12 shows an amplifier/filter driven by a current source, such as a photodiode or photomultiplier tube. R_L is the dc load resistor required to complete the dc bias circuit for the source, and I_{sh} is the source's intrinsic shot-noise current.

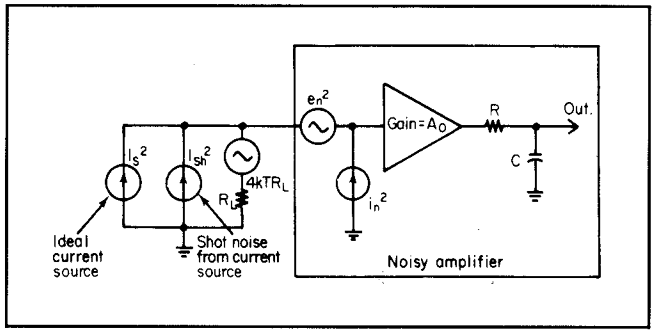

FIGURE 12. Equivalent circuit for noisy amplifier fed from current signal I_s with load resistance R_L.

What can we say about the $(SNR)_o$ and its optimization? The amplifier's input signal voltage is directly proportional to the parallel combination of R_L and the amplifier's ac input impedance; however, for simplicity we assume that the input impedance is much larger than R_L. Therefore, by the use of Equation (5), the voltage $(SNR)_o$ can be shown to be

$$(SNR)_o = \frac{I_s R_L}{[(I_{sh}^2 + i_n^2)R_L^2 + e_n^2 + 4kTR_L]^{1/2} f_n^{1/2}} \qquad (26)$$

Note from Equation (26) that the voltage of the signal varies directly with R_L, which leads to the next statement.

Statement 4. $(SNR)_o$ is maximized asymptotically as R_L approaches infinity.

In many respects, the situation is analogous to the one in which the source resistance is very much lower than R_{opt}. When operation is from a source resistance that is either lower or higher than R_{opt}, the situation with regard to noise can, in principle, be improved by using a transformer to match the source resistance to R_{opt} of the amplifier. As shown earlier, this is indeed true for working from R_s values that are much smaller than R_{opt}. However, practical transformer design limitations usually prevent one from improving SNR by using a step-down transformer to match current-source resistances to R_{opt}. As discussed previously, when working from a low source resistance, the situation is only worsened by connecting a resistor in series with the source to make R_s equal to R_{opt}; similarly, it can only be worsened when working from a current source by connecting a resistor in parallel with the input to make R_s equal to R_{opt}. The load resistor R_L is such a parallel resistor. However, by making it as large as possible, the shunting effect is minimized and the best possible SNR is obtained.

Hence, for a given preamplifier, the experimenter should use the highest value of R_L possible (regardless of the noise-figure contours for that amplifier). In practice, the input cable's capacitive reactance and the amplifier's finite input impedance constrain the size of R_L.

Where the signal derives from a current source, noise-figure contours should be consulted only to determine the system noise or SNR or to select the preamplifier to be used.

Maximum SNR when working from a current source will generally be achieved by:

(1) Restricting the system bandwidth in accordance with maximum tolerable signal distortion.

(2) Keeping R_L as large as possible.

(3) Using an extremely high input impedance amplifier having low e_n and i_n. Amplifiers with FET (field-effect transistor) input stages are most appropriate.

(4) Keeping the source shot noise to a minimum.

(5) Keeping the input cables as short as possible.

Summary

The reader should now be in a position to reap his well-deserved reward — namely, to take the raw noise data (e_n and i_n, R_e and R_i, NF or F, or T_e) furnished by the preamplifier manufacturer, and to convert the data into some useful numbers that compare amplifiers and allow estimation of the probable noise performance in a given experiment. The paragraphs dealing with transformer impedance matching show that when working with signals from a low source impedance, a great improvement in signal-to-noise ratio can be achieved by matching the source impedance to the optimum noise resistance of the amplifier by means of a step-up signal transformer.

One factor to bear in mind in using most of the equations provided here is that they are based solely on a consideration of source thermal noise and amplifier noise. As pointed out at the beginning, there are many other sources of noise as well, and all too often they also degrade the SNR and cannot be neglected. Nevertheless, even though the computed SNR may exceed the actual SNR due to other sources of noise, the equations do define the best operating conditions.

Readers should realize that this discussion is primarly confined to "front-end" considerations. By using the appropriate method of additional signal processing, enormous additional improvements in signal-to-noise ratio can be obtained. These include the use of lock-in amplifiers, signal averagers, or in some instances, general signal correlators.

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