

# Radiometry and photometry FAQ

by

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*“When I use a word, it means just what I choose it to mean -  
neither more nor less.”*

Lewis Carroll (Charles Lutwidge Dodgson)

Effective technical communication demands a system of symbols, units and nomenclature (SUN) that is reasonably consistent and that has widespread acceptance. Such a system is the International System of Units (SI). There is no area where words are more important than radiometry and photometry. This document is an attempt to provide necessary and correct information to become conversant.

1. What is the motivation for this FAQ?
2. What is radiometry? What is photometry? How do they differ?
3. What is projected area? What is solid angle?
4. What are the quantities and units used in radiometry?
5. How do I account for spectral quantities?
6. What are the quantities and units used in photometry?
7. What is the difference between lambertian and isotropic?
8. When do the properties of the eye get involved?
9. How do I convert between radiometric and photometric units?
10. Where can I learn more about this stuff?

## 1. What is the motivation for this FAQ?

There is so much misinformation and conceptual confusion regarding photometry and radiometry, particularly on the WWW by a host of “authorities”, it is high time someone got it straight. So here it is, with links to the responsible agencies.

**Background:** It all started over a century ago. An organization called the General Conference on Weights and Measures (CGPM) was formed by a diplomatic treaty called the Metre Convention. This treaty was signed in 1875 in Paris by representatives from 17 nations (including the USA). There are now 48 member nations. Also formed were the International Committee for Weights and Measures (CIPM) and the International Bureau of Weights and Measures (BIPM). The CIPM, along with a number of sub-committees, suggests modifications to the CGPM. In our arena, the subcommittee is the CCPR, Consultative Committee on Photometry and Radiometry. The BIPM is the physical facility responsible for dissemination of standards, the international metrology institute.

The SI was adopted by the CGPM in 1960. It currently consists of seven base units and a larger number of derived units. The base units are a choice of seven well-defined units that by convention are regarded as independent. The seven are: metre, kilogram, second, ampere, kelvin, mole and **candela**. The derived units are those formed by various combinations of the base units.

International organizations involved in the promulgation of SUN include the International Commission on Illumination (CIE), the International Union of Pure and Applied Physics (IUPAP), and the International Standards Organization (ISO). In the USA, the American National Standards Institute (ANSI) is the primary documentary (protocol) standards organization. Many other scientific and technical organizations publish recommendations concerning the use of SUN for their learned publications. Examples are the International Astronomical Union (IAU) and the American Institute of Physics (AIP).

Read all about the SI, its history and application, at [physics.nist.gov/cuu/](http://physics.nist.gov/cuu/) or at [www.bipm.fr](http://www.bipm.fr).

## 2. What is radiometry? What is photometry? How do they differ?

Radiometry is the measurement of optical radiation, which is electromagnetic radiation within the frequency range between  $3 \times 10^{11}$  and  $3 \times 10^{16}$  Hz. This range corresponds to wavelengths between 0.01 and 1000 micrometres ( $\mu\text{m}$ ), and includes the regions commonly called the ultraviolet, the visible and the infrared. Two out of many typical units encountered are  $\text{watts/m}^2$  and  $\text{photons/s-steradian}$ .

Photometry is the measurement of light, which is defined as electromagnetic radiation that is detectable by the human eye. It is thus restricted to the wavelength range from about 360 to 830 nanometers (nm;  $1000 \text{ nm} = 1 \mu\text{m}$ ).

Photometry is just like radiometry except that everything is weighted by the spectral response of the eye. Visual photometry uses the eye as a comparison detector, while physical photometry uses either optical radiation detectors constructed to mimic the spectral response of the eye, or spectroradiometry coupled with appropriate calculations to do the eye response weighting. Typical photometric units include lumens, lux, candelas, and a host of other bizarre ones.

The only real difference between radiometry and photometry is that radiometry includes the entire optical radiation spectrum, while photometry is limited to the visible spectrum as defined by the response of the eye. In my forty years of experience, photometry is more difficult to understand, primarily because of the arcane terminology, but is fairly easy to do, because of the limited wavelength range. Radiometry, on the other hand, is conceptually somewhat simpler, but is far more difficult to actually do.

### 3. What is projected area? What is solid angle?

Projected area is defined as the rectilinear projection of a surface of any shape onto a plane normal to the unit vector. The differential form is  $dA_{proj} = \cos(\beta) dA$  where  $\beta$  is the angle between the local surface normal and the line of sight. We can integrate over the (perceptible) surface area to get

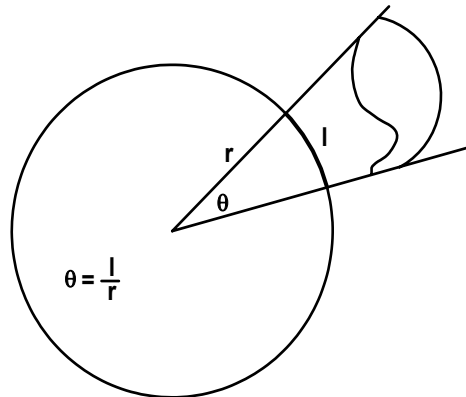
$$A_{proj} = \int_A \cos \beta dA$$

Some common examples are shown in the table below:

SHAPE	AREA	PROJECTED AREA
Flat rectangle	$A = L \times W$	$A_{proj} = L \times W \cos \beta$
Circular disc	$A = \pi r^2 = \pi d^2 / 4$	$A_{proj} = \pi r^2 \cos \beta = \pi d^2 \cos \beta / 4$
Sphere	$A = 4 \pi r^2 = \pi d^2$	$A_{proj} = A/4 = \pi r^2$

Plane angle and solid angle are two derived units on the SI system. The following definitions are from NIST SP811.

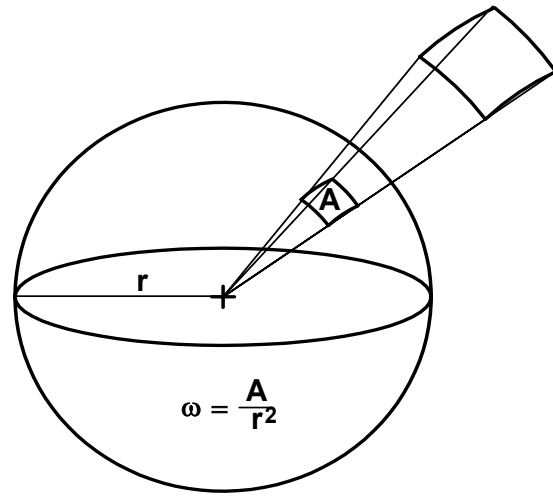
*“The radian is the plane angle between two radii of a circle that cuts off on the circumference an arc equal in length to the radius.”*



The abbreviation for the radian is **rad**. Since there are  $2\pi$  radians in a circle, the conversion between degrees and radians is  $1 \text{ rad} = (180/\pi)$  degrees.

A solid angle extends the concept to three dimensions.

*“One steradian (sr) is the solid angle that, having its vertex in the center of a sphere, cuts off an area on the surface of the sphere equal to that of a square with sides of length equal to the radius of the sphere.”*



The solid angle is thus ratio of the spherical area to the square of the radius. The spherical area is a projection of the object of interest onto a unit sphere, and the solid angle is the surface area of that projection. If we divide the surface area of a sphere by the square of its radius, we find that there are  $4\pi$  steradians of solid angle in a sphere. One hemisphere has  $2\pi$  steradians. Accepted symbols for solid angle are  $\omega$ , the lowercase Greek letter omega, and  $\Omega$ , the uppercase omega. I use  $\omega$  exclusively for solid angle, reserving  $\Omega$  for the advanced concept of projected solid angle ( $\omega \cos\theta$ ).

Both plane angles and solid angles are dimensionless quantities, and they can lead to confusion when attempting dimensional analysis.

#### 4. What are the quantities and units used in radiometry?

Radiometric units can be divided into two conceptual areas: those having to do with power or energy, and those that are geometric in nature. The first two are:

**Energy** is an SI derived unit, measured in joules (J). The recommended symbol for energy is  $Q$ . An acceptable alternate is  $W$ .

**Power** (a.k.a. radiant flux) is another SI derived unit. It is the rate of flow (derivative) of energy with respect to time,  $dQ/dt$ , and the unit is the watt (W). The recommended symbol for power is  $\Phi$  (the uppercase Greek letter phi). An acceptable alternate is  $P$ .

Energy is the integral over time of power, and is used for integrating detectors and pulsed sources. Power is used for non-integrating detectors and continuous (in a time sense) sources. Even though we patronize the power utility, what we are actually buying is energy in watt hours.

Now we become more specific and incorporate power with the geometric quantities area and solid angle.

**Irradiance** (a.k.a. flux density) is another SI derived unit and is measured in  $W/m^2$ . Irradiance is power per unit area incident from all directions in a hemisphere onto a surface that coincides with the base of that hemisphere. A similar quantity is **radiant exitance**, which is power per unit area leaving a surface into a hemisphere whose base is that surface. The symbol for irradiance is  $E$  and the symbol for radiant exitance is  $M$ . Irradiance (or radiant exitance) is the derivative of power with respect to area,  $d\Phi/dA$ . The integral of irradiance or radiant exitance over area is power.

**Radiant intensity** is another SI derived unit and is measured in  $W/sr$ . Intensity is power per unit solid angle. The symbol is  $I$ . Intensity is the derivative of power with respect to solid angle,  $d\Phi/d\omega$ . The integral of radiant intensity over solid angle is power.

**Radiance** is the last SI derived unit we need and is measured in  $W/m^2\text{-sr}$ . Radiance is power per unit projected area per unit solid angle. The symbol is  $L$ . Radiance is the derivative of power with respect to solid angle and projected area,  $d\Phi/d\omega dA \cos(\theta)$  where  $\theta$  is the angle between the surface normal and the specified direction. The integral of radiance over area and solid angle is power.

A great deal of confusion concerns the use and misuse of the term *intensity*. Some folks use it for  $W/sr$ , some use it for  $W/m^2$  and others use it for  $W/m^2\text{-sr}$ . It is quite clearly defined in the SI system, in the definition of the base unit of luminous intensity, the candela. Some attempt to justify alternate uses by adding adjectives like *optical* (used for  $W/m^2$ ) or *specific* (used for  $W/m^2\text{-sr}$ ), but this practice only adds to the confusion. The underlying concept is (quantity per unit solid angle). For an extended discussion, see my paper entitled “Getting Intense on Intensity” for Metrologia (official journal of the BIPM) and a letter to OSA’s “Optics and Photonics News”, with a modified version available on the web at [www.optics.Arizona.EDU/Palmer/intenopn.html](http://www.optics.Arizona.EDU/Palmer/intenopn.html).

Photon quantities are also common. They are related to the radiometric quantities by the relationship  $Q_p = hc/\lambda$  where  $Q_p$  is the energy of a photon at wavelength  $\lambda$ ,  $h$  is Planck’s constant and  $c$  is the velocity of light. At a wavelength of  $1 \mu m$ , there are approximately  $5 \times 10^{18}$  photons per second in a watt. Conversely, a single photon has an energy of  $2 \times 10^{-19}$  joules ( $W \text{ s}$ ) at  $1 \mu m$ . Common units include  $s^{-1}\text{-m}^{-2}\text{-sr}^{-1}$  for photon radiance.

## 5. How should I represent spectral quantities?

Most sources of optical radiation are spectrally dependent, and the terms radiance, intensity, etc. give no information about the distribution of these quantities over wavelength. Spectral quantities, like spectral radiance, spectral power, etc. are

defined as the quotient of the quantity in an infinitesimal range of wavelength divided by that wavelength range. In other words, spectral quantities are derivative quantities, per unit wavelength, and have an additional ( $\lambda^{-1}$ ) in their units. When integrated over wavelength they yield the total quantity. These spectral quantities are denoted by using a subscript  $\lambda$ , e.g.,  $L_\lambda$ ,  $E_\lambda$ ,  $\Phi_\lambda$ , and  $I_\lambda$ .

Some other quantities (examples include spectral transmittance, spectral reflectance, spectral responsivity, etc.) vary with wavelength but are not used as derivative quantities. These quantities should **not** be integrated over wavelength; they are only weighting functions, to be included with the above derivative quantities. To distinguish them from the derivative quantities, they are denoted by a parenthetical wavelength, i.e.  $\mathfrak{R}(\lambda)$  or  $\tau(\lambda)$ .

## 6. What are the quantities and units used in photometry?

They are basically the same as the radiometric units except that they are weighted for the spectral response of the human eye and have funny names. A few additional units have been introduced to deal with the amount of light reflected from diffuse (matte) surfaces. The symbols used are identical to those radiometric units, except that a subscript “v “ is added to denote “visual. “The following chart compares them.

QUANTITY	RADIOMETRIC	PHOTOMETRIC
power	watt (W)	lumen (lm)
power per unit area	W/m <sup>2</sup>	lm/m <sup>2</sup> = lux (lx)
power per unit solid angle	W/sr	lm/sr = candela (cd)
power per unit area per unit solid angle	W/m <sup>2</sup> -sr	lm/m <sup>2</sup> -sr = cd/m <sup>2</sup> = nit

Now we can get more specific about the details.

**Candela** (unit of luminous intensity). The candela is one of the seven base units of the SI system. It is defined as follows:

*The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency  $540 \times 10^{12}$  hertz and that has a radiant intensity in that direction of 1/683 watt per steradian.*

The candela is abbreviated as **cd** and its symbol is  $I_v$ . The above definition was adopted by the 16th CGPM in 1979.

The candela was formerly defined as the luminous intensity, in the perpendicular direction, of a surface of 1/600 000 square metre of a black body at the temperature of freezing platinum under a pressure of 101 325 newtons per square metre. This earlier definition was initially adopted in 1946 and later modified by the 13th CGPM (1967). It was abrogated in 1979 and replaced by the current definition.

The current definition was adopted because of several reasons. First, the freezing point of platinum ( $\approx 2042\text{K}$ ) was tied to another base unit, the kelvin. If the best estimate of this point were changed, it would then impact the candela. The uncertainty of the thermodynamic temperature of this fixed point created an unacceptable uncertainty in the value of the candela. Second, the realization of the Pt blackbody was extraordinarily difficult; only a few were ever built. Third, if the temperature were slightly off, possibly because of temperature gradients or contamination, the freezing point might change or the temperature of the cavity might differ. The sensitivity of the candela to a slight change in temperature is significant. At a wavelength 555 nm, a change in temperature of only 1K results in a luminance change approaching 1%. Fourth, the relative spectral radiance of blackbody radiation changes drastically (some three orders of magnitude) over the visible range. Finally, recent advances in radiometry offered new possibilities for the realization of the candela.

The value 683 lm/W was selected based upon the best measurements with existing platinum freezing point blackbodies and the best estimates for the thermodynamic temperature of the platinum point. . It has varied over time from 620 to nearly 700 lm/W, depending largely upon the assigned value of the Pt freezing point on the best available temperature scale. The value of 1/600 000 square metre was chosen to maintain consistency with prior flame standards.

Also note that in the definition there is no specification for the spatial distribution of intensity. Luminous intensity, while often associated with an isotropic (see section 7 below) point source, is a valid specification for characterizing highly directional light sources such as automotive lighting, searchlights and LEDs.

One other issue before we press on. Since the candela is now defined in terms of other SI derived quantities, there is really no need to retain it as an SI base quantity. It remains so for reasons of history and continuity.

**Lumen** (unit of luminous flux). The lumen is an SI derived unit for luminous flux. The abbreviation is lm and the symbol is  $\Phi_v$ . The lumen is derived from the candela and is the luminous flux emitted into unit solid angle (1 sr) by an isotropic (see section 7 below) point source having a luminous intensity of 1 candela. The lumen is the product of luminous intensity and solid angle, cd-sr. It is analogous to the unit of radiant flux (watt), differing only in the eye response weighting. If a light source is isotropic, the relationship between lumens and candelas is  $1 \text{ cd} = 4\pi \text{ lm}$ . In other words, an isotropic source having a luminous intensity of 1 candela emits  $4\pi$  lumens into space, which just happens to be  $4\pi$  steradians. We can also state that  $1 \text{ cd} = 1 \text{ lm/sr}$ , analogous to the equivalent radiometric definition.

If a source is not isotropic, the relationship between candelas and lumens is empirical. A fundamental method used to determine the total flux (lumens) is to measure the luminous intensity (candelas) in many directions using a goniophotometer, and then numerically integrate over the entire sphere. Later on,

we can use this “calibrated” lamp as a reference in an integrating sphere for routine measurements of luminous flux.

Lumens are what we get from the hardware store when we purchase a light bulb. We want a high number of lumens with a minimum of power consumption and a reasonable lifetime. Projection devices are also characterized by lumens to indicate how much luminous flux they can deliver to a screen. The ANSI standard delineates the method for determining average luminous flux; it is not a different lumen.

Recently, marketeers for digital projectors have introduced a new unit, the “market comparable lumen,” which is somewhere between 1.1 and 1.25 ANSI lumens. This appears to be a ploy to jack up the numbers to compete with those who are not inclined to use ANSI lumens.

**Lux** (unit of luminous flux density, or illuminance). **Illuminance** is another SI derived unit which denotes luminous flux density. The unit has a special name, the **lux**, which is lumens per square metre, or  $\text{lm}/\text{m}^2$ . The symbol is  $E_v$ . Most light meters measure this quantity, as it is of great importance in illuminating engineering. The IESNA Lighting Handbook has some sixteen pages of recommended illuminances for various activities and locales, ranging from morgues to museums. Typical values range from 100 000 lx for direct sunlight to 20-50 lx for hospital corridors at night.

**Nit** (unit of luminance). Luminance should probably be included on the official list of derived SI units, but is not. It is analogous to radiance, differentiating the lumen with respect to both area and direction. This unit also has a special name, the **nit**, which is  $\text{cd}/\text{m}^2$  or  $\text{lm}/\text{m}^2 \text{sr}$  if you prefer. The symbol is  $L_v$ . It is most often used to characterize the “brightness “ of flat emitting or reflecting surfaces. A typical use would be the luminance of your laptop computer screen. They have between 100 and 250 nits, and the sunlight readable ones have more than 1000 nits. Typical CRT monitors have between 50 and 125 nits.

### **Other photometric units**

We have other photometric units (boy, do we have some strange ones). Photometric quantities should be reported in SI units as given above. However, the literature is filled with now obsolete terminology and we must be able to interpret it. So here are a few terms that have been used in the past.



## Illuminance:

$$1 \text{ metre-candle} = 1 \text{ lux}$$

$$1 \text{ phot} = 1 \text{ lm/cm}^2 = 10^4 \text{ lux}$$

$$1 \text{ foot-candle} = 1 \text{ lumen/ft}^2 = 10.76 \text{ lux}$$

$$1 \text{ milliphot} = 10 \text{ lux}$$

**Luminance:** Here we have two classes of units. The first is conventional, easily related to the SI unit, the  $\text{cd/m}^2$  (nit).

$$1 \text{ stilb} = 1 \text{ cd/cm}^2 = 10^4 \text{ cd/m}^2 = 10^4 \text{ nit}$$

$$1 \text{ cd/ft}^2 = 10.76 \text{ cd/m}^2 = 10.76 \text{ nit}$$

The second class was designed to “simplify” characterization of light reflected from diffuse surfaces by including in the definitions the concept of a perfect (reflectance  $\rho = 1$ ), diffuse reflector (Lambertian, see section 7 below.) If one unit of illuminance falls upon this hypothetical reflector, then 1 unit of luminance is reflected. The perfect diffuse reflector emits  $1/\pi$  units of luminance per unit illuminance. If the reflectance is  $\rho$ , then the luminance is  $\rho$  times the illuminance. Consequently, these units all have a factor of  $(1/\pi)$  built in.

$$1 \text{ lambert} = (1/\pi) \text{ cd/cm}^2 = (10^4/\pi) \text{ cd/m}^2$$

$$1 \text{ apostilb} = (1/\pi) \text{ cd/m}^2$$

$$1 \text{ foot-lambert} = (1/\pi) \text{ cd/ft}^2 = 3.426 \text{ cd/m}^2$$

$$1 \text{ millilambert} = (10/\pi) \text{ cd/m}^2$$

$$1 \text{ skot} = 1 \text{ milliblondel} = (10^{-3}/\pi) \text{ cd/m}^2$$

Photometric quantities are already the result of integration over wavelength. It therefore makes no sense to speak of spectral luminance or the like.

## 7. What is the difference between Lambertian and isotropic?

Both terms mean “the same in all directions” and are unfortunately sometimes used interchangeably.

**Isotropic** implies a spherical source that radiates the same in all directions, i.e., the intensity ( $\text{W/sr}$ ) is the same in all directions. We often encounter the phrase “isotropic point source.” There can be no such thing; because the energy density would have to be infinite. But a small, uniform sphere comes very close. The best example is a globular tungsten lamp with a milky white diffuse envelope, as used in dressing room lighting. From our vantage point, a distant star can be considered an isotropic point source.

**Lambertian** refers to a flat radiating surface. It can be an active surface or a passive, reflective surface. Here the intensity falls off as the cosine of the observation angle with respect to the surface normal (Lambert's law). The radiance ( $\text{W/m}^2\text{-sr}$ ) is independent of direction. A good example is a surface painted with a

good “matte” or “flat” white paint. If it is uniformly illuminated, like from the sun, it appears equally bright from whatever direction you view it. Note that the flat radiating surface can be an elemental area of a curved surface.

The ratio of the radiant exitance ( $\text{W}/\text{m}^2$ ) to the radiance ( $\text{W}/\text{m}^2\text{-sr}$ ) of a lambertian surface is a factor of  $\pi$  and not  $2\pi$ . We integrate radiance over a hemisphere, and find that the presence of the factor of  $\cos(\theta)$  in the definition of radiance gives us this interesting result. It is not intuitive, as we know that there are  $2\pi$  steradians in a hemisphere.

A lambertian sphere illuminated by a distant point source will display the maximum radiance at the surface where the local normal coincides with the incoming beam. The radiance will fall off with a cosine dependence to zero at the edge of the lit area (terminator.) If the intensity (integrated radiance over area) is unity when viewing from the source, then the intensity when viewing from the side is  $1/\pi$ . Think about this and consider whether or not our Moon is lambertian. Take a look at [www.optics.Arizona.EDU/Palmer/moon/lunacy.htm](http://www.optics.Arizona.EDU/Palmer/moon/lunacy.htm).

## 8. Where do the properties of the eye get involved?

We know that the eye does not see all wavelengths equally. The eye has two general classes of photosensors, cones and rods.

**Cones:** The cones are responsible for light-adapted vision; they respond to color and have high resolution in the central foveal region. The light-adapted relative spectral response of the eye is called the spectral luminous efficiency function for photopic vision,  $V(\lambda)$ . This empirical curve, first adopted by the International Commission on Illumination (CIE) in 1924, has a peak of unity at 555 nm, and decreases to levels below  $10^{-5}$  at about 370 and 785 nm. The 50% points are near 510 nm and 610 nm, indicating that the curve is slightly skewed. The  $V(\lambda)$  curve looks very much like a Gaussian function; in fact a Gaussian curve can easily be fit and is a good representation under some circumstances. I used a non-linear regression technique to obtain the following equation:

$$V(\lambda) \cong 1.019e^{-285.4(\lambda-0.559)^2}$$

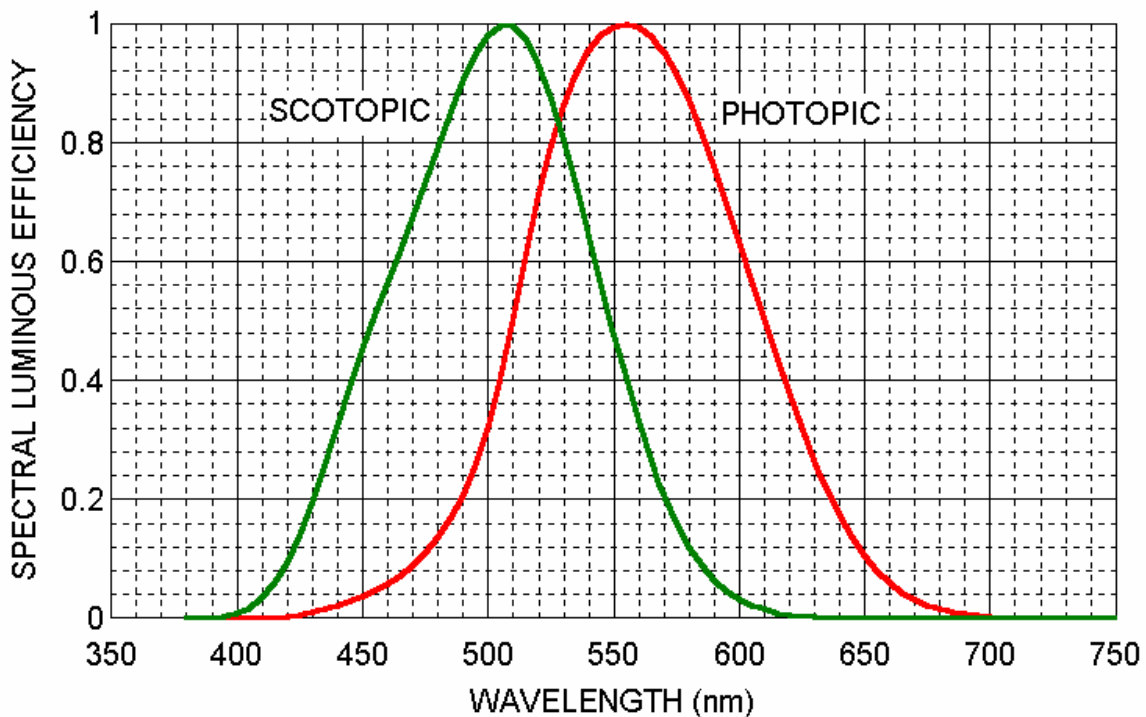
More recent measurements have shown that the 1924 curve may not best represent typical human vision. It appears to underestimate the response at wavelengths shorter than 460 nm. Judd (1951), Vos (1978) and Stockman and Sharpe (1999) have made incremental advances in our knowledge of the photopic response. Go to [www.cvrl.org](http://www.cvrl.org) for details.

**Rods:** The rods are responsible for dark-adapted vision, with no color information and poor resolution when compared to the foveal cones. The dark-adapted relative spectral response of the eye is called the spectral luminous efficiency function for scotopic vision,  $V'(\lambda)$ . This is another empirical curve, adopted by the CIE in 1951.

It is defined between 380 nm and 780 nm. The  $V'(\lambda)$  curve has a peak of unity at 507 nm, and decreases to levels below  $10^{-3}$  at about 380 and 645 nm. The 50% points are near 455 nm and 550 nm. This scotopic curve can also be fit with a Gaussian, although the fit is not quite as good as the photopic curve. My best fit is

$$V'(\lambda) \cong 0.992e^{-321.9(\lambda-0.503)^2}$$

Photopic (light adapted cone) vision is active for luminances greater than 3 cd/m<sup>2</sup>. Scotopic (dark-adapted rod) vision is active for luminances lower than 0.01 cd/m<sup>2</sup>. In between, both rods and cones contribute in varying amounts, and in this range the vision is called mesopic. There are currently efforts under way to characterize the composite spectral response in the mesopic range for vision research at intermediate luminance levels.



The Color Vision Research Lab ([www.cvrl.org](http://www.cvrl.org)) has an impressive collection of the data files, including  $V(\lambda)$  and  $V'(\lambda)$ , that you need to do this kind of work.

## 9. How do I convert between radiometric and photometric units?

We know from the definition of the candela that there are 683 lumens per watt at a frequency of 540THz, which is 555 nm (in vacuum or air). This is the wavelength that corresponds to the maximum spectral responsivity of the human eye. The conversion from watts to lumens at any other wavelength involves the product of the power (watts) and the  $V(\lambda)$  value at the wavelength of interest. As an example, we can compare laser pointers at 670 nm and 635 nm. At 670 nm,  $V(\lambda)$  is 0.032 and

a 5 mW laser has  $0.005\text{W} \times 0.032 \times 683 \text{ lm/W} = 0.11 \text{ lumens}$ . At 635 nm,  $V(\lambda)$  is 0.217 and a 5 mW laser has  $0.005\text{W} \times 0.217 \times 683 \text{ lm/W} = 0.74 \text{ lumens}$ . The shorter wavelength (635 nm) laser pointer will create a spot that is almost 7 times as bright as the longer wavelength (670 nm) laser (assuming the same beam diameter).

In order to convert a source with non-monochromatic spectral distribution to a luminous quantity, the situation is decidedly more complex. We must know the spectral nature of the source, because it is used in an equation of the form:

$$X_v = K_m \int_0^\infty X_\lambda V(\lambda) d\lambda$$

where  $X_v$  is a luminous term,  $X_\lambda$  is the corresponding **spectral** radiant term, and  $V(\lambda)$  is the photopic spectral luminous efficiency function. For  $X$ , we can pair luminous flux (lm) and spectral power (W/nm), luminous intensity (cd) and spectral radiant intensity (W/sr-nm), illuminance (lx) and spectral irradiance (W/m<sup>2</sup>-nm), or luminance (cd/m<sup>2</sup>) and spectral radiance (W/m<sup>2</sup>-sr-nm). This equation represents a weighting, wavelength by wavelength, of the radiant spectral term by the visual response at that wavelength. The constant  $K_m$  is a scaling factor, the maximum spectral luminous efficiency for photopic vision, 683 lm/W. The wavelength limits can be set to restrict the integration to only those wavelengths where the product of the spectral term  $X_\lambda$  and  $V(\lambda)$  is non-zero. Practically, this means we only need integrate from 360 to 830 nm, limits specified by the CIE  $V(\lambda)$  table. Since this  $V(\lambda)$  function is defined by a table of empirical values, it is best to do the integration numerically. Use of the Gaussian equation given above is only an approximation. I compared the Gaussian equation with the tabulated data using blackbody curves and found the differences to be less than 1% for temperatures between 1500K and 20000K. This result is acceptable for smooth curves, but don't try it for narrow wavelength sources, like LEDs.

There is nothing in the SI definitions of the base or derived units concerning the eye response, so we have some flexibility in the choice of the weighting function. We can use a different spectral luminous efficacy curve, perhaps one of the newer ones. We can also make use of the equivalent curve for scotopic (dark-adapted) vision for studies at lower light levels. This  $V'(\lambda)$  curve has its own constant  $K'_m$ , the maximum spectral luminous efficiency for scotopic vision.  $K'_m$  is 1700 lm/W at the peak wavelength for scotopic vision (507 nm) and this value was deliberately chosen such that the absolute value of the scotopic curve at 555 nm coincides with the photopic curve, at the value 683 lm/W. Some workers are referring to "scotopic lumens", a term which should be discouraged because of the potential for misunderstanding. In the future, we can also expect to see spectral weighting to represent the mesopic region.

The International Commission on Weights and Measures (CGPM) has approved the use of the CIE  $V(\lambda)$  and  $V'(\lambda)$  curves for determination of the value of photometric quantities of luminous sources.

Now about converting from lumens to watts. The conversion from watts to lumens that we saw just above required that the spectral function  $X_\lambda$  of the radiation be known over the spectral range from 360 to 830 nm, where  $V(\lambda)$  is non-zero. Attempts to go in the other direction, from lumens to watts, are far more difficult. Since we are trying to back out a quantity that was weighted and placed inside of an integral, we must know the spectral function  $X_\lambda$  of the radiation over the entire spectral range where the source emits, not just the visible region.

Also please note that there is no such thing as a spectral lumen, spectral lux, spectral nit, spectral candela, etc. The photometric terms are all the result of integration over wavelength; they are never subject to integration.

## 10. Where can I learn more about this stuff?

### Books, significant journal articles:

DeCusatis, C., “Handbook of Applied Photometry” AIP Press (1997).  
Authoritative, with pertinent chapters written by technical experts at BIPM, CIE and NIST. Skip chapter 4!

Rea, M., ed. “Lighting Handbook: Reference and Application” 8th edition,  
Illuminating Engineering Society of North America (1993).

“Symbols, Units and Nomenclature in Physics” International Union of Pure and Applied Physics (1987).

“American National Standard Nomenclature and Definitions for Illuminating Engineering” ANSI Standard ANSI/IESNA RP-16 96.

“The Basis of Physical Photometry” CIE Technical Report 18.2 (1983)

### Publications available on the World Wide Web

All you ever wanted to know about the SI is contained at BIPM and at NIST.  
Available publications (highly recommended) include:

“The International System of Units (SI)” 7th edition (1998), direct from BIPM. The official document is in French; this is the English translation). Available on-line at [www.bipm.fr/pdf/si-brochure.pdf](http://www.bipm.fr/pdf/si-brochure.pdf).

NIST Special Publication SP330 “The International System of Units (SI)” The US edition of the above BIPM publication. Available on-line at [physics.nist.gov/Pubs/SP330/sp330.pdf](http://physics.nist.gov/Pubs/SP330/sp330.pdf).

NIST Special Publication SP811 “Guide for the Use of the International System of Units (SI)” Available on-line at [physics.nist.gov/Document/sp811.pdf](http://physics.nist.gov/Document/sp811.pdf).

Papers published in recent issues of the NIST Journal of Research are also available on-line at [nvl.nist.gov/pub/nistpubs/jres/jres.htm](http://nvl.nist.gov/pub/nistpubs/jres/jres.htm). Of particular relevance is "The NIST Detector-Based Luminous Intensity Scale," Vol. 101, page 109 (1996).

### **Useful Web sites**

BIPM International Bureau of Weights and Measures - [www.bipm.fr/](http://www.bipm.fr/)

NIST Nat'l Inst. of Standards and Technology - [physics.nist.gov/cuu/](http://physics.nist.gov/cuu/)

ISO International Standards Organization - [www.iso.ch/](http://www.iso.ch/)

ANSI American Nat'l Standards Institute - [www.ansi.org/](http://www.ansi.org/)

CIE International Commission on Illumination - [www.cie.co.at/cie/](http://www.cie.co.at/cie/)

IESNA Illuminating Engineering Society of North America - [www.iesna.org/](http://www.iesna.org/)

IUPAP International Union of Pure and Applied Physics -  
[www.physics.umanitoba.ca/iupap/](http://www.physics.umanitoba.ca/iupap/)

Color Vision Research Lab – [www.cvrl.org](http://www.cvrl.org)

AIP American Institute of Physics - [www.aip.org](http://www.aip.org)

SPIE - International Society for Optical Engineering - [www.spie.org](http://www.spie.org)

OSA Optical Society of America - [www.osa.org](http://www.osa.org)

CORM Council of Optical Radiation Measurements - [www.corm.org](http://www.corm.org)

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WWW: [www.optics.arizona.edu/Palmer/](http://www.optics.arizona.edu/Palmer/)