Étendue

This article is taken from parts of Chapter 2 of *Illumination Engineering: Design with Nonimaging Optics*.[1] To learn more about étendue, please see this book, especially the electronic version that you can get from the UA Library. Additionally, it is a focus of discussion in OPTI 306 and OPTI 485/585.

Étendue is one of the most basic yet important concepts in the design of nonimaging and illumination optics. First, it explains the flux transfer characteristics of the optical system, and, second, it plays an integral role in the ability to shape the distribution of radiation at the target. Interestingly, the concept of étendue was only formally accepted in the 1970s through a series of letters to a journal.[2,3,4,5] Ref. 2 asked for input to a proposal to be submitted to the Nomenclature Sub-Committee of the International Commission for Optics in order to standardize the name for what ultimately became étendue.[6] Initially the author of this journal letter favored the term "optical extent," with the term étendue not likely due to "the typing disadvantage of é, (and) the pronunciation difficulty of the French u."[6] Through the series of replies, both in the journal and by private communications, the term étendue gained favor "since the accent will be dropped anyway."[5] The dropping of the accent is not applicable today since modern computer technology makes it simple to include. This recent history of varied opinions and potential confusion illustrates the complexity inherent in the term étendue, indicating that it has the potential to be interpreted in many ways

Étendue is a French word that means, as a verb, extended, and, as a noun, reach.[2] In the field of optics étendue is a quantity arising out of the geometrical characteristics of flux propagation in an optical system. As will be seen the use of étendue in the field of optics is in good agreement with the French root. It describes both the angular and spatial propagation of flux through the system, so it obviously relates to the radiance propagation characteristics of a system. In a lossless system, which is one without absorption, scatter, gain, or Fresnel reflection losses, all flux that is transmitted by the entrance pupil of the system is emitted from the exit pupil. The étendue of a system is defined as[2,3]

$$\xi = n^2 \iint_{pupil} \cos\theta dA_s d\Omega , \tag{1.1}$$

where n is the index of refraction in source space and the integrals are performed over the entrance pupil. The total flux that propagates through this system is found from $L(\mathbf{r}, \hat{\mathbf{a}})$, which is

[1] R. J. Koshel, editor, *Illumination Engineering: Design with Nonimaging Optics*, author of three chapters (IEEE-Wiley, New York, NY, 2013).
[2] W. H. Steel, "Luminosity, Throughput, or Etendue?," *Appl. Opt.* **13**, pp. 704-705 (1974).
[3] C. W. McCutchen, "Optical exxtent," *Appl. Opt.* **13**, p 1537 (1974).
[4] G. G. Shepherd, "How About Radiance Response?," *Appl. Opt.* **13**, p 1734 (1974).
[5] W. H. Steel, "Luminosity, Throughput, or Etendue? Further Comments," *Appl. Opt.* **14**, p 252 (1975).
[6] Minutes of the 9th Session of ICO, section 10(c), *J. Opt. Soc. Am.* **63**, 906 (1973).

the source radiance at the point **r** in the direction of the unit vector **â**. By integrating we obtain the flux,

$$\begin{aligned}\Phi &= \iint_{\text{pupil}} L(\mathbf{r},\hat{\mathbf{a}})\, dA_{s,proj}\, d\Omega \\ &= \iint_{\text{pupil}} L(\mathbf{r},\hat{\mathbf{a}}) \cos\theta\, dA_s\, d\Omega,\end{aligned} \tag{1.2}$$

where the spatial integral is over the source area which is in view of the entrance pupil and the angular integral is over the source emission that is within the field of view of the pupil, i.e., that subtended by the pupil from the source point **r**. If a spatially uniform, Lambertian source is assumed, then the source radiance function is given by L_s, which upon substitution in Eq. (1.2) gives

$$\Phi = L_s \iint_{pupil} \cos\theta\, dA_s\, d\Omega\,. \tag{1.3}$$

So, the total flux that is transmitted by an optical system with a spatially uniform, Lambertian emitter in terms of the étendue is given by

$$\Phi = \frac{L_s \xi}{n^2}. \tag{1.4}$$

Note that the étendue is a geometric quantity that describes the flux propagation characteristics for a lossless system. The term lossless puts constraints on the interpretation of a system étendue. The term implies that absorption, scatter, and reflection losses are not considered, but, more importantly, it restricts the integration of Eqs. (1.1) to (1.3) to that of the entrance pupil criterion. In realistic systems there are not only absorption, aberration, diffraction, and reflection losses, but also losses associated to manufacturing tolerances and lack of capture of source flux by the entrance pupil. Therefore, the geometric étendue provides a theoretical limit to flux transfer capability of a real-world optical system.

Reference 1 then goes into detail about the conservation of étendue – which means the integral product of the projected area and the solid angle is maintained in a lossless optical system. This says that the étendue in Eq. (1.4) is constant in this lossless optical system. Additionally, it can be seen that the term L/n^2 is also conserved, such that the power, Φ, is constant in a lossless optical system (i.e., conservation of energy). Figure 1 depicts how one can do this proof using generalized radiance – see Ref. 1 for the gory details. The question to ask yourself is, "Are optical systems lossless?" Nope – not at all. Here are some loss methods in optical systems:

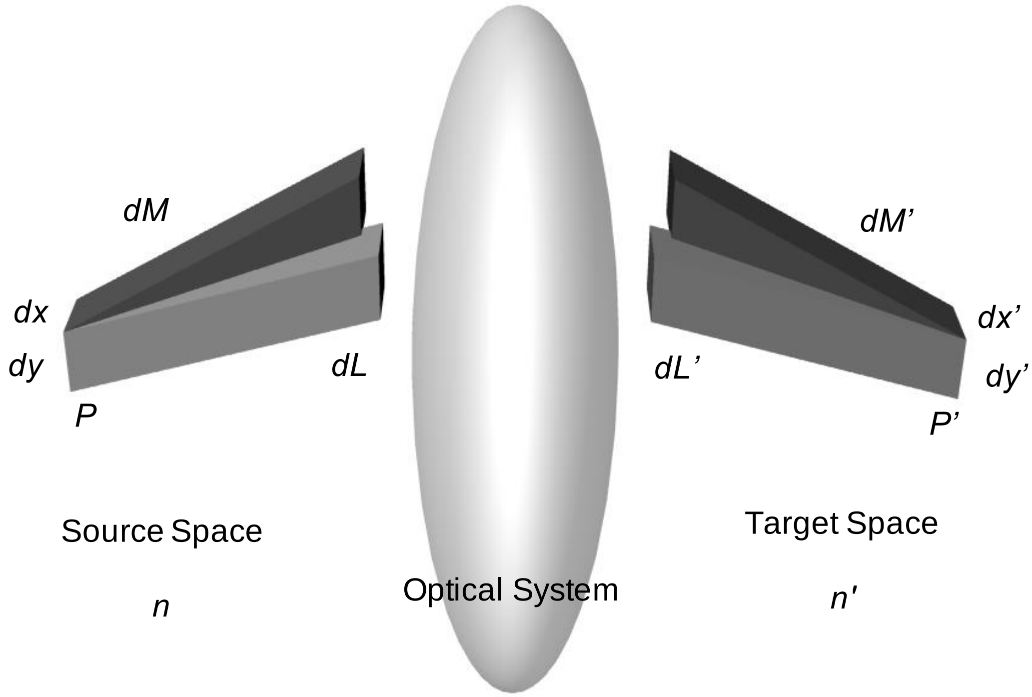

Figure 1. Representation of beam propagation from source space (**r**) to target space (**r'**) via an intermediate optical system. This figure is used in Ref. 1 to prove generalized radiance: $dxdydpdq = dx'dy'dp'dq'$.

- Vignetting,
- Aberrations,
- Scatter,
- Fresnel reflections,
- Absorption, and
- Diffraction.

There are others. Thus, étendue and its conservation are purely theoretical constructs, but it does provide an upper limit to the capabilities of an optical system. Additionally, the conservation of étendue also means that you can only keep étendue constant or it can increase (i.e., either a not-well designed optical system or you allow it to increase for tolerance reasons) in the optical system while maintaining the theoretical lossless aspect.

Etendue is one of the most important aspects to describing and designing efficient optical systems. It is important to remember that étendue is solely a geometric factor – it does not include any physical properties of the light. However, due to that one can seemingly break the limits of étendue by using such optical phenomena as the spectrum, polarization, coherence, and even mixing to increase the flux in a beam. A short explanation of each method to increase flux without affecting étendue is warranted:

- Spectral methods: a dichroic optical element can be used to add two separate beams with the same geometrical parameters. The two beams have spectra of $\Delta\lambda_1$ and $\Delta\lambda_2$ and fluxes of Φ_1 and Φ_2, and these spectra do not overlap. One beam is reflected by the dichroic window, while the other is transmitted, so the total power is $\Phi_{tot} = \Phi_1 + \Phi_2$ with the original beam parameters.
- Polarization methods: a polarization element can be used to add two orthogonally polarized beams with the same geometrical parameters. Like the spectral case the total power is $\Phi_{tot} = \Phi_1 + \Phi_2$ with the original beam parameters.
- Coherence methods: interference can be used to provide a higher flux than delineated by conservation of étendue. This is done through constructive and destructive interference.
- Mixing methods: the arguments presented herein assumed that each ray was seeing the same optical system as the other rays. However, you can have an optical system where some rays see m_1 elements while another set of rays see m_2 elements. An illustrative example is a source coupled to a retroreflector over half its emission space. The light that does not interact with the retroreflector propagates out with the expected étendue, while the retroreflected light matches this étendue (assuming the source emission is symmetric in the forward and backward directions). You have doubled your flux while the étendue is half the expected value. Realistically, there is source geometry obstructing perfect performance. For example a filament lamp will essentially have the entire retroreflected radiation incident on the coil. Some of this radiation will be absorbed, but some will scatter off the source geometry and add to the flux in the forward direction at no expense to the étendue.

In conclusion, étendue is one of the most important parameters in the design and analysis of optical systems. By understanding it (and also trying to break its limits) will help you as you go forward in your optics career, in fact, I have heard a number of companies will ask you about it during the interview process. Simply, you have been using and will continue to use étendue and its conservation throughout your studies and career. Explore it, try to break it, learn its nuances, and you will ultimately come to have a better love-hate relationship with it.

We want to hear your thoughts about all of these proposals and welcome suggestions for others. How useful will they be? Will they be educational? What? So, we created a survey at: https://forms.gle/4Ka1AQJyhi2Ri6sd8.

Photon Snacks is a column for Light Bytes edited by John Koshel, Associate Dean for Undergraduate Affairs in the Wyant College of Optical Sciences. You can find the previously written articles at https://wp.optics.arizona.edu/jkoshel/photon-snacks/. Additionally, make suggestions for articles (or even write one!) by emailing jkoshel@optics.arizona.edu or by visiting the survey anytime at https://forms.gle/ibC9LhPemeniJwhv9.