735 W CLINTON ST, ITHACA N Y 14850 607-272-7640 TWX 510-255-9307 ITHACO

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MEASURING NOISE SPECTRA WITH VARIABLE ELECTRONIC FILTERS

DONALD F. CHANDLER

ABSTRACT

A simple general method of measuring a noise spectrum with variable electronic filters is described, and all the necessary information for measuring a noise spectrum with an ITHACO Variable Electronic Filter is provided. Errors associated with making noise measurements are discussed in the appendices.

INTRODUCTION

Although random signals occur in many forms such as voltage, current, charge, force, velocity, acceleration, temperature, etc., measuring the spectrum of a random signal is usually accomplished by converting the signal to a signal voltage, and then measuring the spectrum of the signal voltage. The spectrum of a signal voltage can be described in terms of the spectral density functions: power spectral density (mean-square voltage per unit bandwidth) and amplitude spectral density (RMS voltage per $\sqrt{Hz}$). Since the converted signal is a voltage, it is convenient to deal with the amplitude spectral density from which the power spectral density can be obtained by squaring.

A number of noise spectra are shown in figure 1. White noise (curve a) has a constant spectral density for all frequencies. Band-limited white noise (curve b) has a constant spectral density up to some frequency above which the spectral density rolls off. 1/f noise (curve c) has a spectral density which increases below some frequency. A typical amplifier noise spectrum (curve d) might possess 1/f noise at low frequencies, constant spectral density at mid frequencies, slightly increasing spectral density at higher frequencies, and band-limiting of the noise at the highest frequencies.

A measurement of the spectral density of a noise signal can be obtained at one frequency by converting the noise signal to a signal voltage; passing the signal voltage through an ideal band-pass filter with one hertz bandwidth; and measuring the filter output with a true RMS voltmeter as in figure 2. The voltmeter reading is the amplitude spectral density of the signal voltage at the filter frequency inasmuch as the meter reading is the voltage per $\sqrt{Hz}$. To obtain the complete spectrum of the noise, the measurement must be repeated at each frequency. To obtain the original noise spectrum the readings can be divided by the converter sensitivity.

There are a number of practical limitations associated with measuring a noise spectrum. The spectral density must be constant over the filter pass-band so that a one hertz band-pass filter may be too coarse at the lowest frequencies and unnecessarily fine at high frequencies. Ideal band-pass filters are not available, and for practical filters the –3 dB bandwidth and noise bandwidth may be significantly different so the noise bandwidth of the filter must be determined (appendix I). Noise introduced by the measuring instruments may alter the reading enough to require correction (see appendix II). Noise signals have larger peak factors than sinusoidal signals and failure to allow for larger peak factors will result in measurement errors (see appendix III). The standard deviation (RMS error) of a single measurement of the spectral density of a random signal is dependent on filter bandwidth and sampling time so it is likely that measurements of spectral noise at low frequencies will require a compromise between filter bandwidth (frequency resolution), sampling time (time required to obtain a measurement), and measurement error (see appendix IV).

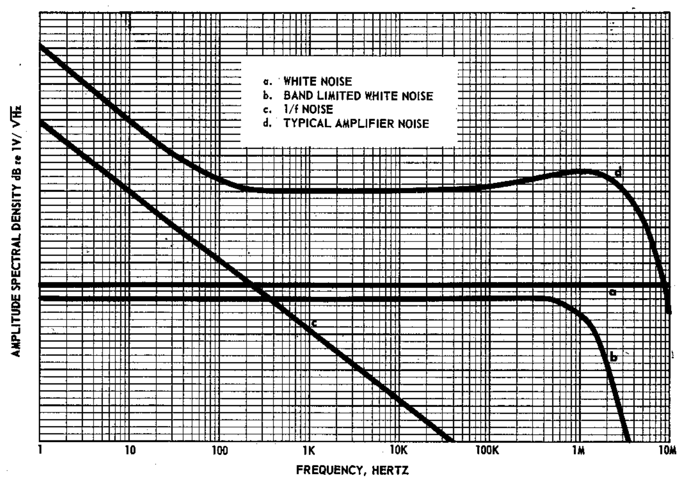


FIGURE 1 TYPICAL NOISE SPECTRA

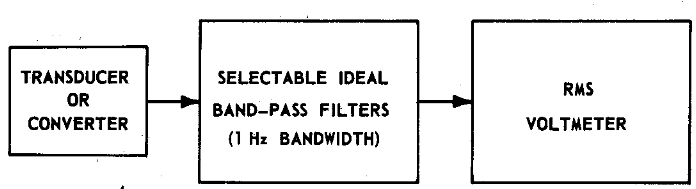


FIGURE 2 MEASURING A NOISE SPECTRUM

Finally, measurements made with other than a true RMS meter, such as a meter measuring the average absolute value but calibrated to read the RMS of a sine wave, will require corrections to obtain the true spectral density. Even so, these corrections are available for only a few types of noise signals (see appendix V).

A METHOD OF MEASURING A NOISE SPECTRUM

A method of measuring the spectral density of a noise signal is shown in figure 3. The preamplifier raises the signal level so that it is well above the filter noise. The bandpass filter selects a portion of the spectrum to measure, and for the measurement to be accurate it is necessary that the spectral density be constant over the filter pass-band. The post amp raises the signal to a level which the RMS meter can measure, and a true RMS voltmeter is used so that the spectral density measurements obtained can be used without further qualification.

The amplitude spectral density at a frequency f_o is given by formula 2 in figure 3:

$$\xi_n(f_o) = \frac{e_o}{A_1 A_2 A_3 \sqrt{B_n}} \quad (V/\sqrt{Hz})$$

If the RMS meter reading, amplifier gain, filter gain, and filter noise bandwidth are known, the amplitude spectral density can be computed. For example if:

$e_o = .01$ volts	$A_1 = 10$
$f_o = 1{,}000$ Hz	$A_2 = 1$
$B_n = 100$ Hz	$A_3 = 1{,}000$

Then the amplitude spectral density is given by

$$\xi_n(1{,}000\ Hz) = \frac{.01}{(10)(1)(1{,}000)\ \sqrt{100}} = 10^{-7} V/\sqrt{Hz}$$

It is often easier to compute the amplitude spectral density in terms of decibels referred to $1V/\sqrt{Hz}$ as given by formula 3 in figure 3.

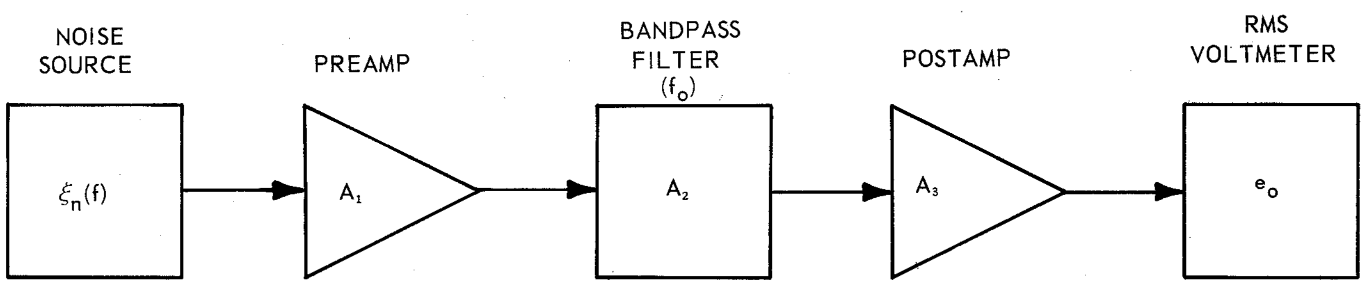
Amplitude Spectral Density (dB re $1V/\sqrt{Hz}$) =

$$e'_o - [A'_1 + A'_2 + A'_3] - 10 \log_{10} B_n$$

where the RMS meter reading is given in dB V(dB re 1 V) and amplifier and filter gains are given in dB. For for example:

$e'_o = -40$ dBV	$A'_1 = 20$ dB
$10 \log_{10} B_n = 20$	$A'_2 = 0$ dB
$f_o = 1{,}000$ Hz	$A'_3 = 60$ dB

Amplitude spectral density at 1000 Hz =
$-40 - [80] - 20 = -140$[dB re 1 $V/\sqrt{Hz}$]



IF THE SPECTRAL DENSITY IS CONSTANT OVER THE FILTER PASS– BAND:

1 $e_o = A_1 A_2 A_3 \xi_n(f_o) \sqrt{B_n}$

AMPLITUDE SPECTRAL DENSITY AT f_o, $\xi_n(f_o)$ in $V/\sqrt{Hz}$

2 $\xi_n(f_o) = \dfrac{e_o}{A_1 A_2 A_3 \sqrt{B_n}}$

WHERE
A_1 = PREAMP GAIN
A_2 = FILTER GAIN
A_3 = POSTAMP GAIN
B_n = FILTER NOISE BANDWIDTH
e_o = RMS VOLTMETER READING
$\xi_n(f_o)$ = AMPLITUDE SPECTRAL DENSITY AT f_o IN $V/\sqrt{Hz}$

3 AMPLITUDE SPECTRAL DENSITY, dB re $1V/\sqrt{Hz}$ at f_o

$$= 20 \log_{10} \xi_n(f_o)$$
$$= 20 \log_{10} e_o - 20 \log_{10} A_1 A_2 A_3 - 10 \log_{10} B_n$$
$$= e'_o - [A'_1 + A'_2 + A'_3] - 10 \log_{10} B_n$$

WHERE
e'_o = RMS READING IN dBV
A'_1 = PREAMP GAIN IN dB
A'_2 = FILTER GAIN IN dB
A'_3 = POSTAMP GAIN IN dB
f_o = FILTER CENTER FREQUENCY

FIGURE 3 A METHOD OF MEASURING AMPLITUDE SPECTRAL DENSITY

Chart I is a simple form for recording data and computing the amplitude spectral density. Column 1 is the frequency of the measurement (center frequency of the filter). Columns 2 and 3 are the high-pass and low-pass filter settings if a variable electronic filter is used. Column 4 is the filter gain (insertion loss) in decibels. Column 5 is $10 \log_{10} B_n$ where B_n is the noise bandwidth. Columns 6 and 7 are the preamp and postamp gain in decibels. Column 8 is the RMS meter reading in dB V (decibels referred to 1 Volt). Column 9 is the amplitude spectral density in dB re $1V/\sqrt{Hz}$. Column 10 is the amplitude spectral density in Volts/$\sqrt{Hz}$. For example:

① Center frequency, Hz	=	1,000
② High-pass setting, Hz	=	800
③ Low-pass setting, Hz	=	1,250
④ Filter gain, dB	=	−1.28
⑤ $10 \log_{10} B_n$	=	28.32
⑥ Preamp gain, dB	=	40.00
⑦ Postamp gain, dB	=	40.00
④+⑤+⑥+⑦	=	107.04
⑧ RMS meter reading, dB V	=	−33.0
⑨ Amplitude Spectral Density at 1000 Hz (dB) re $1V/\sqrt{Hz}$	=	⑧−[④+⑤+⑥+⑦]
	=	−140
⑩ Amplitude Spectral Density at 1,000 Hz (Volt/$\sqrt{Hz}$)	=	Anti $\log_{10}$ ⑨
	=	10^{-7}

MEASURING A NOISE SPECTRUM WITH A VARIABLE ELECTRONIC FILTER

A variable electronic filter, as defined in this note, consists of cascaded high-pass and low-pass filters with independent control of the cut-off frequencies. The cut-off frequency is determined by a frequency setting switch (or dial) and a frequency multiplier switch. If the frequency setting switches are not changed, changing the multiplier switches by a factor of ten will change the cut-off frequencies, center frequency, bandwidth, and noise bandwidth by a factor of ten. It is convenient therefore, to tabulate the filter characteristics for various filter settings with the high-pass multiplier set to unity and consider the effects of the high-pass multiplier separately.

Chart II lists the −3 dB frequencies, center frequency, bandwidth, noise bandwidth, and filter gain for ITHACO Variable Electronic Filters with the high-pass multiplier set to unity and the low-pass setting less than a decade above the high-pass setting.

For filter separation greater than ten:

$$\frac{F_{LP}}{F_{HP}} \geq 10 \qquad \begin{array}{ll} \text{Noise bandwidth, Hz} = 1.023\left(F_{LP} - F_{HP}\right) \\ \text{Filter Gain, dB} = 0.00 \end{array}$$

For high-pass multiplier settings other than unity all frequencies in Chart II are multiplied by the high-pass multiplier.

Chart III provides a means of obtaining $10 \log_{10} B_n$ for ITHACO Variable Electronic Filters with 0, 1/3, 2/3 and 1 octave separation of the high-pass and low-pass filters. Again the tables show $10 \log_{10} B'_n$, where B'_n is the noise bandwidth for that setting with the high-pass multiplier set to unity:

Noise Bandwidth, $B_n = (B'_n)$ (H.P. Multiplier) so that,

$$10 \log_{10}(B_n) = 10 \log_{10}\left[B'_n \text{ (H.P. Mult)}\right]$$
$$= 10 \log_{10} B'_n + 10 \log_{10}(\text{H.P. Mult})$$

For example:

High-Pass cut-off frequency, $F_{HP} = 1.00 \times 10^3$Hz

Low-Pass cut-off frequency, $F_{LP} = 2.00 \times 10^3$Hz

$$\text{Separation } \frac{F_{LP}}{F_{HP}} = \frac{2.00 \times 10^3}{1.00 \times 10^3} = 2.00 \qquad \text{(1 octave)}$$

④ (Column 4 in table for 1 octave separation)

$10 \log_{10} B'_n$ = .66

⑥ (Column 6 in table for H.P. Multiplier)

$10 \log_{10}(\text{H.P. Mult}) = 10 \log_{10} 10^3$ = 30.00

$10 \log_{10} B_n$ = ④ + ⑥ = 30.66

Similarly:

High-Pass cut-off frequency, $F_{HP} = 2.00 \times 10^2$Hz

Low-Pass cut-off frequency, $F_{LP} = 2.50 \times 10^2$Hz

$$\text{Separation } \frac{F_{LP}}{F_{HP}} = \frac{2.50 \times 10^2}{2.00 \times 10^2} = 1.25 \qquad \text{(1/3 octave)}$$

④ (Column 4 in table for 1/3 octave separation)

$10 \log_{10} B'_n$ = .88

⑥ (Column 6 in table for H.P. Multiplier)

$10 \log_{10} (\text{H.P. Mult}) = 10 \log_{10}(10^2)$ = 20.00

$10 \log_{10} B_n$ = ④ + ⑥ = 20.88

The procedure for measuring a noise spectrum with a variable electronic filter is identical to that described above if the noise bandwidth and filter gain are known. The noise bandwidth and filter gain for ITHACO Variable Electronic Filters can be obtained from Chart II or Chart III. The measurements can then be recorded, and the amplitude spectral density computed on Chart I.

A special case worthy of discussion is the measurement of spectral noise in one octave frequency steps. Such

CHART I
AMPLITUDE SPECTRAL DENSITY

MEASUREMENTS OF____________________ DATE__________
MEASUREMENT BY__________

①	②	③	④	⑤	⑥	⑦	⑧ *	⑨ **	⑩ ***
CENTER FREQUENCY Hz	HIGH-PASS SETTING Hz	LOW-PASS SETTING Hz	FILTER GAIN dB	$10 \log_{10} B_n$	PREAMP GAIN dB	POSTAMP GAIN dB	RMS VOLTAGE dBV	AMPLITUDE SPECTRAL DENSITY dB **	AMPLITUDE SPECTRAL DENSITY $V/\sqrt{Hz}$

* RMS VOLTAGE, dBV (dB REFERRED TO 1 VOLT)

** AMPLITUDE SPECTRAL DENSITY, dB re $1V/\sqrt{Hz}$ = ⑧ − [④ + ⑤ + ⑥ + ⑦]

*** AMPLITUDE SPECTRAL DENSITY, $V/\sqrt{Hz}$ = ANTILOG ⑨

–3dB FREQUENCIES – CENTER FREQUENCY – BANDWIDTH – NOISE BANDWIDTH – FILTER GAIN FOR ITHACO VARIABLE ELECTRONIC FILTERS FORMED BY CASCADING 4 POLE BUTTERWORTH LOW-PASS AND HIGH-PASS FILTERS

RESPONSE OF SEPARATE FILTER SECTIONS

NORMAL { HIGH–PASS FREQUENCY SETTING: –3 dB AND +180° (BUTTERWORTH)
LOW–PASS FREQUENCY SETTING: –3 dB AND –180° (BUTTERWORTH)

PULSE { HIGH–PASS FREQUENCY SETTING: –3 dB AND +180° (BUTTERWORTH)
LOW–PASS FREQUENCY SETTING: –8.6 dB AND –187° (BESSEL)

RESPONSE OF BAND-PASS FILTER (NORMAL MODE)

–3 dB FREQUENCIES, CENTER FREQUENCY, –3 dB BANDWIDTH, NOISE BANDWIDTH, FILTER GAIN

NORMALIZED HIGH-PASS SETTING HERTZ		NORMALIZED LOW-PASS SETTING, HERTZ																				
		1.00	1.25	1.60	2.00	2.50	3.15	4.00	5.00	6.30	8.00	10.0	12.5	16.0	20.0	25.0	31.5	40.0	50.0	63.0	80.0	100
1.00	UPPER –3dB FREQ., Hz	1.245	1.418	1.699	2.057	2.530	3.165	4.000	5.000	6.300	8.000	10.00	12.50	16.00	20.00	25.00	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz	1.000	1.118	1.265	1.414	1.581	1.774	2.000	2.236	2.510	2.828	3.162	3.536	4.000	4.472	5.000	5.612	6.325	7.071	7.937	8.944	10.00
	LOWER –3dB FREQ., Hz	0.803	0.881	0.942	0.972	0.988	0.995	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	–3dB BANDWIDTH, Hz	0.442	0.537	0.758	1.084	1.542	2.170	3.002	4.000	5.300	7.000	9.000	11.50	15.00	19.00	24.00	30.50	39.00	49.00	62.00	79.00	99.00
	NOISE BANDWIDTH, Hz	.513	.612	.848	1.163	1.620	2.252	3.105	4.097	5.414	7.178	9.226	11.79	15.38	19.48	24.60	31.26	39.99	50.23	63.53	81.00	101.5
	FILTER GAIN, dB	-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.25	UPPER –3dB FREQ., Hz		1.556	1.773	2.124	2.571	3.163	3.956	5.000	6.300	8.000	10.00	12.50	16.00	20.00	25.00	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz		1.250	1.414	1.581	1.768	1.984	2.236	2.500	2.806	3.162	3.536	3.953	4.472	5.000	5.590	6.275	7.071	7.906	8.874	10.00	11.18
	LOWER –3dB FREQ., Hz		1.004	1.101	1.177	1.215	1.235	1.245	1.247	1.250	1.250	1.250	1.250	1.250	1.250	1.250	1.250	1.250	1.250	1.250	1.250	1.250
	–3dB BANDWIDTH, Hz		0.552	0.671	0.947	1.355	1.928	2.713	3.753	5.050	6.750	8.750	11.25	14.75	18.75	23.75	30.25	38.75	48.75	61.75	78.75	98.75
	NOISE BANDWIDTH, Hz		.641	.766	1.059	1.454	2.025	2.815	3.877	5.123	6.769	8.974	11.53	15.12	19.21	24.35	31.01	39.72	49.95	63.31	80.72	101.3
	FILTER GAIN, dB		-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.60	UPPER –3dB FREQ., Hz			1.992	2.269	2.718	3.291	4.048	5.064	6.400	8.000	10.00	12.50	16.00	20.00	25.00	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz			1.600	1.789	2.000	2.245	2.530	2.828	3.175	3.578	4.000	4.472	5.060	5.657	6.325	7.099	8.000	8.944	10.04	11.31	12.65
	LOWER –3dB FREQ., Hz			1.284	1.410	1.506	1.556	1.581	1.592	1.597	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600	1.600
	–3dB BANDWIDTH, Hz			0.707	0.859	1.212	1.734	2.467	3.472	4.803	6.400	8.400	10.90	14.40	18.40	23.40	29.90	38.40	48.40	61.40	78.40	98.40
	NOISE BANDWIDTH, Hz			.821	.979	1.357	1.860	2.591	3.602	4.966	6.561	8.660	11.48	14.76	18.86	23.99	30.65	39.36	49.60	62.95	80.35	100.8
	FILTER GAIN, dB			-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.00	UPPER –3dB FREQ., Hz				2.480	2.836	3.398	4.113	5.060	6.330	8.000	10.00	12.50	16.00	20.00	25.00	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz				2.000	2.236	2.510	2.828	3.162	3.550	4.000	4.472	5.000	5.657	6.325	7.071	7.937	8.944	10.00	11.22	12.65	14.14
	LOWER –3dB FREQ., Hz				1.606	1.762	1.882	1.945	1.976	1.990	1.996	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000	2.000
	–3dB BANDWIDTH, Hz				0.884	1.074	1.515	2.168	3.084	4.340	6.004	8.000	14.00	14.00	18.00	29.50	29.50	38.00	48.00	61.00	78.00	98.00
	NOISE BANDWIDTH, Hz				1.026	1.225	1.696	2.325	3.240	4.503	6.209	8.204	10.83	14.35	18.45	23.58	30.23	38.95	49.20	62.52	79.98	100.5
	FILTER GAIN, dB				-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
2.50	UPPER –3dB FREQ., Hz					3.113	3.545	4.247	5.142	6.325	7.913	10.00	12.50	16.00	20.00	25.00	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz					2.500	2.806	3.162	3.536	3.969	4.472	5.000	5.590	6.325	7.071	7.906	8.874	10 00	11.18	12.55	14.14	15.81
	LOWER –3dB FREQ., Hz					2.008	2.203	2.354	2.431	2.470	2.487	2.495	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500	2.500
	–3dB BANDWIDTH, Hz					1.105	1.343	1.894	2.710	3.855	5.425	7.505	10.00	13.50	17.50	22.50	29 00	37.50	4.750	60.50	77.50	97.00
	NOISE BANDWIDTH, Hz					1.282	1.531	2.246	2.907	4.050	5.630	7.754	10.24	13.54	17.95	23.07	29.72	38.46	48.70	62.02	79.43	99.88
	FILTER GAIN, dB					-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
3.15	UPPER –3dB FREQ., Hz						3.921	4.467	5.352	6.478	7.970	9.970	12.60	16.00	20.00	25.00	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz						3.150	3.550	3.969	4.455	5.020	5.612	6.275	7.099	7.937	8.874	9.961	11.22	12.55	14.09	15.87	17.75
	LOWER –3dB FREQ., Hz						2.523	2.775	2.966	3.063	3.144	3.146	3.148	3.150	3.150	3.150	3.150	3.150	3.150	3.150	3.150	3.150
	–3dB BANDWIDTH, Hz						1.392	1.691	2.386	3.415	4.826	6.824	9.452	12.85	12.85	21.85	28.35	36.85	46.85	59.85	76.85	96.85
	NOISE BANDWIDTH, Hz						1.616	1.930	2.670	3.664	5.105	7.096	9.772	12.91	17.06	22.62	29.07	37.76	48.03	61.38	78.80	99.31
	FILTER GAIN, dB						-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
4.00	UPPER –3dB FREQ., Hz							4.980	5.672	6.796	8.226	10.12	12.66	16.00	20.00	25.00	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz							4.000	4.472	5.020	5.657	6.325	7.071	8.000	8.944	10.00	11.22	12.65	14.14	15.87	17.89	20.00
	LOWER –3dB FREQ., Hz							3.212	3.524	3.766	3.890	3.952	3.980	3.992	4.000	4.000	4.000	4.000	4.000	4.000	4.000	4.000
	–3dB BANDWIDTH, Hz							1.768	2.148	3.030	4.336	6.168	8.680	12.01	16.00	21.00	27.50	36.00	46.00	59.00	76.00	96.00
	NOISE BANDWIDTH, Hz							2.051	2.449	3.392	4.651	6.479	9.005	12.42	16.41	21.65	28.71	36.90	47.15	60.46	77.89	98.40
	FILTER GAIN, dB							-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
5.00	UPPER –3dB FREQ., Hz								6.225	7.090	8.495	10.28	12.65	16.08	20.00	25.00	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz								5.000	5.612	6.325	7.071	7.906	8.944	10.00	11.18	12.55	14.14	15.81	17.75	20.00	22.36
	LOWER –3dB FREQ., Hz								4.015	4.405	4.708	4.862	4.940	4.975	4.990	5.000	5.000	5.000	5.000	5.000	5.000	5.000
	–3dB BANDWIDTH, Hz								2.210	2.685	3.788	5.420	7.710	11.11	15.01	20.00	26.50	45.00	45.00	58.00	75.00	95.00
	NOISE BANDWIDTH, Hz								2.564	3.062	4.241	5.814	8.100	11.26	15.51	20.49	27.07	35.89	46.13	59.43	76.91	97.39
	FILTER GAIN, dB								-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
6.30	UPPER –3dB FREQ., Hz									7.843	8.933	10.70	12.96	15.94	19.94	25.20	31.50	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz									6.300	7.099	7.937	8.874	10.04	11.22	12.55	14.09	15.87	17.75	19.92	22.45	25.10
	LOWER –3dB FREQ., Hz									5.059	5.550	5.932	6.126	6.224	6.268	6.287	6.300	6.300	6.300	6.300	6 300	6.300
	–3dB BANDWIDTH, Hz									2.785	3.383	4.773	6.829	9.715	13.67	18.91	25.20	33.70	43.70	56.70	73.70	93.70
	NOISE BANDWIDTH, Hz									3.232	3.898	5.339	7.328	10.21	14.19	19.54	25.82	34.12	45.24	58.14	75.51	96.05
	FILTER GAIN, dB									-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00	0.00
8.00	UPPER –3dB FREQ., Hz										9.960	11.34	13.59	16.45	20.24	25.32	32.00	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz										8.000	8.944	10.00	11.31	12.65	14.14	15.87	17.89	20.00	22.45	25.30	28.28
	LOWER –3dB FREQ., Hz										6.424	7.048	7.533	7.779	7.904	7.960	7.984	8.000	8.000	8.000	8.000	8.000
	–3dB BANDWIDTH, Hz										3.536	4.296	6.061	8.672	12.34	17.36	24.02	32.00	42.00	55.00	72.00	92.00
	NOISE BANDWIDTH, Hz										4.102	4.898	6.784	9.300	12.96	18.01	24.83	32.81	43.30	57.41	73.79	94.30
	FILTER GAIN, dB										-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00	0.00
10.0	UPPER –3dB FREQ., Hz											12.45	14.18	16.99	20.57	25.30	31.65	40.00	50.00	63.00	80.00	100.0
	CENTER FREQ., Hz											10.00	11.18	12.65	14.14	15.81	17.75	20.00	22.36	25.10	28.28	31.62
	LOWER –3dB FREQ., Hz											8.030	8.810	9.416	9.724	9.880	9.950	9.980	10.00	10.00	10.00	10.00
	–3dB BANDWIDTH, Hz											4.420	5.370	7.576	10.84	15.42	21.70	30.02	40.00	53.00	70.00	90.00
	NOISE BANDWIDTH, Hz											5.129	6.124	8.482	11.63	16.20	27.05	31.05	41.02	54.14	71.78	92.26
	FILTER GAIN, dB											-6.02	-2.98	-1.23	-0.53	-0.22	-0.09	-0.03	0.00	0.00	0.00	0.00

DIRECTIONS:

1. DIVIDE THE CUT-OFF FREQUENCY SETTINGS BY HIGH-PASS MULTIPLIER TO OBTAIN NORMALIZED SETTINGS.
2. OBTAIN THE FILTER GAIN AND NORMALIZED FREQUENCIES FROM TABLE USING NORMALIZED SETTINGS.
3. MULTIPLY NORMALIZED FREQUENCIES OBTAINED FROM TABLE BY HIGH-PASS MULTIPLIER.

EXAMPLE:

HIGH–PASS CUT–OFF FREQUENCY = 6.30×10^3 → (Divide by High-Pass Multiplier) → NORMALIZED HIGH–PASS SETTING = 6.30

LOW–PASS CUT–OFF FREQUENCY = 1.00×10^4 → NORMALIZED LOW–PASS SETTING = 10.0

	NORMALIZED (From table)		ACTUAL
UPPER –3 dB FREQUENCY, Hz	10.70	(Multiply by High-Pass Multiplier)	10.70×10^3
CENTER FREQUENCY, Hz	7.937		7.937×10^3
LOWER –3 dB FREQUENCY, Hz	5.932		5.932×10^3
–3 dB BANDWIDTH, Hz	4.773		4.773×10^3
NOISE BANDWIDTH, Hz	5.339		5.339×10^3
FILTER GAIN, dB	–1.23	(Gain not normalized)	–1.23

$\frac{F_{LP}}{F_{HP}} \geq 10$

UPPER –3 dB FREQUENCY = LOW–PASS CUT–OFF FREQUENCY
CENTER FREQUENCY = $\sqrt{F_{HP} \cdot F_{LP}}$
LOW –3 dB FREQUENCY = HIGH–PASS CUT–OFF FREQUENCY

–3 dB BANDWIDTH = $F_{LP} - F_{HP}$
NOISE BANDWIDTH = $1.023\,(F_{LP} - F_{HP})$
FILTER GAIN = 0.0 dB

spectral noise measurements will provide adequate resolution for practically all noise spectra. The suggested method is the simplest, most direct way of setting a variable electronic filter, since the high-pass frequency setting, low-pass frequency setting, and center frequency are identical, and the filter gain is –6.0 dB. Chart IV is a simple form for recording the measurements and computing the amplitude spectral density. Chart IV is identical to Chart I except that the filter settings, noise bandwidth, and filter gain are tabulated for the internationally preferred octave frequencies used in ITHACO Variable Electronic Filters.

APPENDIX I

NOISE BANDWIDTH

An ideal band-pass filter is defined as a filter with no attenuation in the pass-band and infinite attenuation outside the pass-band. The noise bandwidth of a filter is defined as the bandwidth of an ideal filter which has the same value of absolute transmittance in its pass-band as the maximum absolute transmittance of the filter and delivers the same mean square output voltage from a white noise source as the filter. This definition can be stated mathematically:

IDEAL FILTER	MEAN SQUARE OUTPUT	PRACTICAL FILTER
$\int_0^\infty \xi_n^2 Y_o^2 df$	$= e_o^2$	$= \int_0^\infty \xi_n^2 \lvert Y(f)\rvert^2 df$
$\xi_n^2 Y_o^2 \int_{f_1}^{f_2} df$	$= e_o^2$	$= \xi_n^2 \int_0^\infty \lvert Y(f)\rvert^2 df$
$\xi_n^2 Y_o^2 B_n$	$= e_o^2$	$= \xi_n^2 \int_0^\infty \lvert Y(f)\rvert^2 df$

Solving for B_n:

Noise Bandwidth, $B_n = \frac{1}{Y_o^2} \int_0^\infty |Y(f)|^2 df$

where $B_n = (f_2 - f_1)$ = filter noise bandwidth

ξ_n = amplitude spectral density of the white noise source $V/\sqrt{Hz}$

$Y(f)$ = filter transmittance function

Y_o = Maximum absolute value of $Y(f)$

It follows that if the amplitude spectral density is constant across the filter pass-band, the amplitude spectral density at the filter frequency is given by:

$$\xi_n(f_o),\ V/\sqrt{Hz} = \frac{e_o}{Y_o \sqrt{B_n}}$$

The ratio of noise bandwidth to –3 dB bandwidth is 1.57, 1.11, 1.05, 1.025, 1.02, 1.01, --- 1.00 for a 1,2,3,4,5,6, —— ∞ pole low-pass Butterworth filter. The noise bandwidths of cascaded 4 pole Butterworth high-pass and low-pass filters are tabulated in Chart II.

APPENDIX II

ADDITION AND SUBTRACTION OF NOISE LEVEL IN DECIBELS

ADDITION

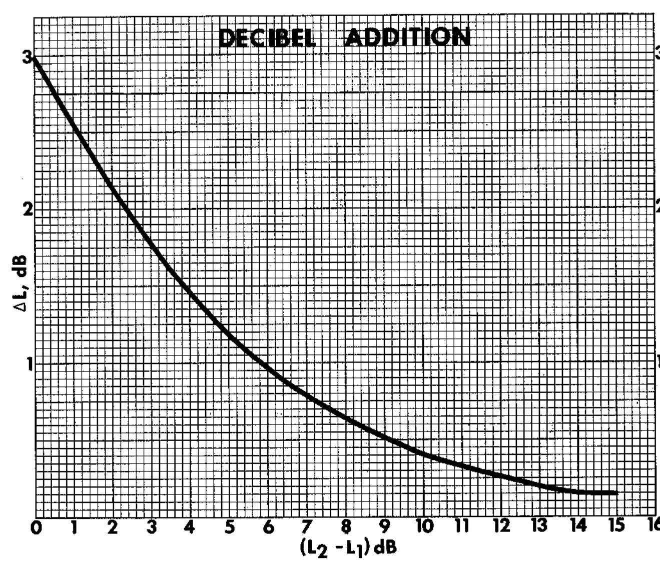
If two non coherent noise signals are combined, the resulting noise level in dB can be calculated from the dB values of the separate noise signals as follows:

1. Calculate the difference (in dB) between the two noise signals $L_2 - L_1$.
2. Find ΔL in the graph below.
3. Add ΔL to the highest of the two signals. The result is the noise level in dB of the combined signals.

EXAMPLE:

Two non coherent noise signals, –80 dB V and –85 dB V are to be combined.

1. $L_2 - L_1$ = (–80 dBV) – (–85 dBV) = 5 dB
2. From the graph below, ΔL = 1.2 dB
3. Combined signal level = (–80 dBV) + 1.2 dB = –78.8 dBV



SUBTRACTION

Measurements made in the presence of instrument noise can be corrected by decibel subtraction. If the signal to noise ratio is greater than 20 dB the effect of the interfering noise can be ignored. Specifically when the spectral noise to be measured is not 20 dB larger than the preamplifier input noise, corrections must be made to obtain the true spectral noise level.

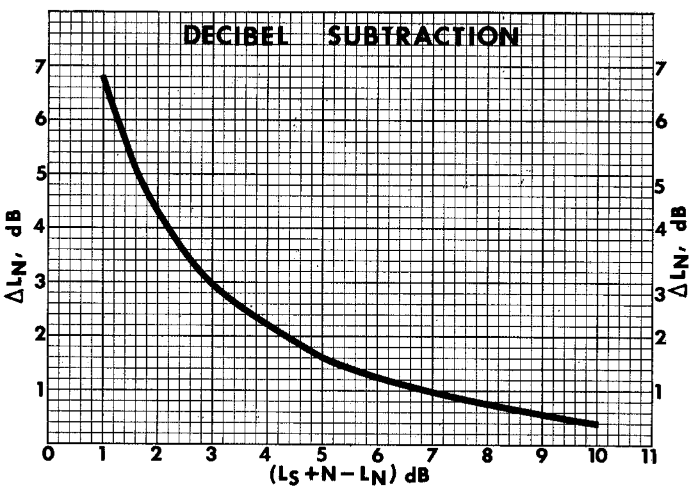
1. Calculate the difference (in dB) between the signal plus noise, L_{S+N} and noise level, L_N:
   $(L_{S+N} - L_N)$.
2. Find ΔL in the graph below.
3. Subtract ΔL from the signal plus noise $(L_{S+N} - \Delta L)$. The result is the signal level in dB.

EXAMPLE:

Given: $L_{S+N} = -85$ dBV

$L_N = -90$ dBV

1. $L_{S+N} - L_N = (-85 \text{ dBV}) - (-90 \text{ dBV})$
   $= 5$ dB
2. $\Delta L = 1.7$ dB
3. $L_N = L_{S+N} - \Delta L$
   $= -85 - 1.7$
   $= -86.7$ dB V



APPENDIX III

QUALIFIED PEAK FACTORS AND CLIPPING ERRORS

Gaussian noise has a probability greater than zero of exceeding any finite magnitude, no matter how large, with the probability falling off rapidly for large values. In practical experiments, however, large values are limited by non-linearities in either the noise source or the measuring instruments, so it is important to know how a measurement would be affected by limiting the noise peaks.

A fixed peak factor (ratio of PEAK to RMS) cannot be assigned to a Gaussian noise, since, if sufficient time is allowed and the measuring system doesn't limit the measurement, any value can be expected. A qualified peak factor can be assigned for a specified probability that the corresponding peak will be exceeded. A table of qualified peak factors is given below for Gaussian and Rayleigh noise. Rayleigh noise occurs when Gaussian noise is passed through a filter with narrow bandwidth compared with the filter center frequency. The resultant noise possesses a low-frequency envelope which has a Rayleigh distribution (for more information, reader is referred to *Electrical Noise*, W.R. Bennett, McGraw-Hill).

$$\text{QUALIFIED PEAK FACTOR} = \frac{\text{PEAK}}{\text{RMS}}$$

PERCENT OF TIME PEAK IS EXCEEDED	GAUSSIAN NOISE		RAYLEIGH NOISE	
	$\frac{\text{PEAK}}{\text{RMS}}$	$\left(\frac{\text{PEAK}}{\text{RMS}}\right)_{dB}$	$\frac{\text{PEAK}}{\text{RMS}}$	$\left(\frac{\text{PEAK}}{\text{RMS}}\right)_{dB}$
10	1.65	4.3	1.52	3.6
1	2.58	8.2	2.15	66
.1	3.29	10.4	2.56	8.4
.01	3.89	11.8	3.03	9.6
.001	4.42	12.9	3.39	10.6
.0001	4.89	13.8	3.68	11.3

Errors associated with limiting in the measuring instruments will vary according to whether the limiting is "hard" or "soft" as well as the distribution of the noise being measured. It is not usually practical to attempt to correct for limiting error, but a bench mark can be provided by tabulating the errors associated with measuring the mean-square, RMS, and average absolute value of a Gaussian noise process with various clipping levels.

	PERCENT ERROR		
CLIPPING LEVEL σ	AVERAGE ABSOLUTE VALUE	RMS VALUE	MEAN–SQUARE VALUE
1.9	2.3	5.1	9.9
2.1	1.6	3.2	6.3
2.3	.92	1.9	3.8
2.5	.50	1.1	2.2
2.7	.27	.63	1.3
2.9	.14	.35	.69
3.1	.07	.18	.36
3.3	.03	.09	.18
3.5	<.01	.04	.09
3.7	<.01	.02	.01
3.9	<.01	.01	.02
4.1	<.01	<.01	.01

APPENDIX IV

SAMPLING ERRORS ASSOCIATED WITH MEASURING A RANDOM PROCESS

The RMS value of a continuous ergodic random process can be determined from the power spectral density:

$$\text{(1)} \quad (e_{rms})^2 = \int_0^\infty P_{SD}(f)df,$$

where P_{SD} = power spectral density,
from the probability density function:

$$\text{(2)} \quad (e_{rms})^2 = \int_{-\infty}^{+\infty} V^2 p(v)dv,$$

where p(v) = probability density function
V = signal level,
or from the time integral:

$$\text{(3)} \quad (e_{rms})^2 = \lim_{T\to\infty} \frac{1}{T}\int_0^T e^2(t)dt,$$

where e(t) = time function of the random process.

The most common method of determining the RMS value is to compute the time integral, but the infinite time interval in the definition is replaced with a finite time interval, and the computation is then an estimate of true mean-squared value. If samples of the signal are taken of duration T, and the RMS value of the samples calculated, the calculated values will vary around the true RMS value. The deviation of the sampled values around the true RMS value will vary with the bandwidth and and sampling time. If the noise is squared and sampled over periods of time T and if the product $B_nT>>1$ where B_n is the noise bandwidth, the standard deviation of the energy fluctuation is:

$$\text{(4)} \quad \frac{\sigma}{(e_{rms})^2} = \frac{1}{\sqrt{B_nT}} \quad \text{where } \sigma = \text{standard deviation}$$

When the sampling process is replaced by continuous averaging process such as an RC network:

$$\text{(5)} \quad \frac{\sigma}{(e_{rms})^2} = \frac{1}{\sqrt{2B_nRC}} .$$

The effective sampling time of the RC network is twice the RC time constant.

Formulas (4) and (5) were derived for energy fluctuations so they are valid for energy fluctuation. For small energy fluctuations:

$$\text{(6)} \quad \frac{1}{\sqrt{B_nT}} = \frac{1}{\sqrt{B_n2RC}} = \frac{\sigma}{(e_{rms})^2} \approx \frac{2\sigma'}{e_{rms}} \approx \frac{2\sigma''}{e_{ave}}$$

where B_n = Noise bandwidth of the measurement

e_{rms} = True RMS value of the noise

σ = Standard deviation around the true mean-square value

σ' = Standard deviation around the true RMS value

σ'' = Standard deviation around the average absolute value

The practical aspects of formula 6 become more apparent when the averaging time constant required to keep the RMS value to less than 1% of the true RMS value is considered:

$$\text{(7)} \quad \frac{\sigma'}{e_{rms}} \leqslant .01$$

The time constant required is 1,250, 125 and 12.5 seconds for noise bandwidths of 1, 10 and 100 Hertz. Even with these time constants there is no guarantee that a single measurement will be less than 1% of the true RMS measurement. It is only more probable that it will be less than 1% deviation from the true RMS value. The Chart below gives the RC time constant required to keep the standard deviation to less than 1%, 2%, 3%, 5% and 10% of the true RMS value for noise bandwidths of 1,3, 10,30 and 100 Hz.

		RC, SECONDS				
$\left(\frac{\sigma}{e_{rms}}\right)$	RC B_n	B_n = 1 Hz	B_n = 3	B_n = 10 Hz	B_n = 30	B_n = 100
.01	1.25 X 10^3	1,250	417	125	41.7	12.5
.02	3.13 X 10^2	313	104	31.3	10.4	3.1
.03	1.4 X 10^2	140	46.7	14	4.7	1.4
.05	50	50	16.7	5	1.7	.5
.1	125	12.5	4.2	12.5	.4	.1

SAMPLING ERRORS

APPENDIX V

ERRORS ASSOCIATED WITH MEASURING A RANDOM PROCESS WITH AN INSTRUMENT MEASURING THE AVERAGE ABSOLUTE VALUE AND CALIBRATED TO READ RMS FOR A SINEWAVE

The spectral density measurements described in this paper require RMS or Mean-Square values of the noise, and these can be obtained by using a true RMS meter.

The RMS value of a random process can also be obtained from measurements made by an instrument measuring the average absolute value and calibrated to read the RMS of a sine wave if the form factor of the noise is known:

$$\text{FORM FACTOR} = \frac{\text{RMS}}{\text{AVE, ABSOLUTE VALUE}}$$

The form factor and the measurement error of an instrument which measures the average absolute value but calibrated to the RMS value of a sine wave are tabulated below for a sine wave, Gaussian noise, and Rayleigh noise.

In many cases the noise signal cannot be classified and the form factor is not known. Use of a true RMS meter would elinimate these corrections and the measurement need not be limited to a particular type of noise.

USEFUL FACTS FOR METERS WHICH MEASURE AVERAGE ABSOLUTE VALUE BUT ARE CALIBRATED TO READ RMS VALUE OF A SINE WAVE

	* FORM FACTOR = $\frac{\text{RMS}}{\text{AVE ABS}}$		** ERROR OF AVERAGE READING INSTRUMENT	
	PER UNIT	dB	PERCENT	dB
SINE WAVE	1.111	.91	0	0
GAUSSIAN NOISE	1.253	1.96	−11.3	−1.05
RAYLEIGH NOISE	1.128	1.04	−1.5	−.13

* FORM FACTOR = RATIO RMS TO THE AVERAGE ABSOLUTE VALUE
** ERROR OF AN AVERAGE READING INSTRUMENT CALIBRATED TO READ THE RMS VALUE OF A SINE WAVE