Lens of Varying Thickness

A biconvex lens is formed by polishing identical convex surfaces of radius \( R = 50 \) mm on both ends of a glass rod of index \( n = 1.5 \).

a) For an object at infinity, sketch plots of the system focal length and paraxial image location as a function of the length of the rod (measure the image location relative to the rear vertex, and consider positive thicknesses from 0 to 500 mm only).

Hint: To simplify the calculations, first solve for the focal length and image location in terms of the reduced thickness of the rod.

b) As the rod length changes, several classes of two-element optical instruments are generated. Identify these classes as well as specific systems and qualitatively explain their image-forming operation in terms of an equivalent pair of thin lenses. Ignore all reflections.

Solution:

Biconvex lens of variable thickness

\[ n = 1.5 \quad R = 50mm \quad (R_1 = 50mm \quad R_2 = -50mm) \]

a) Focal length and BFD:

\[ \phi = \phi_1 + \phi_2 - \phi \phi_2 t \]
\[ f = f'_R = \frac{1}{\phi} \quad (n' = 1) \]
\[ d' = \delta' = -\frac{\phi_1}{\phi} \tau \quad \text{BFD} = f' + d' \]

\[ \phi_1 = \frac{(n - 1)}{R_1} = \frac{0.5}{50mm} = 0.01/\text{mm} \]
\[ \phi_2 = \frac{(1 - n)}{R_2} = \frac{-0.5}{-50mm} = 0.01/\text{mm} \]

\[ \phi_1 = \phi_2 = 0.01/\text{mm} \]
<table>
<thead>
<tr>
<th>$\tau$</th>
<th>$\phi$</th>
<th>$f$</th>
<th>$d'$</th>
<th>BFD</th>
<th>$t = \tau n$</th>
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The diagrams show the changes in focal length and BFD over time for zones A, B, and C.

Zone ① ② ③
b) Instruments:

Note: the rear focal point of the first surface is 150mm from the first surface

The BFD is measured from the second surface.

Ⓐ (t=0) Thin Lens.

Ⓑ (t=150 mm) Second Surface at the Rear Focal Point of the First Surface. This is analogous to the second surface serving as a Field Lens.

Ⓒ (t=300 mm) Telescope (the intermediate image is at the front focal point of the rear surface).

Zones:

① (0 < t < 150 mm) Thick Lens – a real image is formed outside the lens

② (150 mm < t < 300 mm) An intermediate image formed inside the glass rod at the rear focal point of the first surface. This image is inside the front focal point of the second surface. The second surface acts as a magnifier and a virtual image is produced.

③ (t > 300 mm) The intermediate image at the rear focal point of the first surface is now outside (to the left of) the front focal point of the second surface. A real image is formed. This is equivalent to a relay system. Note that the system focal length is negative since P’ and F’ for the system are reversed.