Keplerian – Vignetting and Telecentric

A 5x Keplerian telescope has a 200 mm focal length objective. The objective lens serves as the system stop and has a diameter of 40 mm.

a) What is the required eye lens diameter for the telescope to have an unvignetted field of view of +/- 2 degrees?

b) Convert the telescope into a doubly telecentric system. Where is the stop location?

c) Two objects are located 400 mm and 100 mm to the left of the objective lens. Where are the respective image planes (relative to the second lens)?

Solution:

\[ f_{OBJ} = 200 \text{mm} \quad \phi_{OBJ} = 0.005 \text{ / mm} \]

a) Telescope Design

\[ MP = -5 = -f_{OBJ} / f_{EYE} \]

\[ f_{EYE} = 40 \text{mm} \]

\[ L = f_{OBJ} + f_{EYE} = 240 \text{mm} \]

\[ D_{OBJ} = D_{STOP} = 40 \text{mm} \]

\[ FOV = \pm 2^\circ \quad \bar{u} = \tan \left( 2^\circ \right) = 0.0349 \]

Trace marginal and chief rays to the eye lens:

\[ u = 0 \]

\[ y_1 = \frac{D_{OBJ}}{2} = 20 \]

\[ u' = u - y_1 \phi_1 = -0.1 \]
\[ y_2 = y_1 + u'L = -4.0\text{mm} \]
\[ \bar{u} = \bar{u'} = 0.0349 \]
\[ \bar{y}_1 = 0 \]
\[ \bar{y}_2 = \bar{u}'L = 8.38\text{mm} \]

The eye lens diameter is determined so that the system is unvignetted over the specified FOV:

\[ a_{\text{EYE}} \geq |y_2| + |\bar{y}_2| \]
\[ a_{\text{EYE}} \geq 12.38\text{mm} \]
\[ D_{\text{EYE}} \geq 24.76\text{mm} \]

b) For the system to be doubly telecentric, the stop is located at the common focal point of the telescope. The Stop is 200 mm to the right of the objective lens.

c) Imaging - \( \Delta z \) from \( F \) of objective.
\( \Delta z' \) from \( F' \) of eye lens.

\[ m = \frac{\Delta z'}{\Delta z} = m^2 = \frac{1}{MP^2} = \frac{1}{25} \]

Object 400 mm left of the objective lens:

\[ s = -400\text{mm} = -f_{\text{OBJ}} + \Delta z = -200\text{mm} + \Delta z \]
\[ \Delta z = -200\text{mm} \quad \text{from} \quad F_{\text{OBJ}} \]
\[ \Delta z' = -8\text{mm} \quad \text{from} \quad F'_{\text{EYE}} \]

Image: \( s' = f_{\text{EYE}} + \Delta z' = 40\text{mm} - 8\text{mm} = 32\text{mm} \) to the right of the eye lens.
Object 100 mm left of the objective lens:

\[ s = -100\text{mm} = -f_{\text{OBJ}} + \Delta z = -200\text{mm} + \Delta z \]

\[ \Delta z = 100\text{mm} \quad \text{from } F_{\text{OBJ}} \]

\[ \Delta z' = 4\text{mm} \quad \text{from } F'_{\text{EYE}} \]

Image: \( s' = f'_{\text{EYE}} + \Delta z' = 40\text{mm} + 4\text{mm} = 44\text{mm} \) to the right of the eye lens.