Objective plus Relay

The combination of an objective lens and a relay lens can be thought of as a single compound objective. If the objective focal length is 100 mm, the relay focal length is 25 mm, and the separation of these two elements is 150 mm, what is the focal length of the combination of the two lenses? Explain your result in terms of the rear focal point and principal planes of this two-element system and why this relative order occurs.

Solution:

\[
\begin{align*}
  f_{\text{OBJ}} &= 100 \text{mm} \\
  \phi_{\text{OBJ}} &= 0.01 / \text{mm} \\
  t &= 150 \text{mm} \\
  f_{\text{RELAY}} &= 25 \text{mm} \\
  \phi_{\text{RELAY}} &= 0.04 / \text{mm}
\end{align*}
\]

Assuming that the object is at infinity, an intermediate image is produced 50 mm to the left of the relay lens. Since the focal length of the relay lens is 25 mm, the relay lens is operating at 1:1 conjugates. A second real intermediate image is formed 50 mm to the right of the relay lens – at the rear focal point of the system.

System power and focal length:

\[
\phi = \phi_{\text{OBJ}} + \phi_{\text{RELAY}} - \phi_{\text{OBJ}}\phi_{\text{RELAY}} t \\
\phi = -0.01 / \text{mm} \\
f = f'_{R} = -100 \text{mm}
\]
The system focal length is negative!

Look at \( P' \):

\[
d' = -\frac{\phi_{\text{OBJ}}}{\phi} l
\]

\[d' = +150\text{mm}\]

The relay lens changes the sign of the marginal ray angle, causing \( P' \) to be to the right of \( F' \)